Online Appendix: Geopolitical Oil Price Risk and Economic Fluctuations*

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ABSTRACT

This appendix describes the methodology for constructing a time series of oil price uncertainty, the data sources, and the solution method for our general equilibrium model. It plots the time series of the real price of oil underlying our analysis, shows the robustness to an alternative specification of the production function, presents responses to a growth and oil production disaster, and compares the responses to a positive and negative growth disaster probability shock.

*The views expressed in this paper are our own and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

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A MEASURING UNCERTAINTY

Our method of constructing quarterly measures of uncertainty builds on Jurado et al. (2015). We first summarize the key steps of the estimation process before discussing the data used in the estimation.

A.1 METHODOLOGY Let $\mathbf{Y}_t = (y_{1,t}, \ldots, y_{N_y,t})'$ be a vector of data containing N_y variables. Our objective is to estimate the 1-quarter ahead uncertainty about select elements of $\mathbf{Y}_t$, defined as

$$\mathcal{U}_t^j \equiv \sqrt{E[(y_{j,t+1} - E[y_{j,t+1}|I_t])^2|I_t]},$$

where the expectation is taken with respect to the information set I_t and j refers to the variable of interest. There are four steps:

1. Generate forecast errors for $y_{j,t+1}$ using a forecasting model that includes lags of the variable y_j, estimated factors extracted from a panel of predictor variables, $\hat{\mathbf{F}}_t$, and a set of additional predictors contained in a vector $\mathbf{W}_t$.
2. Fit autoregressive models for the factors in $\hat{\mathbf{F}}_t$ and the variables in $\mathbf{W}_t$ and generate residuals for each variable.
3. Estimate a stochastic volatility model for each residual.
4. Calculate $\mathcal{U}_t^j$.

Factors Let $\mathbf{X}_t = (X_{1,t}, \ldots, X_{N_x,t})'$ be a vector of predictors that are available for forecasting. These data are transformed to be stationary. It is assumed that the transformed variables have an approximate factor structure,

$$X_{i,t} = \Lambda_i^{F'} \mathbf{F}_t + e_{i,t}^X,$$

where $\mathbf{F}_t$ is a $r_F \times 1$ vector of latent factors, $\Lambda_i^{F'}$ is a $1 \times r_F$ vector of loadings for variable i and the idiosyncratic errors are given by $e_{i,t}^X$. The estimated factors, denoted as $\hat{\mathbf{F}}_t$, are estimated using principal components and the number of factors is selected using the criterion of Bai and Ng (2002). Each of the factors is assumed to follow an autoregressive process with two lags,

$$F_t = \Phi^F(L) F_{t-1} + v_t^F,$$
$$v_t^F = \sigma_t^F \epsilon_t^F, \quad \epsilon_t^F \sim \mathbb{N}(0,1),$$
$$\ln(\sigma_t^F)^2 = \alpha^F + \beta^F \ln(\sigma_{t-1}^F)^2 + \tau^F \eta_t^F, \quad \eta_t^F \sim \mathbb{N}(0,1),$$

where $\Phi^F(L)$ is a lag polynomial. As with the other lag order choices made below, our results are robust to reasonable variation in the lag order.

Additional predictors The $r_W \times 1$ vector $\mathbf{W}_t$ includes the squared values of the first factor in $\hat{F}_t$ and a set of N_G factors estimated using principal components on the squared values of the variables in $\mathbf{X}_t$. Each variable in $\mathbf{W}_t$ is assumed to follow an autoregressive process with two lags,

$$W_t = \Phi^W(L)W_{t-1} + v_t^W,$$
$$v_t^W = \sigma_t^W \epsilon_t^W, \quad \epsilon_t^W \sim \mathbb{N}(0,1),$$
$$\ln(\sigma_t^W)^2 = \alpha^W + \beta^W \ln(\sigma_{t-1}^W)^2 + \tau^W \eta_t^W, \quad \eta_t^W \sim \mathbb{N}(0,1),$$

where $\Phi^W(L)$ is a lag polynomial.

Forecasting Model A forecast for $y_{j,t+1}$ is produced with the factor-augmented forecasting model,

$$y_{j,t+1} = \phi_j^Y(L)y_{j,t} + \gamma_j^F(L)\hat{\mathbf{F}}_t + \gamma_j^W(L)\mathbf{W}_t + \nu_{j,t+1}^Y,$$
$$\nu_t^y = \sigma_t^y \epsilon_t^y, \quad \epsilon_t^y \sim \mathbb{N}(0,1)$$
$$\ln(\sigma_t^y)^2 = \alpha^y + \beta^y \ln(\sigma_{t-1}^y)^2 + \tau^y \eta_t^y, \quad \eta_t^y \sim \mathbb{N}(0,1),$$

where $\phi_j^Y(L)$, $\gamma_j^F(L)$, and $\gamma_j^W(L)$ are lag polynomials of orders 2, 1, and 1, respectively. As in Jurado et al. (2015, footnote 10), a hard threshold is applied to remove any variables from the forecasting model that do not have incremental predictive power.

Uncertainty Define $\mathbf{Z}_t \equiv (\hat{\mathbf{F}}_t', \mathbf{W}_t')'$ as a vector that collects the estimated factors and the additional predictors contained in $\mathbf{W}_t$. Then let $\mathcal{Z}_t \equiv (\mathbf{Z}_t', \ldots, \mathbf{Z}_{t-q+1}')'$ and $Y_{j,t} = (y_{j,t}, \ldots, y_{j,t-q+1})'$, where $q = 2$. The FAVAR model can be written in companion form as

$$\begin{pmatrix} \mathcal{Z}_t \\ Y_{j,t} \end{pmatrix} = \begin{pmatrix} \Phi^{\mathcal{Z}} & 0 \\ \Lambda_j' & \Phi_j^Y \end{pmatrix} \begin{pmatrix} \mathcal{Z}_{t-1} \\ Y_{j,t-1} \end{pmatrix} + \begin{pmatrix} \mathcal{V}_t^{\mathcal{Z}} \\ \mathcal{V}_{j,t}^Y \end{pmatrix} \Longleftrightarrow \mathcal{Y}_{j,t} = \Phi_j^{\mathcal{Y}} \mathcal{Y}_{j,t-1} + \mathcal{V}_{j,t}^{\mathcal{Y}}.$$

