A NEW INSTRUMENT FOR SOLVING THE QUADRATIC

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Throughout the history of mathematics the quadratic equation has been a sore thumb. Either the cumbersome quadratic formula has been used, or the shoot and miss method of eye-balling the equation has been forced into use. No quick, accurate method has ever been found. To develop a new system for finding the roots of a quadratic I have used the properties of the sum and the products of the roots. I have incorporated these with the slide rule to fashion an instrument for speedy discovery of the roots of the quadratic.

I soon decided to use the slide rule to assist me in solving the quadratic. The slide rule is ideal for showing the factors of any number, as shown in Figure 1 below.

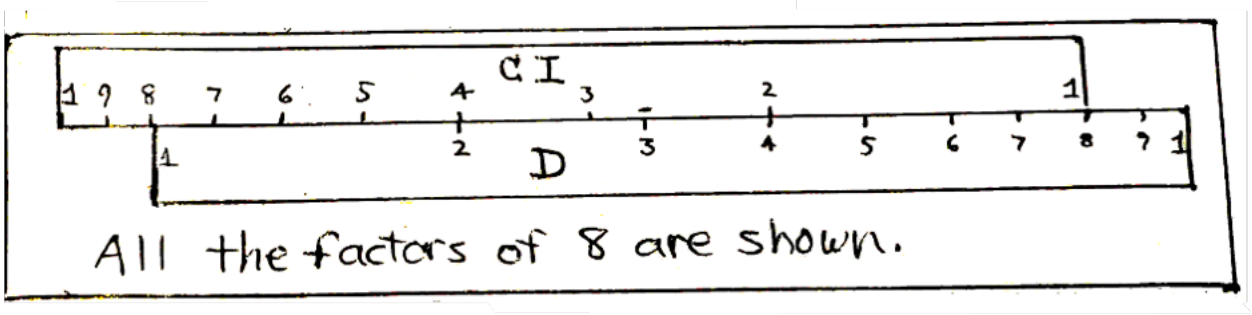


Figure 1.

By using the quadratic formula the product of the roots of the quadratic can be shown to equal **c/a**. In a similar fashion the sum of the roots can be shown to equal **-b/a**.

I endeavored to use these relationships in the following manner: first, place 1 of the **CI** scale over your value for **c/a**. As an example I shall use $3x^2-13X+14=0$. In

this case, the value for **c/a** is 14/3, or 4.6. On the imaginary **G** scale the sum of the two numbers, directly below it, is read. This hypothetical situation is pictured in Figure 2 below. By moving over to the value of **|-b/a|** on the **G** scale, in this case 4.3, the roots are read directly below this number; the quadratic is solved. The roots are 2.3 and 2. The roots are positive since **-b/a** is positive.

But problems are soon encountered when trying to enlarge the system. The problem is that the calibrations on the **G** scale are not constant for different numbers. This problem is illustrated in Figure 2 below.

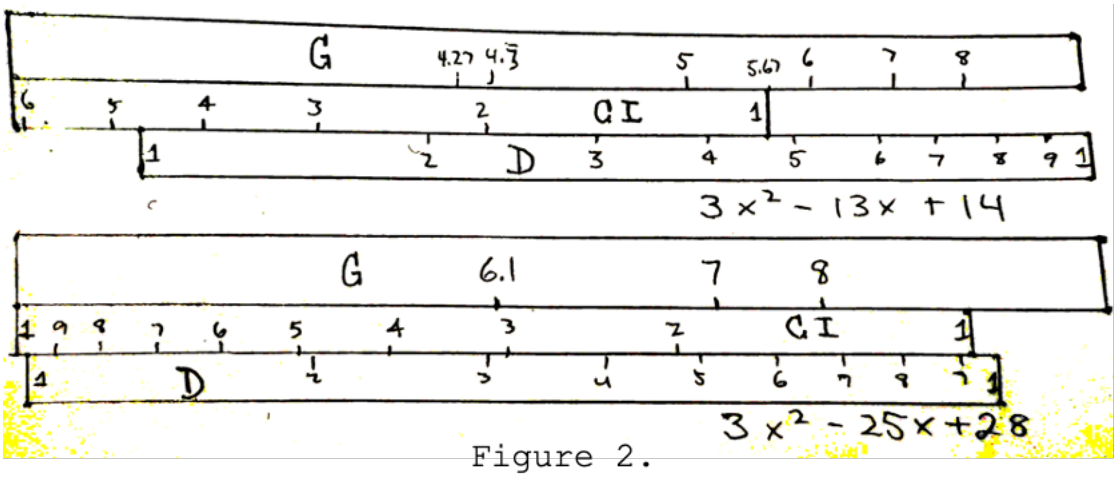


Figure 2.

