Cyclical Net Entry and Exit

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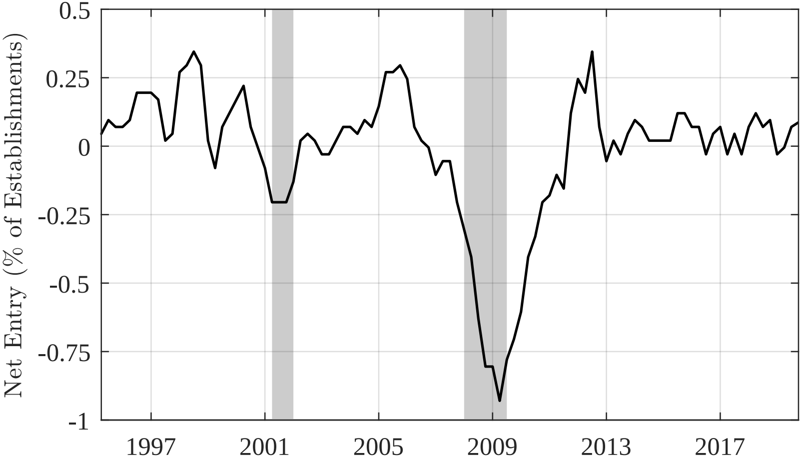
Motivation

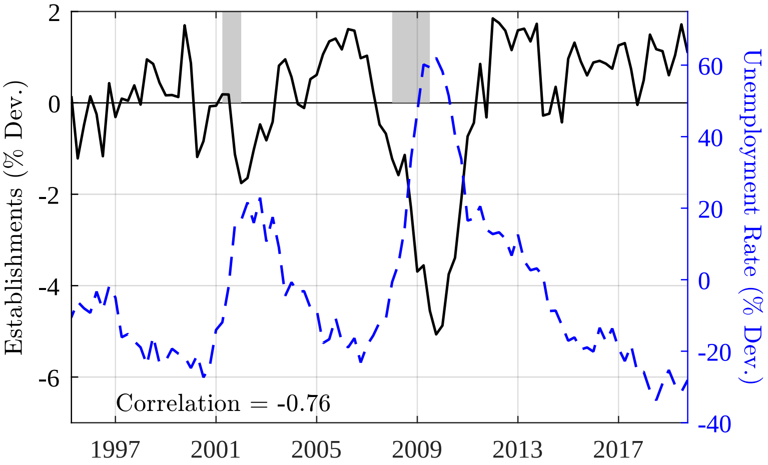
- Fluctuations in net entry and exit can be significant



Source: BLS, Business Employment Dynamics; NBER

Motivation

- Number of establishments move with unemployment rate



Source: BLS, Business Employment Dynamics; Current Population Survey; NBER

Motivation

- Many business cycle models abstract from net entry and exit of firms, especially those with labor market frictions
- Data show infrequent but severe episodes of net exit that are highly correlated with labor market dynamics
- What are the effects of cyclical net entry and exit?

Contributions

Methods:

- Extend RBC model with endogenous net entry and exit and unemployment à la Diamond-Mortensen-Pissarides
- Highlight the key mechanisms driven by their interaction
- Quantify the effects on macroeconomic dynamics

Results:

- Cyclical net entry and exit generates a 20% increase in volatility and a 40% increase in skewness
- Generates fast-slow unemployment dynamics in bad times
- Leads to a 55% higher welfare cost of business cycles
- Model endogenously matches the volatility of uncertainty

Related Literature

- RBC labor search and matching without entry and exit
  (Merz, 1995; Andolfatto, 1996; Den Haan et al., 2000)
- Cyclical entry and exit without labor search
  (Campbell, 1998; Jaimovich and Floetotto, 2008;
  Bilbiie, Ghironi, and Melitz, 2012)
- Explanations of non-normal labor market dynamics
  (McKay and Reis, 2008; Ilut et al., 2018;
  Ferraro, 2018; Dupraz et al., 2019)
- Empirical departures from normality
  (Neftci, 1984; Sichel, 1993; Acemoglu et al., 2017)

Outline

- Environment
- Mechanisms
- Quantitative Results

Environment

- Textbook search and matching model with capital
- Introduce decreasing returns to scale, profits, and endogenous net entry and exit
- Agents in the model:
  - Risk averse representative household
  - Z perfectly competitive incumbent firms
  - Representative employment agency