The forecast error variance is

$$\Omega_{j,t}^{\mathcal{Y}}(1) \equiv E_t[(\mathcal{Y}_{j,t+1} - E_t\mathcal{Y}_{j,t+1})(\mathcal{Y}_{j,t+1} - E_t\mathcal{Y}_{j,t+1})'],$$

where $E_t\mathcal{Y}_{j,t+1} = \Phi_j^{\mathcal{Y}}\mathcal{Y}_{j,t}$. The forecast error variances can be calculated as

$$\Omega_{j,t}^{\mathcal{Y}}(1) = E_t[\mathcal{V}_{j,t+1}^{\mathcal{Y}} \mathcal{V}_{j,t+1}^{\mathcal{Y}'}].$$

The uncertainty of $y_{j,t+1}$ is

$$\mathcal{U}_t^j = \sqrt{1_j' \Omega_{j,t}^{\mathcal{Y}}(1) 1_j},$$

where 1 is a selection vector and j refers to the growth rate of real GDP and the growth rate of the inflation-adjusted U.S. refiners' acquisition cost of imported crude oil, respectively.

A.2 DATA

Our dataset includes most of the financial and macroeconomic variables listed in the data appendix of Ludvigson et al. (2021) plus U.S. real GDP and the inflation-adjusted U.S. refiners' acquisition cost of imported crude oil.

The macroeconomic variables are from the April 2024 vintage of the FRED-MD database with the following modifications.

- We linearly interpolate the missing values of $UMCSENTx$ that occur through 1977.
- We set the missing value of $CP3Mx$ for 4/1/2020 to its value on 3/1/2020.
- We set the missing value of $COMPAPFFx$ for 4/1/2020, to its value on 3/1/2020.

Monthly data are averaged by quarter and transformed to stationarity using the code in the FRED-MD database. Both real GDP and the real price of oil are log-differenced. The data set starts in 1974Q1. The sample begins in 1974Q2, because we lose one observation due to differencing.

The financial variables are obtained from FRED-MD, CRSP and the Fama-French database. Returns are aggregated by summing the monthly values by quarter.

B DATA SOURCES

We use the following time-series provided by Haver Analytics:

1. **Consumer Price Index for All Urban Consumers**: Not seasonally Adjusted, Monthly, Index (PCUN@USECON)
2. **World Production of Crude Oil Including Lease Condensate**
   Not Seasonally Adjusted, Thousands of Barrels per Day
   (Monthly, AWOACAUF@ENERGY; Quarterly, BWOACAUF@ENERGY)
3. **United States: Petroleum Products Expenditures**
   Annual, Millions of Dollars (ZUSPATCV@USENERGY)
4. **US Crude Oil Imported Acquisition Cost by Refiners**
   Not Seasonally Adjusted, Quarterly, Dollars per Barrel (CUSIQABF@USENERGY)
5. **Civilian Noninstitutional Population: 16 Years & Over**
   Not Seasonally Adjusted, Quarterly, Thousands (LN16N@USECON)
6. **Gross Domestic Product: Implicit Price Deflator**
   Seasonally Adjusted, Quarterly, 2012=100 (DGDP@USNA)
7. **Gross Domestic Product**
   Seasonally Adjusted, Quarterly, Billions of Dollars (GDP@USECON)
8. **Gross Domestic Product**
   Annual, Millions of Dollars (GDPY@USNA)

9. **Personal Consumption Expenditures: Nondurable Goods**
   Seasonally Adjusted, Quarterly, Billions of Dollars (CN@USECON)
10. **Personal Consumption Expenditures: Services**
   Seasonally Adjusted, Quarterly, Billions of Dollars (CS@USECON)
11. **Personal Consumption Expenditures: Durable Goods**
   Seasonally Adjusted, Quarterly, Billions of Dollars (CD@USECON)
12. **Private Fixed Investment**
   Seasonally Adjusted, Quarterly, Billions of Dollars (F@USECON)
13. **Total Economy: Labor share**
   Seasonally Adjusted, Quarterly, Percent (LXEBL@USNA)
14. **Net Stock: Private Fixed Assets**, Annual, Billions of Dollars (EPT@CAPSTOCK)
15. **Net Stock: Durable Goods**, Annual, Billions of Dollars (EDT@CAPSTOCK)
16. **Depreciation: Private Fixed Assets**, Annual, Billions of Dollars (KPT@CAPSTOCK)
17. **Depreciation: Durable Goods**, Annual, Billions of Dollars (KDT@CAPSTOCK)
18. **CBOE Crude Oil Volatility Index (OVX)**, Daily, Index (SPOVX@DAILY)

We also use the following data sources:

1. **FRED-MD**, Monthly Databases for Macroeconomic Research. The data is available at https://research.stlouisfed.org/econ/mccracken/fred-databases (McCracken, 2024). Under Monthly Data, we use the April 2024 vintage.
2. **Fama-French**, Database. The data is available at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
   (Fama and French, 2024).
3. **WRDS**, Stock Market Indexes. The data is available at https://wrds-www.wharton.upenn.edu (Wharton Research Data Services, 2024).
4. **Geopolitical Risk Index**, Historical series (GPRH). The data is available at https://www.matteoiacoviello.com/gpr.htm (Caldara and Iacoviello, 2024).
5. **Global Oil Inventories** Monthly, Millions of Barrels per Day (Inv_t) from Kilian (2022).

