THE ZERO LOWER BOUND: FREQUENCY, DURATION, AND NUMERICAL CONVERGENCE

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Introduction

- Popular monetary policy rule due to Taylor (1993)

$$\hat{r}_t = \phi\hat{\pi}_t + \varepsilon_t, \quad \varepsilon_t \text{ bounded support}$$

- Taylor principle requires $\phi > 1$ (active monetary policy)
  - Necessary and sufficient for unique bounded equilibrium
- Three key assumptions
  1. Fiscal policy is passive
  2. Policy parameters are fixed
  3. Zero lower bound (ZLB) never binds
- Leeper (1991) relaxes the first assumption and Davig and Leeper (2007) relaxes the second assumption
- This paper relaxes the third assumption

Main Findings

- We adopt a textbook New Keynesian model with two alternative stochastic processes:
  1. 2-state Markov process governing monetary policy
  2. Persistent discount factor or technology shocks
- Convergence is *not* guaranteed even if the Taylor principle is satisfied when the ZLB does not bind.
- The boundary of the convergence region imposes a clear tradeoff between the expected frequency and average duration of ZLB events
  - Household can expect frequent—but brief—ZLB events or infrequent—but prolonged—ZLB events
  - Parameters of the stochastic process affect convergence

A Little Background

- Davig and Leeper (2007): Fisherian Economy

$$\phi(s_t)\pi_t = E_t\pi_{t+1} + \nu_t, \quad \nu \sim \mathrm{AR}(1)$$

$p_{ij} = \Pr[s_t = j | s_{t-1} = i]$ and $\phi(s_t = j) = \phi_j$, $s_t \in \{1, 2\}$

- Integration over s_t

$$E[\pi_{t+1} | s_t = i, \Omega_t^{-s}] = p_{i1}E[\pi_{1t+1} | \Omega_t^{-s}] + p_{i2}E[\pi_{2t+1} | \Omega_t^{-s}],$$

where $\Omega_t^{-s} = \{\nu_t, \nu_{t-1}, \ldots, s_{t-1}, s_{t-2}, \ldots\}$

- Define $\pi_{jt} = \pi_t(s_t = j, \nu_t)$. The system is

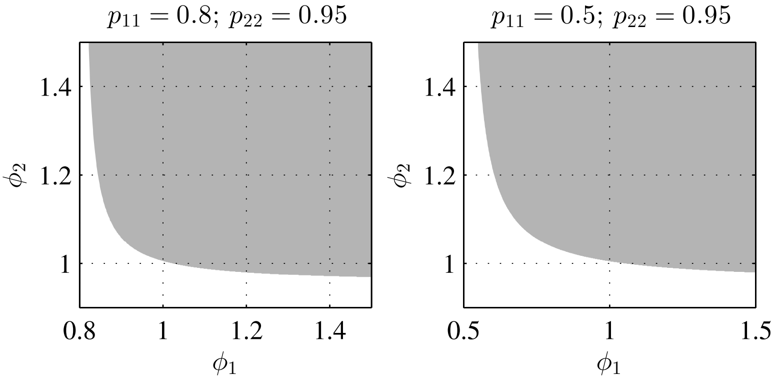
$$\begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix} \begin{bmatrix} \pi_{1t} \\ \pi_{2t} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} E_t\pi_{1t+1} \\ E_t\pi_{2t+1} \end{bmatrix} + \begin{bmatrix} \nu_t \\ \nu_t \end{bmatrix}$$

Determinacy: Fisherian Economy

- The existence of a unique bounded MSV solution requires

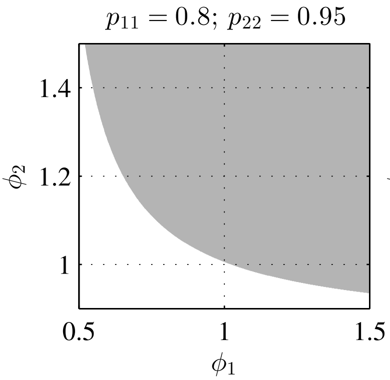
$$p_{11}(1-\phi_2) + p_{22}(1-\phi_1) + \phi_1\phi_2 > 1 \quad \text{(LRTP)}$$

- Example determinacy/convergence regions



Determinacy: NK Economy

- Example determinacy regions



- ZLB is similar to DL with $\phi_1 = 0$ and $\phi_2 > 1$, but with a truncated distribution on the nominal interest rate

