# Online Appendix: Geopolitical Oil Price Risk and Economic Fluctuations*

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September 17, 2025


ABSTRACT

This appendix describes the methodology for constructing a time series of oil price uncertainty, our data sources, and the solution method for our general equilibrium model. It plots the time series of the real price of oil underlying our analysis as well as the U.S. oil expenditure share. It presents the responses to a macroeconomic disaster and to positive and negative macroeconomic disaster probability shocks. It provides the results of several robustness exercises, including an alternative CES production function, an alternative utility function that allows households to consume oil, and a higher average oil expenditure share. It concludes by comparing the CES production technology to the putty-clay technology.


*The views expressed in this paper are our own and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.


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# A MEASURING UNCERTAINTY

Our method of constructing quarterly measures of uncertainty builds on Jurado et al. (2015). We first summarize the key steps of the estimation process before discussing the data used in the estimation.

**A.1 METHODOLOGY** Let $\mathbf{Y}_t = (y_{1,t}, \ldots, y_{N_y,t})'$ be a vector of data containing $N_y$ variables. Our objective is to estimate the 1-quarter ahead uncertainty about select elements of $\mathbf{Y}_t$, defined as

$$\mathcal{U}_t^j \equiv \sqrt{E[(y_{j,t+1} - E[y_{j,t+1}|I_t])^2|I_t]},$$

where the expectation is taken with respect to the information set $I_t$ and $j$ refers to the variable of interest. There are four steps:

1. Generate forecast errors for $y_{j,t+1}$ using a forecasting model that includes lags of the variable $y_j$, estimated factors extracted from a panel of predictor variables, $\hat{\mathbf{F}}_t$, and a set of additional predictors contained in a vector $\mathbf{W}_t$.
2. Fit autoregressive models for the factors in $\hat{\mathbf{F}}_t$ and the variables in $\mathbf{W}_t$ and generate residuals for each variable.
3. Estimate a stochastic volatility model for each residual.
4. Calculate $\mathcal{U}_t^j$.

**Factors** Let $\mathbf{X}_t = (X_{1,t}, \ldots, X_{N_x,t})'$ be a vector of predictors that are available for forecasting. These data are transformed to be stationary. It is assumed that the transformed variables have an approximate factor structure,

$$X_{i,t} = \Lambda_i^{F'} \mathbf{F}_t + e_{i,t}^X,$$

where $\mathbf{F}_t$ is a $r_F \times 1$ vector of latent factors, $\Lambda_i^{F'}$ is a $1 \times r_F$ vector of loadings for variable $i$ and the idiosyncratic errors are given by $e_{i,t}^X$. The estimated factors, denoted as $\hat{\mathbf{F}}_t$, are estimated using principal components and the number of factors is selected using the criterion of Bai and Ng (2002). Each of the factors is assumed to follow an autoregressive process with two lags,

$$F_t = \Phi^F(L) F_{t-1} + v_t^F,$$
$$v_t^F = \sigma_t^F \epsilon_t^F, \quad \epsilon_t^F \sim \mathbb{N}(0,1),$$
$$\ln(\sigma_t^F)^2 = \alpha^F + \beta^F \ln(\sigma_{t-1}^F)^2 + \tau^F \eta_t^F, \quad \eta_t^F \sim \mathbb{N}(0,1),$$

where $\Phi^F(L)$ is a lag polynomial. As with the other lag order choices made below, our results are robust to reasonable variation in the lag order.

**Additional predictors** The $r_W \times 1$ vector $\mathbf{W}_t$ includes the squared values of the first factor in $\hat{F}_t$ and a set of $N_G$ factors estimated using principal components on the squared values of the variables in $\mathbf{X}_t$. Each variable in $\mathbf{W}_t$ is assumed to follow an autoregressive process with two lags,

$$
\begin{gathered}
W_t = \Phi^W(L) W_{t-1} + v_t^W, \\
v_t^W = \sigma_t^W \epsilon_t^W, \quad \epsilon_t^W \sim \mathbb{N}(0,1), \\
\ln(\sigma_t^W)^2 = \alpha^W + \beta^W \ln(\sigma_{t-1}^W)^2 + \tau^W \eta_t^W, \quad \eta_t^W \sim \mathbb{N}(0,1),
\end{gathered}
$$

where $\Phi^W(L)$ is a lag polynomial.

**Forecasting Model** A forecast for $y_{j,t+1}$ is produced with the factor-augmented forecasting model,

$$
\begin{gathered}
y_{j,t+1} = \phi_j^Y(L) y_{j,t} + \gamma_j^F(L)\hat{\mathbf{F}}_t + \gamma_j^W(L)\mathbf{W}_t + \nu_{j,t+1}^Y, \\
\nu_t^y = \sigma_t^y \epsilon_t^y, \quad \epsilon_t^y \sim \mathbb{N}(0,1) \\
\ln(\sigma_t^y)^2 = \alpha^y + \beta^y \ln(\sigma_{t-1}^y)^2 + \tau^y \eta_t^y, \quad \eta_t^y \sim \mathbb{N}(0,1),
\end{gathered}
$$

where $\phi_j^Y(L)$, $\gamma_j^F(L)$, and $\gamma_j^W(L)$ are lag polynomials of orders 2, 1, and 1, respectively. As in Jurado et al. (2015, footnote 10), a hard threshold is applied to remove any variables from the forecasting model that do not have incremental predictive power.

**Uncertainty** Define $\mathbf{Z}_t \equiv (\hat{\mathbf{F}}_t', \mathbf{W}_t')'$ as a vector that collects the estimated factors and the additional predictors contained in $\mathbf{W}_t$. Then let $\mathcal{Z}_t \equiv (\mathbf{Z}_t', \ldots, \mathbf{Z}_{t-q+1}')'$ and $Y_{j,t} = (y_{j,t}, \ldots, y_{j,t-q+1})'$, where $q = 2$. The FAVAR model can be written in companion form as

$$
\begin{pmatrix} \mathcal{Z}_t \\ Y_{j,t} \end{pmatrix} = \begin{pmatrix} \Phi^{\mathcal{Z}} & 0 \\ \Lambda_j' & \Phi_j^Y \end{pmatrix} \begin{pmatrix} \mathcal{Z}_{t-1} \\ Y_{j,t-1} \end{pmatrix} + \begin{pmatrix} \mathcal{V}_t^{\mathcal{Z}} \\ \mathcal{V}_{j,t}^Y \end{pmatrix} \Longleftrightarrow \mathcal{Y}_{j,t} = \Phi_j^{\mathcal{Y}} \mathcal{Y}_{j,t-1} + \mathcal{V}_{j,t}^{\mathcal{Y}}.
$$

The forecast error variance is

$$
\Omega_{j,t}^{\mathcal{Y}}(1) \equiv E_t[(\mathcal{Y}_{j,t+1} - E_t \mathcal{Y}_{j,t+1})(\mathcal{Y}_{j,t+1} - E_t \mathcal{Y}_{j,t+1})'],
$$

where $E_t \mathcal{Y}_{j,t+1} = \Phi_j^{\mathcal{Y}} \mathcal{Y}_{j,t}$. The forecast error variances can be calculated as

$$
\Omega_{j,t}^{\mathcal{Y}}(1) = E_t[\mathcal{V}_{j,t+1}^{\mathcal{Y}} \mathcal{V}_{j,t+1}^{\mathcal{Y}\prime}].
$$

The uncertainty of $y_{j,t+1}$ is

$$
\mathcal{U}_t^j = \sqrt{1_j' \Omega_{j,t}^{\mathcal{Y}}(1) 1_j},
$$

where $1$ is a selection vector and $j$ refers to the growth rate of real GDP and the growth rate of the inflation-adjusted U.S. refiners' acquisition cost of imported crude oil, respectively.

**A.2 DATA** Our dataset includes most of the financial and macroeconomic variables listed in the data appendix of Ludvigson et al. (2021) plus U.S. real GDP and the inflation-adjusted U.S. refiners' acquisition cost of imported crude oil.

The macroeconomic variables are from the April 2024 vintage of the FRED-MD database with the following modifications.

- We linearly interpolate the missing values of $UMCSENTx$ that occur through 1977.
- We set the missing value of $CP3Mx$ for 4/1/2020 to its value on 3/1/2020.
- We set the missing value of $COMPAPFFx$ for 4/1/2020, to its value on 3/1/2020.