We apply the following data transformations:

1. **Per Capita Real Output**: $Y_t = 10^9 \times GDP_t/((DGDP_t/100)(1000 \times LN16N_t))$.
2. **Per Capita Real Consumption**: $C_t = 10^9(CN_t+CS_t)/((DGDP_t/100)(1000\times LN16N_t))$.
3. **Per Capita Real Investment**: $I_t = 10^9(F_t + CD_t)/((DGDP_t/100)(1000 \times LN16N_t))$.

4. **Depreciation Rate**: $\delta = (1 + \frac{1}{T/4}\sum_{t=1}^{T/4}(KPT_t + KDT_t)/(EPT_{t-1} + EDT_{t-1}))^{1/4} - 1$.
5. **Capital Services Share**: $\xi = 1 - \frac{1}{T}\sum_{t=1}^{T} LXEBL/100$.
6. **Real Price of Oil**: $p_t^o = CUSIQABF_t/(DGDP_t/100)$.
7. **Expenditure Share of Oil**: $ZUSPATCV_t/GDPY_t$.
8. **Oil Consumption**: $o_t =$ Days per Month $\times\ AWOACAUF_t/1000 - (INV_t - INV_{t-1})$.
9. **Inventory-Oil Consumption Share**: $INV_t/\sum_{j=t-2}^{t} o_t$ for $t = 3, 6, \ldots, 3T$.
10. **CPI Inflation Rate**: $\pi_t^{cpi} = 100 \times (PCUN_t/PCUN_{t-1} - 1)$.
11. **Asset Returns**: We use two time series from the Fama-French data library:

    - Net nominal risk-free rate, monthly, percent (RF)
    - Net nominal excess market return, monthly, percent ($MKTmRF$)

    Define the market return as

    $$RM_t \equiv MKTmRF_t + RF_t.$$

    The gross quarterly analogues of the Fama-French series and CPI inflation are given by

    $$RF_t^Q \equiv \prod_{j=t-2}^{t}(1 + RF_j/100),\ RM_t^Q \equiv \prod_{j=t-2}^{t}(1 + RM_j/100),\ \pi_t^Q \equiv \prod_{j=t-2}^{t}(1 + \pi_j^{cpi}/100)$$

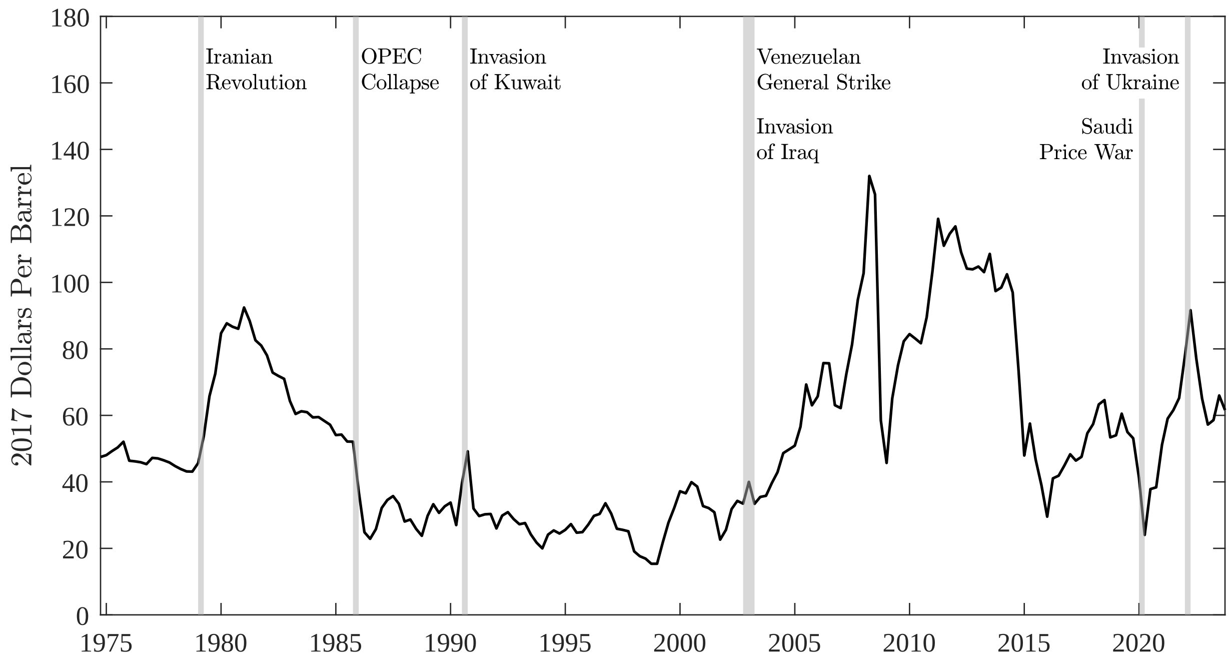
    for $t = 3, 6, \ldots, 3T$, so the quarterly real risk-free rate and equity premium are

    $$r_t = 100 \times (RF_t^Q/\pi_t^Q - 1),\ r_t^{ex} = 100 \times (RM_t^Q/\pi_t^Q - 1) - r_t.$$

All empirical targets are computed using quarterly data, except the expenditure share of oil which is based on annual data.

C OIL MARKET DATA

Figure 1: Real U.S. refiners' acquisition cost crude oil imports, 1974Q4-2023Q4



D SOLUTION METHOD

The equilibrium system of the DSGE model is summarized by $E[g(\mathbf{x}_{t+1}, \mathbf{x}_t, \varepsilon_{t+1})|\mathbf{z}_t, \vartheta] = 0$, where g is a vector-valued function, $\mathbf{x}_t$ is the vector of model variables, ε_t is the vector of shocks, $\mathbf{z}_t = [k_t, s_t, v_t^g, v_t^e, \ln p_t^g, \ln p_t^e, \epsilon_t]$ is the vector of states, and ϑ is the vector of parameters.