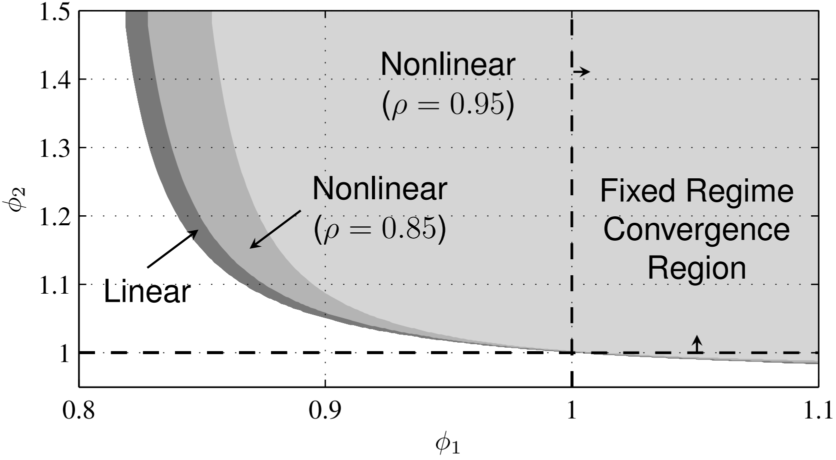
Nonlinear Fisherian Economy

A second-order approximation of the Euler equation around the deterministic steady state implies

$$\hat{r}_t + \underbrace{(\hat{r}_t - E_t[\hat{\pi}_{t+1}] + E_t[\hat{\beta}_{t+1}])^2}_{=0 \text{ (First Order)}} =$$

$$E_t[\hat{\pi}_{t+1}] - E_t[\hat{\beta}_{t+1}] - \underbrace{(E_t[(\hat{\pi}_{t+1} - \hat{\beta}_{t+1})^2] - (E_t[\hat{\pi}_{t+1} - \hat{\beta}_{t+1}])^2)}_{=0 \text{ (First Order, Jensen's Inequality)}}$$

Nonlinear Fisherian Economy



Numerical Procedure

- We compute global nonlinear solutions to each setup using policy function iteration on a dense grid
  - Linear interpolation and Gauss-Hermite quadrature
  - Duration of ZLB events is stochastic
  - Expectational effects of hitting and leaving ZLB
- Algorithm is *non-convergent* whenever
  - a policy function continually drifts from steady state
  - the iteration step (max distance between policy function values on successive iterations) diverges for $100+$ iterations
- Algorithm is *convergent* whenever
  - the iteration step is less than 10^{-13} for $10+$ iterations

Numerical Procedure

- Our algorithm yields the same determinacy regions Davig and Leeper analytically derive in their Fisherian economy and New Keynesian economy
  - When the LRTP is satisfied (not satisfied), our algorithm converges (diverges)
- Within the class of MSV solutions, there is a link between the convergent solution and determinate equilibrium
  - Non-MSV solutions with fundamental or non-fundamental components may still exist
  - Finding locally unique MSV solutions with a ZLB is helpful since most research is based on MSV solutions.
- Not a proof, but it provides confidence that our algorithm accurately captures MSV solutions

Literature

- Linearized models with a singular ZLB event
  - Eggertsson and Woodford (2003), Christiano (2004), Braun and Waki (2006), Eggertsson (2010, 2011), Erceg and Linde (2010), Christiano et al. (2011), Gertler and Karadi (2011), and many others
- Nonlinear models with recurring ZLB events
  - Judd et al. (2011), Fernández Villaverde et al. (2012), Gust et al. (2012), Basu and Bundick (2012), Mertens and Ravn (2013), Aruoba and Schorfheide (2013), Gavin et al. (2014)
- Determinacy in Markov-switching models
  - Davig and Leeper (2007), Farmer et al. (2009,2010), Cho (2013), Barthélemy and Marx (2013)
- Determinacy in models with a ZLB constraint
  - Benhabib et al (2001a), Alstadheim and Henderson (2006)

Key Model Features

- Representative Household
  - Values consumption and leisure with preferences

$$E_0 \sum_{t=0}^{\infty} \widetilde{\beta}_t \{\log(c_t) - \chi n_t^{1+\eta}/(1+\eta)\}$$