Incumbent Firm Production

- Productivity (TFP):

$$\ln a_{t+1} = (1-\rho_a)\ln \bar{a} + \rho_a \ln a_t + \sigma_a \varepsilon_{a,t+1}, \;\; \varepsilon_a \sim \mathbb{N}(0,1)$$

- Profit maximization for firm $j \in [0, Z_t]$:

$$J^F_{A,j,t} = \max_{k_{j,t}, n_{j,t}} y_{j,t} - w_t n_{j,t} - r_{k,t} k_{j,t} - \psi_y + E_t \left[x_{t+1} J^F_{X,j,t+1} \right]$$

$$y_{j,t} = a_t \left(k^{\alpha}_{j,t} n^{1-\alpha}_{j,t} \right)^{\vartheta}$$

$$J^F_{X,j,t} = \max\{ J^F_{A,j,t}, 0 \}$$

- Symmetry with $K_{t-1} = Z_t k_t$ and $N_t = Z_t n_t$ implies:

$$Y_t = a_t Z_t^{1-\vartheta} (K^{\alpha}_{t-1} N^{1-\alpha}_t)^{\vartheta}$$

$$w_t = (1-\alpha)\vartheta Y_t / N_t$$

$$r_{k,t} = \alpha \vartheta Y_t / K_{t-1}$$

Firm Entry and Exit

- Value of an active firm:

$$J^F_{A,t} = (1-\vartheta)Y_t/Z_t - \psi_y + E_t[x_{t+1} J^F_{X,t+1}]$$

- Value of an inactive firm ($\psi_n \geq 0$, entry cost):

$$J^F_{I,t} = \max\{0, J^F_{A,t} - \psi_n\}$$

- Free entry and exit implies:

$$J^F_{A,t} \leq \psi_n, J^F_{A,t} \geq 0, \quad \rightarrow \quad J^F_{I,t} = 0, J^F_{A,t} \in [0, \psi_n]$$

- Fraction of incumbents that remain active:

$$\xi_t = \mathbb{I}(J^F_{A,t} > 0) + (Z_t/Z_{t-1})\mathbb{I}(J^F_{A,t} = 0)$$
$$J^F_{X,t} = \xi_t J^F_{A,t}$$

Employment Dynamics

- Job separation rate:

$$s_t = \bar{s} + (1 - \bar{s})(1 - \xi_t)$$

- Unemployed searching for work:

$$U_t^s = U_{t-1} + \chi s_t N_{t-1}$$

- Labor market tightness:

$$\theta_t \equiv V_t / U_t^s$$

- Job-filling Rate:

$$q_t = 1/(1 + \theta_t^{\iota})^{1/\iota}$$

- Employment:

$$N_t = (1 - s_t) N_{t-1} + q_t V_t$$

Employment Agencies

- Post vacancies and sell labor to active firms at w_t
- Representative agency solves

$$J_t^E = \max_{N_t, V_t} (w_t - w_t^n)N_t - \kappa V_t + E_t[x_{t+1} J_{t+1}^E]$$

subject to $N_t = (1 - s_t)N_{t-1} + q_t V_t$ and $V_t \geq 0$

- Optimal vacancy creation decision:

$$\frac{\kappa - \lambda_{V,t}}{q_t} = w_t - w_t^n + E_t[x_{t+1}(1 - s_{t+1})\frac{\kappa - \lambda_{V,t+1}}{q_{t+1}}]$$

WAGES AND MARKET CLEARING

- w_t^n determined by Nash Bargaining
  - b and η denote worker outside option and bargaining power
  - Calibrate to match labor market volatility and the elasticity of wages with respect to productivity (Ljungqvist and Sargent, 2017; Bernstein et al., 2020)
- Workers' wage rate:

$$w_t^n = \eta(w_t + \kappa E_t[x_{t+1}(1 - \chi s_{t+1})\theta_{t+1}]) + (1 - \eta)b$$

