The Consequences of an Unknown Debt Target

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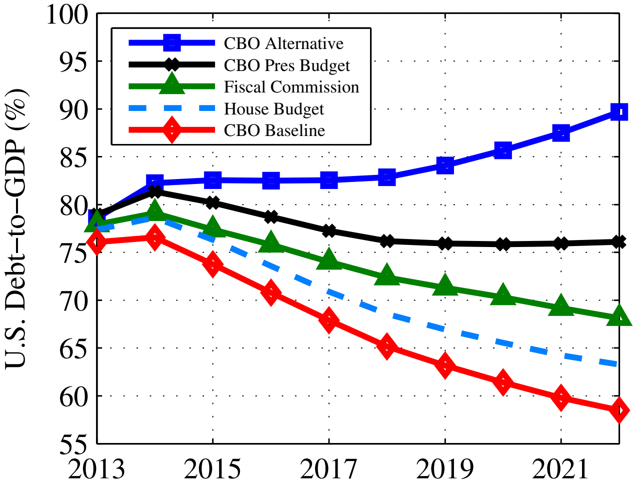
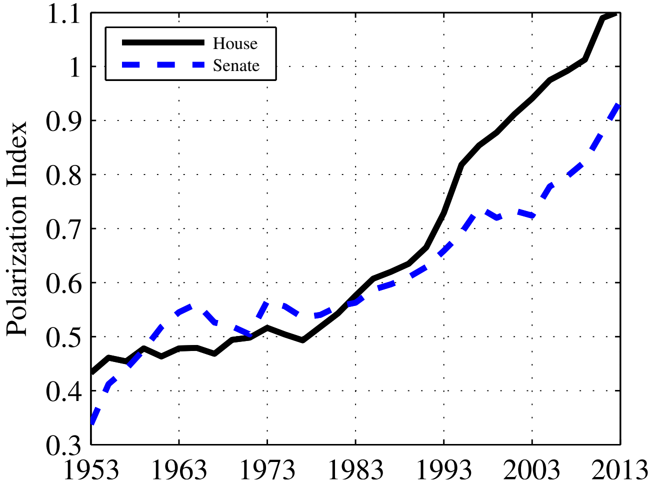
MOTIVATION

- Agreement on benefits of central bank communication
- No consensus about conduct of fiscal policy
- Recently adopted fiscal rules:
    - EU Stability and Growth Pact sets debt target equal to 60%
    - Sweden 2010 Budget Act sets lending target of 1% of GDP
    - NZ Fiscal Responsibility Act requires “prudent” debt level
    - Canada committed to debt-to-GDP ratio of 25% by 2021
    - 1985 U.S. Gramm-Rudman-Hollings Balanced Budget Act

U.S. Budget Proposals



Polarization of the U.S. Congress



Main Results

1. An unknown debt target amplifies the effects of tax shocks.
2. Stark changes in fiscal policy lead to welfare losses.
3. The Bush tax cut debate may have slowed the recovery.

RBC Model

Household chooses $\{c_j, n_j, i_j, b_j\}_{j=t}^{\infty}$ to maximize

$$E_t^{\ell} \sum_{j=t}^{\infty} \beta^{j-t} \left\{ \log c_j - \chi \frac{n_j^{1+\eta}}{1+\eta} \right\}$$

subject to

$$c_t + i_t + b_t = (1-\tau_t)(w_t n_t + r_t^k k_{t-1}) + r_{t-1} b_{t-1} + \bar{z}$$
$$k_t = i_t + (1-\delta) k_{t-1}$$

P.C. firm produces $y_t = \bar{a} k_{t-1}^{\alpha} n_t^{1-\alpha}$, and chooses $\{k_{t-1}, n_t\}$ to maximize $y_t - w_t n_t - r_t^k k_{t-1}$.

Fiscal Policy

- Government budget constraint,

$$b_t + \tau_t(w_t n_t + r_t^k k_{t-1}) = r_{t-1} b_{t-1} + \bar{g} + \bar{z}.$$

- State-dependent income tax rate policy,

$$\tau_t = \bar{\tau}(s_t) + \gamma(b_{t-1}/y_{t-1} - \overline{by}(s_t)) + \varepsilon_t,$$

  where s is an m-state hidden Markov chain with transition matrix P, and $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$.

- Signal extraction problem,

$$\begin{aligned} x_t &\equiv \tau_t - \gamma b_{t-1}/y_{t-1} \\ &= \bar{\tau}(s_t) - \gamma \overline{by}(s_t) + \varepsilon_t, \end{aligned}$$

  which has a mixed PDF of m normal distributions.

