Macroeconomic Responses to Uncertainty Shocks: The Perils of Recursive Orderings*

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Abstract

A common practice in empirical macroeconomics is to examine alternative recursive orderings of the variables in structural vector autoregressive (VAR) models. When the implied impulse responses look similar, the estimates are considered trustworthy. When they do not, the estimates are used to bound the true response without directly addressing the identification challenge. A leading example of this practice is the literature on the effects of uncertainty shocks on economic activity. We prove by counterexample and show by simulation that this practice is invalid, whether the data generating process is a structural VAR model or a dynamic stochastic general equilibrium model. Simulation evidence suggests that the underlying identification challenge can be addressed using an instrumental variables estimator.

Keywords: Cholesky decomposition; orthogonalization; simultaneity; endogeneity; uncertainty; business cycle

JEL Classifications: C32, C51, E32

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1 INTRODUCTION

Structural vector autoregressive (VAR) analysis continues to be widely applied in empirical macroeconomics. In recent decades there has been a proliferation of research on new approaches to the identification of structural VAR models; yet, many applied researchers continue to use much simpler identification strategies that impose a recursive structure on the structural impact multiplier matrix. This allows this matrix to be estimated by applying a lower triangular Cholesky decomposition to the reduced-form VAR residual covariance matrix with the diagonal elements normalized to be positive. Sometimes this approach can be justified on economic grounds, but more often it creates mutually uncorrelated shocks that have no obvious structural interpretation.

The fact that recursive VAR models in general will not recover the population responses when the true model is not recursive has been discussed at length in the econometrics literature (see, e.g., Braun and Mittnik, 1993; Cooley and Leroy, 1985; Kilian and Lütkepohl, 2017; Leamer, 1985). However, this fact has done little to diminish the appeal of ad hoc recursive orderings in applied work, in particular when relating one macroeconomic model variable to another without reference to an explicit structural model. Even having acknowledged the well-known limitations of recursive models, many applied users believe that ad hoc recursive orderings may be used to learn about the quantitative importance of causal effects in the data. A leading example is the literature on the effects of uncertainty shocks on economic activity. For example, Altig et al. (2020) concedes that "drawing causal inferences from VARs is challenging—in part because policy, and policy uncertainty, can respond to current and anticipated future economic conditions", but argues that "despite the challenges, [recursively identified] VARs are useful for ... gauging whether uncertainty innovations foreshadow weaker macroeconomic performance conditional on standard macro and policy variables." This view is widely shared in the literature.

A common practice even in top economics journals has been to report impulse response estimates based on alternative orderings of the model variables with the uncertainty measure ordered first or last. Evidence that the responses are robust to alternative orderings is taken as evidence that the shock of interest has been identified and the impulse response estimates are trustworthy. A closely related argument is that alternative recursive orderings may be used to bound the true impulse response without directly addressing the identification challenge (e.g., Altig et al., 2020; Bachmann et al., 2013; Baker et al., 2016; Basu and Bundick, 2017; Bloom, 2009; Caggiano et al., 2014; Fernández-Villaverde et al., 2015; Leduc and Liu, 2016).

While both of these approaches have been questioned, the validity of these popular arguments has never been formally examined. For example, regarding the first practice, Bernanke (1986) pointed out that the usual practice of trying alternative orderings of the variables still restricts attention to recursive models which (roughly speaking) occupy a set of measure zero within the set of models described by the same reduced-form VAR model. Regarding the second practice, Watson (1994) remarked in a footnote that there is no basis for applied researchers estimating a variety of recursive models in the belief (or hope) that the set of recursive models somehow "brackets" the truth. Neither these warnings nor similar statements elsewhere in the VAR literature, however, have prevented these approaches from being routinely applied by leading practitioners and being published in leading journals in recent years. Clearly, these warnings have been ineffective, perhaps because they were not accompanied by formal analysis. In this paper, we prove by counterexample that both arguments are incorrect, calling into question many empirical results reported in the literature. Our examples are designed to persuade researchers of the dangers of relying on ad hoc recursive orderings and to dispel common misconceptions found in applied work.

What is new about our analysis is not the insight that recursively identified VAR models are questionable if their identifying restrictions are not supported by extraneous evidence. Rather we show analytically based on structural VAR examples and by simulation using data generated from dynamic stochastic general equilibrium (DSGE) models that the robustness of impulse responses to alternative ad hoc recursive orderings does not establish that the true response has been identified. We also prove that alternative orderings cannot be used to bound the true responses, as commonly argued in applied work.

We are not alone in expressing skepticism about the validity of recursively identified models of uncertainty shocks. Several recent studies have proposed alternative VAR approaches to identifying uncertainty shocks that avoid the use of recursive orderings (e.g., Angelini et al., 2019; Benati, 2023; Berger et al., 2020; Brianti, 2023; Caggiano and Castelnuovo, 2023; Caggiano et al., 2021; Caldara et al., 2016; Carriero et al., 2021; Furlanetto et al., 2019; Ludvigson et al., 2021; Piffer and Podstawski, 2018). Ludvigson et al. (2021) also illustrate that recursive impact multiplier matrices are inconsistent with the estimate of their model that does not impose a recursive structure. However, none of these studies formally show why the use of alternative recursive orderings is misleading nor do they address the common practice of bounding the responses based on alternative orderings. Since the bounding approach is based on the premise of a nonrecursive population model, the emergence of nonrecursive identification strategies has done little to discourage the use of alternative recursive orderings, especially since there is no consensus in the literature on how to identify nonrecursive models. Moreover, even authors who published studies based on nonrecursive models have continued to rely on recursive identifications in more recent work, which suggests that these issues require more formal analysis.

The analysis in our paper remains timely and relevant not only because the empirical findings of earlier studies based on alternative recursive orderings continue to shape the academic discourse about the role of uncertainty, but because recursively identified VAR models (or equivalently local projection models) have remained a common empirical approach in the growing literature on uncertainty shocks, notwithstanding their critics. In just the last two years, at least a dozen new studies have used recursive VAR models to identify uncertainty shocks (e.g., Ahir et al., 2022; Beraja and Wolf, 2021; Born and Pfeifer, 2021; Cacciatore and Ravenna, 2021; Caldara and Iacoviello, 2022; Chen and Tillmann, 2021; Gao et al., 2022; Grimme, 2023; Larsen, 2021; Londono et al., 2021; Śmiech et al., 2021; Theophilopoulou, 2022).

The remainder of the paper is organized as follows. In Section 2 we examine the common practice of conducting robustness checks based on alternative recursive orderings. This section also studies the practice of constructing upper and lower bounds on the effects of uncertainty shocks based on alternatively ordering the uncertainty variable first and last in the model without directly addressing the identification challenge. We analytically demonstrate the invalidity of both approaches using stylized VAR processes. Our analysis is not subject to reduced-form model misspecification or estimation error, which facilitates the comparison of alternative models.

Ad hoc recursive VAR orderings fail when real activity and uncertainty are simultaneously determined. The origins of this simultaneity are best illustrated within the context of a fully specified dynamic economic model. In Section 3, we examine the credibility of recursive VAR models when the data are generated by calibrated DSGE models of the determination of aggregate uncertainty. This allows us to quantify the large-sample bias in impulse response estimators from recursively estimated models in a realistic setting, while controlling for the degree of endogeneity. Not only are recursive VAR models invalid, regardless of the ordering, when uncertainty is fully endogenous in the data generating process, but the recursive VAR model is also unable to recover the population response of output to an uncertainty shock when aggregate uncertainty is only partially endogenous. This is true even in a quasi-recursive setting when 90% of the variation in uncertainty is exogenous. Thus, our analysis of DSGE models supports and reinforces our earlier conclusion that recursive VAR models of the effects of uncertainty shocks cannot be trusted and that alternative recursive orderings fail to bound the population response in general. We also discuss why evidence that a VAR model based on a specific recursive ordering can approximately recover population responses from a given DSGE model, as in Basu and Bundick (2017), is not sufficient to defend the use of that VAR model.

The identification challenge is that measured uncertainty in general responds to both level and volatility shocks, as do the macroeconomic aggregates. This makes it difficult to distinguish endogenous and exogenous variation in measured uncertainty for many identification methods, not only for recursive models. We provide simulation evidence that an instrumental variables (IV) estimator may overcome these identification challenges, while many other non-recursive estimators proposed in the literature do not. The concluding remarks are in Section 4.

2 ROBUSTNESS CHECKS FOR ALTERNATIVE AD HOC RECURSIVE ORDERINGS

There are situations in which a fully recursive model can be economically justified, but such situations are rare in applied work. More often, researchers rely on a block recursive model structure. Since the responses of interest are invariant to the identification of the remaining structural shocks, one may estimate such models without loss of generality based on a Cholesky decomposition, retaining only the responses to the structural shock of interest. In contrast, in this paper we are concerned with uses of recursive orderings that lack a compelling economic rationale.[1]

2.1 THE IDENTIFICATION PROBLEM

In general, the response estimates implied by a Cholesky decomposition will differ depending on the ordering of the variables in the structural VAR model. This is not surprising since each of these recursive models assumes a different economic structure. In an n-dimensional VAR process, there are $n!$ alternative Cholesky orderings. Unless the covariances of the reduced-form residuals are zero, each of these orderings will imply different impulse response estimates. However, for low enough error correlations the responses will tend to be similar across alternative orderings.

In a model that is explicitly identified based on economic reasoning, we know *a priori* that one of these orderings is correct and the others are not, so the response estimates are unique. This is not the case when Cholesky decompositions are used in an ad hoc fashion without explicit economic motivation, as is often the case in applied work. It is the latter situation that this paper addresses. A classical example is the literature that examines the effect of uncertainty shocks on economic activity. This question has led to a large literature relying on recursively identified structural VAR models (e.g., Altig et al., 2020; Bachmann et al., 2013; Baker et al., 2016; Basu and Bundick, 2017; Bekaert et al., 2013; Bloom, 2009; Caggiano et al., 2014; Caldara and Iacoviello, 2022; Fernández-Villaverde et al., 2015; Jurado et al., 2015; Leduc and Liu, 2016). The empirical response estimates in turn have stimulated theoretical work on the transmission of uncertainty shocks.

[1] It should be noted that the use of recursive VAR models as auxiliary models for indirect inference on the parameters of DSGE models does not require the population response to be correctly identified and hence is not subject to the concerns expressed in this paper.

