Chapter 9 Solved Exercises

Question 1. (Nearly Exercise 9.3) Let $A = \{1, 2, 3, 4\}$. Each part below is a separate definition for the relation "$a \sim b$." For each, write out all pairs that are related.

(a) $a \sim b$ when $a \leq b$

(b) $a \sim b$ when $2 \mid (a + b)$

Question 2. (Exercise 9.4)

(a) List all partitions of the set $\{1, 2\}$.

(b) List all partitions of the set $\{a, b, c\}$.

Question 3. (Exercise 9.6) Suppose a group of 10 people are in a room, and some of them shake hands with some of the others (but not everyone shakes hands with everyone). If A is the set of 10 people, consider the relation $\sim$ on A where $a \sim b$ if person a shook hands with person b. Given just this information, is $\sim$ guaranteed to be reflexive? Symmetric? Transitive? Give a brief justification for each.

Question 4. (Exercise 9.7) Consider the relation $\sim$ on the set $\{w, x, y, z\}$ such that this is the complete list of related elements:

$$z \sim z \qquad x \sim y \qquad y \sim x$$

$$w \sim w \qquad x \sim x \qquad y \sim y$$

Is $\sim$ reflexive? Symmetric? Transitive? If a property holds, you do not need to justify it. If it doesn't, say why it fails. If all three hold, then $\sim$ is an equivalence relation; in this case, list the equivalence classes.

Question 5. (Exercise 9.12) Let $A = \mathcal{P}(\mathbb{N})$. Let $\sim$ be the relation on A where $a \sim b$ provided $a \subseteq b$. Is $\sim$ reflexive? Symmetric? Transitive? For each property, prove that it holds or find a counterexample. Is $\sim$ an equivalence relation?

Question 6. (Exercise 9.15 part (e)) The rule $a \sim b$ when $2a + b \equiv 0 \pmod 3$ defines a relation on $\mathbb{Z}$. Prove that $\sim$ is an equivalence relation and find its equivalence classes.

Question 7. (Exercise 9.16 part (c)) Consider the rule $a \sim b$ when $a \neq b$. This rule defines a relation on $\mathbb{Z}$. Is $\sim$ reflexive? Symmetric? Transitive? If a property holds, provide a brief justification. If it doesn't, say why it fails.

Question 8. (Exercise 9.23) In this exercise we will put some rigor behind the practice of thinking of fractions in their "lowest terms," which was a central idea in the proof that $\sqrt{2}$ is irrational. We will represent a fraction $\frac{a}{b}$ as an ordered pair (a, b) where $b \neq 0$, and the equality $\frac{a}{b} = \frac{c}{d}$ will be thought of as $ad = bc$.

Let $A = \{(a, b) : a, b \in \mathbb{Z} \text{ and } b \neq 0\}$. Define the relation $\sim$ on A to be

$$(a, b) \sim (c, d) \qquad \text{if} \qquad ad = bc.$$

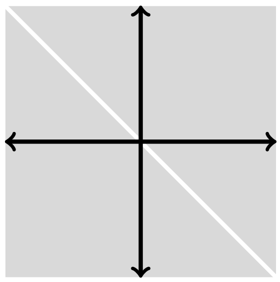
Prove that $\sim$ is an equivalence relation.

Question 9. (Exercise 9.29) Determine a familiar equivalence relation whose equivalence classes are the following:

$$\{\dots, -6, -3, 0, 3, 6, \dots\}\ ,\ \{\dots, -5, -2, 1, 4, 7, \dots\}\ ,\ \{\dots, -4, -1, 2, 5, 8, \dots\}.$$

Question 10. (Exercise 9.35 parts (c) and (d)) Note 9.13 allows us to think about a relation on $\mathbb{R}$ or $\mathbb{Z}$ as a subset of $\mathbb{R} \times \mathbb{R}$ or $\mathbb{Z} \times \mathbb{Z}$. This in turn allows us to graph a relation on the xy-plane, either as a shaded region or points of the integer grid. Each of the following corresponds to a familiar relation on $\mathbb{R}$ or $\mathbb{Z}$. Determine this relation for each.

(c)



(d)