We discretize the continuous shocks, $\{\varepsilon_g, \varepsilon_{go}, \varepsilon_p^g, \varepsilon_p^e\}$ using the Markov chain in Rouwenhorst (1995). The bounds of the six continuous state variables are chosen so they cover at least 99% of the ergodic distribution, reducing the need for extrapolation. Specifically, the bounds on capital, k_t, range from -15% to $+10\%$, the bounds on storage, s_t, range from -50% to $+80\%$, and the bounds on the error correction term, ϵ_t, range from -30% to $+15\%$ of the deterministic steady state. The bounds on the probability of a growth disaster, p_t^g, are set to $[0.00005, 0.8]$, while the bounds on the probability of an oil production disaster, p_t^e, are set to $[0.0000025, 0.8]$. Both are converted to logs, consistent with the specifications of the processes. We discretize k_t, s_t, and ϵ_t each into 7 points, and $\ln p_t^g$ and $\ln p_t^g$ into 15 points given the nonlinearity in the transmission of the probability shocks. All of the grids for the continuous states are evenly spaced. There are also binary indicators for whether the economy is in a growth disaster or an oil production disaster, creating 4 outcomes. The product of the points in each dimension, D, is the total number of nodes in the state space ($D = 308{,}700$).

The realization of $\mathbf{z}_t$ on node d is denoted $\mathbf{z}_t(d)$. The Rouwenhorst method provides integration nodes for the continuous shocks, $[\varepsilon_{g,t+1}(m), \varepsilon_{go,t+1}(m), \varepsilon^g_{p,t+1}(m), \varepsilon^e_{p,t+1}(m)]$. The transition matrices for the discrete states determine the integration weights for their future realizations, $[v^g_{t+1}(m), v^e_{t+1}(m)]$. The weight for a particular realization of the continuous and discrete shocks is $\phi(m)$, where $m \in \{1, \ldots, M\}$ and M is the product of the number of realizations of each shock. The two disaster probability shocks, ε^g_p and ε^e_p, have the same number of realizations as the corresponding state variable (15). Each growth shock, ε_g and ε_{go}, has 7 possible realizations. Each discrete state has two possible outcomes. Thus, $M = 44{,}100$ possible shock realizations.

The vector of policy functions and the realization on node d are denoted by $\mathbf{pf}_t$ and $\mathbf{pf}_t(d)$, where $\mathbf{pf}_t \equiv [n(\mathbf{z}_t), o(\mathbf{z}_t), J(\mathbf{z}_t), p^e(\mathbf{z}_t), r(\mathbf{z}_t)]$. The following steps outline our algorithm:

1. Use the Sims (2002) `gensys` algorithm to solve the log-linear model without any disasters or time-varying probabilities. Then map the solution for the policy functions to the discretized state space, copying the solution on the dimensions that were excluded from the linear model. This provides an initial conjecture, $\mathbf{pf}_0$, for the nonlinear algorithm.
2. On iteration $j \in \{1, 2, \ldots\}$ and each node $d \in \{1, \ldots, D\}$, use Chris Sims' `csolve` to find the $\mathbf{pf}_t(d)$ that satisfies $E[g(\cdot)|\mathbf{z}_t(d), \vartheta] \approx 0$. Guess $\mathbf{pf}_t(d) = \mathbf{pf}_{j-1}(d)$. Then
   (a) Solve for all variables dated at time t, given $\mathbf{pf}_t(d)$ and $\mathbf{z}_t(d)$.
   (b) Linearly interpolate the policy functions, $\mathbf{pf}_{j-1}$, at the updated state variables, $\mathbf{z}_{t+1}(m)$, to obtain $\mathbf{pf}_{t+1}(m)$ on every integration node, $m \in \{1, \ldots, M\}$.
   (c) Given $\{\mathbf{pf}_{t+1}(m)\}_{m=1}^M$, solve for the other elements of $\mathbf{s}_{t+1}(m)$ and compute
   $$E[g(\mathbf{x}_{t+1}, \mathbf{x}_t(d), \varepsilon_{t+1})|\mathbf{z}_t(d), \vartheta] \approx \textstyle\sum_{m=1}^M \phi(m) g(\mathbf{x}_{t+1}(m), \mathbf{x}_t(d), \varepsilon_{t+1}(m)).$$

   When `csolve` has converged, set $\mathbf{pf}_j(d) = \mathbf{pf}_t(d)$.
3. Repeat step 2 until $\text{maxdist}_j < 10^{-5}$, where $\text{maxdist}_j \equiv \max\{|(\mathbf{pf}_j - \mathbf{pf}_{j-1})/\mathbf{pf}_{j-1}|\}$. When that criterion is satisfied, the algorithm has converged to an approximate solution.