Sources of limited information

1. Time-varying mean, not standard deviation
2. Unknown debt target state
   - Bayesian updates conditional probabilities
   - Expectations formation is rational/Bayesian
   - Rational learning is embedded in optimization problem
3. Unknown transition matrix
   - Bayesian updates transition matrix
   - Expectations formation is adaptive
   - Household must reoptimize given estimate

Information Sets

	Full Information	Limited Information	
	Case 0	Case 1	Case 2
Current Debt Target State	Known	Unknown	Unknown
Debt Target Transition Matrix	Known	Known	Unknown

$$\mathbb{E}\left[f(\mathbf{v}_{t+1}, \mathbf{z}^{\ell}_{t+1}, \mathbf{v}_t, \mathbf{z}^{\ell}_t)|\Omega^{\ell}_t\right] = 0$$

$$\mathbf{v}_t \equiv (c_t, n_t, k_t, i_t, b_t)$$

$$\mathbf{z}^{\ell}_t \equiv \begin{cases} (k_{t-1}, r_{t-1}b_{t-1}, \tau_t, s_t), & \text{for } \ell = 0, \\ (k_{t-1}, r_{t-1}b_{t-1}, \tau_t, \mathbf{q}_{t-1}), & \text{for } \ell \in \{1, 2\}, \end{cases}$$

$$\Omega^0_t \equiv \{M, \Theta, \mathbf{z}^0_t, P\}$$

$$\Omega^1_t \equiv \{M, \Theta, \mathbf{z}^1_t, P\} \qquad \Omega^2_t \equiv \{M, \Theta, \mathbf{z}^2_t, \hat{P}_t, \mathbf{x}^t\}$$

$$\Theta \equiv (\beta, \eta, \chi, \delta, \bar{a}, \alpha, \gamma, \{\bar{\tau}(i)\}^m_{i=1}, \{\overline{by}(i)\}^m_{i=1}, \sigma^2_{\varepsilon})$$

Expectations Formation

$$\mathbb{E}\left[f(\mathbf{v}_{t+1},\mathbf{z}^{\ell}_{t+1},\mathbf{v}_t,\mathbf{z}^{\ell}_t)|\Omega^{\ell}_t\right] =$$

$$\begin{cases} \sum_{j=1}^{m} p_{ij} \int_{-\infty}^{+\infty} f(\mathbf{v}_{t+1},\mathbf{z}^{0}_{t+1},\mathbf{v}_t,\mathbf{z}^{0}_t)\phi(\varepsilon_{t+1})\mathrm{d}\varepsilon_{t+1} & \text{for } \ell = 0 \\ \sum_{i=1}^{m} \mathbf{q}_t(i) \sum_{j=1}^{m} p^{\ell}_{ij} \int_{-\infty}^{+\infty} f(\mathbf{v}_{t+1},\mathbf{z}^{\ell}_{t+1},\mathbf{v}_t,\mathbf{z}^{\ell}_t)\phi(\varepsilon_{t+1})\mathrm{d}\varepsilon_{t+1} & \text{for } \ell \in \{1,2\} \end{cases}$$

- For $\ell = 0$, $s_t = i$ is known.
- For $\ell \in \{1,2\}$, $\mathbf{q}_t(i) \equiv \Pr[s_t = i|\mathbf{x}^t]$.
- For $\ell \in \{0,1\}$, $p_{ij} \in P$ is known.
- For $\ell = 2$, $p_{ij} \in \hat{P}_t$ are estimates.

Related Literature

1. Recurring regime change: Aizenman and Marion (1993); Bizer and Judd (1989); Dotsey (1990)
2. Current regime unobserved:
   - Monetary: Andolfatto and Gomme (2003); Leeper and Zha (2003); Schorfheide (2005)
   - Fiscal: Davig (2004)
3. Other policy uncertainty: Davig and Leeper (2011); Davig et al. (2010, 2011); Richter (2012); Davig and Forester (2014); Bi et al. (2013)
4. Learning papers:
   - Adaptive: Kreps (1998); Cogley and Sargent (2008)
   - Bayesian: Schorfheide (2005); Bianchi and Melosi (2012)
5. Stochastic Volatility: Bloom (2009); Bloom et al. (2012)
   SV in Fiscal Policy: Fernández-Villaverde et al. (2013); Born and Pfeifer (2014)

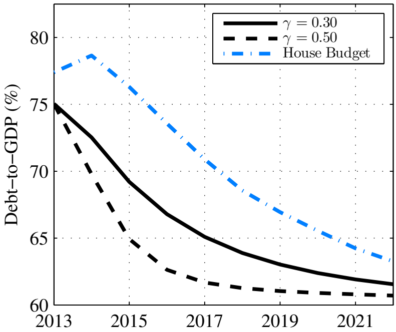
Calibration and Solution

Low Debt Target	$\overline{by}(1)$	0.60
Mid Debt Target	$\overline{by}(2)$	0.75
High Debt Target	$\overline{by}(3)$	0.90
Fiscal Policy Rule Coefficient	γ	0.30
Fiscal Noise Standard Deviation	σ_ε	Estimated
Prior Transition Matrix	$\bar{P}$	Estimated

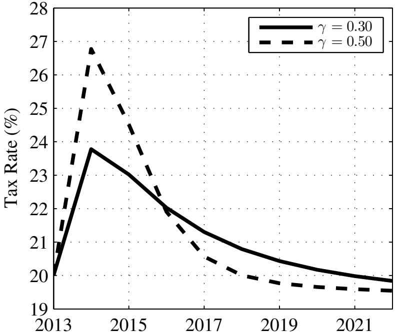
Debt targets are far apart so we use global nonlinear solution:

- Evenly spaced discretization
- Fixed-point policy function iteration
- Linear interpolation
- Gauss-Hermite integration
- 3-state Markov chain Discretization

