THE MATCHING FUNCTION AND NONLINEAR BUSINESS CYCLES

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MOTIVATION

- The matching function is a core component of search models
- A Cobb-Douglas function implies a constant matching elasticity
- But a constant matching elasticity is unlikely to hold empirically
- How does a time-varying matching elasticity affect dynamics?

Results

- Analytically, there are simple conditions for the dynamics of the labor market with a time-varying matching elasticity
- The elasticity of substitution between vacancies and job seekers governs the cyclicality of the matching elasticity
- Quantitatively, the cyclicality of the matching elasticity generates large differences in higher-order business cycle moments
- Normatively, the cyclicality of the matching elasticity determines
  - the cyclicality of the efficiency-restoring vacancy tax wedge
  - the optimal response of the real interest rate to productivity shocks

Matching Elasticity Dynamics

- Consider a matching function, $\mathcal{M}(u_t^s, v_t)$, that satisfies the usual properties and is constant returns to scale, so that the matching elasticity depends only on labor market tightness, $\theta_t = v_t/u_t^s$
- **Proposition 1.** To first order, any constant returns to scale matching function is equivalent to a Cobb-Douglas specification, $\mathcal{M}(u_t^s, v_t) = \phi(u_t^s)^{1-\bar{\epsilon}} v_t^{\bar{\epsilon}}$, where $\bar{\epsilon}$ is a fixed matching elasticity.
- Generally, the elasticity of substitution between vacancies and job seekers, σ_t, might not equal 1 or be constant
- **Proposition 2.** The matching elasticity, $\epsilon_t = \epsilon(\theta_t)$, is increasing in labor market tightness, θ_t, when $\sigma_t > 1$, constant when $\sigma_t = 1$, and decreasing when $\sigma_t < 1$.

U.S. Matching Elasticity Estimates

- Empirically, there is disagreement about the matching elasticity and sparse evidence about its cyclicality
- Cobb-Douglas matching function (various samples)
  - Do not correct for endogeneity (OLS): $\bar{\epsilon} \in [0.28, 0.77]$
  - Endogeneity corrections (GMM with IV): $\bar{\epsilon} \in [0.24, 0.70]$
- CES matching function
  - Blanchard and Diamond (1989): $\bar{\epsilon} = 0.54, \sigma = 0.74$
  - Shimer (2005): $\bar{\epsilon} = 0.28, \sigma = 1.06$
  - Sahin et al. (2014): $\bar{\epsilon} \in [0.24, 0.66], \sigma \in [0.9, 1.2]$
- Non-parametric
  Lange and Papageorgiou (2020): Procyclical $\epsilon_t \in (0.15, 0.3)$

Search and Matching Model

- Entering period t, there are u_{t-1} unemployed workers
- New matches satisfy $m_t = \min\{\mathcal{M}(u_{t-1}, v_t), u_{t-1}, v_t\}$
- Job finding and job filling rates

$$f_t = m_t/u_{t-1}, \quad q_t = m_t/v_t, \quad f_t, q_t \in [0,1]$$

- Laws of motion

$$n_t = (1-\bar{s})n_{t-1} + f_t u_{t-1}$$
$$u_t = u_{t-1} + \bar{s}n_{t-1} - f_t u_{t-1}$$

Firms and Market Clearing

- Labor productivity a_t follows an AR(1) process in levels

$$a_{t+1} = \bar{a} + \rho_a(a_t - \bar{a}) + \sigma_a \varepsilon_{a,t+1},\ 0 \leq \rho_a < 1,\ \varepsilon_a \sim \mathbb{N}(0,1)$$

- The representative firm solves

$$V_t = \max_{v_t, n_t} (a_t - w_t) n_t - \kappa v_t + E_t x_{t+1} V_{t+1}$$

subject to $n_t = (1 - \bar{s}) n_{t-1} + q_t v_t$ and $v_t \geq 0$

- Optimality under Nash Bargaining implies

$$w_t = \eta(a_t + \kappa E_t[x_{t+1}\theta_{t+1}]) + (1 - \eta) b$$

- Aggregate resource constraint: $c_t + \kappa v_t = a_t n_t$

Analytical Solution

- **Assumption 1.**
  $\gamma = 0$ (Risk-neutral) and $\eta = 0$ (i.e., sticky wages $w_t = b$)
- **Proposition 3.** Under Assumption 1, the marginal cost of hiring follows the stochastic process

$$(\kappa - \lambda_{v,t})/q_t = \delta_0 + \delta_1(a_t - \bar{a})$$

where

$$\delta_0 = \frac{\bar{a} - b}{1 - \beta(1 - \bar{s})} > 0, \qquad \delta_1 = \frac{1}{1 - \beta(1 - \bar{s})\rho_a} > 0,$$

and $\lambda_{v,t} > 0$ implies $q_t = 1$.