- Aggregate resource constraint:

$$C_t + I_t + \kappa V_t = Y_t$$

Understanding the Mechanism

- Special case: $K_t = \bar{K}$ and $\psi_n = 0$, so $J^F_{A,t} \equiv 0$
- The number of active firms is increasing in output:

$$Z_t = (1 - \vartheta) Y_t / \psi_y$$

- Output (and hence profit) is increasing in Z_t

$$Y_t = a_t Z_t^{1-\vartheta} (\bar{K}^\alpha N_t^{1-\alpha})^\vartheta$$

- Positive feedback between Z_t and the real economy
- Intuition: Lower Z increases inputs used by each firm, lowering productivity and output due to decreasing returns

Output Amplification

- Differentiate and combine expressions for Z_t and Y_t:

$$d\log Y_t = (1/\vartheta)d\log a_t + (1-\alpha)d\log N_t$$

- Without entry and exit (denoted NE):

$$d\log Y_t^{NE} = d\log a_t + \vartheta(1-\alpha)d\log N_t$$

- Entry and exit causes output to respond more to changes in productivity and employment under decreasing returns
- Mechanism: $a_t \uparrow \Rightarrow Y_t \uparrow \Rightarrow \Pi_t \uparrow \Rightarrow Z_t \uparrow \Rightarrow Y_t \uparrow$

Labor Market Amplification

- Labor market dynamics are governed by wages:

$$d\log w_t = d\log Y_t - d\log N_t$$

- Substitute for Y and Y^{NE}:

$$d\log w_t = (1/\vartheta)d\log a_t - \alpha d\log N_t$$
$$d\log w_t^{NE} = d\log a_t - (1-\vartheta(1-\alpha))d\log N_t$$

- Wages inherit the amplified dynamics of output
- Wage dynamics govern the payoff from vacancy creation
- Amplifies labor market frictions in tandem with other sources of nonlinearities (e.g., gross complementarity in the matching function, law of motion for unemployment)

Output Asymmetry

- Firm exit causes endogenous separations:

$$ds_t = -(1-\bar{s})\tfrac{Z_t}{Z_{t-1}}\mathbb{I}\{d\log Z_t < 0\}d\log Z_t$$

- Combine with employment law of motion:

$$d\log N_t = (1-\bar{s})\tfrac{n_{t-1}}{n_t}\mathbb{I}\{d\log Z_t < 0\}d\log Z_t + \tfrac{d(q_t V_t)}{N_t}$$

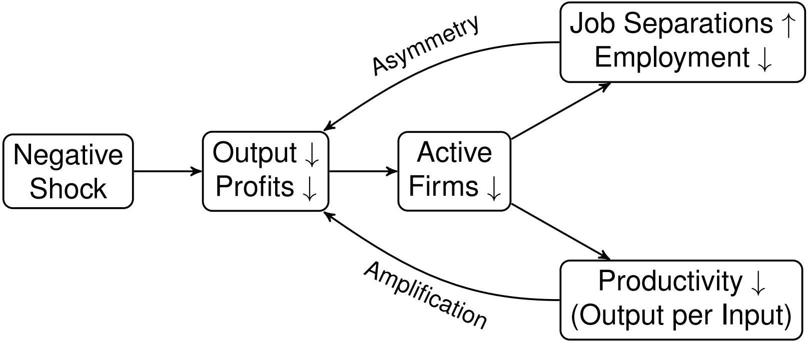
- Combine with the production function:

$$d\log Y_t = \frac{(1/\vartheta)d\log a_t + (1-\alpha)d(q_t V_t)/N_t}{1-(1-\alpha)(1-\bar{s})(n_{t-1}/n_t)\mathbb{I}\{d\log Z_t < 0\}}$$

- Mechanism: $a_t \downarrow \Rightarrow Y_t, \Pi_t \downarrow \Rightarrow Z_t \downarrow \Rightarrow s_t \uparrow \Rightarrow N_t \downarrow \Rightarrow Y_t \downarrow$

Summary of Key Mechanisms

- Amplifies output via input reallocation
- Skews output via endogenous separations



Numerical Methods

- Two innovations to standard policy function iteration
- Recall:

$$\begin{cases} J^F_{A,t} \in (0, \psi_n), & \text{if } \Delta Z = 0 \\ J^F_{A,t} = \psi_n, & \text{if } \Delta Z \geq 0 \\ J^F_{A,t} = 0, & \text{if } \Delta Z \leq 0 \end{cases}$$

