A Simple Explanation of Countercyclical Uncertainty

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Introduction

- Aggregate real uncertainty is countercyclical in U.S. data
- We show search and matching frictions embedded in the Diamond-Mortensen-Pissarides model explain this fact
  - Nonlinearity in the identity for new matches makes employment uncertainty increasing in unemployment
- Countercyclical uncertainty fluctuations are endogenous responses to changes in real activity that do not affect the severity of business cycles or warrant policy intervention

Related Literature

- Search frictions generate nonlinear labor dynamics
  (Petrosky-Nadeau and Zhang, 2017; Petrosky-Nadeau et al., 2018)
  Our paper: Search generates time-varying uncertainty
- Other endogenous uncertainty mechanisms
  (Ilut et al., 2018; Straub and Ulbricht, 2019; Fajgelbaum et al., 2017; Ilut and Schneider, 2014; Van Nieuwerburgh and Veldkamp, 2006)
  Our paper: Simple mechanism in a textbook model
- Impulse responses to exogenous volatility shocks
  (Basu and Bundick, 2017; Mumtaz and Zanetti, 2013; Leduc and Liu, 2016; Fernández-Villaverde et al., 2015, 2011; Born and Pfeifer, 2014)
  Our paper: Correlation of uncertainty and output
- Causality between real activity and uncertainty
  (Ludvigson et al., 2021; Carriero et al., 2021, Berger et al., 2020)
  Our paper: Uncertainty is primarily driven by real activity

Outline

1. Uncertainty Data
2. Model and Mechanism
3. Quantitative Results
4. Implications for VAR Models
5. Model Extensions and Robustness

Empirical Uncertainty Measure

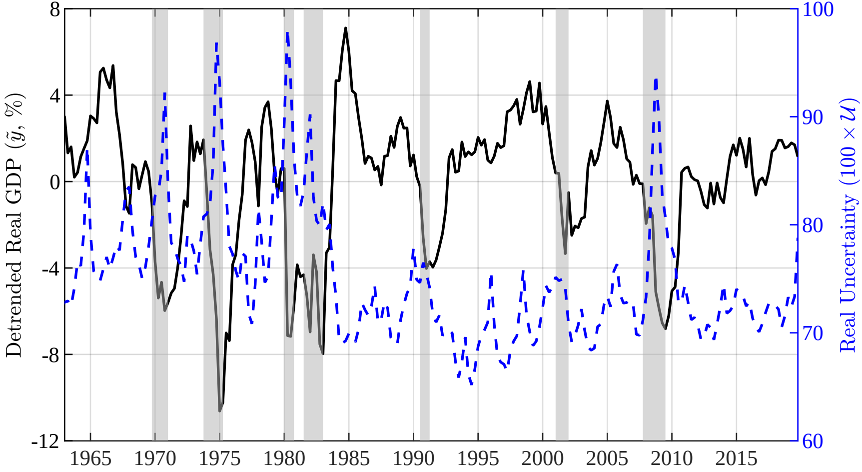
- Uncertainty surrounding variable y_j of horizon h:

$$\mathcal{U}_{j,t}(h) = \sqrt{E_t[(y_{j,t+h} - E_t[y_{j,t+h}])^2]}$$

  Definition follows Jurado, Ludvigson, Ng (AER, 2015)
- Real uncertainty series is the average of $\mathcal{U}_{j,t}(h)$ across 73 monthly measures of real activity in the data
- Benefits of this measure:
  - Strips out the predictable variation in each variable
  - Information set includes 280 macro/financial variables
  - Cleanly maps to business cycle models
- Use a quarterly horizon ($h = 3$) and average across months in each quarter to produce a quarterly series

COUNTERCYCLICAL UNCERTAINTY

$$SD(\mathcal{U}) = 5.93,\ SD(\tilde{y}) = 3.15,\ Corr(\mathcal{U}, \tilde{y}) = -0.60$$



Quantitative Model

- Use a textbook Diamond-Mortensen-Pissarides model (Den Haan et al. 2000, Andolfatto 1996, Merz 1995)
  - Representative household chooses consumption and investment by pooling employed and unemployed incomes
  - Representative firm posts vacancies and hires workers
  - Wage rate is determined by standard Nash bargaining
- Exogenous shocks to first and second moments of TFP

$$\ln a_t = (1-\rho_a)\ln \bar{a} + \rho_a \ln a_{t-1} + \sigma_{a,t-1}\varepsilon_{a,t}$$
$$\ln \sigma_{a,t} = (1-\rho_{sv})\ln \bar{\sigma}_a + \rho_{sv} \ln \sigma_{a,t-1} + \sigma_{sv}\varepsilon_{sv,t}$$