Aggregate uncertainty in this literature is typically measured by the volatility of macroeconomic or financial aggregates. As noted by Ludvigson et al. (2021), empirical studies often differ according to whether aggregate uncertainty is ordered ahead of or after real activity in the VAR model. It is useful to analyze this situation in a bivariate setting. Allowing for additional variables ordered in between the first and the last variable does not change the logic of the arguments below. For example, consider a stylized model of the impact of uncertainty shocks on real GDP growth, where the reduced-form shocks, u_t, are linked to the structural shocks, w_t, through the structural impact multiplier matrix, B_0^{-1} with elements $b_0^{ij} = \partial u_{i,t}/\partial w_{j,t}$, $i \in \{1,2\}$, $j \in \{1,2\}$.[2] There are two recursive models of this relationship. We can postulate that

$$\begin{pmatrix} u_t^{\Delta gdp} \\ u_t^{\text{uncertainty}} \end{pmatrix} = \begin{bmatrix} b_0^{11} & 0 \\ b_0^{21} & b_0^{22} \end{bmatrix} \begin{pmatrix} w_t^{\text{other}} \\ w_t^{\text{uncertainty}} \end{pmatrix} \tag{1}$$

or, alternatively, that

$$\begin{pmatrix} u_t^{\text{uncertainty}} \\ u_t^{\Delta gdp} \end{pmatrix} = \begin{bmatrix} b_0^{11} & 0 \\ b_0^{21} & b_0^{22} \end{bmatrix} \begin{pmatrix} w_t^{\text{uncertainty}} \\ w_t^{\text{other}} \end{pmatrix}. \tag{2}$$

Unlike in properly-identified recursive VAR models, in this setting there is no compelling reason to prefer one ordering over the other. On the one hand, it has been noted that changes in economic activity may affect uncertainty about the economy, which suggests ordering uncertainty last. On the other hand, it seems reasonable to expect uncertainty shocks to affect economic activity within the current month, which argues for ordering uncertainty first. Neither recursive model is consistent with both economic arguments, suggesting that at least one, if not both models are misspecified.

Many studies in this literature seek to resolve this concern by reporting impulse response estimates based on a recursive model in which a measure of uncertainty is ordered first as well as estimates based on an alternative recursive model in which this uncertainty measure is ordered last. If the response estimates are similar across these specifications, this is taken as evidence that response estimates are robust and hence trustworthy. It is this practice that we discuss next.

[2]As is standard, $E[u_t] = E[w_t] = 0$, $\mathrm{Var}(u_t) = \Sigma$, and $\mathrm{Var}(w_t)$ is normalized to an identity matrix.

2.2 ROBUSTNESS TO ALTERNATIVE ORDERINGS DOES NOT MEAN THE TRUE RESPONSE IS IDENTIFIED In general, a finding that the impulse responses to uncertainty shocks are invariant to the ordering would not be expected. The responses will be identical only when the reduced-form error correlation is zero. Even in that case, however, examining alternative recursive orderings is not sufficient. It is entirely possible and indeed plausible that the underlying population model is a simultaneous equations model that allows uncertainty to respond endogenously to economic activity conditional on past data. In fact, the recent literature provides many arguments why uncertainty is simultaneously determined with economic activity rather than determined recursively.

For some choices of the model parameters, a simultaneous equations model may imply zero correlation in the reduced-form errors, yet the responses in the population model need not look anything like the responses from recursively identified models. Uncorrelated reduced-form errors may arise when the off-diagonal elements of the structural impact multiplier matrix are of different signs. For example, consider the population model

$$\begin{aligned} \begin{pmatrix} u_t^{\Delta gdp} \\ u_t^{\text{uncertainty}} \end{pmatrix} &= \begin{bmatrix} 1 & 0.5 \\ -0.75 & 1.5 \end{bmatrix} \begin{pmatrix} w_t^{\text{other}} \\ w_t^{\text{uncertainty}} \end{pmatrix}, \\ \Sigma = B_0^{-1}(B_0^{-1})' = \begin{bmatrix} 1.25 & 0 \\ 0 & 2.8125 \end{bmatrix}&, \quad \text{chol}(\Sigma) = \begin{bmatrix} 1.118 & 0 \\ 0 & 1.6771 \end{bmatrix}, \end{aligned} \tag{3}$$

which has uncorrelated reduced-form errors. In this model, positive shocks to uncertainty increase economic growth on impact, consistent with “growth options” theories, whereas positive shocks to economic activity reduce macroeconomic uncertainty in line with Ludvigson et al. (2021).[3]

Model 3 is a counterexample to the notion that, when Σ is diagonal, B_0^{-1} must also be diagonal. The population impact response of real GDP growth to a structural uncertainty shock in this model is 0.5. Imposing a recursive ordering with real GDP growth ordered first implies an impact response of real GDP growth to the uncertainty shock that is 0. After re-ordering the two variables such that uncertainty is ordered first, the Cholesky estimate yields the same impact response of 0

[3]Bloom (2014) provides a survey of the channels through which uncertainty shocks are transmitted.

to an uncertainty shock. An applied user thus would be tempted to conclude that the robust finding of a zero response for both orderings means that the population response must be zero, when in reality the population response is 0.5. In short, the response being robust within the universe of recursive orderings does not mean it is valid when allowing the true model to be nonrecursive.

A reduced-form error correlation of zero does not require output to respond positively to uncertainty shocks, but may arise more generally when the off-diagonal elements of B_0^{-1} are of the opposite sign. In other words, this example relies on one structural shock driving output and uncertainty in the same direction and the other shock driving them in the opposite direction. If the uncertainty shock lowered real GDP and the other shock lowered uncertainty, in contrast, the reduced-form error correlation would typically be far from zero, and likewise, if both off-diagonal elements were positive.

What invalidates the robustness argument is not that we somehow know that Model 3 actually is the data generating process. Rather the concern is that there are nonrecursive data generating processes with reduced-form error correlations of zero. As a result, even if the response estimates are robust to alternative recursive orderings, we can never know whether we have identified the population response. Thus, empirical evidence that alternative recursive orderings generate similar responses does not lend credence to the identification.

What makes the limiting case of a zero correlation in Model 3 of practical interest is that the lack of identification in this extreme situation carries over to processes with reduced-form error correlations that are only close to zero, as may be easily verified. One may object that a data generating process with nearly uncorrelated reduced-form errors is perhaps not realistic. This point is not self-evident. Clearly, researchers arguing that their impulse response estimates are similar under alternative recursive orderings must be dealing with a situation in which the correlations of the reduced-form errors are at least close to zero or the responses would not be similar, suggesting that Model 3 is empirically relevant.

While one could envision alternative data generating processes for which these error correlations are nontrivial, the responses implied by such processes would not be invariant to alternative recursive orderings, so trying to establish robustness becomes moot. For example, when the uncertainty shock affects real GDP growth, but the uncertainty variable does not respond to the other shock contemporaneously, as implicitly assumed in many studies, the reduced-form error correlation will tend to be far from zero, so the ordering of the variables matters and the model would fail the robustness check. This has led some researchers to provide an alternative justification for the use of alternative recursive orderings that we turn to next.

2.3 ALTERNATIVE RECURSIVE ORDERINGS DO NOT BOUND THE POPULATION RESPONSE

When the reduced-form error correlations are far from zero, the orthogonalized responses based on Models 1 and 2 will change substantially with the ordering of the variables, rendering moot the strategy of establishing robustness to alternative orderings. This fact has not dissuaded practitioners from reporting response estimates based on alternative orderings.

A common argument in the literature is that, when VAR studies order uncertainty before output, the authors are not necessarily claiming that uncertainty is exogenous; Rather they are conditioning on this hypothesis for illustrative purposes. The belief is that this assumption yields an upper bound on the effect of uncertainty shocks (see, e.g., Caggiano et al., 2014). Similarly, by ordering uncertainty last, this argument goes, we give all other shocks a chance to explain the data first and hence end up with a lower bound on the effect of uncertainty shocks. For example, Gao et al. (2022) writes that "to bracket the effect of... volatility shocks on fundamentals, we consider two different schemes: in the first approach, we let [the] implied ... variance be the first among ... the variables in the VAR, while in the second approach, it is ordered last. Thus, ... volatility innovations from the VAR are treated as most exogenous to the system under the first ordering and as least exogenous under the second ordering. By comparing the impulse responses under these two specifications, we can assess the range of possible responses to ... volatility shocks in the data."

Similarly, Jurado et al. (2015) consider alternative recursive orderings. Their reasoning is that "as uncertainty is placed last in the VAR, the effects of uncertainty shocks on the other variables in the system are measured after we have removed all the variation in uncertainty that is attributable to shocks to the other endogenous variables in the system. That the effects of uncertainty shocks are still non-trivial is consistent with the view that uncertainty has important implications for economic activity." This suggests that they view the estimates from models that order uncertainty above the macroeconomic aggregates as an upper bound and the estimate from models with uncertainty ordered last as the lower bound.

The maintained assumption when bounding the responses is that the population model is nonrecursive but unknown. As shown next, it is not possible in general to bound the population response with alternative orderings. Likewise, forecast error variance decompositions cannot be bounded, since they are constructed from the impulse responses. For example, consider the population model

$$\begin{aligned} \begin{pmatrix} u_t^{\Delta gdp} \\ u_t^{\text{uncertainty}} \end{pmatrix} &= \begin{bmatrix} 1 & 0.5 \\ -0.9 & 1 \end{bmatrix} \begin{pmatrix} w_t^{\text{other}} \\ w_t^{\text{uncertainty}} \end{pmatrix}, \\ \Sigma = B_0^{-1}(B_0^{-1})' &= \begin{bmatrix} 1.25 & -0.4 \\ -0.4 & 1.81 \end{bmatrix}, \quad \text{chol}(\Sigma) = \begin{bmatrix} 1.118 & 0 \\ -0.3578 & 1.2969 \end{bmatrix}. \end{aligned} \tag{4}$$

In this case, the impact response of real GDP growth is 0 compared to the population response of 0.5. Reversing the order of the variables yields

$$\begin{aligned} \begin{pmatrix} u_t^{\text{uncertainty}} \\ u_t^{\Delta gdp} \end{pmatrix} &= \begin{bmatrix} 1 & -0.9 \\ 0.5 & 1 \end{bmatrix} \begin{pmatrix} w_t^{\text{uncertainty}} \\ w_t^{\text{other}} \end{pmatrix}, \\ \Sigma = B_0^{-1}(B_0^{-1})' &= \begin{bmatrix} 1.81 & -0.4 \\ -0.4 & 1.25 \end{bmatrix}, \quad \text{chol}(\Sigma) = \begin{bmatrix} 1.3454 & 0 \\ -0.2973 & 1.0778 \end{bmatrix}. \end{aligned} \tag{4'}$$

The impact response of real GDP growth is now -0.2973. The conventional wisdom would suggest that the true response must be between -0.2973 and 0, but the population response is 0.5 in this model, which is of the opposite sign of the range of values bounded by the recursive estimates.[4] In fact, a similarly erroneous conclusion would have been reached in Model 3, which would have suggested that the population response is bounded from above and below by 0 when it is 0.5.