Labor Market Tightness Dynamics

- **Proposition 4.** Labor market tightness, $\theta(a_t)$, is convex at a_t when $\sigma_t > 1/2$, linear at a_t when $\sigma_t = 1/2$, and concave at a_t when $\sigma_t < 1/2$.
- Interpretation:

$$\theta'(a_t) = \frac{\delta_1}{\kappa} \frac{\mathcal{M}(1, \theta(a_t))}{1 - \epsilon_t}$$

Two Channels

1. Higher productivity generates more matches
2. Higher productivity lowers the matching elasticity (when $\sigma_t < 1$)

$$\sigma_t < 1/2 \rightarrow \text{channel } 2 > \text{channel } 1$$
$$\sigma_t = 1/2 \rightarrow \text{channels offset}$$
$$\sigma_t > 1/2 \rightarrow \text{channel } 1 > \text{channel } 2$$

Job Finding Rate Dynamics

- **Proposition 5.** The job finding rate, $f_t = f(a_t)$, is convex at a_t when $\sigma_t > 1/(2\epsilon_t)$, linear at a_t when $\sigma_t = 1/(2\epsilon_t)$, and concave at a_t when $\sigma_t < 1/(2\epsilon_t)$.
- Interpretation:

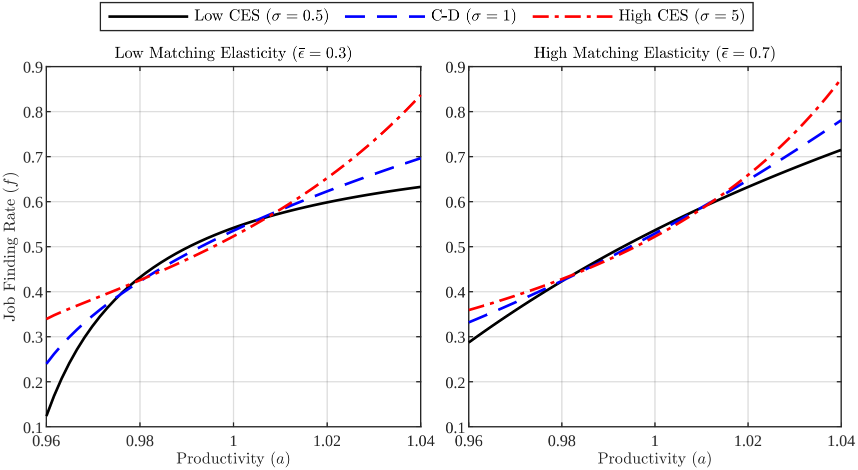
$$f'(a_t) = \mathcal{M}_v(1, \theta(a_t))\theta'(a_t)$$

Two Channels

1. Higher productivity raises labor market tightness and lowers $\mathcal{M}_v(1, \theta(a_t))$
2. Higher productivity affects responsiveness of tightness itself through $\theta'(a_t)$

$$\sigma_t < 1/(2\epsilon_t) \rightarrow \text{channel } 1 > \text{channel } 2$$
$$\sigma_t = 1/(2\epsilon_t) \rightarrow \text{channels offset}$$
$$\sigma_t > 1/(2\epsilon_t) \rightarrow \text{channel } 2 > \text{channel } 1$$

The Job Finding Rate Function



Unemployment Rate Dynamics

- Matching function affects job finding rate, which affects unemployment rate via law of motion

$$u_t = u_{t-1} + \bar{s}n_{t-1} - f_t u_{t-1}$$

- $\partial u_t / \partial a_t = -u_{t-1} f'(a_t)$, the size of the unemployment response to a change in productivity is larger when
  - unemployment is already elevated
  - the job finding rate function is steeper

Quantitative Exercise

- Assume CES matching function, log utility
- Given different $(\sigma, \bar{\epsilon})$ pairs, set other parameters using U.S. data from 1955 to 2019
  - Set discount factor, separation rate, AC/shock SD of productivity
  - Set vacancy posting cost, κ, and flow value of unemployment, b, to match mean and SD of unemployment rate
  - Set bargaining power, η, to match wage-productivity elasticity
- Solve model globally and simulate nonlinear model

