Chapter 5 Solutions to Selected Exercises

Notes:

- The questions are in a separate PDF on LongFormMath.com.

- For most problems there are many correct solutions, so the below are not the only correct ways to solve the problems.

- If you spot an error, please email it to me at LongFormMath@gmail.com. Thanks!

Solution to Question 1. If *everyone* loves your baby, then that includes your baby! So your baby must love their self. But since your baby loves *only* you, that means your baby can only love one person, and that person is you.

Since your baby loves itself, and you are the only person your baby loves, you must be your own baby.

Solution to Question 2.
(a) is a true statement
(b) is a false statement
(c) is not a statement
(d) is a false statement

Solution to Question 3. If $n = 1$, then

$$\frac{2n^2 + 5 + (-1)^n}{2} = \frac{2 + 5 - 1}{2} = \frac{6}{2} = 3 \ \text{ is prime}$$

is a true statement. And if $n = 5$, then

$$\frac{2n^2 + 5 + (-1)^n}{2} = \frac{50 + 5 - 1}{2} = \frac{54}{2} = 27 \ \text{ is prime}$$

is a false statement.

Solution to Question 4.

(f) If a tree has m edges, then it has $m + 1$ vertices.

(g) If an integer is not odd, then it is even.

(h) If a geometric series has ratio r with $|r| \geq 1$, then the series diverges.

Solution to Question 5.

(d) N is a normal subgroup of G if and only if $Ng = gN$ for all $g \in G$.

Solution to Question 6.

(g) There exists an $\epsilon > 0$ such that for all $\delta > 0$, there is some x where $|x - a| < \delta$ and $|f(x) - f(a)| \geq \epsilon$.

(h) If I pass Algebra I and Analysis I this semester, then I will not take Algebra II nor Analysis II next semester.

Solution to Question 7.

P	Q	$\sim P$	$\sim P \vee Q$	$\sim(\sim P \vee Q)$
True	True	False	True	False
True	False	False	False	True
False	True	True	True	False
False	False	True	True	False

Solution to Question 8.

P	Q	R	$P \wedge Q$	$\sim R$	$(P \wedge Q) \vee \sim R$
True	True	True	True	False	True
True	True	False	True	True	True
True	False	True	False	False	False
True	False	False	False	True	True
False	True	True	False	False	False
False	True	False	False	True	True
False	False	True	False	False	False
False	False	False	False	True	True

Solution to Question 9.

P	Q	$P \Rightarrow Q$
True	True	True
True	False	False
False	True	True
False	False	True

P	Q	$\sim P$	$\sim P \vee Q$
True	True	False	True
True	False	False	False
False	True	True	True
False	False	True	True

Since both truth tables align in their final column, the two statements are logically equivalent.

Solution to Question 10.

P	Q	$P \vee Q$	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$	$(P \vee Q) \vee (\sim P \wedge \sim Q)$
True	True	True	False	False	False	True
True	False	True	False	True	False	True
False	True	True	True	False	False	True
False	False	False	True	True	True	True

Since the final column is all "true," this means that the expression $(P \vee Q) \vee (\sim P \wedge \sim Q)$ is true regardless of the truth values of P and Q—meaning this expression is a tautology.