These examples involve reduced-form errors with negative or zero correlations. As an example

[4]Similarly, the share of the one-step ahead forecast error variance of Δgdp explained by $w_t^{\text{uncertainty}}$ is 0.0707 when uncertainty is ordered first and 0 when it is ordered last, which does not bound the population value of 0.2.

of a model with a positive reduced-form error correlation, consider

$$\begin{pmatrix} u_t^{\Delta gdp} \\ u_t^{\text{uncertainty}} \end{pmatrix} = \begin{bmatrix} 1 & 0.5 \\ -0.1 & 1 \end{bmatrix} \begin{pmatrix} w_t^{\text{other}} \\ w_t^{\text{uncertainty}} \end{pmatrix},$$
$$\Sigma = B_0^{-1}(B_0^{-1})' = \begin{bmatrix} 1.25 & 0.4 \\ 0.4 & 1.01 \end{bmatrix}, \quad \text{chol}(\Sigma) = \begin{bmatrix} 1.118 & 0 \\ 0.3578 & 0.9391 \end{bmatrix}. \tag{5}$$

In this case, the impact response of real GDP growth is 0 compared to the population response of 0.5. Reversing the order of the variables implies an impact response of 0.398, implying bounds that exclude the population response.

The point of these examples is to rigorously demonstrate that there is no reason to expect any of the recursive estimates to recover the population response when the population model is not recursive. Nor is there a reason for recursive estimates to bound the population response in general. This is true whether the reduced-form errors are positively correlated, uncorrelated, or negatively correlated. It is also true whether alternative recursive orderings produce the same response or not.

2.4 SUFFICIENT CONDITION FOR THE BOUNDING EXERCISE TO WORK

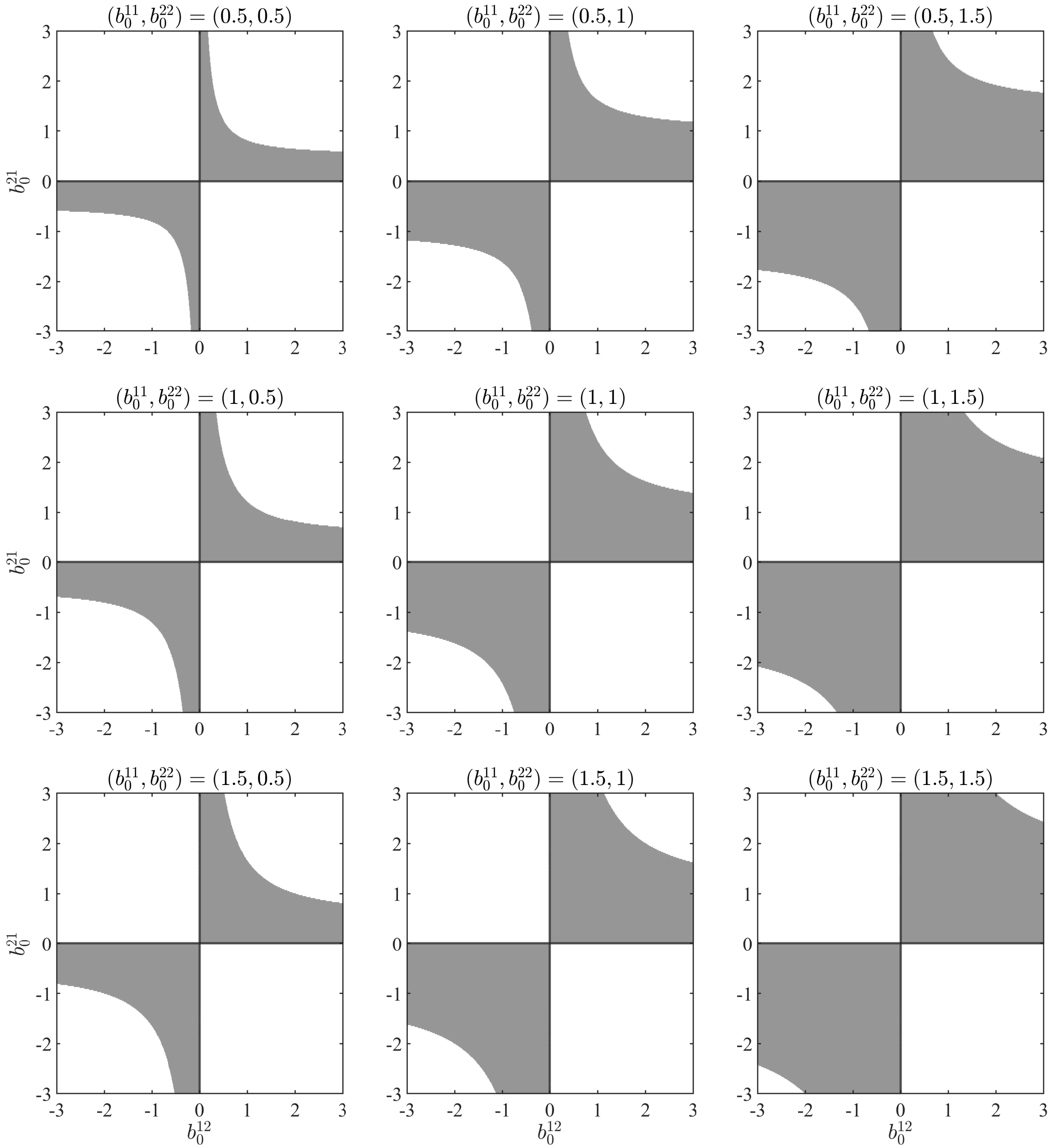
A natural question is how pervasive the failure of the bounding approach is. In this section, we discuss under what conditions bounding fails and when it is expected to work. Recall that the structural impact multiplier matrix and the associated variance-covariance matrix are given by

$$B_0^{-1} = \begin{bmatrix} b_0^{11} & b_0^{12} \\ b_0^{21} & b_0^{22} \end{bmatrix}, \quad \Sigma = B_0^{-1}(B_0^{-1})' = \begin{bmatrix} (b_0^{11})^2 + (b_0^{12})^2 & b_0^{11}b_0^{21} + b_0^{12}b_0^{22} \\ b_0^{11}b_0^{21} + b_0^{12}b_0^{22} & (b_0^{21})^2 + (b_0^{22})^2 \end{bmatrix}.$$

Suppose, without loss of generality, that uncertainty is ordered first in the vector of model variables so b_0^{21} is the true response of GDP growth to an uncertainty shock. A lower triangular Cholesky decomposition of the variance-covariance matrix implies

$$\Sigma = \underbrace{\begin{bmatrix} c_1 & 0 \\ c_2 & c_3 \end{bmatrix}}_{\text{chol}(\Sigma)} \underbrace{\begin{bmatrix} c_1 & c_2 \\ 0 & c_3 \end{bmatrix}}_{\text{chol}(\Sigma)'} = \begin{bmatrix} c_1^2 & c_1c_2 \\ c_1c_2 & c_2^2 + c_3^2 \end{bmatrix}.$$

Figure 1: Parameter regions in which alternative orderings bound the population response



Notes: Shaded regions show parameter combinations in the bivariate VAR model for which the bounding exercise succeeds.

Therefore, the bounds on the population response of GDP growth to an uncertainty shock from alternative recursive orderings are given by $[0, c_2]$, where

$$c_2 = \frac{b_0^{11} b_0^{21} + b_0^{12} b_0^{22}}{\sqrt{(b_0^{11})^2 + (b_0^{12})^2}}.$$

If $b_0^{21} > 0$, then $c_2 > b_0^{21}$ is sufficient for the population response to be bounded. Similarly, if $b_0^{21} < 0$, the sufficient condition is that $c_2 < b_0^{21}$.

For illustrative purposes, assume that $b_0^{11} = b_0^{22} = 1$, as in the earlier examples. Then

$$c_2 = \frac{b_0^{21} + b_0^{12}}{\sqrt{1 + (b_0^{12})^2}}.$$

As illustrated in the middle panel of Figure 1, there are large regions of the parameter space where the bounding condition fails.

It may seem that the bounding condition could be checked before implementing the bounding approach. This is not the case. The bounding condition is nonlinear in the population parameters and hence difficult to characterize even in small-dimensional models. It is also impossible to verify in practice, as it depends on the unknown population parameter values. The problem is not only that we are unable to estimate these parameters consistently without further identifying assumptions, but that there would be no need for bounding the responses if we knew the population parameters with any degree of accuracy. Qualitatively similar results hold when the diagonal elements of B_0^{-1} differ from unity (see Figure 1). Thus, our earlier counterexamples are by no means atypical. The sufficiency conditions are even more complicated as the dimension of the VAR model increases.

3 EXAMINING AD HOC RECURSIVE IDENTIFICATION THROUGH DSGE MODELS

The stylized examples discussed in Section 2 are simple enough to facilitate closed-form solutions, and they are not subject to estimation error or model misspecification. They clearly demonstrate the problems with relying on ad hoc recursive orderings. In this section, we employ a DSGE model of the determination of aggregate uncertainty to further examine the conditions under which one would expect ad hoc recursive VAR models to fail.

Complementing our analytical results with simulation-based results is crucial for four reasons. First, it is essential to formally demonstrate that under reasonable model specifications uncertainty is indeed endogenous, given that it is treated as exogenous in many applications. Second, the analytical examples are static. While the result that the identification fails extends to dynamic simultaneous equations models, the extent of the asymptotic bias will depend on the dynamics of the underlying data generating process. Third, a natural conjecture is that the accuracy of recursive models with uncertainty ordered first would improve when the degree of endogeneity is small. We show that the recursive estimator fails even when exogenous uncertainty shocks account for 90% of the variability of measured uncertainty. Finally, we use the DSGE model to examine the merits of alternative non-recursive identification approaches when uncertainty is endogenous.