Specifying TFP in logs ensures that we do not introduce exogenous curvature into the log production function

Search and Matching

- Mass of unemployed job seekers:

$$u_t^s = u_{t-1} + \chi \bar{s} n_{t-1}$$

- Matching process:

$$\mathcal{M}(u_t^s, v_t) = \xi (u_t^s)^{\phi} v_t^{1-\phi}$$
$$m_t = \min\{\mathcal{M}(u_t^s, v_t), u_t^s, v_t\}$$

- Law of motion for employment:

$$n_t = (1-\bar{s})n_{t-1} + f_t u_t^s$$

where $f_t \equiv m_t/u_t^s = \xi(v_t/u_t^s)^{1-\phi}$ is the job finding rate

Representative Household

- Household solves

$$J_t^H = \max_{c_t, i_t, k_t} \ln c_t + \beta E_t J_{t+1}^H$$

subject to

$$c_t + i_t = w_t n_t + r_t^k k_{t-1} + b u_t - \tau_t + d_t$$

$$k_t = (1-\delta) k_{t-1} + \left(a_1 + \frac{a_2}{1-1/\nu} \left(\frac{i_t}{k_{t-1}} \right)^{1-1/\nu} \right) k_{t-1}$$

- Optimality implies

$$\frac{1}{a_2} \left(\frac{i_t}{k_{t-1}} \right)^{1/\nu} = E_t \left[x_{t+1} \left(r_{t+1}^k + \frac{1}{a_2} \left(\frac{i_{t+1}}{k_t} \right)^{1/\nu} (1 - \delta + a_1) + \frac{1}{\nu - 1} \frac{i_{t+1}}{k_t} \right) \right]$$

Representative Firm

- Firm solves

$$J_t^F = \max_{k_{t-1},n_t,v_t} y_t - w_t n_t - r_t^k k_{t-1} - \kappa v_t + E_t[x_{t+1} J_{t+1}^F]$$

subject to

$$y_t = a_t k_{t-1}^{\alpha} n_t^{1-\alpha}$$
$$n_t = (1-\bar{s})n_{t-1} + q_t v_t$$
$$v_t \geq 0$$

- Optimality implies

$$r_t^k = \alpha y_t / k_{t-1}$$
$$\lambda_{n,t} = (1-\alpha) y_t / n_t - w_t + (1-\bar{s}) E_t[x_{t+1} \lambda_{n,t+1}]$$
$$q_t \lambda_{n,t} = \kappa - \lambda_{v,t}$$
$$\lambda_{v,t} v_t = 0, \quad \lambda_{v,t} \geq 0$$

Wages and Uncertainty

- Wage rate determined by Nash bargaining

$$w_t = \eta((1-\alpha)y_t/n_t + \kappa(1-\chi\bar{s})E_t[x_{t+1}\theta_{t+1}]) + (1-\eta)b$$

  b and η denote worker outside option and bargaining power

- Definition of aggregate uncertainty

$$\mathcal{U}_t = \frac{1}{SD(\Delta y)}\sqrt{E_t[(\ln(y_{t+3}/y_t) - E_t[\ln(y_{t+3}/y_t)])^2]}$$

  Normalize by $SD(\Delta y)$ so units are consistent with the data

Endogenous Uncertainty Mechanism

- *Intuition*: Nonlinearity in the flow of new matches makes employment uncertainty increasing in the mass of people searching for work, generating countercyclical uncertainty
- Consider the law of motion for employment:

$$\hat{n}_{t+1} \equiv n_{t+1}/n_t = 1 - \bar{s} + m_{t+1}/n_t$$
$$\to \sqrt{V_t[\hat{n}_{t+1}]} = \tfrac{1}{n_t}\sqrt{V_t[m_{t+1}]}$$