Higher-Order Moments

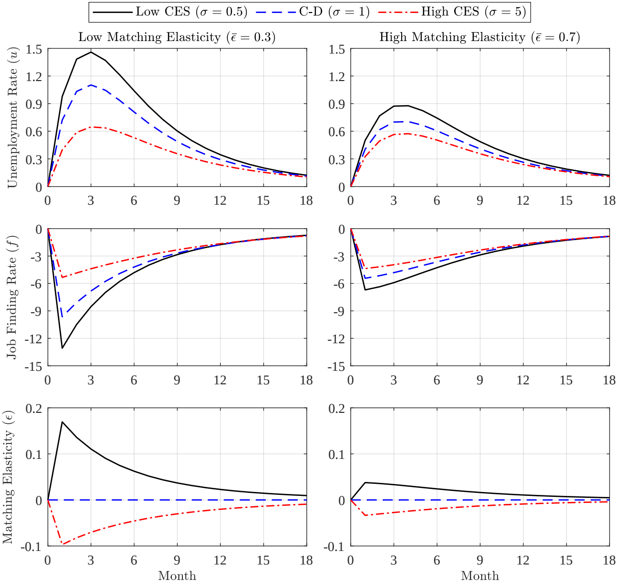
(A) $\bar{\epsilon} = 0.3$

σ	0.5	1.0	5.0
$Skew(f)$	−1.40	−0.58	0.33
$Skew(u)$	2.37	1.35	0.29
$Kurt(f)$	3.35	0.75	0.15
$Kurt(u)$	9.78	3.63	0.04
$SD(\epsilon)$	0.07	0.00	0.07
$Corr(\epsilon, u)$	0.96	0.00	−0.98

(B) $\bar{\epsilon} = 0.7$

σ	0.5	1.0	5.0
$Skew(f)$	−0.29	0.12	0.49
$Skew(u)$	0.95	0.49	0.15
$Kurt(f)$	0.08	−0.06	0.32
$Kurt(u)$	1.55	0.33	−0.08
$SD(\epsilon)$	0.04	0.00	0.03
$Corr(\epsilon, u)$	0.97	0.00	−0.98

GENERALIZED IRFs ($u_0 = 7.5\%$)



Efficient Fiscal Policy

- Equilibrium is inefficient because a new vacancy creates a
  - positive externality for unemployed worker
  - negative externality for other firms

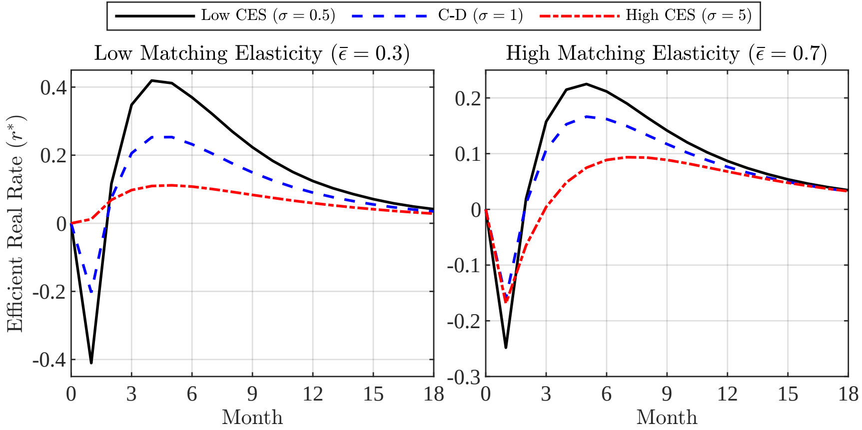
- **Proposition 6.** The efficiency-restoring wedges are given by

$$\tau_v(\theta_t) = (1-\eta)/\epsilon(\theta_t) - 1$$
$$\tau_n(\theta_t) = \theta_t((\kappa - \lambda_{v,t})\tau_v(\theta_t) - \eta\lambda_{v,t})$$

- $\tau_{v,t}$ co-moves negatively with the matching elasticity, e.g., for CES matching function

$$\sigma < 1 \rightarrow \text{countercyclical matching elasticity} \rightarrow \text{procyclical } \tau_{v,t}$$
$$\sigma > 1 \rightarrow \text{procyclical matching elasticity} \rightarrow \text{countercyclical } \tau_{v,t}$$
$$\sigma = 1 \rightarrow \text{constant matching elasticity} \rightarrow \text{constant } \tau_{v,t}$$

Optimal Monetary Policy ($u_0 = 7.5\%$)



Conclusion

- Cobb-Douglas matching function is ubiquitous but implies constant matching elasticity
- We generalize the matching function and derive conditions that determine how the cyclicality of the matching elasticity affects the job finding and unemployment rates
- Those effects are quantitatively large and driven by modest variation in the matching elasticity