Chapter 6 Solved Exercises

Question 1. (Exercise 6.1) Explain in your own words the difference between the contrapositive, the converse and a counterexample.

Question 2. (Exercise 6.2) Give 4 examples of implications, and for each write down their contrapositive. Have two be real-world examples, and two be math examples.

Question 3. (Nearly Exercise 6.3)

(a) What is the contrapositive of "If $2n^2 - 5n + 3$ is odd, then n is even"?

(b) Suppose that $n \in \mathbb{Z}$. Prove that if $2n^2 - 5n + 3$ is odd, then n is even.

Question 4. (Nearly Exercise 6.5 part (c)) Suppose $n \in \mathbb{Z}$. Prove that if $n^2 + 2n + 3$ is even, then n is odd.

Question 5. (Exercise 6.6 part (d)) Suppose $n \in \mathbb{Z}$. Prove that if $3 \nmid (n^2 - 1)$, then $3 \mid n$.

Question 6. (Exercise 6.8 part (e)) Suppose $x \in \mathbb{R}$. Prove that if $x^3 + x > 0$, then $x > 0$.

Question 7. (Exercise 6.10) Define the *Fibonacci sequence* to be the sequence $F_1, F_2, F_3, F_4, \ldots$. where F_1 and F_2 both equal 1, and every term thereafter is the sum of the previous two: $F_n = F_{n-1} + F_{n-2}$ for $n = 3, 4, 5, 6, \ldots$. Thus, the sequence begins

$$1 \ , \ 1 \ , \ 2 \ , \ 3 \ , \ 5 \ , \ 8 \ , \ 13 \ , \ 21 \ , \ 34 \ , \ 55 \ , \ \ldots.$$

Prove that if F_n is not a perfect cube, then $n \notin \{1, 2, 6\}$.

Question 8. (Exercise 6.12 part (c)) Suppose $n \in \mathbb{Z}$. Prove that $(n+1)^2 - 1$ is even if and only if n is even.

Question 9. (Exercise 6.15) Come up with a real-world claim that is false, and prove that it is false by exhibiting a counterexample to the claim.

Question 10. (Exercise 6.16 parts (d), (i) and (o)) The following statements are all false. For each, find a counterexample.

(d) If $x, y \in \mathbb{R}$ and $|x + y| = |x - y|$, then $y = 0$.

(i) If $n \in \mathbb{N}$, then $2n^2 - 4n + 31$ is prime.

(o) If A, B and C are sets, then $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$.