γ Calibration



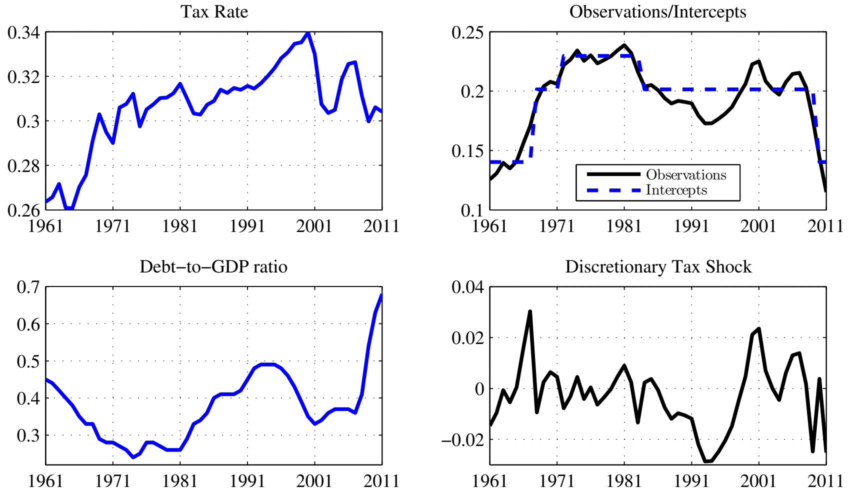
(A) Path to long-run debt target



(B) Short-run income tax adjustment

American Taxpayer Relief Act: Top marginal tax rate increased 4.6 pp, payroll tax increased 2 pp

Data and Tax Rule Fit



Estimation Results

- Estimating with Gibbs sampler gives $\hat{\sigma}_\varepsilon = 0.013$.
- The sampled average transition matrix and 68% credible interval are

$$P_{16} = \begin{bmatrix} 0.78 & 0.11 & 0.05 \\ 0.07 & 0.81 & 0.05 \\ 0.07 & 0.12 & 0.66 \end{bmatrix}$$

$$\bar{P} = \begin{bmatrix} 0.81 & 0.12 & 0.07 \\ 0.08 & 0.84 & 0.08 \\ 0.10 & 0.18 & 0.72 \end{bmatrix}$$

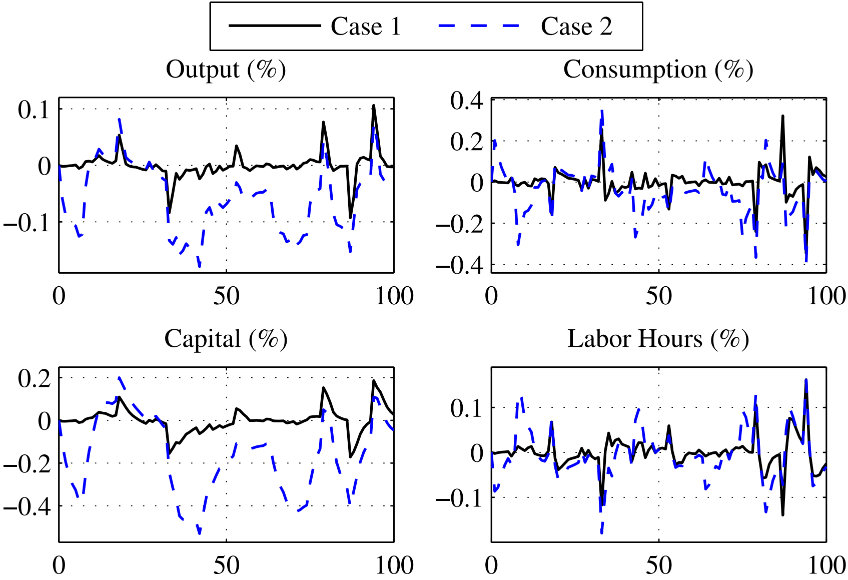
$$P_{84} = \begin{bmatrix} 0.83 & 0.15 & 0.08 \\ 0.10 & 0.87 & 0.11 \\ 0.12 & 0.24 & 0.79 \end{bmatrix}$$