3.1 DSGE MODEL There are many mechanisms that render uncertainty at least partially endogenous, including concavity in decision rules (e.g., Atkinson et al., 2022; Ilut et al., 2018; Straub and Ulbricht, 2019); ambiguity aversion (e.g., Bianchi et al., 2018; Ilut and Schneider, 2014); search and matching frictions (Bernstein et al., 2024), information frictions (e.g., Bachmann and Moscarini, 2012; Benhabib et al., 2016; Fajgelbaum et al., 2017; Saijo, 2017; Straub and Ulbricht, 2023; Van Nieuwerburgh and Veldkamp, 2006); a zero lower bound on the nominal interest rate (Plante et al., 2018); and portfolio diversification (Decker et al., 2016). For expository purposes, we use a textbook real business cycle model augmented to include disaster risk (Gourio, 2012; Shen, 2015) and recursive preferences (Epstein and Zin, 1989). Disaster risk introduces a source of downside risk, while recursive preferences separate risk aversion from the intertemporal elasticity of substitution. These features generate endogenous fluctuations in macroeconomic uncertainty.

The representative household solves the Bellman equation

$$J(b_t) = \max_{c_t, n_t, b_{t+1}} \left[(1-\beta)u_t^{1-1/\psi} + \beta(E_t[J(b_{t+1})^{1-\gamma}])^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}$$

subject to

$$u_t = c_t^{\eta}(1-n_t)^{1-\eta},$$

$$c_t + b_{t+1}/r_t = w_t n_t + b_t + d_t,$$

where $\beta \in (0,1)$ is the discount factor, $\gamma \geq 0$ determines risk aversion, $\psi \geq 0$ is the intertemporal elasticity of substitution, u_t is the utility function, c_t is consumption, n_t is labor hours, b_t is a risk-free bond with return r_t, w_t is the wage rate, and d_t are lump-sum dividends from firm ownership. The term $z_t \equiv (E_t[J_{t+1}^{1-\gamma}])^{\frac{1}{1-\gamma}}$ is the risk-adjusted expectation operator. The Frisch labor supply elasticity, which is used in the calibration of the model, is given by $\eta^\lambda = \frac{1-n}{n}\frac{1-(1-1/\psi)\eta}{1/\psi}$.

The representative firm solves the Bellman equation

$$V(k_t) = \max_{n_t, i_t, k_{t+1}} d_t + E_t[x_{t+1}V(k_{t+1})]$$

subject to

$$d_t = y_t - w_t n_t - i_t,$$

$$y_t = a_t k_t^\alpha n_t^{1-\alpha},$$

$$k_{t+1} = \Theta_{t+1}((1-\delta)k_t + i_t),$$

where x_{t+1} is the pricing kernel derived from the household's optimality conditions, y_t is output, i_t is investment, k_t is the capital stock that depreciates at rate δ, and a_t is technology, which follows

$$\ln a_t = \rho_a \ln a_{t-1} + \sigma_{a,t-1}\varepsilon_{a,t},\ -1 < \rho_a < 1,\ \varepsilon_{a,t} \sim \mathbb{N}(0,1),$$

$$\ln \sigma_{a,t} = (1-\rho_{av})\ln \bar{\sigma}_a + \rho_{av}\ln \sigma_{a,t-1} + \sigma_{av}\varepsilon_{av,t},\ -1 < \rho_{av} < 1,\ \varepsilon_{av,t} \sim \mathbb{N}(0,1).$$

Following Gourio (2012) and Shen (2015), Θ_t is a capital quality shock that is determined by

$$\Theta_t = \mathbb{I}(e_t \geq e^*) + \theta\mathbb{I}(e_t < e^*),$$

where $\mathbb{I}(\cdot)$ is an indicator function that equals one when the condition is true and zero otherwise. When $e_t \geq e^*$, there is no capital quality loss, so $\Theta_t = 1$. When $e_t < e^*$, a disaster causes capital quality loss, so $\Theta_t = \theta < 1$. The likelihood of a disaster, e_t, evolves according to

$$\ln e_t = \rho_e \ln e_{t-1} + \sigma_{e,t-1}\varepsilon_{e,t},\ -1 < \rho_e < 1,\ \varepsilon_{e,t} \sim \mathbb{N}(0,1),$$

$$\ln \sigma_{e,t} = (1-\rho_{ev})\ln \bar{\sigma}_e + \rho_{ev}\ln \sigma_{e,t-1} + \sigma_{ev}\varepsilon_{ev,t},\ -1 < \rho_{ev} < 1,\ \varepsilon_{ev,t} \sim \mathbb{N}(0,1).$$

Table 1: DSGE model calibration at quarterly frequency

(a) Fixed parameters

Parameters	Value	Parameters	Value
Discount Factor (β)	0.995	Size of Disaster (θ)	0.95
Cost Share of Capital (α)	0.333	Disaster Risk Threshold (e^*)	0.97
Capital Depreciation Rate (δ)	0.025	Disaster Risk AC (ρ_e)	0.90
Risk Aversion (γ)	80	Technology Shock Mean ($\bar{a}$)	1
Intertemporal Elasticity (ψ)	1	Technology Shock SD ($\bar{\sigma}_a$)	0.007
Frisch Labor Supply Elasticity (η^λ)	2	Technology AC (ρ_a)	0.90

(b) Specification-specific parameters

	Model 1 ($\varepsilon_e, \varepsilon_a, \varepsilon_{ev}$)		Model 2 ($\varepsilon_e, \varepsilon_a, \varepsilon_{av}$)	
Parameters	Baseline	Quasi-Recursive	Baseline	Quasi-Recursive
Disaster Risk Shock SD ($\bar{\sigma}_e$)	0.0065	0.0045	0.0065	0.0065
Disaster Risk Vol. Shock AC (ρ_{ev})	0.90	0.90	–	–
Disaster Risk Vol. Shock SD (σ_{ev})	0.175	0.365	–	–
Technology Vol. Shock AC (ρ_{av})	–	–	0.90	0.90
Technology Vol. Shock SD (σ_{av})	–	–	0.0275	0.08

The online appendix derives the first-order conditions and defines the competitive equilibrium.

We follow Plante et al. (2018) and Bernstein et al. (2024) and define macroeconomic uncertainty as the conditional volatility of log output growth, which is given by

$$\mathcal{U}_t = \sqrt{E_t[(\ln(y_{t+1}/y_t) - E_t[\ln(y_{t+1}/y_t)])^2]}.$$

This definition is equivalent to the uncertainty surrounding the level of log output because y_t is known at time t and cancels from the definition of $\mathcal{U}_t$. Uncertainty can endogenously fluctuate due to the propagation of the level shocks and exogenously fluctuate due to the volatility shocks.

Table 1 summarizes the parameter values. All of the deep parameters are set to common values in the literature. We normalize the disaster risk threshold, e^*, to 0.97. The size of the disaster, θ, is set to 0.95, so 5% of the capital stock is lost when the disaster state occurs. We set the persistence and standard deviation of technology to match the autocorrelation of utilization-adjusted total factor productivity in the data.[5] We consider two alternative specifications of the DSGE model. Both include the disaster risk and technology levels shocks, but one includes a volatility shock to disaster risk (Model 1) while the other include a volatility shock to technology (Model 2). Within each model, we consider two specifications: in the baseline setting the direct effect of the uncertainty shock explains 50% of the variation in uncertainty, whereas in the quasi-recursive setting that percentage increases to 90%.[6] We set the shock standard deviations to achieve these targets, while matching the standard deviations of detrended output as closely as possible.

We solve the nonlinear model using the policy function iteration algorithm described in Richter et al. (2014), based on the theoretical work in Coleman (1991). Conditional on satisfying the equilibrium system, the algorithm minimizes the Euler equation errors on each node in the state space and computes the maximum change in the policy functions. It then iterates until the maximum change is below a specified tolerance. The online appendix describes the method in more detail.

3.2 SIMULATION RESULTS UNDER RECURSIVE ORDERINGS One of the assumptions in both recursive and nonrecursive VAR models of the transmission of uncertainty shocks is that there exists a second-moment (or volatility) shock in the underlying data generating process. However, one possible scenario is that uncertainty is fully endogenous in the data generating process, in which case fluctuations in uncertainty are entirely explained by the first moment shocks. In this case, recursive VAR models are invalid regardless of the ordering, because the uncertainty shock the VAR model seeks to identify does not exist. This concern also applies to VAR models based on nonstandard identification approaches in which uncertainty is allowed to be determined simultaneously with real activity (e.g., Carriero et al., 2021; Ludvigson et al., 2021).

Next suppose that there exists an exogenous uncertainty shock in the data generating process, along with level shocks driving uncertainty, as would be expected in practice. For each model

[5] We use the Hamilton (2018) filter with 4 lags and a delay of 8 quarters to detrend the data. Hodrick (2020) shows that this method is more accurate than a Hodrick and Prescott (1997) filter when log series are difference stationary.

[6] We use a total variance decomposition, based on the law of total variance, that accounts for both nonlinearities in the model and multiplicative interactions between level and volatility shocks. The decomposition distinguishes between the direct and indirect contributions of each shock to the variance of a particular variable in the model. See Bernstein et al. (2024) for additional details on how the decomposition works, including examples.

specification, we generate time series of log-level data of length $T = 1{,}000{,}000$. For such large T, this approach provides a close approximation to the asymptotic limit of the VAR impulse response functions. Using the simulated data, we estimate a VAR(4) model with intercept, where $\mathbf{y}_t = (y_t, i_t, \mathcal{U}_t)'$, since there are three structural shocks in the DSGE model. Given our focus, output and uncertainty are crucial to include in the VAR model. We choose investment as the third variable since it is an important driver of business cycles, but our results are robust to alternative choices.