  - Cashless economy and bonds are in zero net supply
  - No capital accumulation
- Intermediate and final goods firms
  - Monopolistically competitive intermediate firms produce differentiated inputs
  - Rotemberg (1982) quadratic costs to adjusting prices
  - A competitive final goods firm combines the intermediate inputs to produce the consumption good

Exogenous ZLB Events

- The monetary authority follows

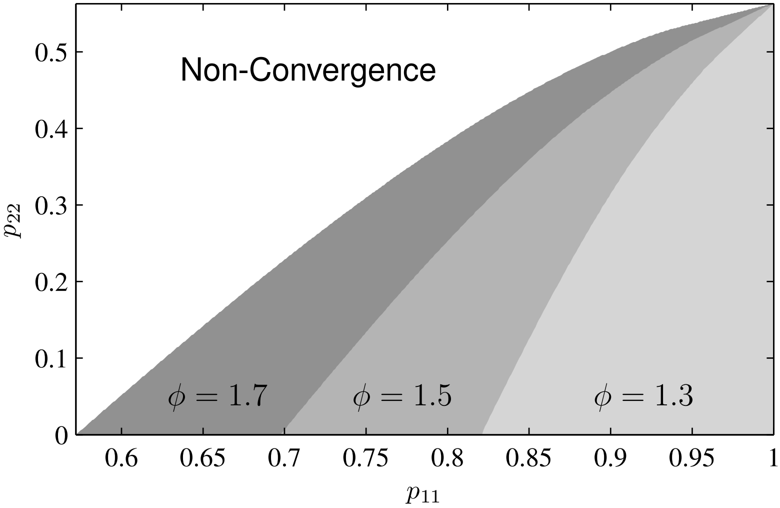
$$r_t = \begin{cases} \bar{r}(\pi_t/\pi^*)^\phi & \text{for} \quad s_t = 1 \\ 1 & \text{for} \quad s_t = 2 \end{cases}$$

  Baseline: $\bar{r} = 1.015$, $\pi^* = 1.005$, and $\phi \in \{1.3, 1.5, 1.7\}$

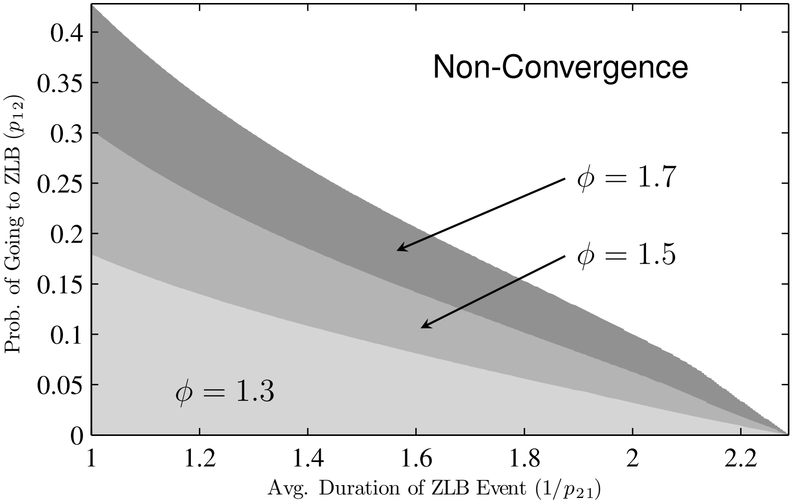
- s_t follows a 2-state Markov chain with transition matrix

$$\begin{bmatrix} \Pr[s_t = 1 | s_{t-1} = 1] & \Pr[s_t = 2 | s_{t-1} = 1] \\ \Pr[s_t = 1 | s_{t-1} = 2] & \Pr[s_t = 2 | s_{t-1} = 2] \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