The next idea was to make a scale that would show all the addends of a number, much like the **CI** and **D** scales show all the factors. By combining these two sets a method of solving any quadratic may be found.

This scale was simply made be placing two evenly calibrated scales opposite each other. These scales are shown in Figure 3 (next page).

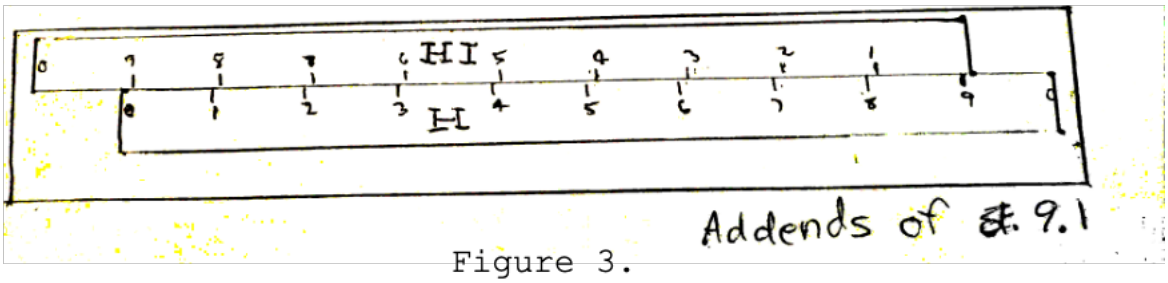


Figure 3.

The only problem encountered with this is that the addends, as well as the factors, run out at the end of scales. To solve this, a continuous scale is needed: a circular scale is perfectly suited.

Here are the scales when put together (Figure 4).