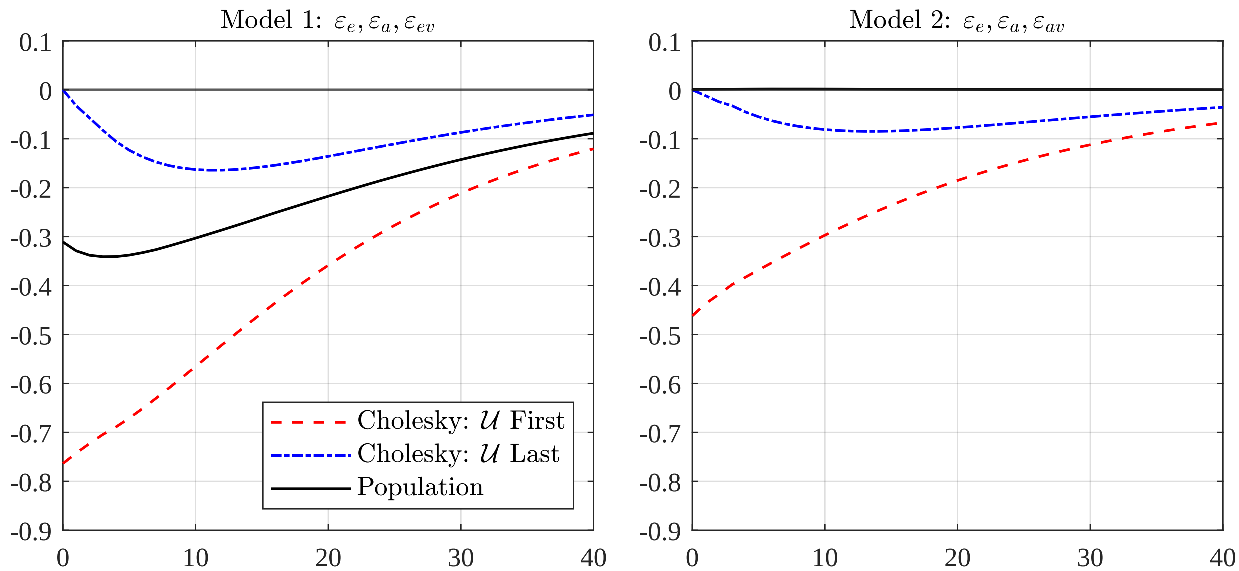
Figure 2a plots the responses of output to a 1 standard deviation positive uncertainty shock in the baseline calibration under alternative recursive orderings. In Model 1, the population response indicates that an uncertainty shock causes a 31 basis point drop in output on impact. The impact response in the recursive VAR is over-estimated when uncertainty is ordered first but under-estimated when uncertainty is ordered last. The reason is that the VAR shocks are linear combinations of the level and volatility shocks in the DSGE model.

One might conclude from this example that the responses from the two recursive VARs may still be used to bound the impact of uncertainty on output, but this result is not guaranteed. In Model 2, the population response shows that a positive uncertainty shock causes a small increase in output. However, the responses from both recursive VARs indicate a decline in output. The decline is particularly large when uncertainty is ordered first, in which case output falls on impact by 46 basis points. These results show that recursively identified VARs based on simulated data from calibrated macro models with time-varying endogenous uncertainty do not robustly bound the effects of uncertainty shocks, mirroring the conclusion in Section 2.

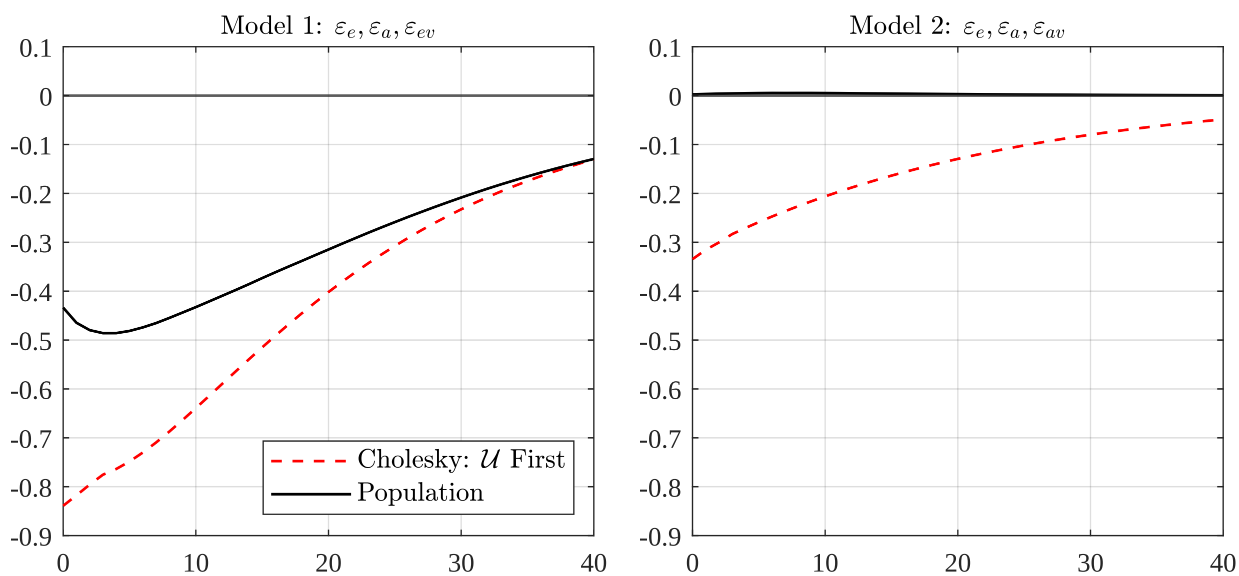
This raises the question of whether a recursive model with uncertainty ordered first would be more reliable when the degree of endogeneity is small. We do not consider the limiting case of the exogenous uncertainty shock explaining 100% of the variability of uncertainty. In general, level shocks in a nonlinear DSGE model will generate some time-varying endogenous uncertainty. Thus, achieving 100% exogeneity requires the level shocks in the DSGE model to be turned off. In that case, the DSGE model described in Section 3.1 no longer has a structural VAR representation, because there is only one structural shock in the model, so estimating a VAR model would not

Figure 2: Output responses to a 1 standard deviation positive uncertainty shock

(a) Baseline Calibration: 50% of uncertainty variation exogenous



(b) Quasi-Recursive Calibration: 90% of uncertainty variation exogenous



make econometric sense. However, we can make the degree of that endogeneity small. Figure 2b shows the impulse response functions in the quasi-recursive case when 90% of the variability in uncertainty is exogenous. Regardless of the choice of model, the recursive estimate does not come close to the population response. This example illustrates that the recursive estimator remains a poor approximation, even when the exogenous uncertainty shock plays a dominant role in driving aggregate uncertainty.[7]

The DSGE models our examples are based on focus on macroeconomic uncertainty. It may seem that the point we are making would be less relevant when focusing on the effects of financial uncertainty (e.g., Carriero and Volpicella, 2024; Ludvigson et al., 2021). It is plausible that financial uncertainty would be less endogenous than macroeconomic uncertainty, which may suggest that ordering financial uncertainty first in a recursive model would provide a good approximation. However, even financial uncertainty is clearly endogenous with respect to the macroeconomy. For example, banking crises are more likely to occur during economic downturns. We could have specified a DSGE model with financial uncertainty to make this point. Given that we have already shown that the recursive model with uncertainty ordered first remains inaccurate, even when macro uncertainty is largely exogenous, one would expect similar results for financial uncertainty.[8]

3.3 ALTERNATIVES TO RECURSIVE ORDERINGS

Our analysis highlights the importance of employing nonrecursive models of the transmission of uncertainty shocks that are consistent with a broad class of alternative data generating processes, including processes that allow uncertainty to be at least partially endogenous with respect to the macroeconomy. Examples of this strategy include Angelini et al. (2019), Benati (2023), Berger et al. (2020), Brianti (2023), Caggiano et al. (2021), Caggiano and Castelnuovo (2023), Caldara et al. (2016), Carriero et al. (2021), Furlanetto et al. (2019), Ludvigson et al. (2021), and Piffer and Podstawski (2018).

Some of these nonrecursive models have their own limitations, however. For example, as noted by Caggiano et al. (2021), it is difficult to identify uncertainty shocks based on sign restrictions on the impulse responses, since uncertainty and financial shocks often have the same qualitative effects on prices and quantities. Likewise, the penalty function method used in Caldara et al. (2016) to identify the effects of uncertainty shocks is problematic. As shown in Arias et al. (2018), this

[7] It should be noted that a nonlinear DSGE model does not in general have a linear VAR representation. Thus, even if the identification problem could be resolved, one would expect the structural VAR shocks to be contaminated by reduced-form model approximation error. This fact does not explain the failure of the recursive models in Figure 2. As shown in Section 3.3, the reduced-form approximation error must be small, because an alternative non-recursive estimator based on a linear reduced-form VAR model comes close to recovering the population responses.

[8] In related work, Berger et al. (2020) show that financial uncertainty, as measured by the VIX, is affected by both level and volatility shocks, invalidating any attempt to identify the uncertainty shock using a recursive ordering.

approach tends to imply unintended sign restrictions on the impulse responses not supported by extraneous evidence.

The performance of other nonrecursive identification aproaches, such as max share estimators and IV estimators, is less understood. Our goal in this section is to examine the extent to which these approaches improve upon ad hoc recursive orderings when uncertainty is partially endogenous with respect to the macroeconomy. We provide evidence that max share estimators are not suitable in these settings, but IV estimators are a promising alternative.

Max share estimators One alternative to recursive models of uncertainty shocks has been the use of max share estimators (e.g., Carriero and Volpicella, 2024).[9] Discussing this approach requires some additional notation for the VAR model. As before, we define the structural impact multiplier matrix as B_0^{-1} with $B_0^{-1}(B_0^{-1})' = \Sigma$. Let P denote the lower triangular Cholesky decomposition of Σ with the diagonal elements normalized to be positive, and let Q be a $K \times K$ orthogonal matrix. Since $Q'Q = QQ' = I_K$ and hence $(PQ)(PQ)' = PP' = \Sigma$, we can express the set of possible solutions for B_0^{-1} as PQ. Identification involves pinning down some or all columns of Q.