Monthly data are averaged by quarter and transformed to stationarity using the code in the FRED-MD database. Both real GDP and the real price of oil are log-differenced. The data set starts in 1974Q1. The sample begins in 1974Q2, because we lose one observation due to differencing.

The financial variables are obtained from FRED-MD, CRSP and the Fama-French database. Returns are aggregated by summing the monthly values by quarter.

# B DATA SOURCES

We use the following time-series provided by Haver Analytics:

1. **Consumer Price Index for All Urban Consumers**: Not seasonally Adjusted, Monthly, Index (PCUN@USECON)
2. **World Production of Crude Oil Including Lease Condensate** Not Seasonally Adjusted, Thousands of Barrels per Day (Monthly, AWOACAUF@ENERGY; Quarterly, BWOACAUF@ENERGY)
3. **United States: Petroleum Products Expenditures** Annual, Millions of Dollars (ZUSPATCV@USENERGY)
4. **US Crude Oil Imported Acquisition Cost by Refiners** Not Seasonally Adjusted, Quarterly, Dollars per Barrel (CUSIQABF@USENERGY)
5. **Civilian Noninstitutional Population: 16 Years & Over** Not Seasonally Adjusted, Quarterly, Thousands (LN16N@USECON)
6. **Gross Domestic Product: Implicit Price Deflator** Seasonally Adjusted, Quarterly, 2012=100 (DGDP@USNA)
7. **Gross Domestic Product** Seasonally Adjusted, Quarterly, Billions of Dollars (GDP@USECON)
8. **Gross Domestic Product** Annual, Millions of Dollars (GDPY@USNA)

9. **Personal Consumption Expenditures: Nondurable Goods**
   Seasonally Adjusted, Quarterly, Billions of Dollars (CN@USECON)
10. **Personal Consumption Expenditures: Services**
    Seasonally Adjusted, Quarterly, Billions of Dollars (CS@USECON)
11. **Personal Consumption Expenditures: Durable Goods**
    Seasonally Adjusted, Quarterly, Billions of Dollars (CD@USECON)
12. **Private Fixed Investment**
    Seasonally Adjusted, Quarterly, Billions of Dollars (F@USECON)
13. **Total Economy: Labor share**
    Seasonally Adjusted, Quarterly, Percent (LXEBL@USNA)
14. **Net Stock: Private Fixed Assets**, Annual, Billions of Dollars (EPT@CAPSTOCK)
15. **Net Stock: Durable Goods**, Annual, Billions of Dollars (EDT@CAPSTOCK)
16. **Depreciation: Private Fixed Assets**, Annual, Billions of Dollars (KPT@CAPSTOCK)
17. **Depreciation: Durable Goods**, Annual, Billions of Dollars (KDT@CAPSTOCK)
18. **CBOE Crude Oil Volatility Index (OVX)**, Daily, Index (SPOVX@DAILY)

We also use the following data sources:

1. **FRED-MD**, Monthly Databases for Macroeconomic Research. The data is available at `https://research.stlouisfed.org/econ/mccracken/fred-databases` (McCracken, 2024). Under Monthly Data, we use the April 2024 vintage.
2. **Fama-French**, Database. The data is available at `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`
   (Fama and French, 2024).
3. **WRDS**, Stock Market Indexes. The data is available at `https://wrds-www.wharton.upenn.edu` (Wharton Research Data Services, 2024).
4. **Geopolitical Risk Index**, Historical series (GPRH). The data is available at `https://www.matteoiacoviello.com/gpr.htm` (Caldara and Iacoviello, 2024).
5. **Global Oil Inventories** Monthly, Millions of Barrels per Day ($Inv_t$) from Kilian (2022).

We apply the following data transformations:

1. **Per Capita Real Output**: $Y_t = 10^9 \times GDP_t/((DGDP_t/100)(1000 \times LN16N_t))$.
2. **Per Capita Real Consumption**: $C_t = 10^9(CN_t+CS_t)/((DGDP_t/100)(1000\times LN16N_t))$.
3. **Per Capita Real Investment**: $I_t = 10^9(F_t + CD_t)/((DGDP_t/100)(1000 \times LN16N_t))$.

4. **Depreciation Rate**: $\delta = (1 + \frac{1}{T/4}\sum_{t=1}^{T/4}(KPT_t + KDT_t)/(EPT_{t-1} + EDT_{t-1}))^{1/4} - 1$.
5. **Capital Services Share**: $\xi = 1 - \frac{1}{T}\sum_{t=1}^{T} LXEBL/100$.
6. **Real Price of Oil**: $p_t^o = CUSIQABF_t/(DGDP_t/100)$.
7. **Expenditure Share of Oil**: $ZUSPATCV_t/GDPY_t$.
8. **Oil Consumption**: $o_t =$ Days per Month $\times\, AWOACAUF_t/1000 - (INV_t - INV_{t-1})$.
9. **Inventory-Oil Consumption Share**: $INV_t/\sum_{j=t-2}^{t} o_t$ for $t = 3, 6, \ldots, 3T$.
10. **CPI Inflation Rate**: $\pi_t^{cpi} = 100 \times (PCUN_t/PCUN_{t-1} - 1)$.
11. **Asset Returns**: We use two time series from the Fama-French data library:

    - Net nominal risk-free rate, monthly, percent ($RF$)
    - Net nominal excess market return, monthly, percent ($MKTmRF$)

    Define the market return as $RM_t \equiv MKTmRF_t + RF_t$. The gross quarterly analogues of the Fama-French series and CPI inflation are given by

    $$RF_t^Q \equiv \prod_{j=t-2}^{t}(1 + RF_j/100),\ RM_t^Q \equiv \prod_{j=t-2}^{t}(1 + RM_j/100),\ \pi_t^Q \equiv \prod_{j=t-2}^{t}(1 + \pi_j^{cpi}/100)$$

    for $t = 3, 6, \ldots, 3T$, so the quarterly real risk-free rate and equity premium are

    $$r_t = 100 \times (RF_t^Q/\pi_t^Q - 1),\ r_t^{ex} = 100 \times (RM_t^Q/\pi_t^Q - 1) - r_t.$$

Data moments are computed with quarterly data. The average oil expenditure share is based on annual data.

# C Solution Method

The equilibrium system of the DSGE model is summarized by $E[g(\mathbf{x}_{t+1}, \mathbf{x}_t, \varepsilon_{t+1})|\mathbf{z}_t, \vartheta] = 0$, where $g$ is a vector-valued function, $\mathbf{x}_t$ is the vector of model variables, $\varepsilon_t$ is the vector of shocks, $\mathbf{z}_t = [k_t, s_t, v_t^g, v_t^e, \ln p_t^g, \ln p_t^e, \epsilon_t]$ is the vector of states, and $\vartheta$ is the vector of parameters.

We discretize the continuous shocks, $\{\varepsilon_g, \varepsilon_{go}, \varepsilon_p^g, \varepsilon_p^e\}$ using the Markov chain in Rouwenhorst (1995). The bounds of the six continuous state variables are chosen so they cover at least $99\%$ of the ergodic distribution, reducing the need for extrapolation. Specifically, the bounds on capital, $k_t$, range from $-15\%$ to $+10\%$, the bounds on storage, $s_t$, range from $-50\%$ to $+80\%$, and the bounds on the error correction term, $\epsilon_t$, range from $-30\%$ to $+15\%$ of the deterministic steady state. The bounds on the probability of a growth disaster, $p_t^g$, are set to $[0.00005, 0.8]$, while the bounds on the probability of an oil production disaster, $p_t^e$, are set to $[0.0000025, 0.8]$. Both are converted to logs. We discretize $k_t$, $s_t$, and $\epsilon_t$ each into 7 points, and $\ln p_t^g$ and $\ln p_t^g$ into 15 points given the nonlinearity in the transmission of the probability shocks. All of the grids for the continuous states are evenly spaced. There are also binary indicators for whether the economy is in a growth disaster or an oil production disaster, creating 4 outcomes. The product of the points in each dimension, $D$, is the total number of nodes in the state space ($D = 308{,}700$).

