Proposition Suppose $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $12a \not\equiv 12b \pmod{n}$, then $n \nmid 12$.

Proof. (Contrapositive) Suppose $n \mid 12$, so there is an integer c for which $12 = nc$. Now reason as follows.

$$\begin{aligned} 12 &= nc \\ 12(a-b) &= nc(a-b) \\ 12a - 12b &= n(ca - cb) \end{aligned}$$

Since $ca - cb \in \mathbb{Z}$, the equation $12a - 12b = n(ca - cb)$ implies $n \mid (12a - 12b)$. This in turn means $12a \equiv 12b \pmod{n}$. ■

5.3 Mathematical Writing

Now that you have begun writing proofs, it is the right time to address issues concerning writing. Unlike logic and mathematics, where there is a clear-cut distinction between what is right or wrong, the difference between good and bad writing is sometimes a matter of opinion. But there are some standard guidelines that will make your writing clearer. Some of these are listed below.

1. **Never begin a sentence with a mathematical symbol.** The reason is that sentences begin with capital letters, but mathematical symbols are case sensitive. Since x and X can have entirely different meanings, putting such symbols at the beginning of a sentence can lead to ambiguity. Following are some examples of bad usage (marked with ×) and good usage (marked with ✓).

   A is a subset of B. ×
   The set A is a subset of B. ✓

   x is an integer, so $2x+5$ is an integer. ×
   Since x is an integer, $2x+5$ is an integer. ✓

   $x^2 - x + 2 = 0$ has two solutions. ×
   $X^2 - x + 2 = 0$ has two solutions. × (and silly too)
   The equation $x^2 - x + 2 = 0$ has two solutions. ✓

2. **End each sentence with a period.** Do this even when the sentence ends with a mathematical symbol or expression.

   Euler proved that $\sum_{k=1}^{\infty} \frac{1}{k^s} = \prod_{p \in P} \frac{1}{1 - \frac{1}{p^s}}$ ×

   Euler proved that $\sum_{k=1}^{\infty} \frac{1}{k^s} = \prod_{p \in P} \frac{1}{1 - \frac{1}{p^s}}$. ✓

Mathematical statements (equations, etc.) are like English phrases that happen to contain special symbols, so use normal punctuation.

3. **Separate mathematical symbols and expressions with words.** Failure to do this can cause confusion by making distinct expressions appear to merge into one. Compare the clarity of the following examples.

   Because $x^2-1=0$, $x=1$ or $x=-1$. ×
   Because $x^2-1=0$, it follows that $x=1$ or $x=-1$. ✓

   Unlike $A\cup B$, $A\cap B$ equals $\emptyset$. ×
   Unlike $A\cup B$, the set $A\cap B$ equals $\emptyset$. ✓

4. **Avoid misuse of symbols.** Symbols such as $=$, $\leq$, $\subseteq$, $\in$, etc., are not words. While it is appropriate to use them in mathematical expressions, they are out of place in other contexts.

   Since the two sets are $=$, one is a subset of the other. ×
   Since the two sets are equal, one is a subset of the other. ✓

   The empty set is a $\subseteq$ of every set. ×
   The empty set is a subset of every set. ✓

   Since a is odd and x odd $\Rightarrow x^2$ odd, a^2 is odd. ×
   Since a is odd and any odd number squared is odd, then a^2 is odd. ✓

5. **Avoid using unnecessary symbols.** Mathematics is confusing enough without them. Don't muddy the water even more.

   No set X has negative cardinality. ×
   No set has negative cardinality. ✓

6. **Use the first person plural.** In mathematical writing, it is common to use the words "we" and "us" rather than "I," "you" or "me." It is as if the reader and writer are having a conversation, with the writer guiding the reader through the details of the proof.

7. **Use the active voice.** This is just a suggestion, but the active voice makes your writing more lively.

   The value $x=3$ is obtained through the division of both sides by 5. ×
   Dividing both sides by 5, we get the value $x=3$. ✓

8. **Explain each new symbol.** In writing a proof, you must explain the meaning of every new symbol you introduce. Failure to do this can lead to ambiguity, misunderstanding and mistakes. For example, consider the following two possibilities for a sentence in a proof, where a and b have been introduced on a previous line.

Since $a \mid b$, it follows that $b = ac$. ×
Since $a \mid b$, it follows that $b = ac$ for some integer c. ✓

If you use the first form, then a reader who has been carefully following your proof may momentarily scan backwards looking for where the c entered into the picture, not realizing at first that it came from the definition of divides.

9. **Watch out for "it."** The pronoun "it" can cause confusion when it is unclear what it refers to. If there is any possibility of confusion, you should avoid the word "it." Here is an example:

   Since $X \subseteq Y$, and $0 < |X|$, we see that it is not empty. ×

   Is "it" X or Y? Either one would make sense, but which do we mean?

   Since $X \subseteq Y$, and $0 < |X|$, we see that Y is not empty. ✓

10. **Since, because, as for, so.** In proofs, it is common to use these words as conjunctions joining two statements, and meaning that one statement is true and as a consequence the other true. The following statements all mean that P is true (or assumed to be true) and as a consequence Q is true also.

Q since P	Q because P	Q, as P	Q, for P	P, so Q
Since P, Q	Because P, Q	as P, Q		

   Notice that the meaning of these constructions is different from that of *"If P, then Q,"* for they are asserting not only that P implies Q, but **also** that P is true. Exercise care in using them. It must be the case that P and Q are both statements **and** that Q really does follow from P.

   $x \in \mathbb{N}$, so $\mathbb{Z}$ ×
   $x \in \mathbb{N}$, so $x \in \mathbb{Z}$ ✓

11. **Thus, hence, therefore consequently.** These adverbs precede a statement that follows logically from previous sentences or clauses. Be sure that a statement follows them.

   Therefore $2k+1$. ×
   Therefore $a = 2k+1$. ✓

Your mathematical writing will get better with practice. One of the best ways to develop a good mathematical writing style is to read other people's proofs. Adopt what works and avoid what doesn't.