Convergence (Shaded) Regions



Convergence (Shaded) Regions



Endogenous ZLB Events

- The monetary authority follows

$$r_t = \max\{1, \bar{r}(\pi_t/\pi^*)^{\phi}\}$$

- Discount Factor (β) or Technology (a) follows

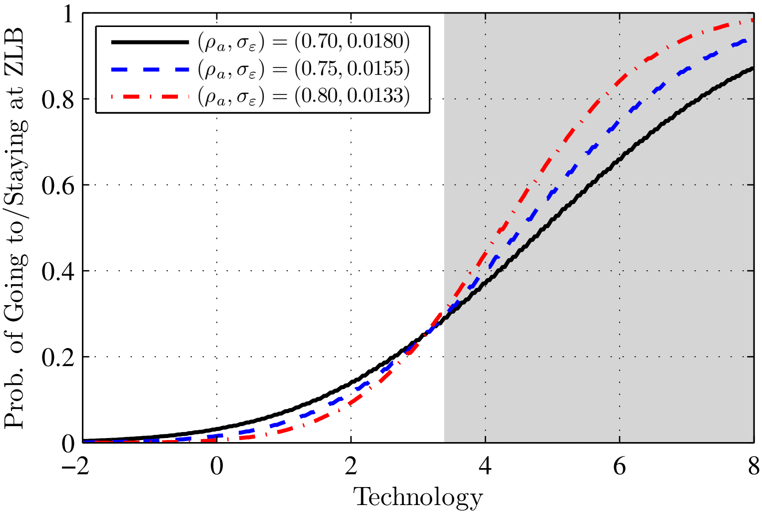
$$z_t = \bar{z}(z_{t-1}/\bar{z})^{\rho_z}\exp(\varepsilon_t), \quad \varepsilon_t \sim \mathbb{N}(0, \sigma_\varepsilon^2)$$

- Let $z_{t-1} \in \{z_1, \ldots, z_N\}$. Probability of going to or staying at the ZLB given z_{t-1} is

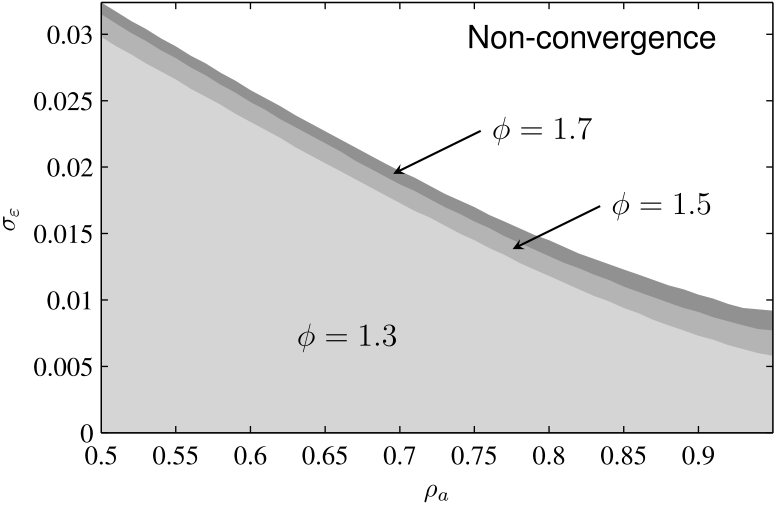
$$\Pr\{s_t = 2 | z_{t-1} = z_i\} = \pi^{-1/2} \sum_{j \in \mathcal{J}_{2,t}(i)} \phi(\varepsilon_j | 0, \sigma_\varepsilon)$$

$\mathcal{J}_{2,t}(i)$ is the set of indices where the ZLB binds given the state $z_{t-1} = z_i$. ϕ are the Gauss-Hermite weights.