Let the reduced-form moving average representation of the VAR model be given by $\mathbf{y}_t = \Phi(L)\mathbf{u}_t$, where $\Phi(L) = I_K + \Phi_1 L + \Phi_2 L^2 + \cdots$, I_K is a K-dimensional identity matrix, and L is a lag operator. Then the h-step ahead forecast error is given by

$$\mathbf{y}_{t+h} - E_{t-1}\mathbf{y}_{t+h} = \sum_{\tau=0}^{h} \Phi_\tau PQ\mathbf{w}_{t+h-\tau},$$

where Φ_τ is the reduced-form matrix for the moving average coefficients, which may be constructed following Kilian and Lütkepohl (2017) with $\Phi_0 = I_K$. As a result, the share of the forecast error variance of variable i that is attributed to shock j at horizon h is given by

$$\Omega_{i,j}(h) = \frac{\sum_{\tau=0}^{h} \Phi_{i,\tau} P\gamma_j\gamma_j' P'\Phi_{i,\tau}'}{\sum_{\tau=0}^{h} \Phi_{i,\tau}\Sigma\Phi_{i,\tau}'},$$

where $\Phi_{i,\tau}$ is the ith row of the lag polynomial at lag τ and γ_j is the jth column of Q. A unique

[9] Max share estimators were popularized by Uhlig (2003, 2004), Barsky and Sims (2011), and Francis et al. (2014), and have been applied in a variety of settings.

estimate of the impact effect of structural shock j may be obtained by choosing the values of γ_j to maximize $\Omega_{i,j}(h)$. Assume without loss of generality that uncertainty is ordered first, output second, and some other variable third. Then the first column of the Q matrix, denoted by γ_1, is associated with the identified "uncertainty" shock and the value of γ_1 is determined by

$$\gamma_1 = \text{argmax } \Omega_{1,1}(h), \quad \Omega_{1,1}(h) \equiv \frac{\sum_{\tau=0}^{h} \Phi_{1,\tau} P \gamma_1 \gamma_1' P' \Phi_{1,\tau}'}{\sum_{\tau=0}^{h} \Phi_{1,\tau} \Sigma \Phi_{1,\tau}'}. \tag{6}$$

The standard max share estimator, defined in (6), only identifies one shock. Carriero and Volpicella (2024) generalize the max share approach by introducing an estimator that can simultaneously identify up to K structural shocks. Their objective function is given by

$$Q_{1:K}^{*} = \text{argmax} \sum_{i=1}^{K} \Omega_{i,i}(h_i), \quad \Omega_{i,i}(h_i) \equiv \frac{\sum_{\tau=0}^{h_i} \Phi_{i,\tau} P \gamma_i \gamma_i' P' \Phi_{i,\tau}'}{\sum_{\tau=0}^{h_i} \Phi_{i,\tau} \Sigma \Phi_{i,\tau}'}, \tag{7}$$

which is subject to a series of inequality constraints,

$$\Omega_{i,i}(h_i) \geq \Omega_{j,i}(h_i), \quad j = 1, \ldots, K, \quad \forall i \neq j,$$

and the constraint that $QQ' = I$. In other words, each shock i is identified so that its contribution to the variance of variable i is greater than its contribution to the variance of each of the other K variables in the VAR model. For example, the uncertainty shock must explain a greater share of the variance of uncertainty than it does for the variance of the other variables.

To operationalize these estimators, we need to select target horizons. For the standard max share estimator, we target uncertainty and set $h = 0$. For the generalized max share estimator, we follow Carriero and Volpicella (2024) and set $h_i = 0$ for each shock. In both cases, we use investment as the third variable in the VAR, as in the recursive model, but our qualitative results are robust to alternative choices.

Table 2 shows that the standard max share estimator is no more accurate than the recursive model whether uncertainty is ordered first or last. Intuitively, this estimator is bound to fail when uncertainty is endogenous because the identified shock is contaminated with level shocks. The generalized max share estimator of Carriero and Volpicella (2024) is more accurate than the stan-

Table 2: RMSE of the response of output to an uncertainty shock for alternative estimators

	Model 1 ($\varepsilon_e, \varepsilon_a, \varepsilon_{ev}$)		Model 2 ($\varepsilon_e, \varepsilon_a, \varepsilon_{av}$)	
VAR Estimator	Baseline	Quasi-Recursive	Baseline	Quasi-Recursive
Cholesky: $\mathcal{U}$ First	7.1	5.0	8.7	6.2
Cholesky: $\mathcal{U}$ Last	4.6	–	2.5	–
Standard Max Share	7.1	5.0	8.7	6.2
Generalized Max Share	0.7	2.4	5.1	3.6
Internal Instrument	0.7	2.1	0.1	0.1

Notes: VAR(4) with $T = 1{,}000{,}000$. RMSE is a sum over 40 quarters.

dard max share estimator, but not necessarily more accurate than the recursive estimators. The problem is again that in our population model, as in all models of endogenous uncertainty, the level shocks are important contributors to the variability of uncertainty. As a result, there is no single shock driving uncertainty, causing the procedure to conflate uncertainty and level shocks.

We also show the performance of the max share estimators in the quasi-recursive case. We continue to find that the standard max share estimator is no more accurate than the recursive estimator. The performance of the generalized max share estimator is case specific, with a higher root mean squared error (RMSE) than in the baseline calibration for Model 1, but a lower RMSE for Model 2. Overall, our results suggest that max share estimators do not work well in general.

Instrumental variable methods An alternative is to use instruments to identify uncertainty shocks, as pioneered in Carriero et al. (2015). A natural baseline is to treat the exogenous uncertainty in the DSGE model ($\ln \sigma_{e,t}$ in Model 1 and $\ln \sigma_{a,t}$ in Model 2) as an instrument for the observed uncertainty ($\mathcal{U}_t$). If the IV strategy does not work in this setting, it will not work more generally. After showing that this IV approach accommodates the endogeneity of observed uncertainty, we then contaminate this instrument with varying degrees of additive Gaussian measurement error and show that similar results hold more generally.

To implement this estimator, we augment the original VAR model with the instrumental variable, resulting in a block recursive VAR model with the instrument ordered first, uncertainty second, and output last. The IV estimator can be constructed from a Cholesky decomposition. This approach is equivalent to using $\ln \sigma_{a,t}$ or $\ln \sigma_{e,t}$ as an internal instrument. As shown in Plagborg-Møller and Wolf (2021), the advantage of this strategy is that it yields valid impulse response estimates even if the shock of interest is non-invertible. In contrast, the proxy VAR approach of using $\ln \sigma_{a,t}$ or $\ln \sigma_{e,t}$ as an external instrument is invalid in that case.

The bottom row of Table 2 reports the RMSE of the IV estimator. When there is no measurement error, the IV estimator generates dramatic improvements in accuracy in both models. In the baseline calibration, the RMSE of the IV estimator is 0.1 and 0.7 compared to 7.1 and 8.7 for the Cholesky model with uncertainty ordered first and 2.5 and 4.6 for the Cholesky model with uncertainty ordered last. The IV estimator also generates improvements under the quasi-recursive calibration, particularly in Model 2.

In practice, uncertainty can only be measured with error. We examine this situation by replacing the instrument $\ln \sigma_{i,t}$ with $\ln \sigma^n_{i,t} \equiv \ln \sigma_{i,t} + \sigma_n \epsilon_{n,t}$ for $i \in \{a, e\}$, where $\epsilon_{n,t} \sim \mathbb{N}(0, 1)$. We find that the IV estimator continues to be more accurate than the recursive estimators, even when 50% of the variability in the instrument is measurement error. This evidence suggests that the internal IV estimator is a promising alternative to recursive estimators, when suitable instruments are available.[10]

3.4 USING DSGE MODELS TO DEFEND PARTICULAR RECURSIVE ORDERINGS

We made the case that recursive VAR models are unable to recover the population response even asymptotically when uncertainty is partially endogenous. This result does not hold universally, however. There are alternative DSGE models for which recursively identified VAR models can approximately recover the population response. For example, Basu and Bundick (2017) show that a VAR model with uncertainty ordered first can approximately recover the population response in their DSGE model.[11] Bernstein et al. (2024) provide an example in which uncertainty is ordered last.

[10] An alternative approach would have been to use the innovation to exogenous uncertainty ($\varepsilon_{e,t}$ or $\varepsilon_{a,t}$) as an instrument rather than the exogenous uncertainty variable. With and without measurement error, using the shock as an IV also systematically outperforms the recursive model with endogenous uncertainty ordered first or last, but it is systematically less accurate than the IV estimator based on log levels.

[11] Typically, generating nontrivial real effects from a volatility shock requires meaningful nonlinearity, breaking the recursive structure. A counterexample is Basu and Bundick (2017), who adopt a standard New Keynesian model with uncertainty modeled as a volatility shock to Epstein and Zin (1991) preferences. The variation in uncertainty in their

It may seem that evidence that a particular recursive ordering yields responses that are similar to the population responses would be enough to defend the recursive approach. This is not the case. There is no doubt that it is possible to write down a DSGE model in which uncertainty shocks are the main driver of the uncertainty variable on impact, allowing recursive models with the uncertainty measure ordered first to approximately recover the population responses. Likewise, it is possible to write down a model in which uncertainty is almost entirely endogenous, in which case a VAR model with uncertainty ordered last may come close to recovering the population response. This does not mean these specific recursive VAR models are justified when working with actual data, however, since any DSGE model is merely one among many possible DSGE models and need not be a good approximation to the actual data generating process.

To make the case for a particular recursive ordering one would have to show not only that there exists some empirically plausible DSGE model for which the proposed recursive ordering provides a good approximation, but also that this ordering provides a good approximation for all other empirically plausible DSGE models. Such a result is unlikely to hold, given the numerous economically plausible mechanisms that render uncertainty at least partially endogenous.

3.5 RECURSIVE IDENTIFICATION IN OTHER CONTEXTS

Our analysis does not imply that there is no role for recursive models in applied work more generally. There are several settings in which a specific recursive ordering may be justified, depending on the economic context (see Kilian and Lütkepohl, 2017). For example, as already illustrated, block recursive models arise naturally when incorporating exogenous instruments as internal instruments in a VAR model (see Plagborg-Møller and Wolf, 2021). A common source of such instruments in empirical work has been shifts in market price expectations around policy shifts (e.g., Gertler and Karadi, 2015; Kilian, 2023).

Block recursive structures also arise when one of the VAR model variables can be shown to be predetermined with respect to macroeconomic aggregates. For example, Kilian and Vega (2011) provide empirical evidence based on the responses of daily oil and gasoline prices to U.S. macroe-

model is almost entirely exogenous, which motivates their approach of ordering uncertainty first in a recursive VAR. However, this result is highly dependent on how the preference shock enters the utility function and is not robust to small changes in the parameterization or the preference specification (see de Groot et al., 2018).

conomic news that these prices are predetermined with respect to the macroeconomy in that they do not respond to macroeconomic news within twenty business days of the news. This supports the common practice of ordering energy prices first in monthly VAR models of the transmission of energy price shocks to the macroeconomy.

Similarly, Davis and Kilian (2011) rely on the fact that monthly excise gasoline taxes do not respond instantaneously to the state of the economy because lawmakers move at a slow pace. This institutional knowledge helps defend a block recursive VAR model of the link from excise gasoline taxes to gasoline consumption.

Another common identifying assumption in empirical work is that there is no feedback from a small open economy to the rest of the world. This market structure has been used, for example, to motivate treating U.S. interest rates in VAR models as predetermined with respect to macroeconomic aggregates in small open economies such as Canada (e.g., Cushman and Zha, 1997).

In contrast, Inoue et al. (2009) exploit differences in the timing of data releases to motivate treating inflation expectations in the Survey of Professional Forecasters as predetermined with respect to macroeconomic data released later in the month. A similar argument is invoked by Sims (1998) who suggests that monetary policymakers react immediately only to variables that they can observe without a delay. The latter argument has also been used in the literature on uncertainty shocks. For example, Leduc and Liu (2016) and Istrefi and Mouabbi (2018) argue that at the time that household or professional survey expectations about economic outcomes are formed, the current realizations of the macroeconomic aggregates are unknown, suggesting a natural recursive identification structure for identifying uncertainty shocks.