▸ CBO Projections

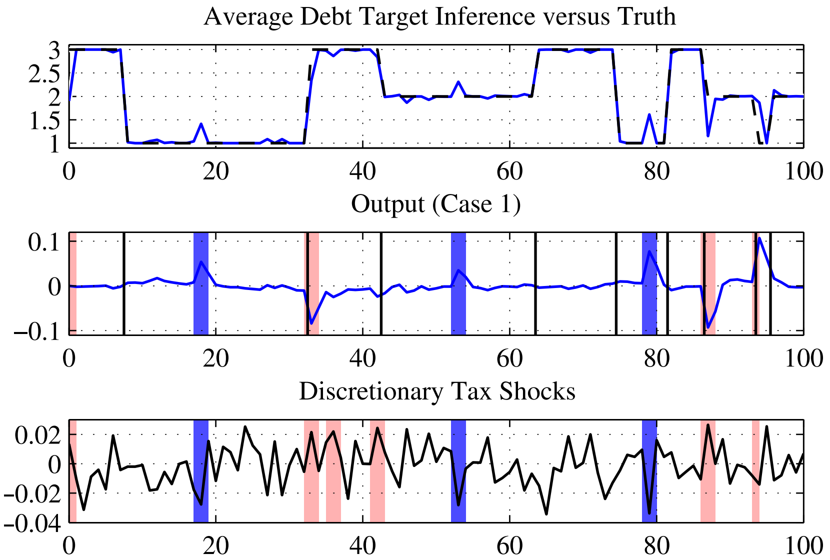
Simulation Procedure

1. Fiscal authority chooses s_t and ε_t to set τ_t given b_{t-1}/y_{t-1}
2. HH observes $x_t = \tau_t - \gamma b_{t-1}/y_{t-1}$ and in
   - Case 1 updates $\mathbf{q}_{t-1}$ given x_t with Bayes' rule ▸ more
   - Case 2 also updates $\hat{P}$ given $\mathbf{x}^t$ with Gibbs sampler ▸ more
3. In case 2, HH updates policy functions given $\hat{P}$
4. HH makes decisions conditional on information set, which updates b_{t-1}/y_{t-1}

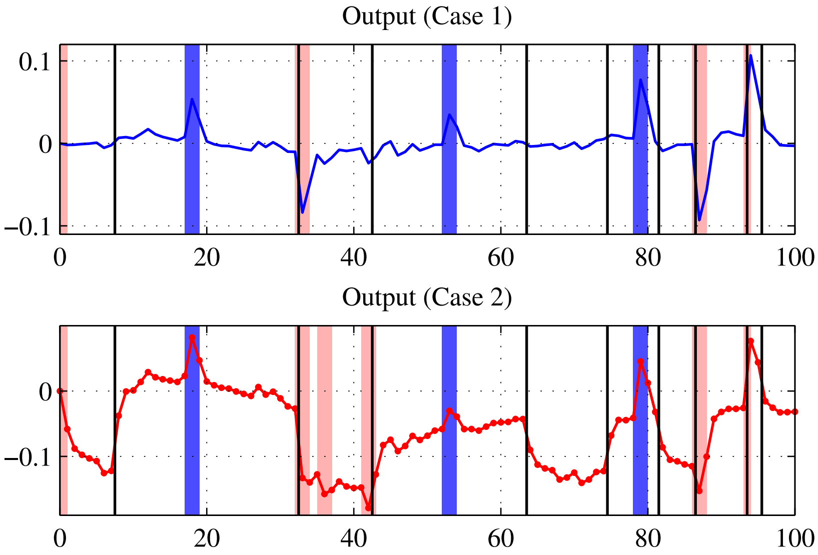
SIMULATION PATHS



Effects of Unknown State



Differences in Output



MACROECONOMIC UNCERTAINTY

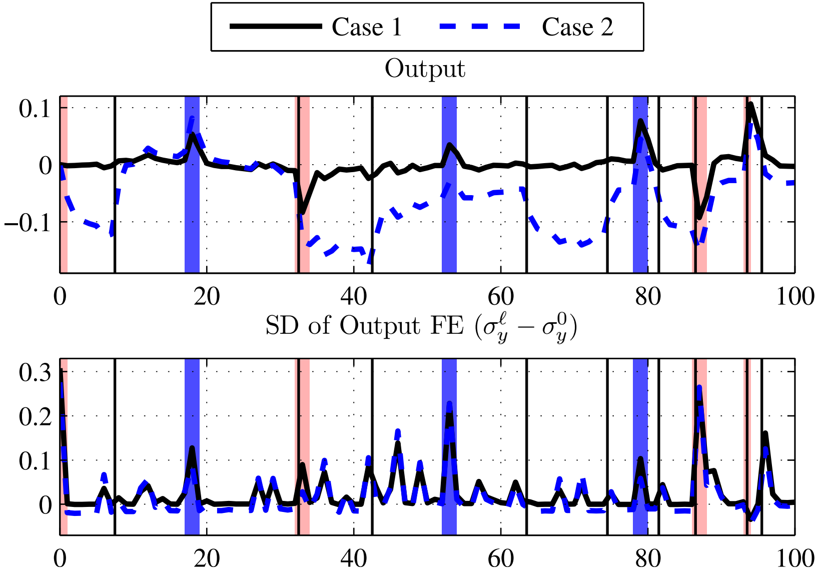
- y_t represents output in the model
- The expected value of the forecast error is given by

$$E_t[FE^{\ell}_{y,t+1}] = E_t[y_{t+1} - E_t y_{t+1} | \Omega^{\ell}_t]$$

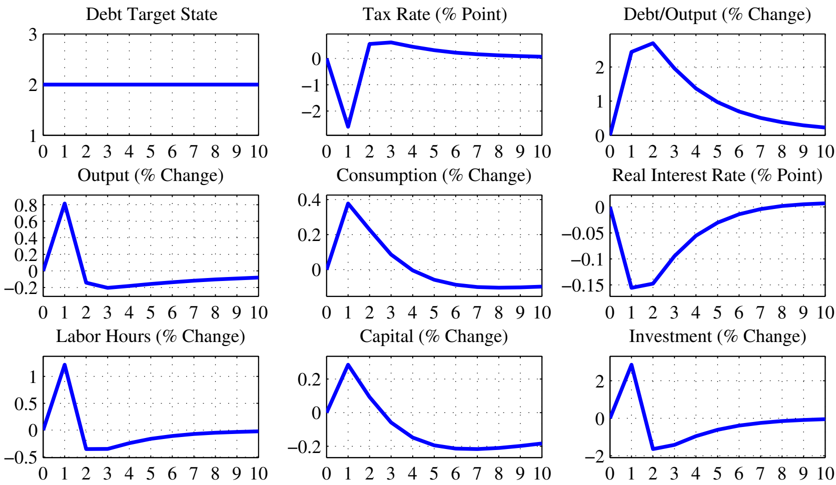
- The expected volatility of the forecast error is