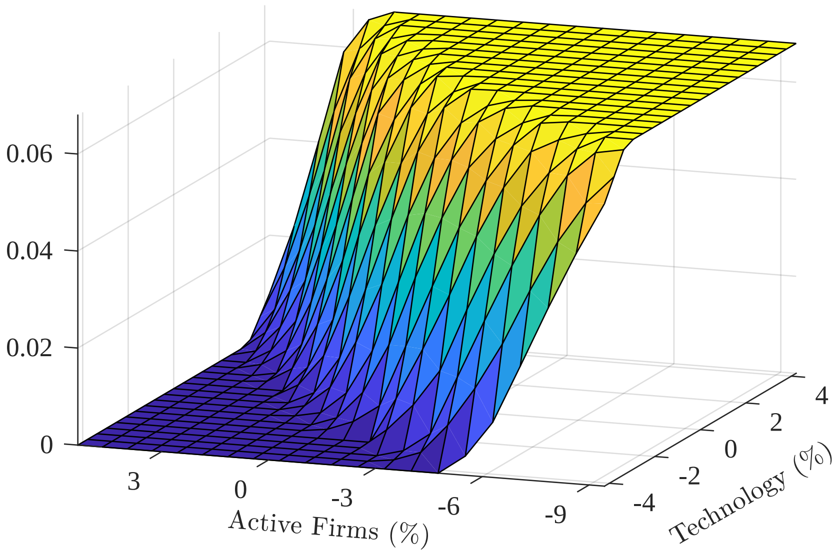
- Introduce auxiliary variable $\mu_{A,t}$ to impose these conditions

$$J^F_{A,t} = \min\left\{\max\left\{0, \mu_{A,t}\right\}, \psi_n\right\}, \quad \Delta Z_t = \mu_{A,t} - J^F_{A,t}$$

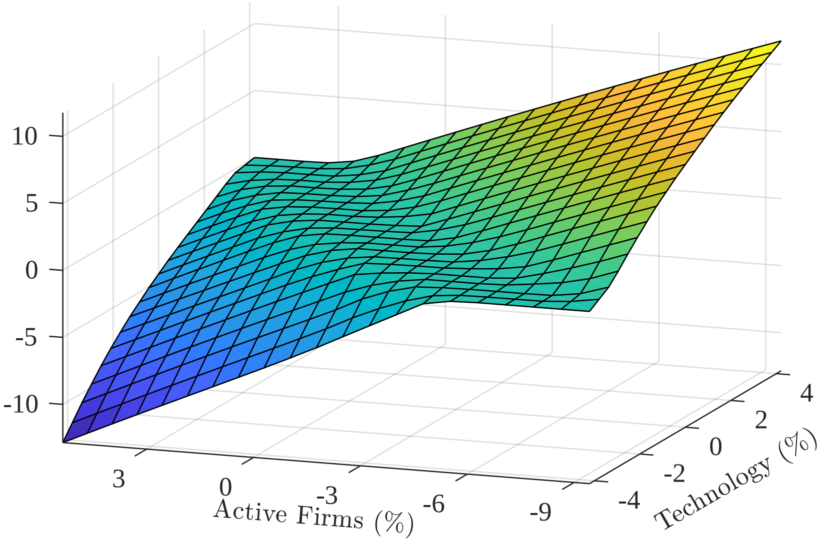
- Introduce second auxiliary variable $\mu_{V,t}$ to ensure $V_t \geq 0$

$$V_t = \max\{0, \mu_{V,t}\}^2, \quad \lambda_{V,t} = \max\{0, -\mu_{V,t}\}^2.$$

Value of Active Firms (J_A^F)



Change in Incumbents (ΔZ)



Monthly Calibration

Parameter		Value	Target	Data	Model
Returns to Scale	ϑ	0.87	Avg. Profit Share	13.44	13.44
Capital Share	α	0.29	Avg. Labor Share	61.55	61.55
Search Duration	χ	0.52	Avg. Unemployment	5.89	5.93
Vacancy Cost	κ	0.11	Avg. Finding Rate	42.15	42.38
Exog. Sep. Rate	$\bar{s}$	0.032	Avg. Sep. Rate	3.27	3.25
Bargaining Weight	η	0.100	Wage-TFP Elasticity	0.60	0.59
Outside Option	b	0.970	Unemployment SD	22.28	22.20
Matching Curvature	ι	0.69	Vacancies SD	23.03	22.76
Inv. Adj. Cost	ν	7.11	Inv. Growth SD	2.13	2.48
Entry Cost	ψ_n	0.068	Entry Share of JC	35.92	35.94
Fixed Cost	ψ_y	0.206	Exit Share of JD	33.38	33.76
TFP Persistence	ρ_a	0.947	Output Growth AC	0.31	0.22
Shock SD	σ_a	0.004	Output Growth SD	0.86	0.95

Note: Discount rate, $\beta = 0.9983$, is set to imply a 2% annual real interest rate. $\delta = 0.0077$.