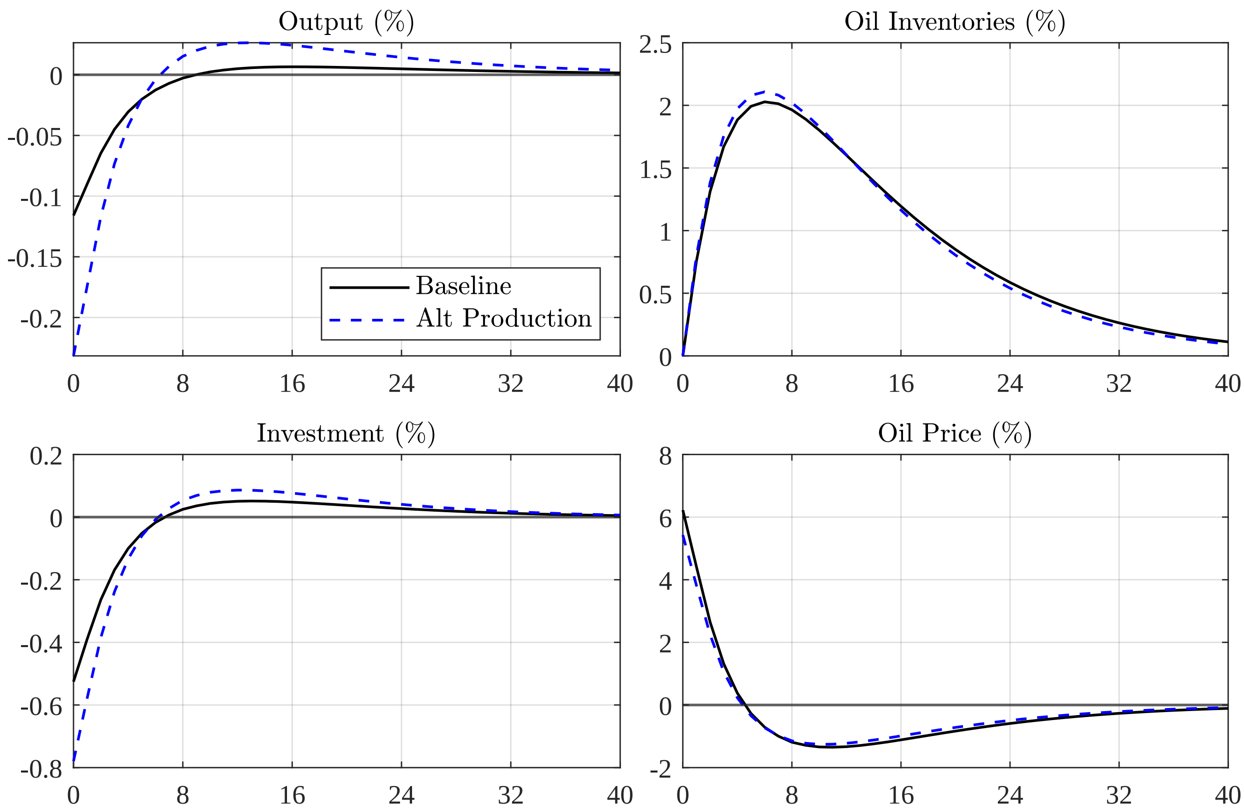
E DETRENDED EQUILIBRIUM

We detrend the model by defining $\tilde{x}_t = x_t/a_t$. The equilibrium system of equations is given by

$$\tilde{w}_t = (1-\xi)\tilde{y}_t/n_t$$
$$p_t^o = \xi\alpha\frac{(\tilde{o}_t/o_0)^{1-1/\sigma}}{(1-\alpha)(\tilde{k}_t/k_0)^{1-1/\sigma}+\alpha(\tilde{o}_t/o_0)^{1-1/\sigma}}\frac{\tilde{y}_t}{\tilde{o}_t}$$
$$E_t[x_{t+1}r_{t+1}^i] = 1$$
$$r_t^i = e^{-\zeta v_t}\frac{1}{p_{t-1}^k}(r_t^k + (1-\delta+a_1+\tfrac{a_2}{\nu-1}(\tilde{\imath}_t/\tilde{k}_t)^{1-1/\nu})p_t^k)$$
$$p_t^k = \tfrac{1}{a_2}(\tilde{\imath}_t/\tilde{k}_t)^{1/\nu}$$
$$r_t^k = \xi(1-\alpha)\frac{(\tilde{k}_t/k_0)^{1-1/\sigma}}{(1-\alpha)(\tilde{k}_t/k_0)^{1-1/\sigma}+\alpha(\tilde{o}_t/o_0)^{1-1/\sigma}}\frac{\tilde{y}_t}{\tilde{k}_t}$$
$$E_t[x_{t+1}r_{t+1}^s] = 1$$
$$r_t^s = \frac{1}{p_{t-1}^o}(1-\omega+\pi\tilde{s}_t^{-3})p_t^o$$
$$\chi\tilde{w}_t\ell_t = (1-\chi)\tilde{c}_t$$
$$x_t = (\beta/g_t^\gamma)(\tilde{u}_t/\tilde{u}_{t-1})^{1-1/\psi}(\tilde{c}_{t-1}/\tilde{c}_t)(\tilde{J}_t/\tilde{z}_{t-1})^{1/\psi-\gamma}$$
$$\tilde{u}_t = \tilde{c}_t^\chi\ell_t^{1-\chi}$$
$$\tilde{z}_t = (E_t[(g_{t+1}J_{t+1})^{1-\gamma}])^{1/(1-\gamma)}$$
$$\tilde{J}_t = \left((1-\beta)\tilde{u}_t^{1-1/\psi} + \beta\tilde{z}_t^{1-1/\psi}\right)^{\frac{1}{1-1/\psi}}$$
$$\tilde{y}_t = y_0 n_t^{1-\xi}\left((1-\alpha)(\tilde{k}_t/k_0)^{1-1/\sigma} + \alpha(\tilde{o}_t/o_0)^{1-1/\sigma}\right)^{\xi/(1-1/\sigma)}$$
$$g_{t+1}\tilde{k}_{t+1} = e^{-\zeta_g v_{t+1}^g}(1-\delta+a_1+\tfrac{a_2}{1-1/\nu}(\tilde{\imath}_t/\tilde{k}_t)^{1-1/\nu})\tilde{k}_t$$
$$g_{t+1}\tilde{s}_{t+1} = (1-\omega)\tilde{s}_t + \tilde{o}_t^s - \tilde{o}_t - \tfrac{\pi}{2}\tilde{s}_t^{-2}$$
$$\tilde{o}_t^s = e_t/\epsilon_t$$
$$\tilde{c}_t + \tilde{\imath}_t = \tilde{y}_t$$
$$n_t + \ell_t = 1$$
$$\ln g_{o,t} = \ln\kappa_0 + \kappa_1\ln g_t + \kappa_2\ln\epsilon_{t-1} + \sigma_{go}\varepsilon_{go,t}$$
$$\ln\epsilon_t = \ln g_t - \ln g_{o,t} + \ln\epsilon_{t-1}$$
$$\ln g_t = \ln\bar{g} + \sigma_g\varepsilon_{g,t} - \zeta_g(v_t^g - \bar{\pi}_1^g)$$
$$\ln e_t = \ln\bar{e} - \zeta_e(v_t^e - \bar{\pi}_1^e)$$
$$Pr(v_{t+1}^g = 1|v_t^g = 1) = \bar{q}^g, \quad Pr(v_{t+1}^g = 1|v_t^g = 0) = p_t^g$$
$$Pr(v_{t+1}^e = 1|v_t^e = 1) = \bar{q}^e, \quad Pr(v_{t+1}^g = 1|v_t^g = 0) = p_t^e$$
$$\ln p_t^g = (1-\rho_p^g)\ln\bar{p}^g + \rho_p^g\ln p_{t-1}^g + \sigma_p^g\varepsilon_{p,t}^g$$
$$\ln p_t^e = (1-\rho_p^e)\ln\bar{p}^e + \rho_p^e\ln p_{t-1}^e + \sigma_p^e\varepsilon_{p,t}^e$$
$$E_t[x_{t+1}r_t] = 1$$
$$1 = E_t[x_{t+1}r_{t+1}^e]$$
$$r_t^e = g_t(\tilde{p}_t^e + \tilde{d}_t^e)/\tilde{p}_{t-1}^e$$
$$\tilde{d}_t^e = \tilde{y}_t - \tilde{\imath}_t - \tilde{w}_t n_t - \vartheta(\tfrac{1}{g_t}E_{t-1}[g_t\tilde{k}_t] - \tfrac{1}{r_t}E_t[g_{t+1}\tilde{k}_{t+1}])$$