The realization of $\mathbf{z}_t$ on node $d$ is denoted $\mathbf{z}_t(d)$. The Rouwenhorst method provides integration nodes for the continuous shocks, $[\varepsilon_{g,t+1}(m), \varepsilon_{go,t+1}(m), \varepsilon_{p,t+1}^g(m), \varepsilon_{p,t+1}^e(m)]$. The transition matrices for the discrete states determine the integration weights for their future realizations, $[v_{t+1}^g(m), v_{t+1}^e(m)]$. The weight for a particular realization of the continuous and discrete shocks is $\phi(m)$, where $m \in \{1, \ldots, M\}$ and $M$ is the product of the number of realizations of each shock. The two disaster probability shocks, $\varepsilon_p^g$ and $\varepsilon_p^e$, have the same number of realizations as the corresponding state variable (15). Each growth shock, $\varepsilon_g$ and $\varepsilon_{go}$, has 7 possible realizations. Each discrete state has two possible outcomes. Thus, $M = 44{,}100$ possible shock realizations.

The vector of policy functions and the realization on node $d$ are denoted by $\mathbf{pf}_t$ and $\mathbf{pf}_t(d)$, where $\mathbf{pf}_t \equiv [n(\mathbf{z}_t), o(\mathbf{z}_t), J(\mathbf{z}_t), p^e(\mathbf{z}_t), r(\mathbf{z}_t)]$. The following steps outline our algorithm:

1. Use the Sims (2002) `gensys` algorithm to solve the log-linear model without any disasters or time-varying probabilities. Then map the solution for the policy functions to the discretized state space, copying the solution on the dimensions that were excluded from the linear model. This provides an initial conjecture, $\mathbf{pf}_0$, for the nonlinear algorithm.
2. On iteration $j \in \{1, 2, \ldots\}$ and each node $d \in \{1, \ldots, D\}$, use Sims' `csolve` code to find the $\mathbf{pf}_t(d)$ that satisfies $E[g(\cdot)|\mathbf{z}_t(d), \vartheta] \approx 0$. Guess $\mathbf{pf}_t(d) = \mathbf{pf}_{j-1}(d)$. Then
   (a) Solve for all variables dated at time $t$, given $\mathbf{pf}_t(d)$ and $\mathbf{z}_t(d)$.
   (b) Linearly interpolate the policy functions, $\mathbf{pf}_{j-1}$, at the updated state variables, $\mathbf{z}_{t+1}(m)$, to obtain $\mathbf{pf}_{t+1}(m)$ on every integration node, $m \in \{1, \ldots, M\}$.
   (c) Given $\{\mathbf{pf}_{t+1}(m)\}_{m=1}^M$, solve for the other elements of $\mathbf{s}_{t+1}(m)$ and compute
   $$E[g(\mathbf{x}_{t+1}, \mathbf{x}_t(d), \varepsilon_{t+1})|\mathbf{z}_t(d), \vartheta] \approx \textstyle\sum_{m=1}^M \phi(m) g(\mathbf{x}_{t+1}(m), \mathbf{x}_t(d), \varepsilon_{t+1}(m)).$$
   When `csolve` has converged, set $\mathbf{pf}_j(d) = \mathbf{pf}_t(d)$.
3. Repeat step 2 until $\text{maxdist}_j < 10^{-5}$, where $\text{maxdist}_j \equiv \max\{|(\mathbf{pf}_j - \mathbf{pf}_{j-1})/\mathbf{pf}_{j-1}|\}$. When that criterion is satisfied, the algorithm has converged to an approximate solution.

**Ergodic Probabilities** Since the probability of entering an oil production or macroeconomic disaster is time varying, there is no closed-form solution for the ergodic probabilities of these disasters. We therefore compute $\pi_1^g$ and $\pi_1^e$ by simulating $v_t^g$ and $v_t^e$ for 1 million periods and then computing the fraction of periods where $v_t^g = 1$ and the fraction of periods where $v_t^e = 1$.

# D DETRENDED EQUILIBRIUM OF THE BASELINE MODEL

We detrend the model by defining $\tilde{x}_t = x_t/a_t$. The equilibrium system of equations is given by

$$\tilde{w}_t = (1-\xi)\tilde{y}_t/n_t$$
$$p_t^o = \xi\alpha \frac{(\tilde{o}_t/o_0)^{1-1/\sigma}}{(1-\alpha)(\tilde{k}_t/k_0)^{1-1/\sigma}+\alpha(\tilde{o}_t/o_0)^{1-1/\sigma}}\frac{\tilde{y}_t}{\tilde{o}_t}$$
$$E_t[x_{t+1}r_{t+1}^i] = 1$$
$$r_t^i = e^{-\zeta v_t}\frac{1}{p_{t-1}^k}(r_t^k + (1-\delta+a_1+\tfrac{a_2}{\nu-1}(\tilde{\imath}_t/\tilde{k}_t)^{1-1/\nu})p_t^k)$$
$$p_t^k = \tfrac{1}{a_2}(\tilde{\imath}_t/\tilde{k}_t)^{1/\nu}$$
$$r_t^k = \xi(1-\alpha)\frac{(\tilde{k}_t/k_0)^{1-1/\sigma}}{(1-\alpha)(\tilde{k}_t/k_0)^{1-1/\sigma}+\alpha(\tilde{o}_t/o_0)^{1-1/\sigma}}\frac{\tilde{y}_t}{\tilde{k}_t}$$
$$E_t[x_{t+1}r_{t+1}^s] = 1$$
$$r_t^s = \frac{1}{p_{t-1}^o}(1-\omega+\pi\tilde{s}_t^{-3})p_t^o$$
$$\chi\tilde{w}_t\ell_t = (1-\chi)\tilde{c}_t$$
$$x_t = (\beta/g_t^\gamma)(\tilde{u}_t/\tilde{u}_{t-1})^{1-1/\psi}(\tilde{c}_{t-1}/\tilde{c}_t)(\tilde{J}_t/\tilde{z}_{t-1})^{1/\psi-\gamma}$$
$$\tilde{u}_t = \tilde{c}_t^\chi\ell_t^{1-\chi}$$
$$\tilde{z}_t = (E_t[(g_{t+1}J_{t+1})^{1-\gamma}])^{1/(1-\gamma)}$$
$$\tilde{J}_t = \left((1-\beta)\tilde{u}_t^{1-1/\psi}+\beta\tilde{z}_t^{1-1/\psi}\right)^{\frac{1}{1-1/\psi}}$$
$$\tilde{y}_t = y_0 n_t^{1-\xi}\left((1-\alpha)(\tilde{k}_t/k_0)^{1-1/\sigma}+\alpha(\tilde{o}_t/o_0)^{1-1/\sigma}\right)^{\xi/(1-1/\sigma)}$$
$$g_{t+1}\tilde{k}_{t+1} = e^{-\zeta_g v_{t+1}^g}(1-\delta+a_1+\tfrac{a_2}{1-1/\nu}(\tilde{\imath}_t/\tilde{k}_t)^{1-1/\nu})\tilde{k}_t$$
$$g_{t+1}\tilde{s}_{t+1} = (1-\omega)\tilde{s}_t+\tilde{o}_t^s-\tilde{o}_t-\tfrac{\pi}{2}\tilde{s}_t^{-2}$$
$$\tilde{o}_t^s = e_t/\epsilon_t$$
$$\tilde{c}_t+\tilde{\imath}_t = \tilde{y}_t$$
$$n_t+\ell_t = 1$$
$$\ln g_{o,t} = \ln\kappa_0+\kappa_1\ln g_t+\kappa_2\ln\epsilon_{t-1}+\sigma_{go}\varepsilon_{go,t}$$
$$\ln\epsilon_t = \ln g_t-\ln g_{o,t}+\ln\epsilon_{t-1}$$
$$\ln g_t = \ln\bar{g}+\sigma_g\varepsilon_{g,t}-\zeta_g(v_t^g-\bar{\pi}_1^g)$$
$$\ln e_t = \ln\bar{e}-\zeta_e(v_t^e-\bar{\pi}_1^e)$$
$$Pr(v_{t+1}^g=1|v_t^g=1)=\bar{q}^g,\quad Pr(v_{t+1}^g=1|v_t^g=0)=p_t^g$$
$$Pr(v_{t+1}^e=1|v_t^e=1)=\bar{q}^e,\quad Pr(v_{t+1}^g=1|v_t^g=0)=p_t^e$$
$$\ln p_t^g = (1-\rho_p^g)\ln\bar{p}^g+\rho_p^g\ln p_{t-1}^g+\sigma_p^g\varepsilon_{p,t}^g$$
$$\ln p_t^e = (1-\rho_p^e)\ln\bar{p}^e+\rho_p^e\ln p_{t-1}^e+\sigma_p^e\varepsilon_{p,t}^e$$
$$E_t[x_{t+1}r_t] = 1$$
$$1 = E_t[x_{t+1}r_{t+1}^e]$$
$$r_t^e = g_t(\tilde{p}_t^e+\tilde{d}_t^e)/\tilde{p}_{t-1}^e$$
$$\tilde{d}_t^e = \tilde{y}_t-\tilde{\imath}_t-\tilde{w}_t n_t-\vartheta(\tfrac{1}{g_t}E_{t-1}[g_t\tilde{k}_t]-\tfrac{1}{r_t}E_t[g_{t+1}\tilde{k}_{t+1}])$$