$$\sigma^{\ell}_{y,t} \equiv \sqrt{E_t[(FE^{\ell}_{y,t+1} - E_t[FE^{\ell}_{y,t+1}])^2 | \Omega^{\ell}_t]}$$

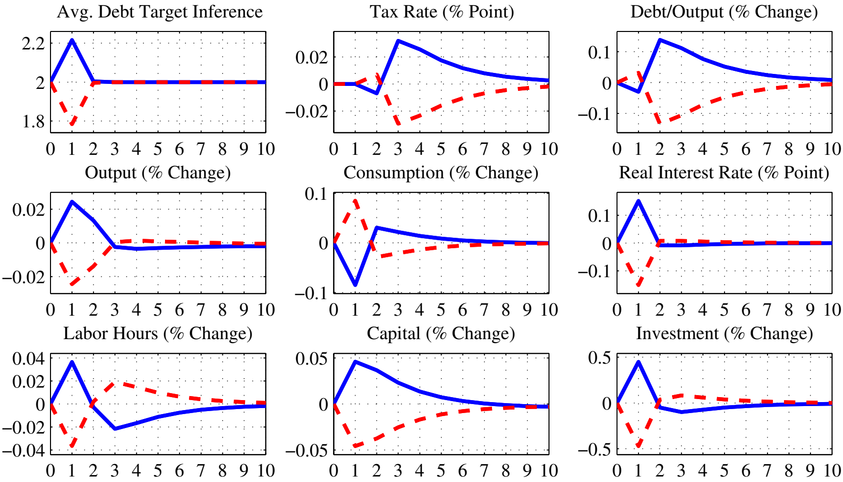
Expected Volatility of Output



FULL INFORMATION IRFS



Unknown State IRFs



UPDATED ESTIMATE OF P

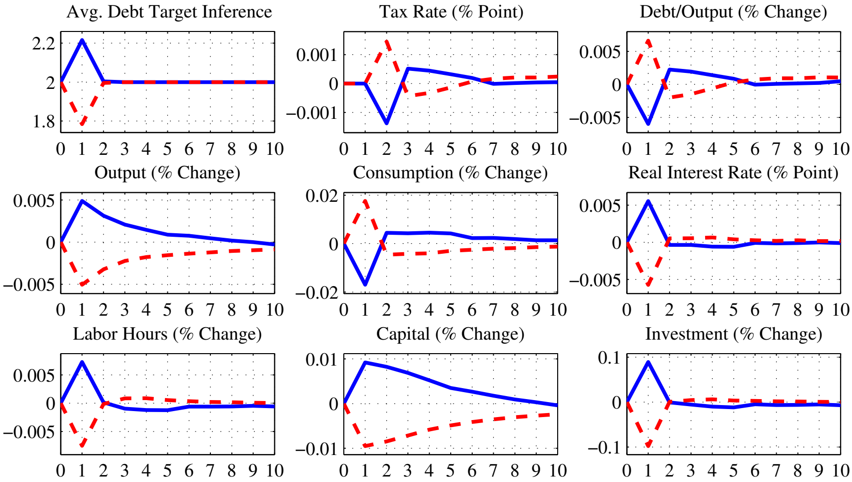
- In Case 2, HH updates estimate of P each period
- In period 1, their estimate is updated from

$$P = \hat{P}_0 = \begin{bmatrix} 0.90 & 0.05 & 0.05 \\ 0.05 & 0.90 & 0.05 \\ 0.05 & 0.05 & 0.90 \end{bmatrix}$$

to

$$\hat{P}_1 = \begin{bmatrix} 0.8947 & 0.0506 & 0.0547 \\ 0.0492 & 0.8970 & 0.0538 \\ 0.0454 & 0.0459 & 0.9087 \end{bmatrix}.$$

Case 2 IRFs



Avg. Debt Target Inference
Tax Rate (% Point)
Debt/Output (% Change)
Output (% Change)
Consumption (% Change)
Real Interest Rate (% Point)
Labor Hours (% Change)
Capital (% Change)
Investment (% Change)