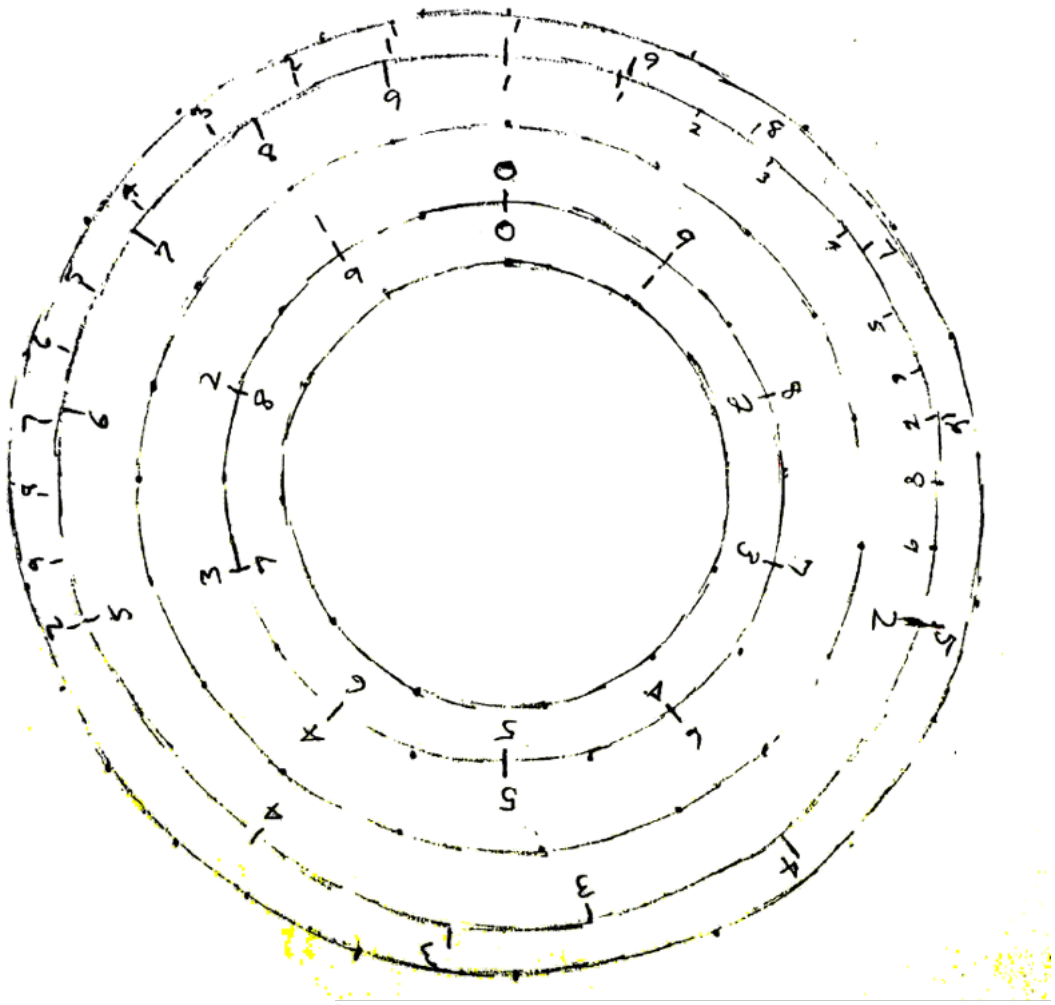


Figure 4.

There are many ways in which to find the roots of the quadratic using these scales. I will show one way, but many others may be found by experimenting. I will first tell how to find the roots when **c** is positive.

When both **b** and **c** are positive the roots are negative since **-b/a** is negative. Throughout this discussion **a** is assumed to be positive. When only **b** is negative the roots are positive. The situation when **c** is negative will be taken up later. The information about the signs of the roots becomes helpful when the absolute values of the roots are known and the sign must be decided upon. The actual procedure for finding the roots is illustrated below. The sample quadratic used is $x^2-8x-12=0$.

First, 0, on the **HI** scale is placed underneath the value for **|-b/a|**. For this equation it is 8, on the **H** scale. These steps can be seen graphically on page 6. After this, 1, of the **D** scale is placed under **c/a** on the **CI** scale. In this case the value is 12. Position the **H** scales to the place where the **H** and **DI** values are equal to one half the **b/a** value, in this case, 4. Place this point under a similar area on **CI** and **D** scales. Remember that there are two such places on the scales; use the one that approximates the square root of **c/a**, in this case about 3.4, not the square root of 120, about 10.9. This position is shown in Figure 5a (page 6). If these four numbers had been the same, the equation itself would have been a perfect square trinomial, and the roots would be the numbers. But the numbers are not equal. The **H** values are

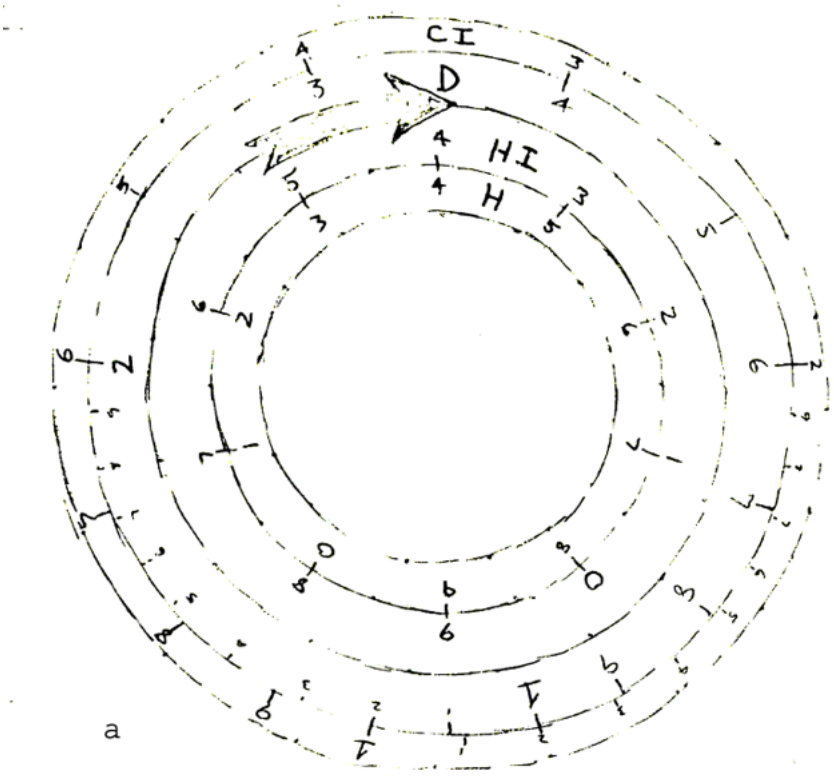
larger. This means there are real roots to the equation. If the **H** values had been smaller there would only be imaginary roots. This position serves as a discriminant of sorts for the equation. Since the **H** values are larger the search for the real roots continues.