# E Oil Market Data

**Figure 1:** Real U.S. refiners' acquisition cost of crude oil imports, 1974Q4-2023Q4

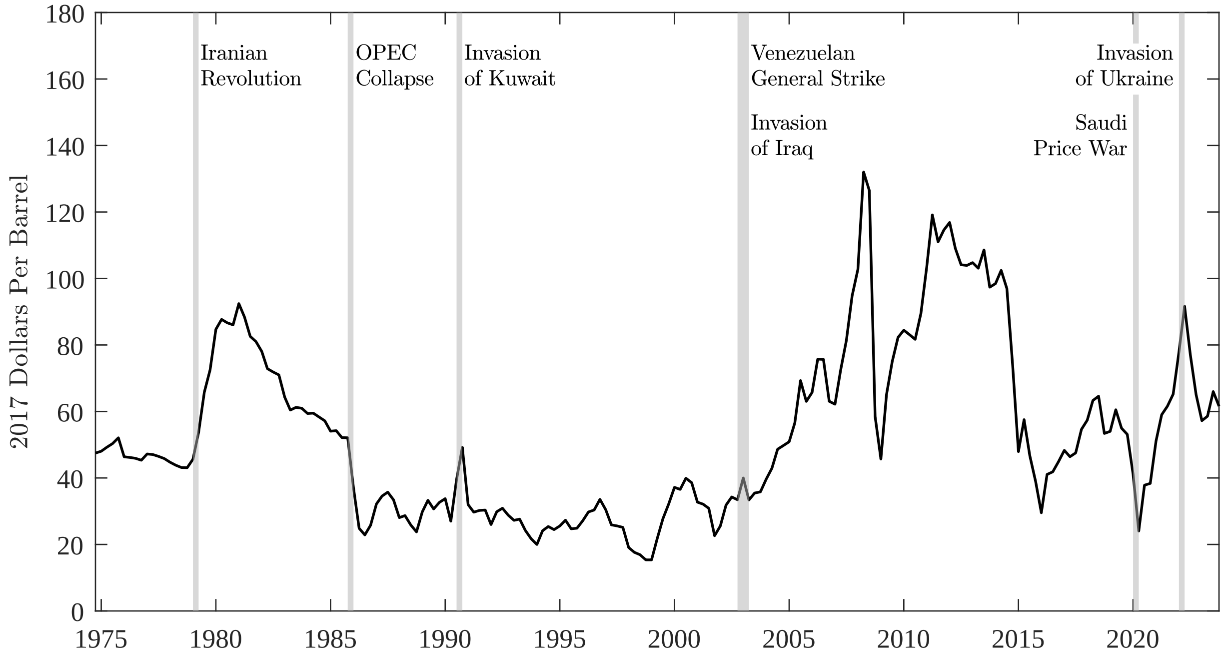
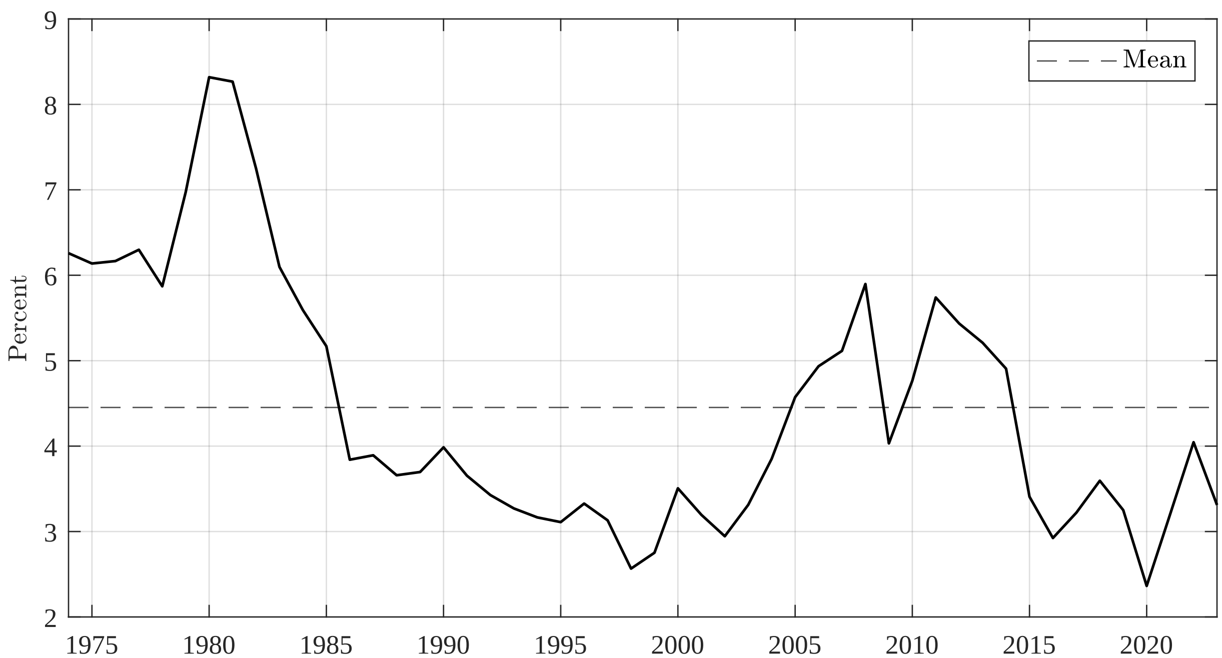


