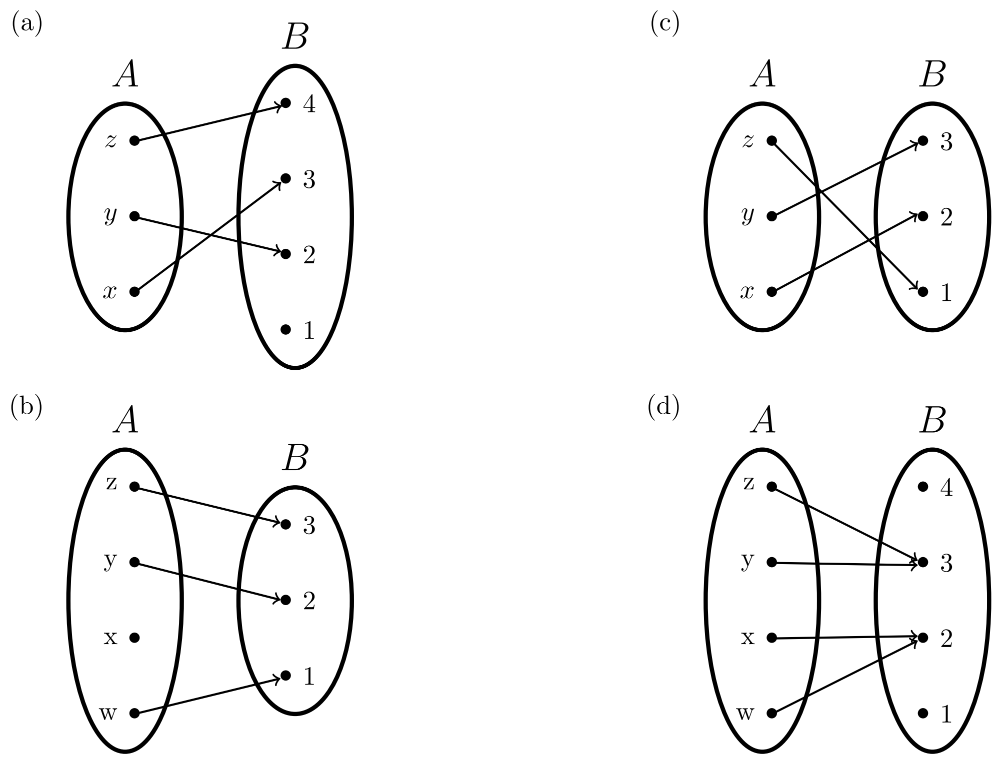
Chapter 8 Solved Exercises

Question 1. (Exercise 8.1) For each of the diagrams below, determine whether the diagram represents a function. If it does, determine whether the function is injective, surjective, bijective, or none of these.



Question 2. (Exercise 8.3) In pre-calculus you may have written things like this:

$$\frac{x^2+x-6}{x+3} = \frac{(x+3)(x-2)}{x+3} = \frac{\cancel{(x+3)}(x-2)}{\cancel{x+3}} = x-2.$$

This seems to suggest that $f(x) = \frac{x^2+x-6}{x+3}$ and $g(x) = x-2$ are the same function. Explain why they are not.

Question 3. (Exercise 8.5) In words, describe the range of the function $f : \mathbb{N}\times\mathbb{N} \to \mathbb{N}$ where $f(m,n) = 2^m3^n$.

Question 4. (Nearly Exercise 8.7 part (a)) Let $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = 4x+3$. Either prove that f is a bijection or prove that it is not.

Question 5. (Exercise 8.7 parts (j) and (k)) Determine whether each of the following is an injection, surjection, bijection or none of these. Prove your answers.

(j) $r : (-\infty, 0) \to (-\infty, 0)$ where $r(x) = -|x+4|$

(k) $s : \mathbb{N} \to \mathbb{N}\times\mathbb{N}$ where $s(x) = (x,x)$

Question 6. (Exercise 8.10) Let A and B be finite sets for which $|A| = |B|$, and suppose $f : A \to B$. Prove that f is injective if and only if f is surjective.

Question 7. (Exercise 8.12 parts (c) and (d)) Determine whether each of the following is invertible. If it is, write down its inverse and show that it satisfies Definition 8.16. If it is not, then that means it is either not injective, not surjective, or both. If it is not injective, give two distinct values x and y from the function's domain for which $f(x) = f(y)$. If it is not surjective, give an element of the codomain which is not hit by the function. You do not need to prove that your answers are correct.

(c) $g : \mathbb{R}^+ \to \mathbb{R}$ where $g(x) = x^4$, where $\mathbb{R}^+$ is the set of nonnegative real numbers

(d) $h : [0, \pi] \to [-1, 1]$ where $h(x) = \cos(x)$

Question 8. (Nearly Exercise 8.16) Let $f : \mathbb{Z} \to \mathbb{Z}$ where $f(x) = x^2 - 2x + 1$, and let $g : \mathbb{Z} \to \mathbb{Z}$ where $g(x) = 2x + 3$. Write down $(f \circ g)(x)$ and $(g \circ f)(x)$ as expanded polynomials and write down their ranges.

Question 9. (Exercise 8.17) Give an example of functions f and g such that $f \circ g$ is injective and g is injective, but f is not injective. Write down f, g and $f \circ g$, but you do not need to prove that your example works.

Question 10. (Exercise 8.20) For functions $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$, prove or disprove each of the following conjectures.

(a) **Conjecture 1:** $(f + g) \circ h = (f \circ h) + (g \circ h)$

(b) **Conjecture 2:** $h \circ (f + g) = (h \circ f) + (h \circ g)$

For this problem, recall that "$f + g$" is the function $(f + g)(x) = f(x) + g(x)$.