The **H** scales are held together and simultaneously rotated so that **H** scale values correspond with **D** scale values. While rotating the **H** scales it is necessary to move the held **H** scales in a clockwise direction. Figures 5a and 5b (next page) illustrate the procedure.

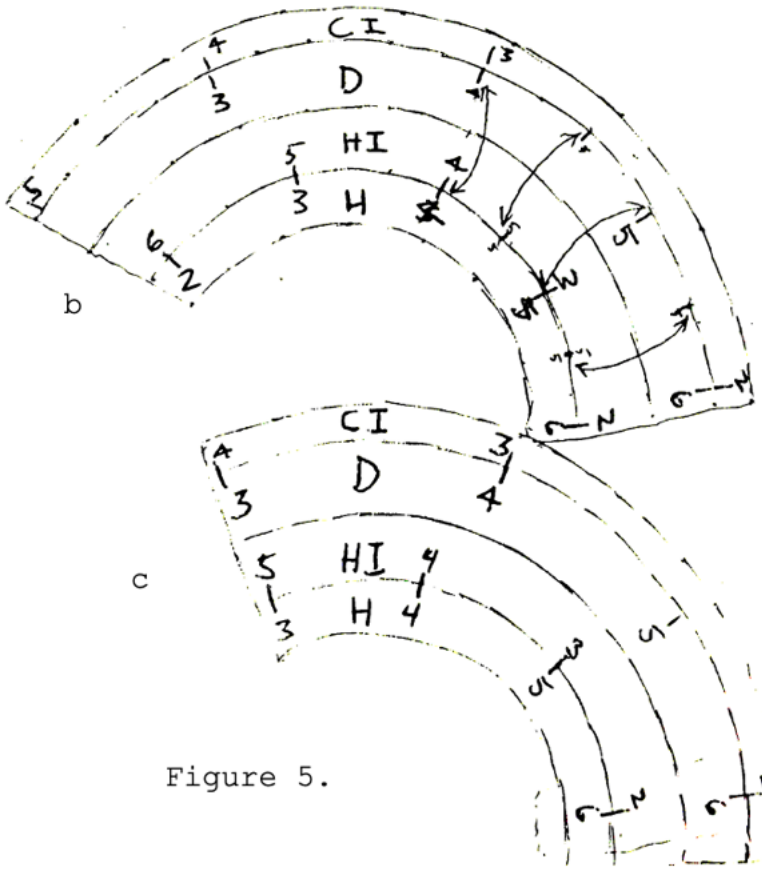
Watch for the change in direction of rotating the **H** scales because the **D** scale calibration becomes smaller at about 4 or 5. Also, while rotating watch for equal values to appear on the **CI** and **D** scales with those on the **H** and **HI** scales. When the values match the missing roots are found. Finally, at 2 and 6 the values match up: the roots are found. The final diagram on page 6 portrays this (Figure 5c).

The system seems painstaking at first, but with practice can eliminate much unneeded work: especially with complex equations which can be done successfully to 3 significant figures.

If **c** is negative there are only a few small changes. If **-b/a** is negative then **HI** is negative and **H** is positive; if **-b/a** is positive then **H** is positive and **HI** is negative. The red numbers are used on the **HI** scale. The other change is that the **H** scales are rotated counter-clockwise. The full procedure is illustrated in Figure 6 (on page 7).



a



b

let D & H values correspond

c

CI = HI

D = H

Figure 5.