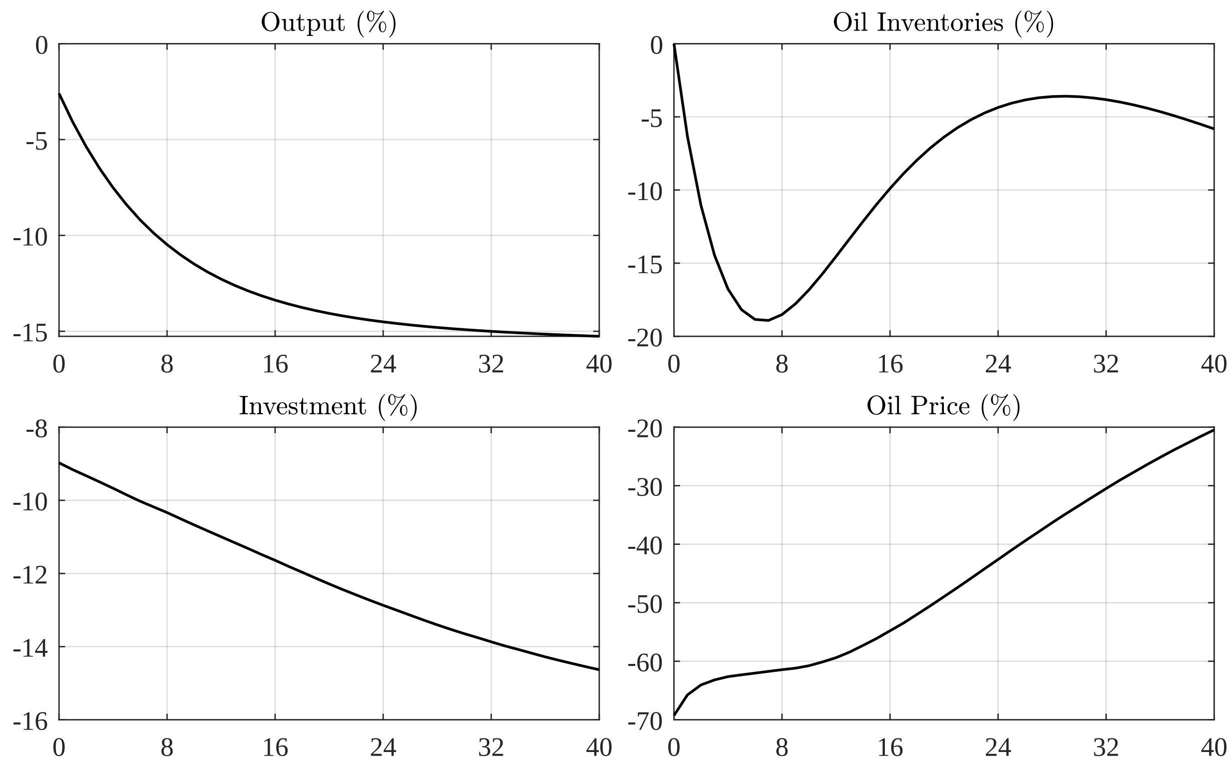

**Figure 2:** U.S. petroleum oil expenditure share, 1974-2023



# F Additional Disaster Risk Results

**F.1 Disaster Realization** Figure 3 shows how the economy responds to a macroeconomic disaster realization. The output and investment responses are broadly in line with the results reported in Gourio (2012). The oil price drops by nearly $70\%$ on impact and oil inventories decline.

**Figure 3:** Responses to a macroeconomic disaster realization



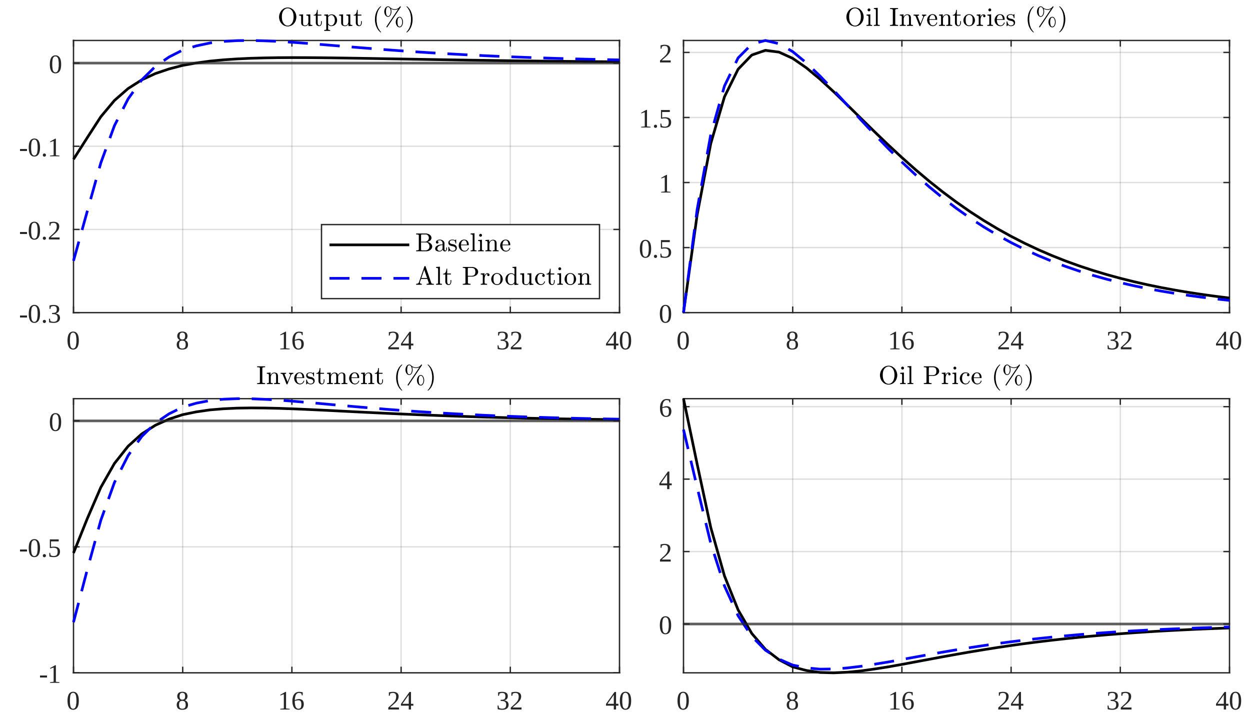

**F.2 Production Function** The specification of the production function in our baseline model follows the seminal work of Kim and Loungani (1992) and Backus and Crucini (2000). Some more recent studies, such as Başkaya et al. (2013), Hassler et al. (2021), Olovsson (2019), and Ready (2018), use the following specification

$$y_t = y_0 \left((1-\alpha)(v_t/v_0)^{1-1/\sigma} + \alpha(o_t/o_0)^{1-1/\sigma}\right)^{1/(1-1/\sigma)},$$

where $v_t = (a_t n_t)^{1-\xi} k_t^{\xi}$ is value added and $\sigma$ is the elasticity of substitution between oil and the capital-labor bundle, rather than just capital. Under this specification, the factor prices are given by

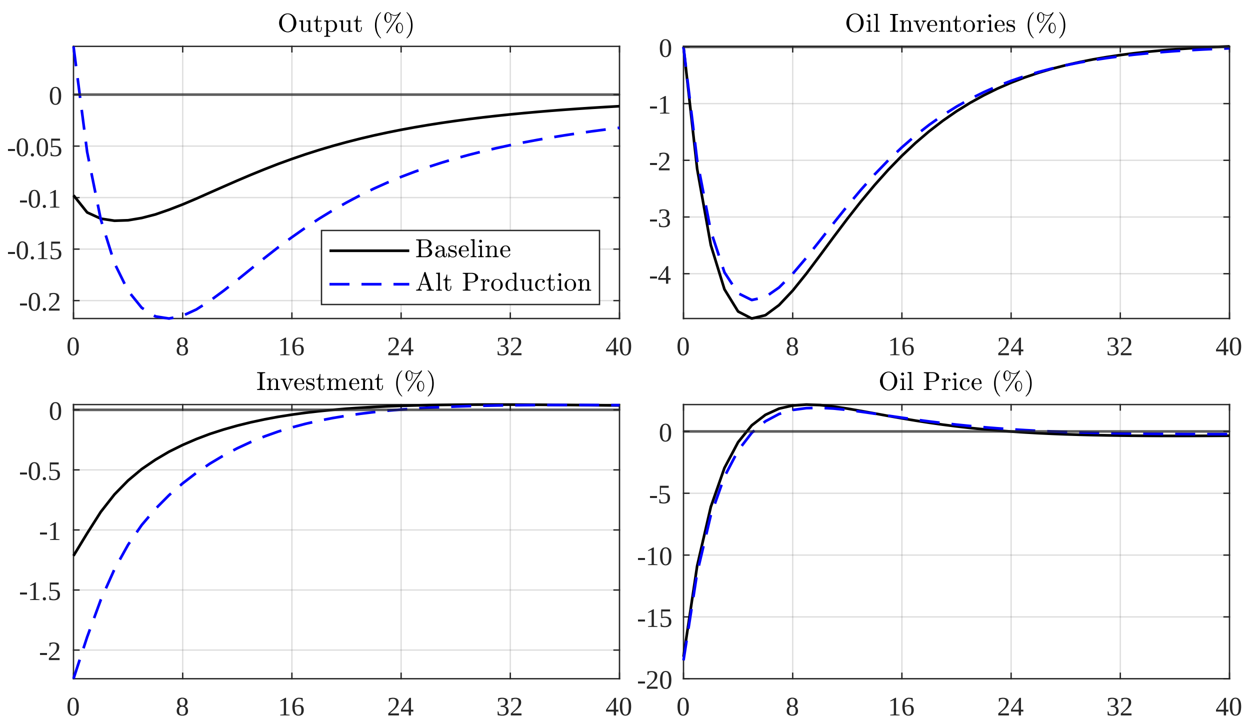
$$\begin{aligned} w_t &= (1-\alpha)(1-\xi)(y_0/n_t)(y_t/y_0)^{1/\sigma}(v_t/v_0)^{1-1/\sigma}, \\ p_t^o &= \alpha(y_0/o_0)(y_t/y_0)^{1/\sigma}(o_t/o_0)^{-1/\sigma}, \\ r_t^k &= (1-\alpha)\xi(y_0/k_t)(y_t/y_0)^{1/\sigma}(v_t/v_0)^{1-1/\sigma}. \end{aligned}$$

**Figure 4:** Responses to a 20pp oil production disaster probability shock




*Notes:* Responses in deviations from the baseline. Simulations assume no disasters are realized.