WELFARE CALCULATION

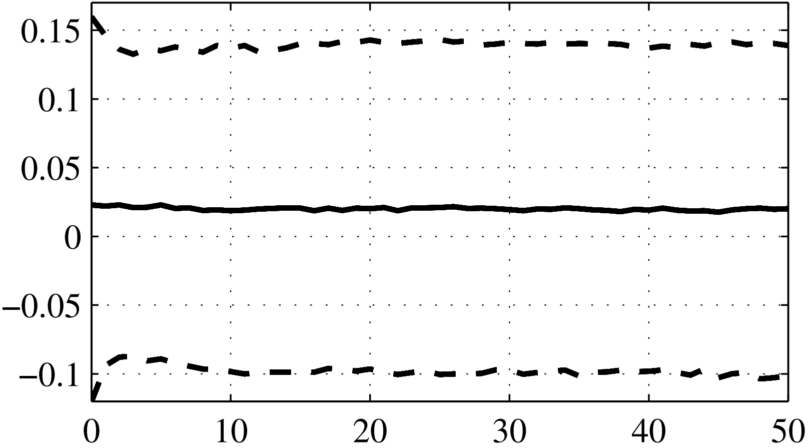
- Treat limited info. cases as alternative to full info.
- Solve for λ^ℓ that satisfies

$$\mathbb{E}_t^\ell W(c_t(\mathbf{z}_{t-1}^1), n_t(\mathbf{z}_{t-1}^1)) =$$

$$\begin{cases} \sum_{i=1}^{m} \mathbf{q}_t(i)\mathbb{E}_t^0 W((1-\lambda^1)c_t(\mathbf{z}_{t-1}^1|s_t=i), n_t(\mathbf{z}_{t-1}^1|s_t=i)) & \text{Case 1} \\ \sum_{i=1}^{m} \mathbf{q}_t(i) \sum_{j=1}^{m} \hat{p}_{ij}\mathbb{E}_t^0 W((1-\lambda^2)c_t(\mathbf{z}_{t-1}^2|s_t, s_{t+1}), n_t(\mathbf{z}_{t-1}^2|s_t, s_{t+1})) & \text{Case 2} \end{cases}$$

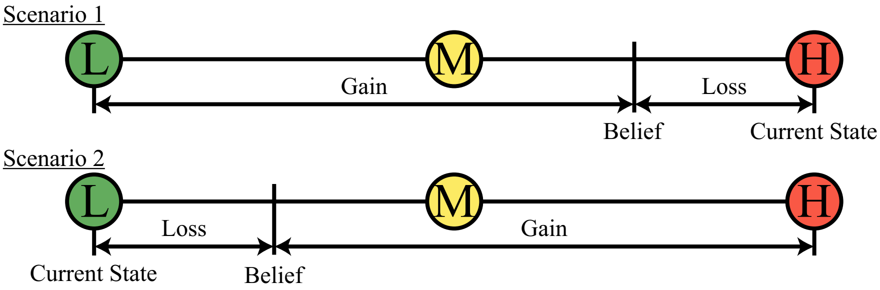
- $\lambda^\ell > 0$ ($\lambda^\ell < 0$) represents a welfare loss (gain) in case ℓ

Case 1 Welfare Distribution
(16-50-84 Bands)

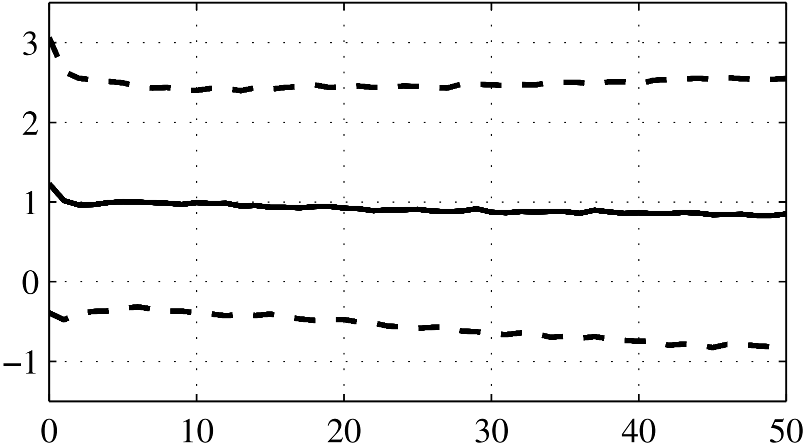


Welfare Gains and Losses

$$\tau_t = \bar{\tau}(s_t) + \gamma(b_{t-1}/y_{t-1} - \overline{by}(s_t)) + \varepsilon_t,$$



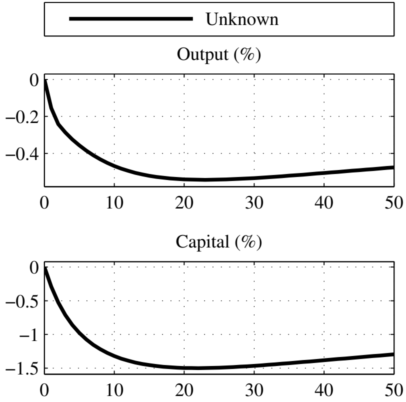
Case 2 Welfare Distribution
(16-50-84 Bands)