There are some concerns with survey-based identification strategies, however. First, most surveys do not elicit responses about agents' uncertainty about economic outcomes. The way uncertainty is typically measured in these studies is to look at the dispersion of forecasts across agents. In general, this dispersion does not correspond to the subjective uncertainty of any given agent, so this approach does not really address the same question as our paper. Second, even if a survey reported subjective uncertainty, the fact that agents do not observe current macroeconomic data is not enough for the identification to work. We also must assume that agents do not have access to forward-looking information correlated with the eventual macroeconomic outcomes. Examples include daily commodity prices that could help predict inflation or daily stock prices that could help predict real output, speeches by Federal Reserve officials foreshadowing the path of interest rates, or simply the news about a pending financial or banking crisis.

Finally, in some cases, it is possible to appeal to theoretical results to justify a block recursive ordering. For example, Anderson et al. (2018) show that oil production does not respond to oil price shocks in the short run. The unresponsiveness of oil production in the short run implicitly reflects high adjustment costs. This motivates setting the impact response of oil production to oil demand shocks in a monthly VAR model of the global oil market to zero, resulting in a block recursive VAR structure with global oil production ordered first. This ordering may also be defended based on conventional estimates of the one-month price elasticity of U.S. oil supply, which are essentially zero (see Kilian, 2022). In related work, Kilian (2009) exploits this fact together with the institutional features of the shipping data used in constructing his index of global real economic activity to derive a fully recursive model of the global oil market (for further discussion see Kilian and Zhou, 2018).

These examples illustrate that there are VAR applications in which a specific recursive structure is justified. Adopting such a structure requires an economic justification, however, that obviates the need to consider alternative recursive models. Thus, the existence of this literature does not detract from our point that there is no justification for the common practice of assessing the robustness of response estimates to alternative orderings (or using alternative recursive orderings to bound the true responses), when no credible justification for a specific recursive ordering is available.

3.6 RELATED APPLICATIONS OF ALTERNATIVE RECURSIVE ORDERINGS While we discussed alternative recursive orderings in the context of one of the leading applications of recursively identified models in the literature, our analysis transcends this illustrative example and applies more generally to other VAR applications as well. The key feature that all these applications have in common is that the variables of interest are simultaneously determined.

One example is the literature on modeling the link between bond yields (and, more generally, financial conditions) and real activity (e.g., Gilchrist et al., 2009). Similar models have been widely used to inform policy debates. For instance, an influential study by Goldman Sachs uses a Cholesky decomposition to identify structural shocks to their financial conditions index (see Abecasis, 2023; Hatzius and Stehn, 2018). The authors note that they "estimate all six possible orderings and average the results [...] as it is difficult to take a strong view on the ordering of the variables in the model." Another example is the relationship between wage inflation and price inflation, which has recently received much attention from policymakers, as the economy recovered from the COVID-19 pandemic (e.g., Chin and Lin, 2023; Peneva and Rudd, 2017). A third example is the literature on semi-structural VAR models of monetary policy. One prominent example of this approach is Eichenbaum and Evans (1995, p. 981) who report responses based on alternative recursive orderings of the policy instrument and macroeconomic aggregates. Similar issues arise when modeling the link between the policy rate and forward-looking indicators, such as commodity prices or stock prices (e.g., Hanson, 2004; Sims, 1992). Finally, alternative recursive orderings are also used in behavioral finance. For example, Cesa-Bianchi and Miranda-Agrippino (2024) explore alternative recursive orderings between firm earnings forecasts, forecast errors, and forecast revisions. Not only are all recursive models inappropriate in these contexts, but our analysis highlights that exploring the sensitivity of the response estimates to alternative orderings is misleading because it ignores the inherent simultaneity of the data.

4 CONCLUSION

The common practice of reporting estimates from alternative recursive identification schemes and verifying the robustness of the conclusions is misleading. Robustness within the class of recursive models does not ensure the validity of the responses when the population model is non-recursive. Nor is the practice of bounding response estimates based on alternative orderings justified. For example, there is no support for the belief that recursive VARs are useful for gauging whether uncertainty innovations foreshadow weaker or stronger macroeconomic performance conditional on other variables. Analogous identification issues also arise in empirical analysis based on local projections. While we focused on the link between real GDP and uncertainty, similar problems arise more generally when modeling variables that are simultaneously determined, such as the relationship between wage inflation and price inflation or between financial conditions and real output. Simulation evidence suggests that the underlying identification challenge can be addressed using an instrumental variables estimator.

REFERENCES

ABECASIS, M. (2023): "The Growth Drag from Tighter Lending Standards," Goldman Sachs U.S. Daily.

AHIR, H., N. BLOOM, AND D. FURCERI (2022): "The World Uncertainty Index," NBER Working Paper 29763.

ALTIG, D., S. BAKER, J. M. BARRERO, N. BLOOM, P. BUNN, S. CHEN, S. J. DAVIS, J. LEATHER, B. MEYER, E. MIHAYLOV, P. MIZEN, AND N. PARKER (2020): "Economic Uncertainty Before and During the COVID-19 Pandemic," *Journal of Public Economics*, 191, 104274.

ANDERSON, S. T., R. KELLOGG, AND S. W. SALANT (2018): "Hotelling under Pressure," *Journal of Political Economy*, 126, 984–1026.

ANGELINI, G., E. BACCHIOCCHI, G. CAGGIANO, AND L. FANELLI (2019): "Uncertainty Across Volatility Regimes," *Journal of Applied Econometrics*, 34, 437–455.

ARIAS, J. E., J. F. RUBIO-RAMÍREZ, AND D. F. WAGGONER (2018): "Inference Based on Structural Vector Autoregressions Identified With Sign and Zero Restrictions: Theory and Applications," *Econometrica*, 86, 685–720.

ATKINSON, T., M. PLANTE, A. W. RICHTER, AND N. A. THROCKMORTON (2022): "Complementarity and Macroeconomic Uncertainty," *Review of Economic Dynamics*, 44, 225–243.

BACHMANN, R., S. ELSTNER, AND E. SIMS (2013): "Uncertainty and Economic Activity: Evidence from Business Survey Data," *American Economic Journal: Macroeconomics*, 5, 217–49.

BACHMANN, R. AND G. MOSCARINI (2012): "Business Cycles and Endogenous Uncertainty," Manuscript, RWTH Aachen University.

BAKER, S. R., N. BLOOM, AND S. J. DAVIS (2016): "Measuring Economic Policy Uncertainty," *Quarterly Journal of Economics*, 131, 1593–1636.

BARSKY, R. B. AND E. R. SIMS (2011): "News Shocks and Business Cycles," *Journal of Monetary Economics*, 58, 273–289.

BASU, S. AND B. BUNDICK (2017): "Uncertainty Shocks in a Model of Effective Demand," *Econometrica*, 85, 937–958.

BEKAERT, G., M. HOEROVA, AND M. LO DUCA (2013): "Risk, Uncertainty and Monetary Policy," *Journal of Monetary Economics*, 60, 771–788.

BENATI, L. (2023): "Economic Policy Uncertainty and the Great Recession," Manuscript, University of Bern.

BENHABIB, J., X. LIU, AND P. WANG (2016): "Endogenous Information Acquisition and Countercyclical Uncertainty," *Journal of Economic Theory*, 165, 601–642.

BERAJA, M. AND C. K. WOLF (2021): "Demand Composition and the Strength of Recoveries," NBER Working Paper 29304.

BERGER, D., I. DEW-BECKER, AND S. GIGLIO (2020): "Uncertainty Shocks as Second-Moment News Shocks," *Review of Economic Studies*, 87, 40–76.

BERNANKE, B. S. (1986): "Alternative Explanations of the Money-Income Correlation," *Carnegie-Rochester Conference Series on Public Policy*, 25, 49–99.

BERNSTEIN, J., M. PLANTE, A. W. RICHTER, AND N. A. THROCKMORTON (2024): "A Simple Explanation of Countercyclical Uncertainty," *American Economic Journal: Macroeconomics*, forthcoming.

BIANCHI, F., C. L. ILUT, AND M. SCHNEIDER (2018): "Uncertainty Shocks, Asset Supply and Pricing over the Business Cycle," *Review of Economic Studies*, 85, 810–854.

BLOOM, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77, 623–685.

——— (2014): "Fluctuations in Uncertainty," *Journal of Economic Perspectives*, 28, 153–176.

BORN, B. AND J. PFEIFER (2021): "Uncertainty Driven Business Cycles: Assessing the Markup Channel," *Quantitative Economics*, 12, 587–623.

BRAUN, P. A. AND S. MITTNIK (1993): "Misspecifications in Vector Autoregressions and Their Effects on Impulse Responses and Variance Decompositions," *Journal of Econometrics*, 59, 319–341.

BRIANTI, M. (2023): "Financial Shocks, Uncertainty Shocks, and Monetary Policy Trade-Offs," Manuscript, University of Alberta.

CACCIATORE, M. AND F. RAVENNA (2021): "Uncertainty, Wages, and the Business Cycle," *Economic Journal*, 131, 2797–2823.

CAGGIANO, G. AND E. CASTELNUOVO (2023): "Global Financial Uncertainty," *Journal of Applied Econometrics*, 38, 432–449.

CAGGIANO, G., E. CASTELNUOVO, S. DELRIO, AND R. KIMA (2021): "Financial Uncertainty and Real Activity: The Good, the Bad, and the Ugly," *European Economic Review*, 136.

CAGGIANO, G., E. CASTELNUOVO, AND N. GROSHENNY (2014): "Uncertainty Shocks and Unemployment Dynamics in U.S. Recessions," *Journal of Monetary Economics*, 67, 78–92.

CALDARA, D., C. FUENTES-ALBERO, S. GILCHRIST, AND E. ZAKRAJŠEK (2016): "The Macroeconomic Impact of Financial and Uncertainty Shocks," *European Economic Review*, 88, 185–207.

CALDARA, D. AND M. IACOVIELLO (2022): "Measuring Geopolitical Risk," *American Economic Review*, 112, 1194–1225.

CARRIERO, A., T. E. CLARK, AND M. MARCELLINO (2021): "Using Time-Varying Volatility for Identification in Vector Autoregressions: An Application to Endogenous Uncertainty," *Journal of Econometrics*, 225, 47–73.