- Express new matches as $m_{t+1} = f_{t+1}u^s_{t+1}$ to obtain

$$\sqrt{V_t[\hat{n}_{t+1}]} = \tfrac{1}{n_t}u^s_{t+1}\sqrt{V_t[f_{t+1}]}$$

Under some assumptions we can solve for the finding rate

Endogenous Uncertainty Mechanism

- Simplifying assumptions:
  - Vacancies are positive ($\lambda_v = 0$)
  - Labor is the only input in production ($\alpha = 0$)
  - Households are risk neutral (linear utility)
  - Wage rate is fixed ($\eta = 0$)
  - TFP process is specified in levels instead of logs
- Guess and verify the solution for the match value:

$$\lambda_{n,t} = \delta_0 + \delta_1(a_t - \bar{a}),$$

$$\delta_0 = \frac{\bar{a} - b}{1 - \beta(1 - \bar{s})} > 0, \qquad \delta_1 = \frac{1}{1 - \beta(1 - \bar{s})\rho_a} > 0$$

- Solution implies:

$$f_t = \xi^{1/\phi}(\lambda_{n,t}/\kappa)^{(1-\phi)/\phi} \rightarrow V_t[f_{t+1}] \propto V_t[\lambda_{n,t+1}^{(1-\phi)/\phi}]$$

Endogenous Uncertainty Mechanism

- Suppose the matching elasticity $\phi = 0.5$
- Employment uncertainty is given by

$$\sqrt{V_t[\hat{n}_{t+1}]} = \tfrac{1}{n_t} u^s_{t+1}(\delta_1 \xi^2/\kappa)\sigma_{a,t}$$

  which is increasing in u^s_{t+1} and therefore countercyclical
- *Intuition*: Suppose the economy is in a recession
  - $\rightarrow$ Larger mass of people looking for work
  - $\rightarrow$ Flow of new matches more sensitive to the job finding rate
  - $\rightarrow$ Wider distribution of new matches
  - $\rightarrow$ Increases employment uncertainty

Estimation Method

- Quantify the endogenous uncertainty mechanism
- Use quarterly data from 1963 to 2019
- Set $\beta = 0.9983$ (2% annual rate), $\delta = 0.0079$, $\alpha = 0.3888$, $\bar{s} = 0.0328$ (data), $\bar{q} = 0.3306$ (Den Haan et al., 2000)
- Estimate remaining 10 parameters to target key moments
- Empirical targets $\hat{\Psi}_T^D$ (and SEs) estimated with GMM
- Apply SMM to the *nonlinear* DMP model to minimize:

$$[\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\mathcal{P},\mathcal{E})]'[\hat{\Sigma}_T^D(1+1/R)]^{-1}[\hat{\Psi}_T^D - \bar{\Psi}_{R,T}^M(\mathcal{P},\mathcal{E})]$$

given parameterization $\mathcal{P}$ and shocks $\mathcal{E}$

▸ Solution Method

Estimated Parameters

Parameters	Targets	Parameters	Targets
b, ϕ	$SD(\tilde{u}), SD(\tilde{v})$	ν	$SD(\tilde{c}), SD(\tilde{\imath}), AC(\tilde{c}), AC(\tilde{\imath})$
η	$Cov(\tilde{w}, \tilde{\ell})/V(\tilde{\ell})$	$\rho_a, \bar{\sigma}_a$	$AC(\tilde{y}), SD(\tilde{y})$
κ, χ	$E(u), E(f)$	ρ_{sv}, σ_{sv}	$AC(\mathcal{U}), SD(\mathcal{U}), Corr(\mathcal{U}, \tilde{y})$

Parameter	Mean	SE
Intra-Period Search Duration (χ)	0.5463	0.0011
Vacancy Posting Cost (κ)	1.1919	0.0090
Outside Option (b)	0.9380	0.0003
Matching Elasticity (ϕ)	0.4940	0.0004
Bargaining Weight (η)	0.1465	0.0007
Investment Adjustment Cost (ν)	5.4153	0.0215
TFP Level Shock AC (ρ_a)	0.9239	0.0006
TFP Level Shock SD ($\bar{\sigma}_a$)	0.0105	0.0000
TFP Volatility Shock AC (ρ_{sv})	0.9438	0.0008
TFP Volatility Shock SD (σ_{sv})	0.0149	0.0001