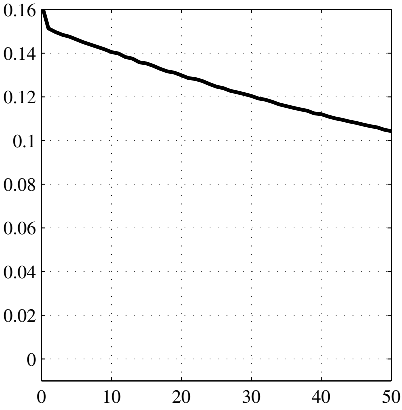
Tax Cut Debate

- Assumption: People expected Bush tax cuts to sunset consistent with the goal of deficit reduction
- Reality: Tax cuts were largely extended (projected to add $360B to annual deficit)
- Suppose true debt target had always been high, despite Congress' rally against debt
- Hypothesis: People's expectations were misaligned with the actual higher long-run debt target, which led to lower investment, output, and welfare loss

Debt Target is Hidden

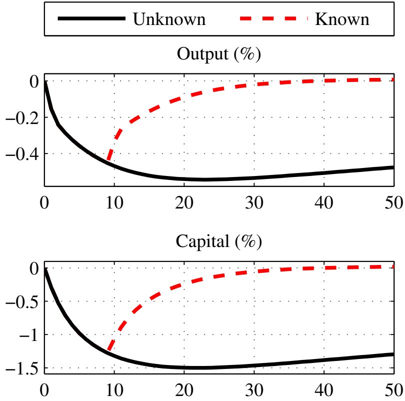


(A) Contractionary paths

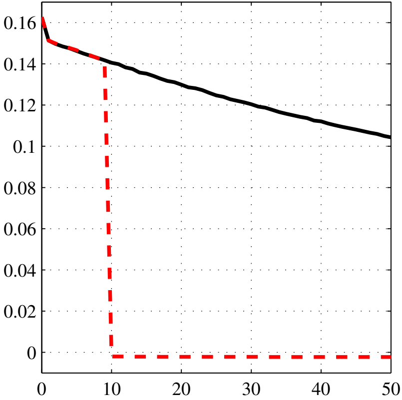


(B) Welfare costs

DEBT TARGET IS REVEALED



(A) Contractionary paths

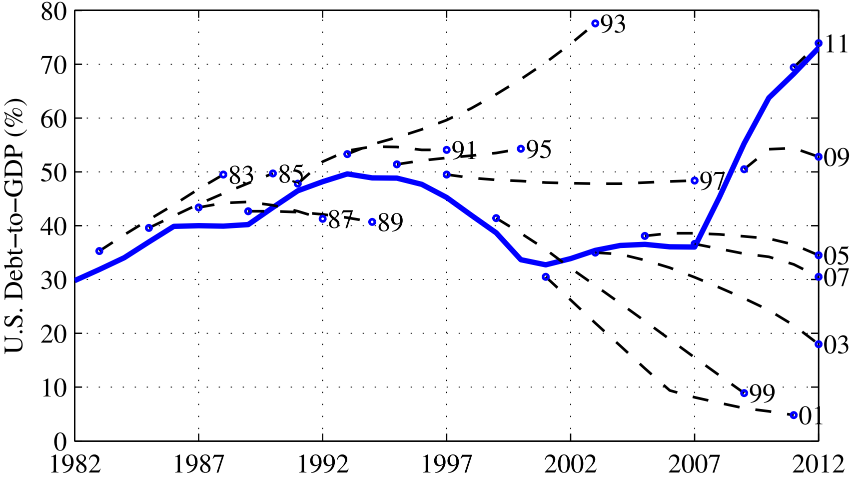


(B) Welfare costs

Conclusion

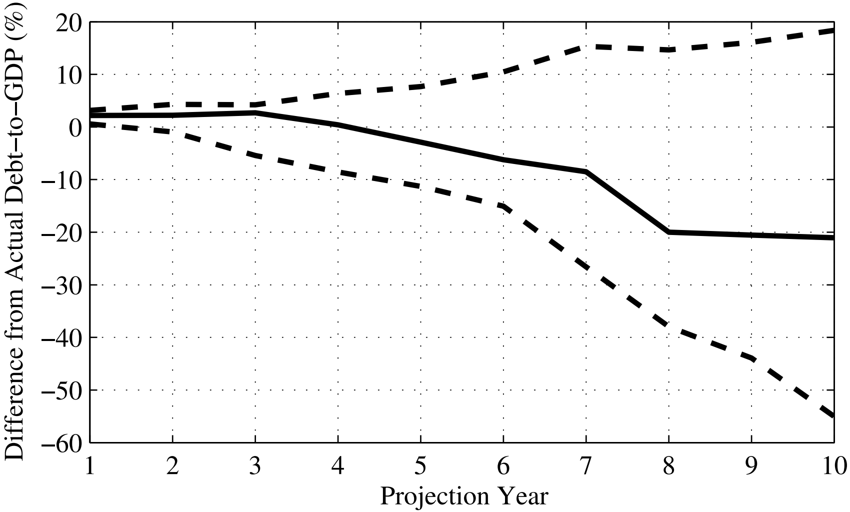
1. An unknown debt target amplifies the effect of tax shocks through changes in expected tax rates
2. Unknown debt target leads to welfare losses on average
3. The Bush tax cut debate may have led to welfare losses

CBO Baseline Projections



Distribution of Differences

(25-50-75 Quantiles)