CARRIERO, A., H. MUMTAZ, K. THEODORIDIS, AND A. THEOPHILOPOULOU (2015): "The Impact of Uncertainty Shocks under Measurement Error: A Proxy SVAR Approach," *Journal of Money, Credit and Banking*, 47, 1223–1238.

CARRIERO, A. AND A. VOLPICELLA (2024): "Max Share Identification of Multiple Shocks: An Application to Uncertainty and Financial Conditions," *Journal of Business & Economic Statistics*, forthcoming.

CESA-BIANCHI, A. AND S. MIRANDA-AGRIPPINO (2024): "On the Aggregate Consequences of Belief Distortions," Manuscript, Bank of England.

CHEN, H. AND P. TILLMANN (2021): "Monetary Policy Uncertainty in China," *Journal of International Money and Finance*, 110.

CHIN, M. AND L. LIN (2023): "The Pass-through of Wages to Consumer Prices in the COVID-19 Pandemic," IMF Working Paper 23/233.

COLEMAN, II, W. J. (1991): "Equilibrium in a Production Economy with an Income Tax," *Econometrica*, 59, 1091–1104.

COOLEY, T. F. AND S. F. LEROY (1985): "Atheoretical Macroeconometrics: A Critique," *Journal of Monetary Economics*, 16, 283–308.

CUSHMAN, D. O. AND T. ZHA (1997): "Identifying Monetary Policy in a Small Open Economy Under Flexible Exchange Rates," *Journal of Monetary Economics*, 39, 433–448.

DAVIS, L. W. AND L. KILIAN (2011): "Estimating the Effect of a Gasoline Tax on Carbon Emissions," *Journal of Applied Econometrics*, 26, 1187–1214.

DE GROOT, O., A. RICHTER, AND N. THROCKMORTON (2018): "Uncertainty shocks in a model of effective demand: Comment," *Econometrica*, 86, 1513–1526.

DECKER, R. A., P. N. D'ERASMO, AND H. M. BOEDO (2016): "Market Exposure and Endogenous Firm Volatility over the Business Cycle," *American Economic Journal: Macroeconomics*, 8, 148–198.

EICHENBAUM, M. AND C. L. EVANS (1995): "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates," *Quarterly Journal of Economics*, 110, 975–1009.

EPSTEIN, L. G. AND S. E. ZIN (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–69.

——— (1991): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis," *Journal of Political Economy*, 99, 263–86.

FAJGELBAUM, P., M. TASCHEREAU-DUMOUCHEL, AND E. SCHAAL (2017): "Uncertainty Traps," *Quarterly Journal of Economics*, 132, 1641–1692.

FERNÁNDEZ-VILLAVERDE, J., P. GUERRÓN-QUINTANA, K. KUESTER, AND J. F. RUBIO-RAMÍREZ (2015): "Fiscal Volatility Shocks and Economic Activity," *American Economic Review*, 105, 3352–84.

FRANCIS, N., M. T. OWYANG, J. E. ROUSH, AND R. DICECIO (2014): "A Flexible Finite-Horizon Alternative to Long-Run Restrictions with an Application to Technology Shocks," *Review of Economics and Statistics*, 96, 638–647.

FURLANETTO, F., F. RAVAZZOLO, AND S. SARFERAZ (2019): "Identification of Financial Factors in Economic Fluctuations," *The Economic Journal*, 129, 311–337.

GAO, L., S. HITZEMANN, I. SHALIASTOVICH, AND L. XU (2022): "Oil Volatility Risk," *Journal of Financial Economics*, 144, 456–491.

GERTLER, M. AND P. KARADI (2015): "Monetary Policy Surprises, Credit Costs, and Economic Activity," *American Economic Journal: Macroeconomics*, 7, 44–76.

GILCHRIST, S., V. YANKOV, AND E. ZAKRAJŠEK (2009): "Credit Market Shocks and Economic Fluctuations: Evidence from Corporate Bond and Stock Markets," *Journal of Monetary Economics*, 56, 471–493.

GOURIO, F. (2012): "Disaster Risk and Business Cycles," *American Economic Review*, 102, 2734–2766.

GRIMME, C. (2023): "Uncertainty and the Cost of Bank versus Bond Finance," *Journal of Money, Credit and Banking*, 55, 143–169.

HAMILTON, J. D. (2018): "Why You Should Never Use the Hodrick-Prescott Filter," *Review of Economics and Statistics*, 100, 831–843.

HANSON, M. S. (2004): "The "Price Puzzle" Reconsidered," *Journal of Monetary Economics*, 51, 1385–1413.

HATZIUS, J. AND S. J. STEHN (2018): "The Case for a Financial Conditions Index," Goldman Sachs Global Economics Paper.

HODRICK, R. J. (2020): "An Exploration of Trend-Cycle Decomposition Methodologies in Simulated Data," NBER Working Paper 26750.

HODRICK, R. J. AND E. C. PRESCOTT (1997): "Postwar U.S. Business Cycles: An Empirical Investigation," *Journal of Money, Credit and Banking*, 29, 1–16.

ILUT, C., M. KEHRIG, AND M. SCHNEIDER (2018): "Slow to Hire, Quick to Fire: Employment Dynamics with Asymmetric Responses to News," *Journal of Political Economy*, 126, 2011–2071.

ILUT, C. L. AND M. SCHNEIDER (2014): "Ambiguous Business Cycles," *American Economic Review*, 104, 2368–2399.

INOUE, A., L. KILIAN, AND F. B. KIRAZ (2009): "Do Actions Speak Louder Than Words? Household Expectations of Inflation Based on Micro Consumption Data," *Journal of Money, Credit and Banking*, 41, 1331–1363.

ISTREFI, K. AND S. MOUABBI (2018): "Subjective Interest Rate Uncertainty and the Macroeconomy: A Cross-Country Analysis," *Journal of International Money and Finance*, 88, 296–313.

JURADO, K., S. C. LUDVIGSON, AND S. NG (2015): "Measuring Uncertainty," *American Economic Review*, 105, 1177–1216.

KILIAN, L. (2009): "Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market," *American Economic Review*, 99, 1053–69.

——— (2022): "Understanding the Estimation of Oil Demand and Oil Supply Elasticities," *Energy Economics*, 107, 105844.

——— (2023): "How to Construct Monthly VAR Proxies Based on Daily Futures Market Surprises," Federal Reserve Bank of Dallas Working Paper 2310.

KILIAN, L. AND H. LÜTKEPOHL (2017): *Structural Vector Autoregressive Analysis*, Cambridge University Press, New York.

KILIAN, L. AND C. VEGA (2011): "Do Energy Prices Respond to U.S. Macroeconomic News? A Test of the Hypothesis of Predetermined Energy Prices," *The Review of Economics and Statistics*, 93, 660–671.

KILIAN, L. AND X. ZHOU (2018): "Modeling Fluctuations in the Global Demand for Commodities," *Journal of International Money and Finance*, 88, 54–78.

LARSEN, V. H. (2021): "Components of Uncertainty," *International Economic Review*, 62, 769–788.

LEAMER, E. E. (1985): "Vector Autoregressions for Causal Inference?" *Carnegie-Rochester Conference Series on Public Policy*, 22, 255–304.

LEDUC, S. AND Z. LIU (2016): "Uncertainty Shocks are Aggregate Demand Shocks," *Journal of Monetary Economics*, 82, 20–35.

LONDONO, J. M., S. MA, AND B. A. WILSON (2021): "The Global Transmission of Real Economic Uncertainty," International Finance Discussion Paper 1317.

LUDVIGSON, S. C., S. MA, AND S. NG (2021): "Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?" *American Economic Journal: Marcoeconomics*, 13, 369–410.

PENEVA, E. V. AND J. B. RUDD (2017): "The Passthrough of Labor Costs to Price Inflation," *Journal of Money, Credit and Banking*, 49, 1777–1802.

PIFFER, M. AND M. PODSTAWSKI (2018): "Identifying Uncertainty Shocks Using the Price of Gold," *Economic Journal*, 128, 3266–3284.

PLAGBORG-MØLLER, M. AND C. K. WOLF (2021): "Local Projections and VARs Estimate the Same Impulse Responses," *Econometrica*, 89, 955–980.

PLANTE, M., A. W. RICHTER, AND N. A. THROCKMORTON (2018): "The Zero Lower Bound and Endogenous Uncertainty," *Economic Journal*, 128, 1730–1757.

RICHTER, A. W., N. A. THROCKMORTON, AND T. B. WALKER (2014): "Accuracy, Speed and Robustness of Policy Function Iteration," *Computational Economics*, 44, 445–476.

SAIJO, H. (2017): "The Uncertainty Multiplier and Business Cycles," *Journal of Economic Dynamics and Control*, 78, 1–25.

SHEN, W. (2015): "News, Disaster Risk, and Time-Varying Uncertainty," *Journal of Economic Dynamics and Control*, 51, 459–479.

SIMS, C. A. (1992): "Interpreting the Macroeconomic Time Series Facts : The Effects of Monetary Policy," *European Economic Review*, 36, 975–1000.

——— (1998): "Comment on Glenn Rudebusch's "Do Measures of Monetary Policy in a VAR Make Sense?"," *International Economic Review*, 39, 933–941.

ŚMIECH, S., M. PAPIEŻ, M. RUBASZEK, AND M. SNARSKA (2021): "The Role of Oil Price Uncertainty Shocks on Oil-Exporting Countries," *Energy Economics*, 93.

STRAUB, L. AND R. ULBRICHT (2019): "Endogenous Second Moments: A Unified Approach to Fluctuations in Risk, Dispersion, and Uncertainty," *Journal of Economic Theory*, 183, 625–660.

——— (2023): "Endogenous Uncertainty and Credit Crunches," *Review of Economic Studies*, forthcoming.

THEOPHILOPOULOU, A. (2022): "The Impact of Macroeconomic Uncertainty on Inequality: An Empirical Study for the United Kingdom," *Journal of Money, Credit and Banking*, 54, 859–884.

UHLIG, H. (2003): "What moves real GNP?" Working paper, Humboldt University.

——— (2004): "Do Technology Shocks Lead to a Fall in Total Hours Worked?" *Journal of the European Economic Association*, 2, 361–371.

VAN NIEUWERBURGH, S. AND L. VELDKAMP (2006): "Learning Asymmetries in Real Business Cycles," *Journal of Monetary Economics*, 53, 753–772.

WATSON, M. W. (1994): "Vector Autoregressions and Cointegration," in *Handbook of Econometrics*, ed. by R. F. Engle and D. L. McFadden, Elsevier, vol. 4, chap. 47, 2843–2915.