▸ Entry-Exit moments

Validation of Firm Dynamics

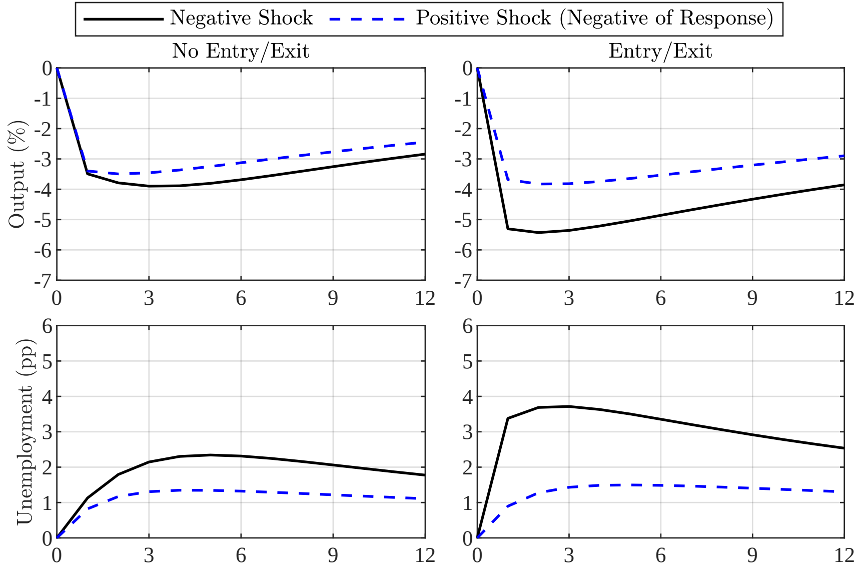
	Quarterly		Annual	
Moment	BED	Model	BDS	Model
$Corr(Z,U)$	-0.76	-0.91	-0.84	-0.92
$Corr(Z,Y)$	0.67	0.89	0.66	0.91
$Corr(Z,s)$	-0.33	-0.13	-0.05	-0.19
$SD(\Delta\tilde{Z})$	0.35	0.43	1.41	1.05
$Skew(\Delta\tilde{Z})$	-0.58	-0.62	-0.10	-0.25

Note: Business Employment Dynamics, 1992-2019; Business Dynamics Statistics, 1978-2018