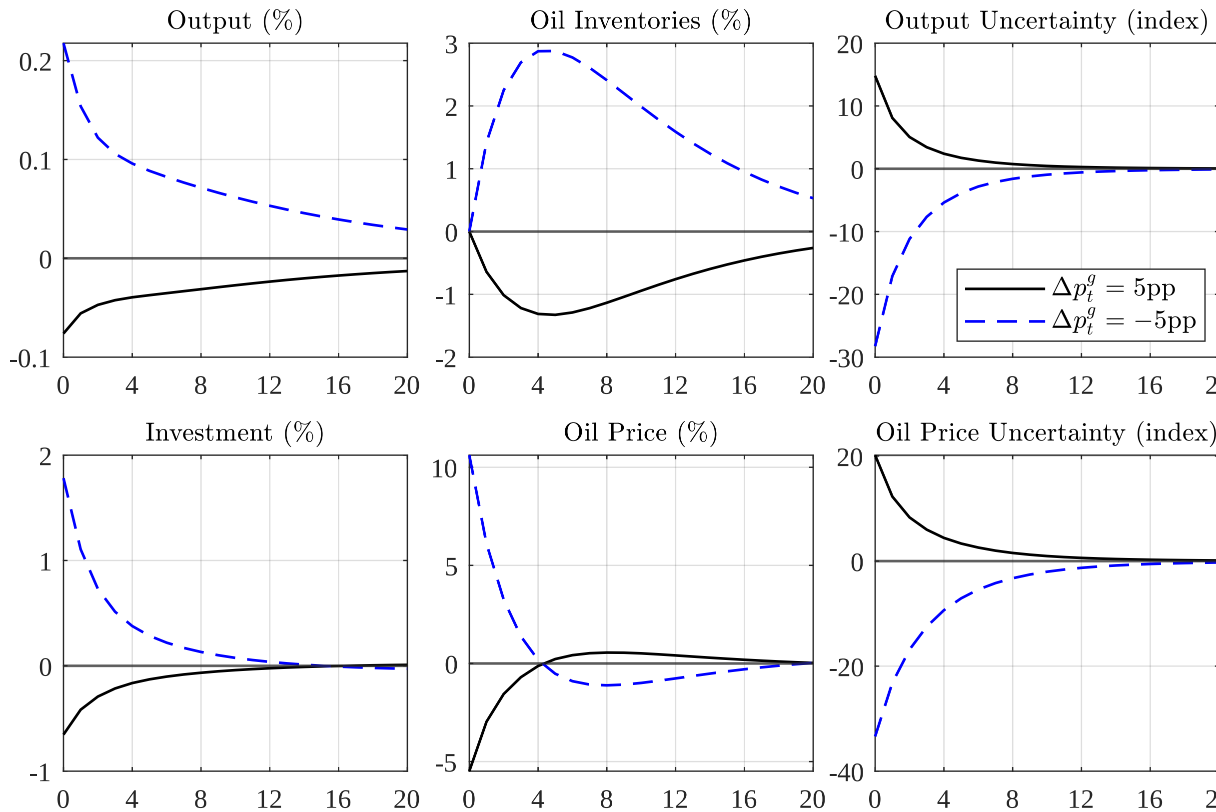
**Figure 5:** Responses to 5pp macroeconomic disaster probability shock




*Notes:* Responses in deviations from the baseline. Simulations assume no disasters are realized.

We recalibrate the model to match the data moments. All of the parameters are unchanged, except that we set $\alpha = 0.063$ in order to match the cost share of oil.[1] Figures 4 and 5 illustrate that our substantive conclusions are robust to this alternative functional form of the production function.

**Figure 6:** Responses to positive and negative macroeconomic disaster probability shocks

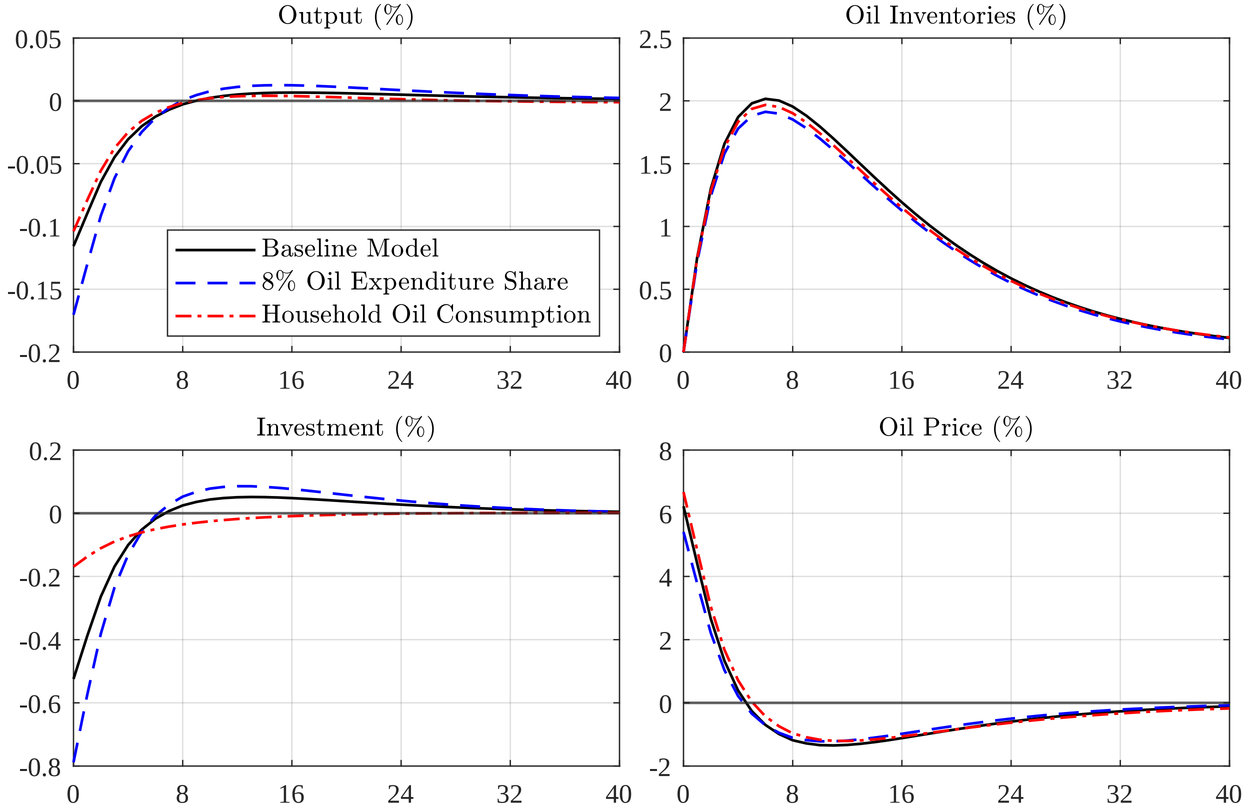



*Notes:* Responses in deviations from the baseline. Simulations assume no disasters are realized.

F.3 SIGN OF THE DISASTER PROBABILITY SHOCK In the paper, we show how the sign of an oil disaster probability shock affects the responses. Figure 6 shows the responses to a $\pm 5$pp macroeconomic disaster probability shock. The simulations are initialized at a $15\%$ disaster probability to permit a positive and negative shock. Consistent with the responses to an oil disaster probability shock, we find that increases and decreases in the price of oil do not have the same effects on uncertainty. A decrease in the probability of a macroeconomic disaster increases the price of oil and reduces oil price uncertainty on impact, while an increase in this probability lowers the oil price and raises oil price uncertainty. This result is inconsistent with a VAR model with GARCH errors.