Estimated Moments

Target	Data	SE	Model	Target	Data	SE	Model
$E(u)$	5.97	0.25	5.93	$SD(\mathcal{U})$	5.93	0.62	6.06
$E(f)$	41.88	1.26	41.92	$AC(\mathcal{U})$	0.89	0.04	0.89
$SD(\tilde{y})$	3.15	0.31	3.65	$Corr(\mathcal{U}, \tilde{y})$	-0.60	0.08	-0.62
$SD(\tilde{c})$	2.06	0.17	2.01	$AC(\tilde{y})$	0.90	0.03	0.88
$SD(\tilde{\imath})$	8.68	0.82	7.30	$AC(\tilde{c})$	0.88	0.03	0.92
$SD(\tilde{u})$	21.36	1.98	21.14	$AC(\tilde{\imath})$	0.89	0.04	0.86
$SD(\tilde{v})$	21.64	2.08	21.65	$Slope(\tilde{w}, \tilde{\ell})$	0.63	0.09	0.63

- Model provides a credible account of business cycle fluctuations in both real activity and uncertainty
- Passes an overidentifying restrictions test at the 5% level

Variance Decomposition Methodology

- Decompose the variances of output and uncertainty into components driven by first and second moment shocks
- Decomposition takes into account:
  - Nonlinearities
  - Interaction effects of the shocks
  - Decomposes total effect into direct and indirect effects
- Linear FEVDs cannot capture nonlinearities
- Generalized FEVDs based on generalized impulse responses (Lanne and Nyberg, OBES, 2016) miss the multiplicative interaction of level and volatility shocks
- Method in Isakin and Ngo (OBES, 2020) does not decompose the total effect into direct and indirect effects

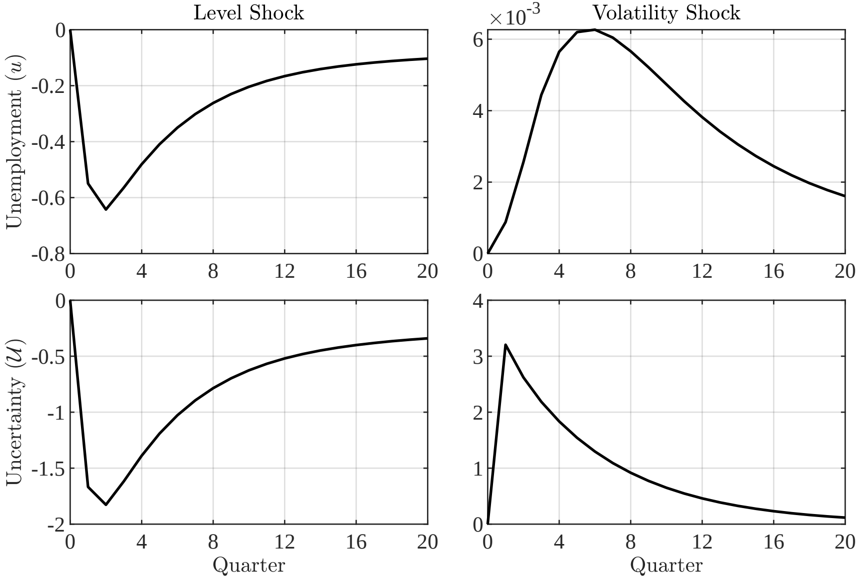
Details

Variance Decomposition Results

	Output	Uncertainty
Level Total	100.00	43.50
Volatility Total	0.20	57.01
Level Direct	99.80	42.99
Volatility Direct	0.00	56.50

- Level shocks explain 43% of the uncertainty variance
- Volatility shocks explain 57% of the uncertainty variance but almost none of the output variance
- Since volatility shocks do not explain output variation, the countercyclicality of uncertainty must be endogenous

Generalized Impulse Responses



Sources of Endogenous Uncertainty

Conditional Standard Deviations

	Ergodic Mode	Recession	% Change
Match Value ($\lambda_n^{(1-\phi)/\phi}$)	9.52	8.94	-6.15
Finding Rate (f)	9.52	8.94	-6.09
Employment ($\ln n$)	0.46	0.91	95.16
Output ($\ln y$)	1.98	2.23	12.45

- Uncertainty about the match value is mainly driven by the marginal product of labor and therefore mildly procyclical
- Uncertainty about the job finding rate is also procyclical because it is proportional to match value uncertainty
- Employment uncertainty is 95% higher in a recession and the primary source of the countercyclicality of uncertainty

Consequences of Uncertainty

Moment	Data	Baseline	Log-Linear	Hosios
$SD(\mathcal{U})$	5.93	6.06	0.00	7.38
$SD(\tilde{y})$	3.15	3.65	3.57	3.79
$SD(\tilde{u})$	21.36	21.14	22.79	22.95
$Corr(\mathcal{U}, \tilde{y})$	-0.60	-0.62	0.00	-0.72