F ADDITIONAL RESULTS

Figure 2: Responses to an oil production disaster probability shock



Notes: Responses in deviations from the baseline. Simulations assume no disasters are realized. The disaster probability shock is 10pp.

F.1 PRODUCTION FUNCTION

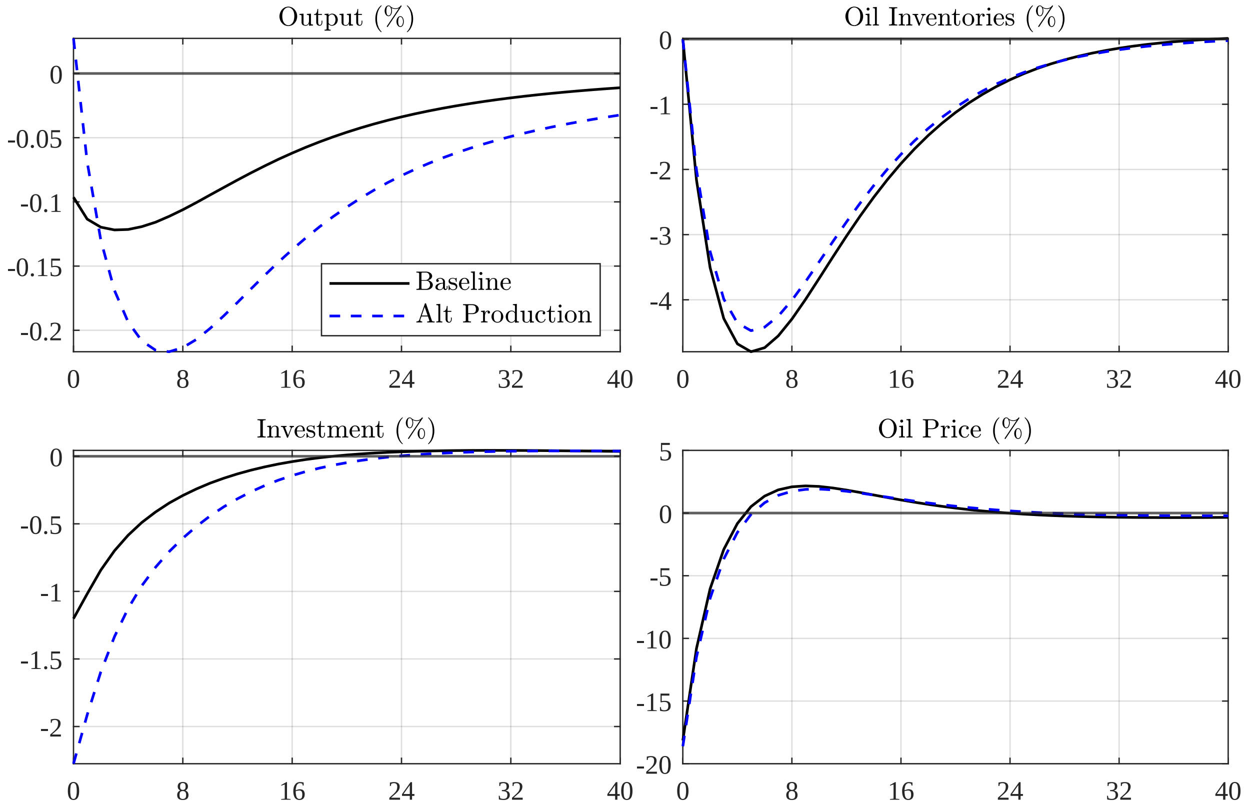
The specification of the production function in our baseline model follows the seminal work of Kim and Loungani (1992) and Backus and Crucini (2000). Some more recent studies, such as Başkaya et al. (2013), Hassler et al. (2021), Olovsson (2019), and Ready (2018), use the following specification

$$y_t = y_0 \left((1-\alpha)(v_t/v_0)^{1-1/\sigma} + \alpha(o_t/o_0)^{1-1/\sigma}\right)^{1/(1-1/\sigma)},$$

where $v_t = (a_t n_t)^{1-\xi} k_t^{\xi}$ is value added and σ is the elasticity of substitution between oil and the capital-labor bundle, rather than just capital. Under this specification, the factor prices are given by

$$\begin{aligned} w_t &= (1-\alpha)(1-\xi)(y_0/n_t)(y_t/y_0)^{1/\sigma}(v_t/v_0)^{1-1/\sigma}, \\ p_t^o &= \alpha(y_0/o_0)(y_t/y_0)^{1/\sigma}(o_t/o_0)^{-1/\sigma}, \\ r_t^k &= (1-\alpha)\xi(y_0/k_t)(y_t/y_0)^{1/\sigma}(v_t/v_0)^{1-1/\sigma}. \end{aligned}$$

Figure 3: Responses to growth disaster probability and stochastic volatility shocks



Notes: Responses in deviations from the baseline. Simulations assume no disasters are realized. The disaster probability shock is 5pp.