Discretization Method

3-state Markov chain

- Define a projection $g : \mathbb{R}^3 \to \mathbb{R}^2$,

$$g(\mathbf{q}_t) \equiv (\mathbf{q}_t - \mathbf{o})\mathbf{B} = \boldsymbol{\xi}_t,$$

  where $\mathbf{o}$ is the origin and $\sum_i \mathbf{q}_t(i) = 1$.
- Apply the Gram-Schmidt process to obtain

$$\mathbf{b}_1 = \tilde{\mathbf{b}}_1 = [0, 1, -1], \quad \mathbf{b}_2 = \tilde{\mathbf{b}}_2 - \text{proj}_{\mathbf{b}_1}\left(\tilde{\mathbf{b}}_2\right) = [1, -1/2, -1/2],$$

  so that $\mathbf{B} \equiv [\mathbf{b}_1^T/||\mathbf{b}_1||, \mathbf{b}_2^T/||\mathbf{b}_2||]$ is an orthonormal basis.
- The mapping becomes

$$\xi_t(1) = \mathbf{q}_t(2)(b_{21} - b_{11}) + \mathbf{q}_t(3)(b_{31} - b_{11})$$
$$\xi_t(2) = \mathbf{q}_t(2)(b_{22} - b_{12}) + \mathbf{q}_t(3)(b_{32} - b_{12}).$$

  where $b_{ij} \in \mathbf{B}$.

back

Hamilton Filter

1. Calculate the joint probability of $(s_t = i, s_{t-1} = j)$,

$$\Pr[s_t = i, s_{t-1} = j|\mathbf{x}^{t-1}] = \Pr[s_t = i, s_{t-1} = j] \Pr[s_{t-1} = j|\mathbf{x}^{t-1}].$$

2. Calculate the joint conditional density-distribution,

$$f(x_t, s_t = i, s_{t-1} = j|\mathbf{x}^{t-1}) = f(x_t|s_t = i, s_{t-1} = j, \mathbf{x}^{t-1}) \Pr[s_t = i, s_{t-1} = j|\mathbf{x}^{t-1}].$$

3. Calculate the likelihood of x_t conditional on its history,

$$f(x_t|\mathbf{x}^{t-1}) = \sum_{i=1}^{m} \sum_{j=1}^{m} f(x_t, s_t = i, s_{t-1} = j|\mathbf{x}^{t-1}).$$

4. Calculate the joint probabilities of $(s_t = j, s_{t-1} = i)$ conditional on $\mathbf{x}^t$,

$$\Pr[s_t = i, s_{t-1} = j|\mathbf{x}^t] = \frac{f(x_t, s_t = i, s_{t-1} = j|\mathbf{x}^{t-1})}{f(x_t|\mathbf{x}^{t-1})}.$$

5. Calculate the output by summing the joint probabilities over the realizations s_{t-1},

$$\Pr[s_t = i|\mathbf{x}^t] = \sum_{j=1}^{m} \Pr[s_t = i, s_{t-1} = j|\mathbf{x}^t].$$

back

Importance Sampler

- Posterior density is product of two independent Dirichlet distributions:

$$f(P|\mathbf{s}^T) \propto \left(\prod_{j=1}^{3} \Pi_j(P)^{\mathbf{1}_j} \right) \left(\prod_{i=1}^{3} \prod_{j=1}^{3} p_{ij}^{a_{ij}+m^o_{ij}-1} \right)$$

where π is the stationary distribution of P and a are the initial shaping parameters.
- Sample L draws, θ^{ℓ}_{ij}, from Dirichlet distribution, then weight them with $w_\ell \equiv \prod_{j=1}^{3} \Pi_j(P^{\ell}_t)^{\mathbf{1}_j}$
- $\hat{p}_{ij}$ result from weighting procedure

$$\hat{p}_{ij} = \frac{\sum_{\ell=1}^{L} w_\ell \theta^{\ell}_{ij}}{\sum_{\ell=1}^{L} w_\ell}.$$

back

Gibbs Sampler

1. Initialize $\mathrm{s}^T = \{s_1, \ldots, s_T\}$ by sampling from the prior, P.
2. For $t \in \{1, \ldots, T\}$ and $j \in \{1, 2, 3\}$, sample s_t
   - If $t = 1$, then $f(s_1 | \mathbf{x}^T, \mathbf{s}_{-1}) \propto \Pi_j(P) p_{jk} f(x_1 | s_1)$, where $s_2 = k$.
   - If $1 < t < T$, then $f(s_t | \mathbf{x}^T, \mathbf{s}_{-t}) \propto p_{ij} p_{jk} f(x_t | s_t)$, where $s_{t-1} = i$ and $s_{t+1} = k$.
   - If $t = T$, then $f(s_T | \mathbf{x}^T, \mathbf{s}_{-T}) \propto \Pi_j(P) p_{ij} f(x_T | s_T)$, where $s_{T-1} = i$.

   $\Pi_j(P)$ is the jth element of the stationary distribution of P, $f(x_t | s_t) = \exp\left\{-\varepsilon_t^2 / (2\sigma^2)\right\} / \sqrt{2\pi\sigma^2}$, where $\varepsilon_t = x_t - (\bar{\tau}(s_t) - \gamma \overline{by}(s_t)))$ is the discretionary *i.i.d.* tax shock.
3. Use the importance sampler to draw P given s^T.
4. Repeat steps 2 and 3 N times.

back