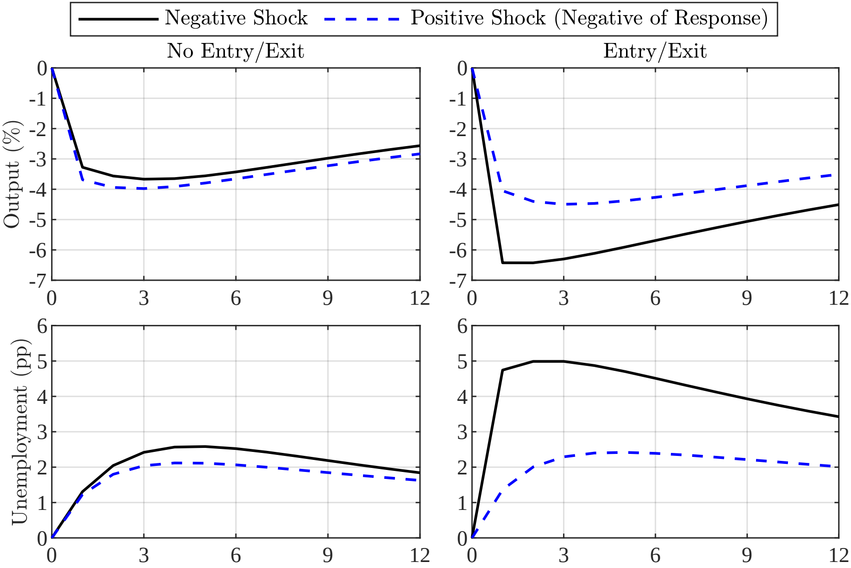
Simulated Moments

	Data		Model	
Moment	Mean	SE	No Entry/Exit	Entry/Exit
$SD(Y)$	3.17	0.26	2.20	2.67
$SD(C)$	2.00	0.16	1.37	1.71
$SD(I)$	8.92	0.77	5.21	6.22
$SD(U)$	22.28	1.85	18.01	22.20
$SD(\Delta \ln U)$	5.56	0.57	6.17	7.37
$Skew(Y)$	-0.59	0.20	-0.31	-0.49
$Skew(C)$	-0.42	0.16	-0.30	-0.41
$Skew(I)$	-0.81	0.21	-0.38	-0.69
$Skew(U)$	0.60	0.20	0.45	0.64
$Skew(\Delta \ln U)$	1.30	0.26	-0.02	0.51

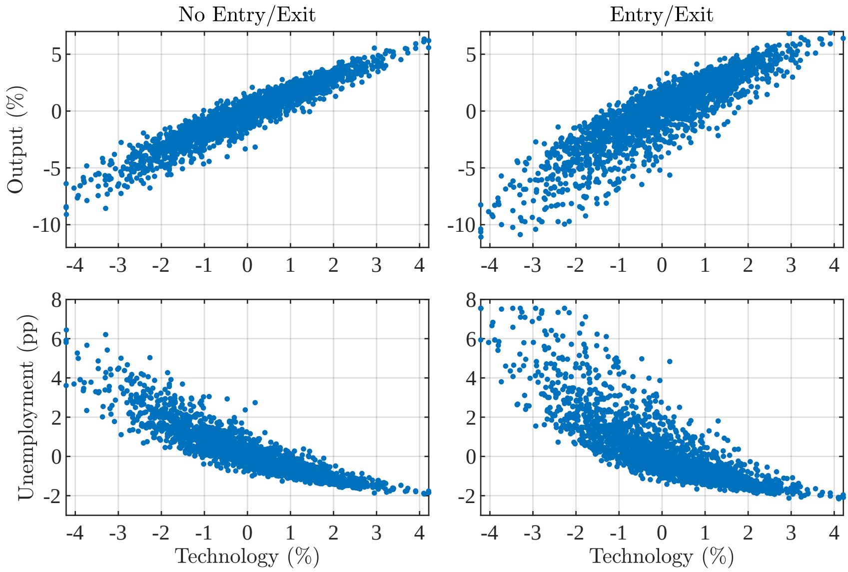
Impulse Responses: Steady State



IMPULSE RESPONSES: RECESSION STATE



Ergodic Distributions



Welfare Cost of Business Cycles

- What $\%$ λ of monthly consumption would households require to make them indifferent between stochastic and non-stochastic paths of consumption?

	No Entry/Exit	Entry/Exit
Welfare Cost ($\lambda, \%$)	0.27	0.42

- Entry/exit increases the cost of fluctuations by 55%
- About 8 times larger than $\lambda = 0.05\%$ in Lucas (2003), which assumes consumption is Gaussian
- Driven by additional skewness imparted by cyclical net entry and exit

Endogenous Uncertainty

- Jurado et al. (2015) Macro Uncertainty Index:

$$\mathcal{U}^Y_{t,t+h} = \sqrt{E_t[(\Delta \log Y_{t+h} - E_t[\Delta \log Y_{t+h}])^2]}$$

	Data		Model	
Moment	Mean	SE	No Entry/Exit	Entry/Exit
$SD(\mathcal{U}^Y)$	5.68	0.51	3.10	6.94
$AC(\mathcal{U}^Y)$	0.84	0.04	0.93	0.78
$Corr(\Delta \ln Y, \mathcal{U}^Y)$	-0.37	0.09	-0.17	-0.42

- Driven by state-dependence of shock transmission
- Net exit creates more uncertainty in bad times

SUMMARY

- This paper studies the effects of cyclical net entry and exit
- Important source of asymmetry and amplification
- Quantitatively the model generates empirically consistent skewness and uncertainty dynamics
- Next steps: introducing heterogeneous firms