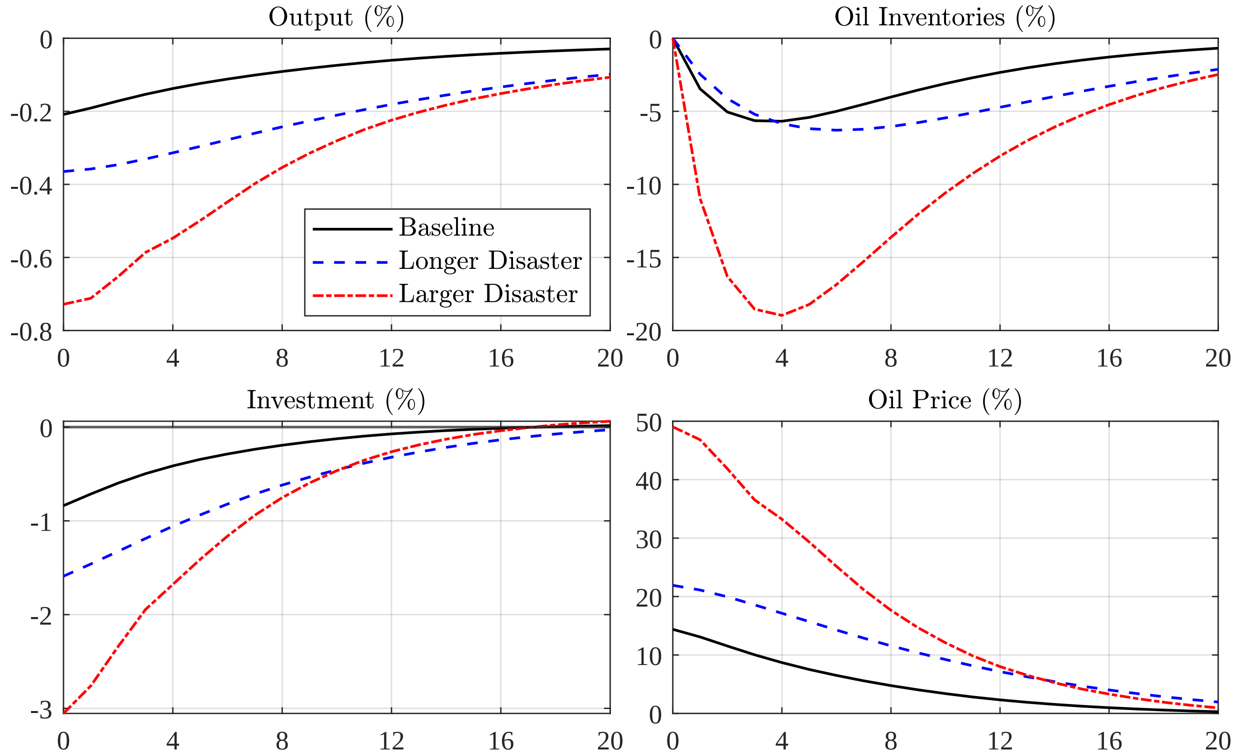
We recalibrate the model to match the data moments. All of the parameters are unchanged, except that we set $\alpha = 0.063$ in order to match the cost share of oil.[1] Figures 2 and 3 illustrate that our substantive conclusions are robust to this alternative functional form of the production function.

F.2 DISASTER REALIZATIONS In the paper, we report the responses to oil production and growth disaster probability shocks, conditional on no disasters being realized. As the probability of a disaster increases, the economic effects converge to those that occur when a disaster is realized. Figure 4a shows how the economy responds to an oil production disaster realization under the baseline calibration (5% drop in global oil production that is expected to last for 3 quarters) and when allowing for a longer disaster (10 quarters on average) and a larger disaster (20% drop in global oil production). Figure 4b shows how the economy responds to a growth disaster realization. In all cases, the disaster occurs in the initial period and then follows its expected path.

[1] We set α to 0.063 so that the average oil expenditure share across simulations matches the share in the data. Similarly, the steady-state inventory share of oil consumption ($\bar{s}/\bar{o}$) is set to 1.18 to match the average share in the data. We set the steady-state labor cost share ($\bar{w}\bar{n}/\bar{y}$), which determines ξ, to 0.5957, in line with the baseline model. The elasticity of substitution, σ, is pinned down by the standard deviation of the price of oil and remains unchanged.