[1] We set $\alpha$ to $0.063$ so that the average oil expenditure share across simulations matches the share in the data. Similarly, the steady-state inventory share of oil consumption ($\bar{s}/\bar{o}$) is set to $1.18$ to match the average share in the data. We set the steady-state labor cost share ($\bar{w}\bar{n}/\bar{y}$), which determines $\xi$, to $0.5957$, in line with the baseline model. The elasticity of substitution, $\sigma$, is pinned down by the standard deviation of the price of oil and remains unchanged.

**Figure 7:** Responses to a 20pp oil production disaster probability shock




*Notes:* Responses in deviations from the baseline. Simulations assume no disasters are realized.

F.4 HIGHER OIL EXPENDITURE SHARE The average oil expenditure share in our baseline model is $4.5\%$, but there was a brief period in the early 1980s when the share reached $8\%$ (see Figure 2). To examine the sensitivity of our results, we also consider a variation of the baseline model with an average oil expenditure share of $8\%$. As shown in Figure 7, even allowing for such an unrealistically high average expenditure share implies only a slightly larger response of real activity to an oil production disaster probability shock. There is little effect on the price of oil and oil inventories. Overall, these results show that our baseline results are robust to changes in the average oil expenditure share.

F.5 HOUSEHOLD OIL CONSUMPTION In our baseline model, all of the oil consumption takes place in the production sector. An alternative assumption would be to follow Bodenstein et al. (2008, 2011) in introducing oil consumption by households. In that case, the household's maximization problem becomes

$$J_t = \max_{c_t, o_{h,t}, n_t, s^e_{t+1}, b_{t+1}} \left((1-\beta)u_t^{1-1/\psi} + \beta(E_t[J_{t+1}^{1-\gamma}])^{\frac{1-1/\psi}{1-\gamma}}\right)^{\frac{1}{1-1/\psi}}$$

subject to

$$u_t = c_{a,t}^{\chi}(a_t(1-n_t))^{1-\chi},$$
$$c_{a,t} = c_{a0}\left((1-\alpha_h)(c_t/c_0)^{1-1/\sigma_h} + \alpha_h(o_{h,t}/o_{h0})^{1-1/\sigma_h}\right)^{\sigma_h/(\sigma_h-1)},$$
$$c_t + p_t^o o_{h,t} + p_t^e s_{t+1}^e + b_{t+1}/r_t = w_t n_t + (p_t^e + d_t^e)s_t^e + b_t,$$

where $c_t$ is goods consumption, $o_{h,t}$ is oil consumption, $c_{a,t}$ is aggregate consumption, and $\sigma_h$ is the elasticity of substitution between goods and oil consumption. Just like in the production sector, we introduce scalars $c_{a0}$, $c_0$, and $o_{h0}$ so that $\alpha_h$ is equal to the household oil expenditure share.

The first-order conditions are given by

$$1 = E_t[x_{t+1}r_{t+1}^e],$$
$$1 = E_t[x_{t+1}r_t],$$
$$(1-\alpha_h)\chi w_t(1-n_t)(c_0 c_{a,t}/c_{a0})^{1/\sigma_h - 1} = (1-\chi)c_t^{1/\sigma_h},$$
$$p_t^o = \tfrac{\alpha_h}{1-\alpha_h}(c_t/o_{h,t})^{1/\sigma_h}(c_0/o_{h0})^{1-1/\sigma_h},$$

where

$$r_{t+1}^e = (p_{t+1}^e + d_{t+1}^e)/p_t^e,$$
$$x_t = \beta(u_t/u_{t-1})^{1-1/\psi}(c_{a,t}/c_{a,t-1})^{1/\sigma_h-1}(c_{t-1}/c_t)^{1/\sigma_h}(J_t/z_{t-1})^{1/\psi-\gamma},$$
$$z_t = (E_t[J_{t+1}^{1-\gamma}])^{1/(1-\gamma)}.$$

The oil storage firm now chooses how much to supply to the household and the final goods firm, but the first-order condition is unchanged. The problem for the final goods firm is unchanged.

We set $\sigma_h = \sigma = 0.105$ and the share of oil consumed by households $\bar{o}_h/(\bar{o}_h+\bar{o})$ to $0.5$ to match the share in U.S. data, which is equal to personal consumption expenditures on gas and other energy goods (CNEA@USNA) divided by petroleum products expenditures (ZUSPATCV@USENERGY). The implied weight on oil in the production function is $\alpha = 0.0667$, while the implied weight on oil in the aggregate consumption function is $\alpha_h = 0.0407$. All other parameters are unchanged.

Figure 7 shows the responses to an oil production disaster probability shock. Intuitively, there is a smaller decline in investment because firms now only represent half of total oil consumption and are therefore less exposed to the risk of a large shortfall in oil production. However, there is a larger decline in goods consumption, given the complementarity with oil consumption in the household's aggregate consumption function. These two competing effects on output roughly cancel out, so the response the response of output is effectively unchanged from our baseline model. We also find little effect on the price of oil and oil inventories. These results show that the distinction between oil consumption by households and by firms has little effect on our baseline results.

## G COMPARISON WITH THE PUTTY-CLAY MODEL

In the Atkeson and Kehoe (1999) model, the real price of oil, $p_t^o$, is exogenous. We postulate that $p_t^o$ follows an autoregressive process with a stochastic volatility shock:

$$\ln p_t^o = (1-\rho_p)\ln \bar{p}^o + \rho_p \ln p_{t-1} + \sigma_{p,t-1}\varepsilon_{p,t},\ 0 < \rho_p < 1,\ \varepsilon_{p,t} \sim \mathbb{N}(0,1),$$
$$\ln \sigma_{p,t} = (1-\rho_{sv})\ln \bar{\sigma}_p + \rho_{sv} \ln \sigma_{p,t-1} + \sigma_{sv}\varepsilon_{sv,t},\ 0 < \rho_{sv} < 1,\ \varepsilon_{sv,t} \sim \mathbb{N}(0,1),$$

where $\varepsilon_{sv,t}$ affects the variance of the oil price and hence uncertainty about the price of oil.

In the production sector, there is a continuum of capital goods characterized by oil intensity $v$. Firms produce output using capital, oil, and labor inputs. Since different capital goods require a fixed oil intensity, existing capital goods use oil in the fixed proportion $1/v$. Thus, there is no substitutability between capital and oil in the short run. However, firms are able to invest in capital with a different energy intensity. Given the infinite number of capital goods, the model is intractable. However, if all types of existing capital are fully utilized, the state space can be represented by two aggregate state variables and each period there is positive investment in at most one type of capital with energy intensity $v_t$. Assuming this condition is satisfied and the production function is Cobb-Douglas in capital and oil, the firm's optimization problem can be written as

$$V_t^f = \max_{n_t, i_t, k_{s,t}, o_t, v_t} y_t - i_t - p_t^o o_{t-1} - w_t n_t + E_t[x_{t+1}V_{t+1}^f]$$

subject to

$$y_t = (a_t n_t)^{1-\xi} k_{s,t-1}^{\xi},$$
$$k_{s,t} = (1-\delta)k_{s,t-1} + i_t v_t^{\phi-1},$$
$$o_t = (1-\delta)o_{t-1} + i_t/v_t,$$

where $n_t$ is labor that receives real wage rate $w_t$, $i_t$ is physical investment, $k_{s,t}$ is aggregate capital services that depreciate at rate $\delta$, $o_t$ is aggregate oil use, $y_t$ is aggregate output, $\phi$ is the cost share of oil, and $\xi$ is the cost share of capital services. Labor productivity, $a_t$, evolves according to

$$\ln a_t = (1-\rho_a)\ln \bar{a} + \rho_a \ln a_{t-1} + \sigma_{a,t-1}\varepsilon_{a,t},\ 0 < \rho_a < 1,\ \varepsilon_{a,t} \sim \mathbb{N}(0,1).$$

The firm's optimality conditions imply

$$w_t = (1-\xi)y_t/n_t,$$
$$\lambda_{k,t} = E_t[x_{t+1}(r_{t+1}^k + (1-\delta)\lambda_{k,t+1})],$$
$$\lambda_{o,t} = E_t[x_{t+1}(-p_{t+1}^o + (1-\delta)\lambda_{o,t+1})],$$

where $\lambda_{k,t} = v_t^{1-\phi}/\phi$, $\lambda_{o,t} = (\phi-1)v_t/\phi$, and $r_t^k = \xi y_t/k_{s,t-1}$.