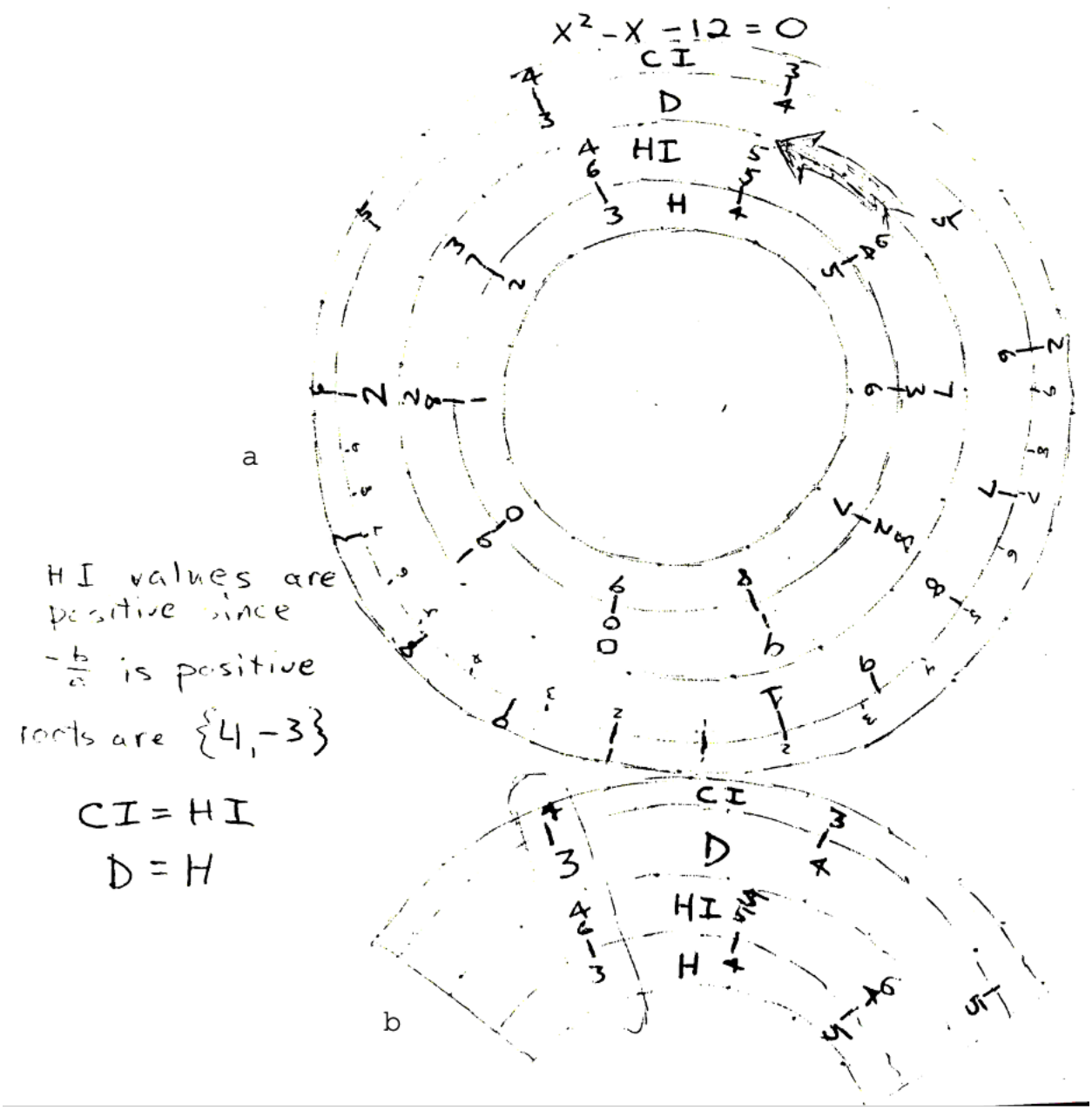

$X^2 - X - 12 = 0$
CI
D
HI
H
a
HI values are positive since $-\frac{b}{a}$ is positive
roots are $\{4, -3\}$
CI = HI
D = H
CI
D
HI
H
b

Figure 6.