1. Linear model produces similar output and unemployment moments $\Rightarrow$ no feedback from uncertainty to real activity
2. Hosios model shows uncertainty fluctuations survive when outcomes are constrained efficient $\Rightarrow$ policy intervention would have only a minor impact on uncertainty dynamics

Large Uncertainty Shocks

Moment	Data	Large Calibration
$SD(\mathcal{U})$	5.93	90.15
$SD(\tilde{y})$	3.15	3.15
$SD(\tilde{u})$	21.36	17.43
$Corr(\mathcal{U},\tilde{y})$	-0.60	-0.04

1. Literature commonly assumes much larger volatility shocks (set σ_{sv} so a 1SD increase in σ_a doubles its mean value)
2. $SD(\mathcal{U})$ is over 14 times larger than it is in the data and the correlation with output almost completely disappears

Large Uncertainty Shocks

Contribution	Output	Uncertainty
Level Total	99.57	2.30
Volatility Total	65.55	99.95
Level Direct	34.45	0.05
Volatility Direct	0.43	97.70

1. Volatility shocks account for around 65% of output variance, suggesting they are a key contributor to business cycles
2. Transmission to output is almost entirely due to their mechanical interaction with level shocks–other channels such as precautionary savings have small effects

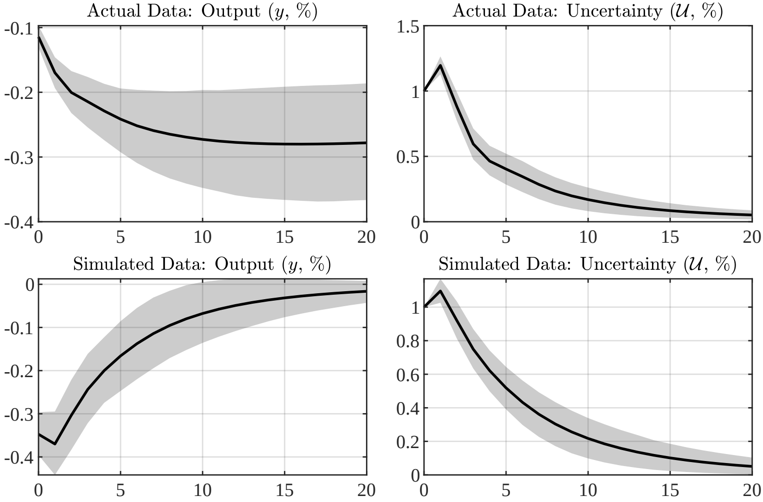
Implications for VARs

- Recursive identification schemes are often used to identify the effect of uncertainty shocks on real activity in VARs (Bloom, 2009; Bachmann et al., 2013; Basu and Bundick, 2017; Leduc and Liu, 2016; Oh, 2020; Fernandez-Villaverde et al., 2015)
- Consider a bivariate VAR:

$$Y_t = \sum_{l=1}^{L} A_l Y_{t-l} + v_t$$

  where Y_t is a vector that includes uncertainty and output
- Estimate the VAR model on actual and simulated data
- Use a Cholesky decomposition to identify structural shocks

Bivariate VAR Responses
(Quarterly Data, Uncertainty Ordered First)



VAR Implications

- The identified "structural" uncertainty shock must be correlated with the level shock from the DMP model
- Estimate the VAR model on simulated monthly data and correlate identified structural shocks with the true shocks

Uncertainty First	**Value**	**Output First**	**Value**
$Corr(\varepsilon_a^{DMP}, \varepsilon_y^{SVAR})$	0.88	$Corr(\varepsilon_a^{DMP}, \varepsilon_y^{SVAR})$	0.99
$Corr(\varepsilon_{sv}^{DMP}, \varepsilon_{\mathcal{U}}^{SVAR})$	0.84	$Corr(\varepsilon_{sv}^{DMP}, \varepsilon_{\mathcal{U}}^{SVAR})$	0.96
$Corr(\varepsilon_a^{DMP}, \varepsilon_{\mathcal{U}}^{SVAR})$	-0.47	$Corr(\varepsilon_a^{DMP}, \varepsilon_{\mathcal{U}}^{SVAR})$	0.00
$Corr(\varepsilon_{sv}^{DMP}, \varepsilon_y^{SVAR})$	0.45	$Corr(\varepsilon_{sv}^{DMP}, \varepsilon_y^{SVAR})$	0.00

