Chapter 7 Solved Exercises

Question 1. (Exercise 7.2 part (b)) Suppose $a, b \in \mathbb{R}$. Prove that if a is rational and ab is irrational, then b is irrational.

Question 2. (Exercise 7.2 part (c)) Give an example of two irrational numbers whose sum is rational.

Question 3. (Nearly Exercise 7.3) Prove that there do not exist integers m and n for which $21m + 35n = 1$.

Question 4. (Exercise 7.6 (c)) Suppose that A and B are sets inside a universal set U. Prove that $A^c \cap (B \cap A) = \emptyset$.

Question 5. (Exercise 7.9 (a)) Prove that $\sqrt{5}$ is irrational.

Question 6. (Exercise 7.9 (d)) Prove that $\sqrt{10}$ is irrational.

Question 7. (Exercise 7.12) Are there infinitely many composite numbers? Prove your answer.

Question 8. (Exercise 7.13) Suppose $a, b, c \in \mathbb{Z}$. Prove that if $a^2 + b^2 = c^2$, then a or b is even.

Question 9. (Exercise 7.15) Prove that if x and y are positive real numbers, then $x + y \geq 2\sqrt{xy}$.

Question 10. (Exercise 7.26) A *magic square* is an $n \times n$ matrix where the sum of the entries in each row, column and diagonal equal the same value. For example,

8	1	6
3	5	7
4	9	2

is a 3×3 matrix whose three rows, three columns, and two diagonals each sum to 15. Thus, this is a magic square.

Prove that the following cannot be completed to form a magic square.

1	2	3	
	4	5	6
7		8	
	9		10