The household's problem is unchanged from the paper. However, following Atkeson and Kehoe (1999), there is no labor productivity growth and no oil storage, so the resource constraint becomes

$$c_t + i_t = y_t - p_t^o o_{t-1}.$$

We compare the Atkeson-Kehoe (AK) putty-clay model to a model that replaces the putty-clay technology by a CES production function that aggregates capital and oil as in our baseline model. In particular, the firm's optimization problem in this CES model is given by

$$V_t = \max_{i_t, k_t, n_t, o_t} y_t - i_t - p_t^o o_t - w_t n_t + E_t[x_{t+1} V_{t+1}]$$

subject to

$$\begin{gathered} y_t = y_0 (a_t n_t)^{1-\xi} \left((1-\alpha)(k_{t-1}/k_0)^{1-1/\sigma} + \alpha(o_t/o_0)^{1-1/\sigma}\right)^{\xi/(1-1/\sigma)} \\ k_t = (1-\delta)k_{t-1} + i_t - \phi(i_t/k_{t-1})k_{t-1}), \end{gathered}$$

where $\sigma$ is the elasticity of substitution between capital and oil. The optimality conditions imply

$$\begin{gathered} w_t = (1-\xi) y_t / n_t, \\ p_t^o = \xi\alpha \tfrac{(o_t/o_0)^{1-1/\sigma}}{(1-\alpha)(k_{t-1}/k_0)^{1-1/\sigma} + \alpha(o_t/o_0)^{1-1/\sigma}} \tfrac{y_t}{o_t}, \\ E_t[x_{t+1} r_{t+1}^i] = 1, \end{gathered}$$

where

$$\begin{gathered} r_{t+1}^i \equiv (r_{t+1}^k + (1-\delta+\mu_1+\tfrac{\mu_2}{\nu-1}(i_{t+1}/k_t)^{1-1/\nu})p_{t+1}^k)/p_t^k, \\ r_t^k \equiv \xi(1-\alpha) \tfrac{(k_{t-1}/k_0)^{1-1/\sigma}}{(1-\alpha)(k_{t-1}/k_0)^{1-1/\sigma} + \alpha(o_t/o_0)^{1-1/\sigma}} \tfrac{y_t}{k_{t-1}}, \\ p_t^k \equiv \tfrac{1}{1-\phi'(i_t/k_{t-1})} = \tfrac{1}{\mu_2}(\tfrac{i_t}{k_{t-1}})^{1/\nu}. \end{gathered}$$

**Calibration** Table 1 summarizes the parameter values used for both the AK and CES models. Table 2 compares the data and simulated moments. In the AK model, we set $\phi = 0.89$ to match the oil expenditure share in the data. In the CES model, $\sigma \in \{0.1, 0.2, 0.3\}$. For each $\sigma$, we set the investment adjustment cost parameter, $\nu$, to achieve the same standard deviation of investment. The moments are almost identical across the four model specifications, providing a credible framework to compare responses to oil price uncertainty shocks.

**Results** Figure 8 shows responses to a 2 standard deviation oil price uncertainty shock. The less substitutable capital and oil are, the better the responses from the CES model approximate those from the AK model. In particular, for $\sigma = 0.1$, which corresponds to the value in our baseline model, the responses are close. This indicates that our model captures the mechanisms in Bernanke (1983) and Atkeson and Kehoe (1999), while remaining tractable in general equilibrium even when the oil price is endogenous.

**Table 1:** Common model parameters

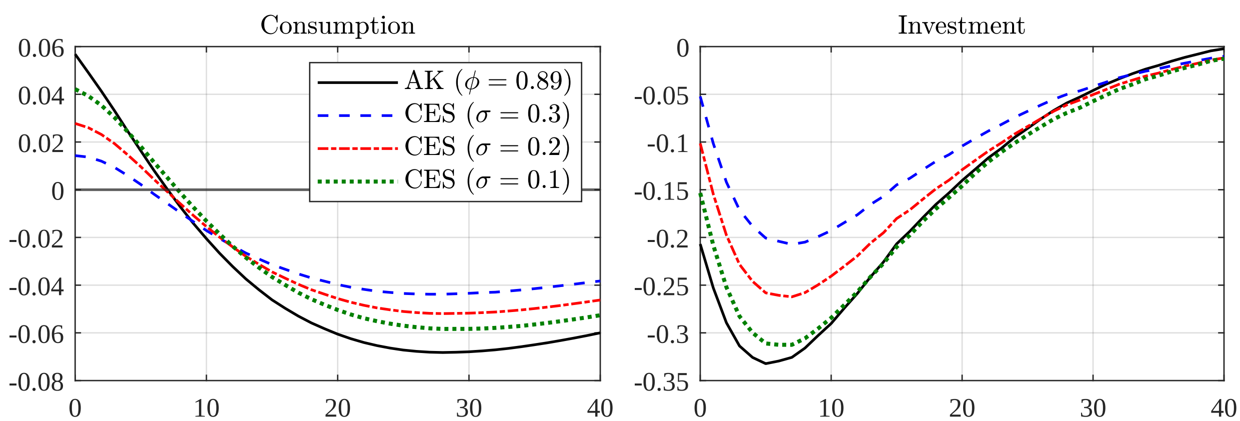
| Parameter | Value | Target |
|---|---|---|
| Discount Factor ($\beta$) | 0.997 | $E(r)$ |
| Risk Aversion ($\gamma$) | 10 | Gao et al. (2022), Croce (2014) |
| Intertemporal Elasticity of Substitution ($\psi$) | 2 | Gao et al. (2022), Croce (2014) |
| Frisch Labor Supply Elasticity ($\eta^\lambda$) | 2 | Peterman (2016), Basu-Bundick (2017) |
| Capital Depreciation Rate ($\delta$) | 0.025 | Depreciation on fixed assets, durables |
| Capital Services Share of Production ($\xi$) | 0.4 | Avg. labor share of income |
| Technology Persistence ($\rho_a$) | 0.9 | $AC(\tilde{y})$ |
| Technology Shock SD ($\sigma_a$) | 0.012 | $SD(\tilde{y})$ |
| Oil Price Persistence ($\rho_{p^o}$) | 0.9 | $AC(\tilde{p}^o)$ |
| Oil Price Shock SD ($\sigma_{p^o}$) | 0.13 | $SD(\tilde{p}^o)$ |
| Oil price volatility shock persistence ($\rho_{sv}$) | 0.95 | $AC(\mathcal{U}_{p^o})$ |
| Oil price volatility shock SD ($\sigma_{sv}$) | 0.115 | $SD(\mathcal{U}_{p^o})$ |

**Table 2:** Data and simulated moments

| | | AK | CES | | |
|---|---|---|---|---|---|
| Moment | Data | $\phi = 0.89$ | $\sigma = 0.30$ | $\sigma = 0.20$ | $\sigma = 0.10$ |
| $E(p^o o/y)$ | 0.043 | 0.043 | 0.043 | 0.043 | 0.043 |
| $SD(\tilde{y})$ | 2.9 | 2.6 | 2.8 | 2.7 | 2.6 |
| $SD(\tilde{i})$ | 8.7 | 9.0 | 8.9 | 8.9 | 8.9 |
| $SD(\tilde{p}^o)$ | 38.1 | 31.4 | 31.4 | 31.4 | 31.4 |
| $SD(\mathcal{U}_{p^o})$ | 29.5 | 29.6 | 29.6 | 29.6 | 29.6 |
| $AC(\tilde{y})$ | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 |
| $AC(\tilde{p}^o)$ | 0.88 | 0.87 | 0.87 | 0.87 | 0.87 |
| $AC(\mathcal{U}_{p^o})$ | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 |

*Notes:* A tilde denotes a variable that was detrended in the data using a Hamilton (2018) filter with 4 lags and a delay of 8 quarters. $SD(\mathcal{U}_{p^o})$ is normalized by $SD(\Delta p^o)$. Standard deviations are percents. The data sample begins in 1980Q1 instead of 1975Q1 due to lost observations from detrending the data.

**Figure 8:** Responses to an oil price uncertainty shock



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