- Ordering uncertainty last removes the contamination only because uncertainty is almost purely endogenous, in which case there is almost no causal effect of uncertainty

DMP Model Extensions

1. Recursive preferences (DMP+EZ)
   (Epstein and Zin, 1989; Petrosky-Nadeau et al., 2018)
2. New Keynesian nominal rigidities (DMP+NK)
   (Leduc and Liu, 2016)
3. Downward wage rigidity (DMP+DWR)
   (Cacciatore and Ravenna, 2021; Dupraz et al., 2019)
4. Inelastic vacancy creation (DMP+IVC)
   (Coles and Kelishomi, 2018)
5. Endogenous job separations (DMP+EJS)
   (Den Haan et al., 2000)

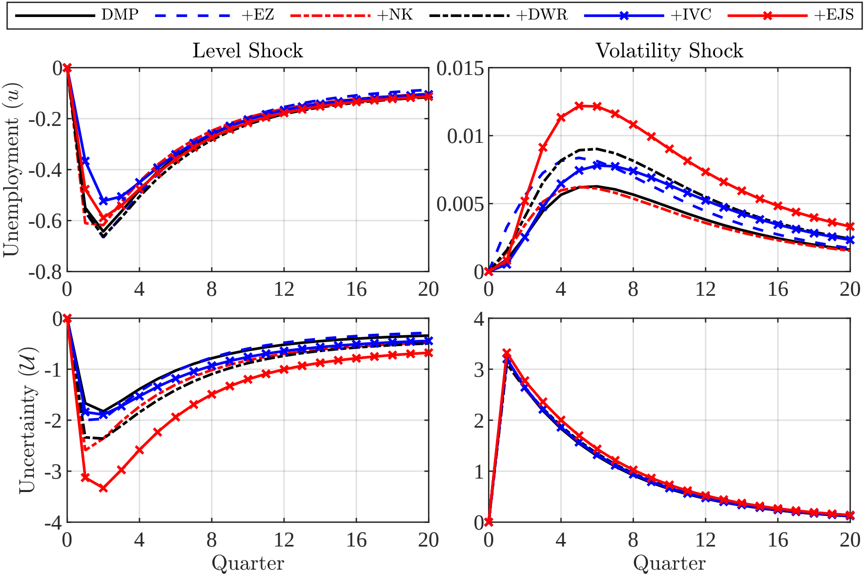
Uncertainty fluctuations remain endogenously countercyclical

▸ Model Descriptions ▸ Other Models

Variance Decompositions

	DMP	+EZ	+NK	+DWR	+IVC	+EJS
Key Moments						
$SD(\mathcal{U})$	6.06	6.12	7.00	7.38	7.20	9.76
$SD(\tilde{y})$	3.65	3.61	3.66	3.76	3.63	3.18
$SD(\tilde{u})$	21.14	20.80	20.83	22.58	19.54	22.21
$Corr(\mathcal{U},\tilde{y})$	-0.62	-0.62	-0.71	-0.71	-0.68	-0.81
$SD(\tilde{s})$	0.00	0.00	0.00	0.00	0.00	8.48
Output Decomposition						
TFP Level Total	100.00	100.00	100.00	100.00	100.00	100.00
TFP Volatility Total	0.20	0.20	0.20	0.21	0.21	0.24
TFP Level Direct	99.80	99.80	99.80	99.79	99.79	99.76
TFP Volatility Direct	0.00	0.00	0.00	0.00	0.00	0.00
Uncertainty Decomposition						
TFP Level Total	43.50	41.84	56.52	60.89	58.06	75.59
TFP Volatility Total	57.01	58.43	43.85	39.53	42.70	25.01
TFP Level Direct	42.99	41.57	56.15	60.47	57.30	74.99
TFP Volatility Direct	56.50	58.16	43.48	39.11	41.94	24.41

DMP Extensions Impulse Responses



Summary

- Search and matching frictions can endogenously explain the negative correlation between output and uncertainty
- Countercyclical uncertainty is an endogenous response to output fluctuations rather than an exogenous impulse
- Uncertainty fluctuations do not feed back into real activity dynamics and remain when the economy is efficient
- Next step is to extend our analysis to financial uncertainty