Figure 4: Responses to a disaster realization

(a) Oil production disaster



(b) Growth disaster

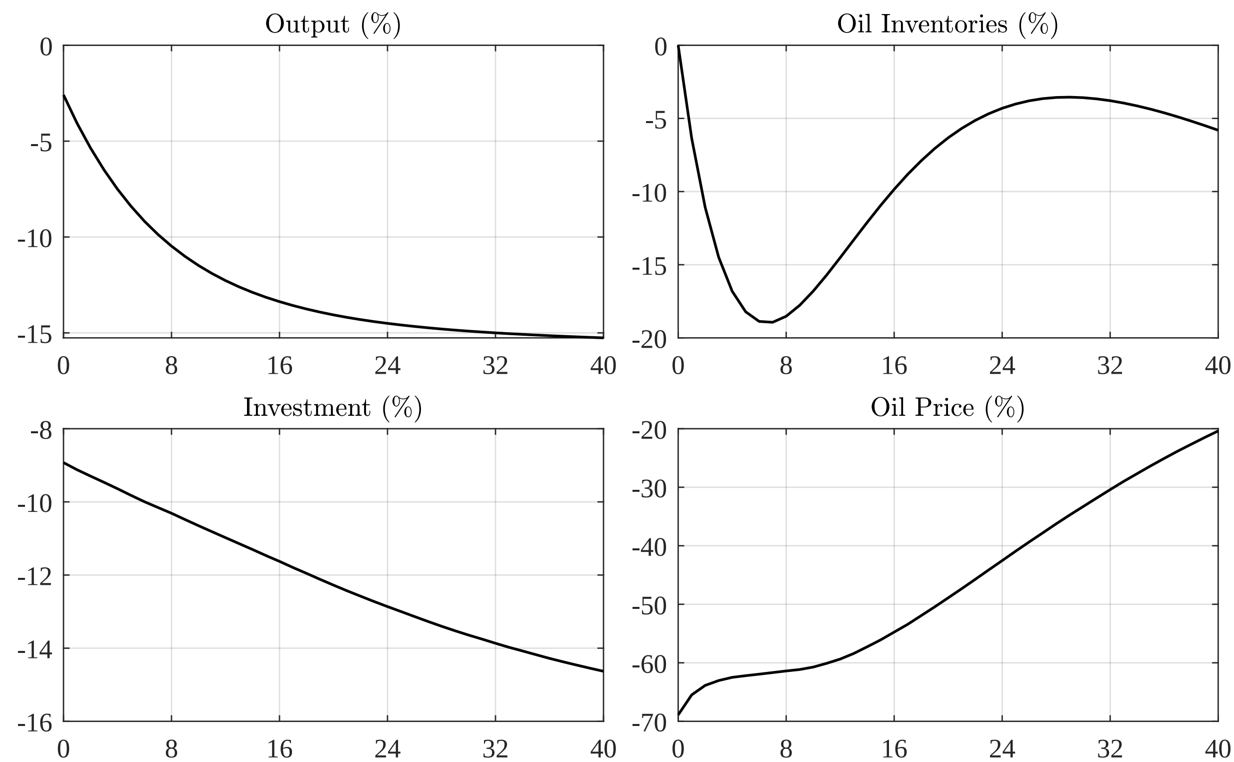
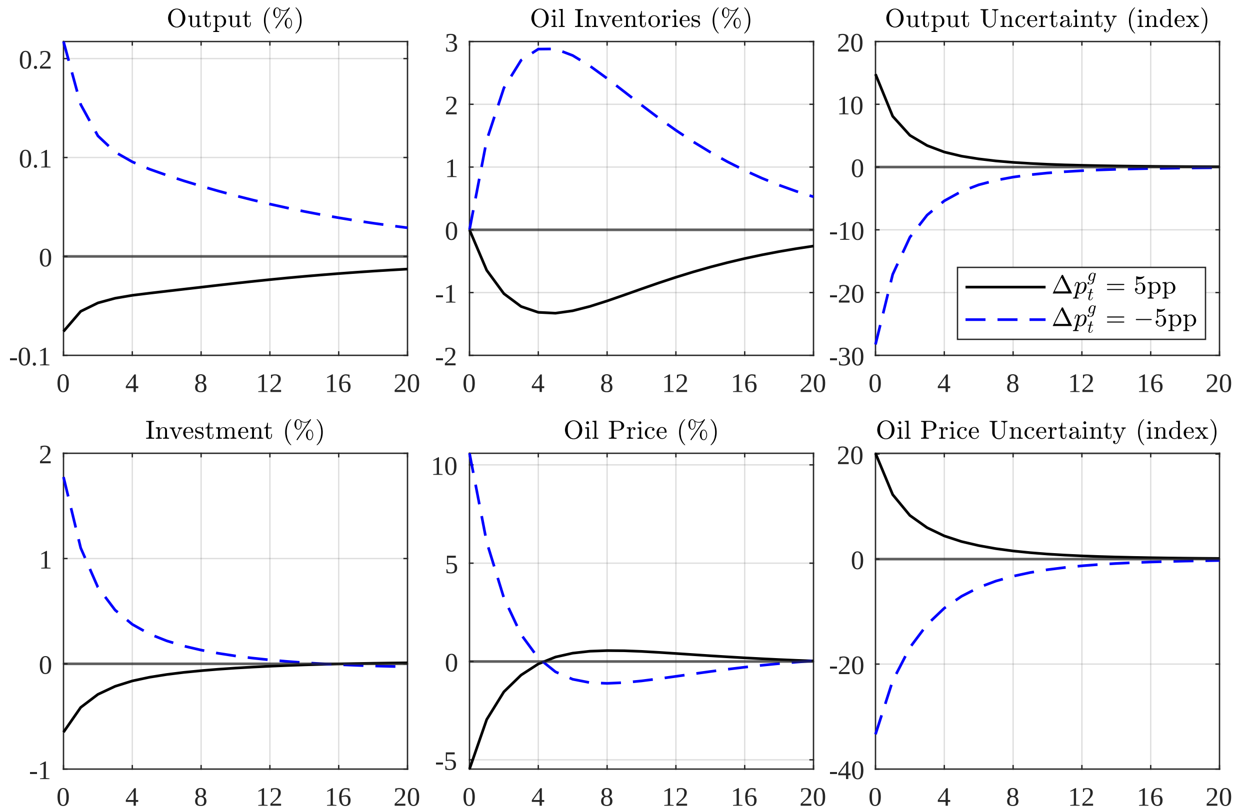


Figure 5: Responses to positive and negative growth disaster probability shocks



Notes: Responses in deviations from the baseline. Simulations assume no disasters are realized.

F.3 SIGN OF THE DISASTER PROBABILITY SHOCK In the paper, we show how the sign of an oil disaster probability shock affects the responses. Figure 5 shows the responses to a ± 5pp growth disaster probability shock. The simulations are initialized at a 15% growth disaster probability to permit a positive and negative shock. Consistent with the responses to an oil disaster probability shock, we find that increases and decreases in the price of oil do not have the same effects on uncertainty. A decrease in the probability of an growth disaster increases the price of oil and reduces oil price uncertainty on impact, while an increase in this probability lowers the oil price and raises oil price uncertainty. This result is inconsistent with a VAR model with GARCH errors.

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