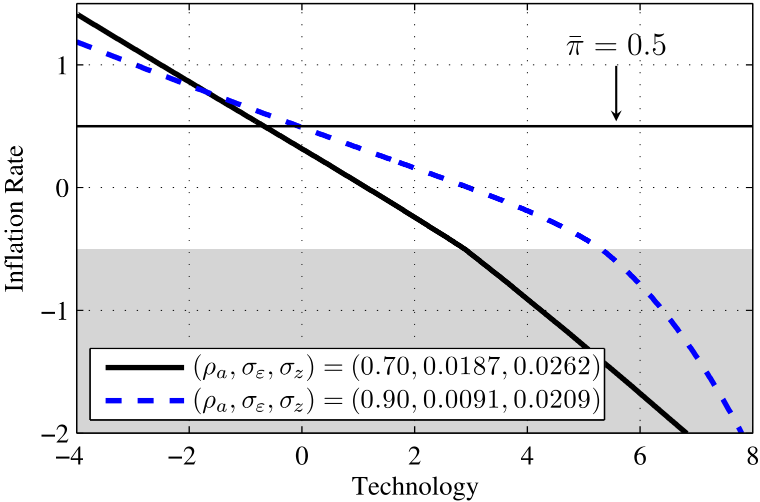
Zero Lower Bound Probabilities



Convergence (Shaded) Regions



Expectational Effect



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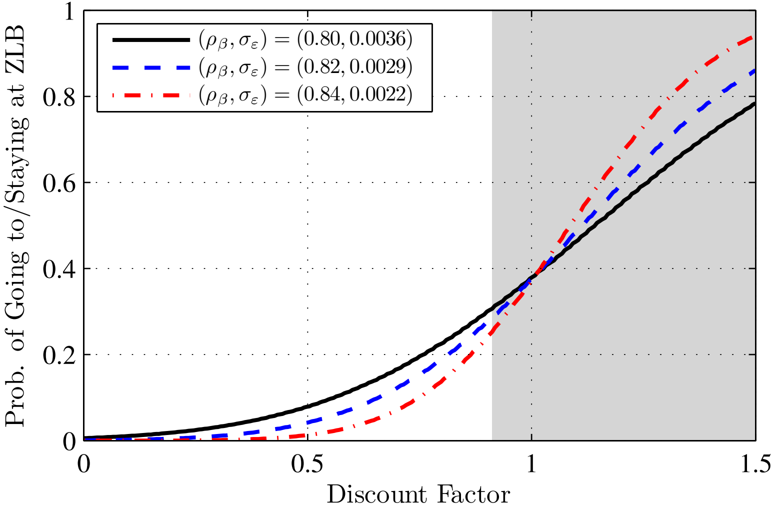
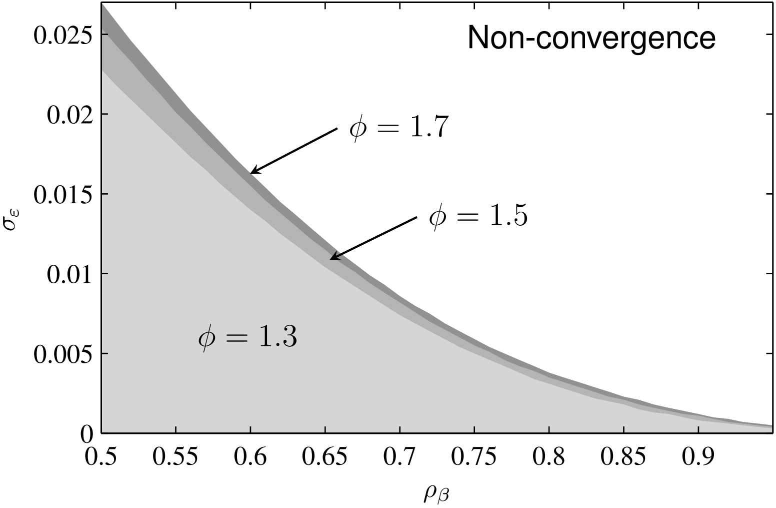


Convergence (Shaded) Regions



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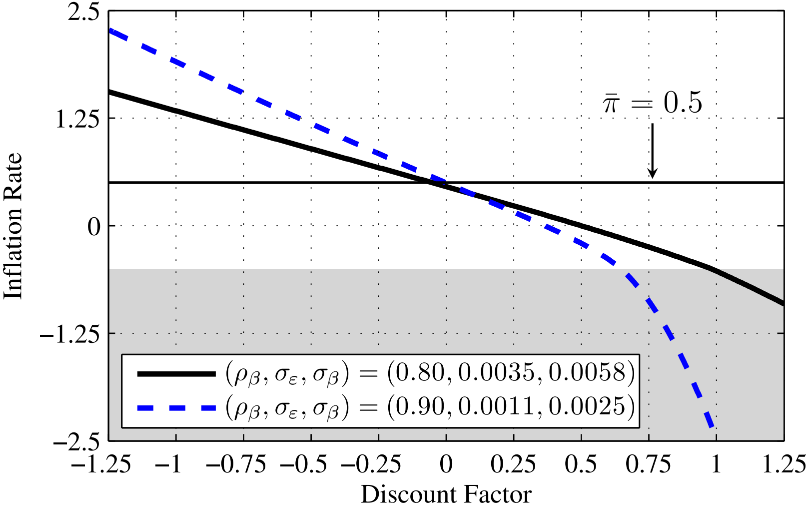

Conclusion

- Convergence region boundary imposes tradeoff between the frequency and duration of ZLB events
- This is important because
  - The central bank can pin down prices when the interest rate is at its ZLB
  - Knowing parameter restrictions is important for estimation and policy analysis
  - Small changes in the parameters impact decision rules and where ZLB first binds