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PGT MATHEMATICS
+2 ZILA SCHOOL DHANBAD
MOB-7717736214

COMPLEX NUMBER

1. Evaluate
   a. i^{998}
   b. i^{-71}
   c. $i^{107} + i^{112} + i^{117} + i^{122}$
   d. $i^{37} \times \frac{1}{i^{67}}$
   e. $(-i)(3i)\left(-\frac{1}{6}i\right)$
   f. $(1+i)^4 \times \left(1+\frac{1}{i}\right)^4$
   g. $4\sqrt{-4} + 5\sqrt{-9} - 3\sqrt{-16}$
   h. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$
   i. $\sqrt{-25} \times \sqrt{-49}$
   j. $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$
   k. $\sqrt{-9} \times \sqrt{-4}$
   l. $6i^{54} + 5i^{37} - 2i^{11} + 6i^{68}$
   m. $\frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4}$
2. Simplify each of the following and put it in the form of (a + ib):
   a. $3(1-2i)— 4 - 5i + (-8 + 3i)$
   b. $(2+3i)(4-5i)$
   c. $\left(-1+\sqrt{3}i\right)^{-1}$
   d. $(4-3i)^3$
3. Find the conjugate of each of the following :
   a. i^3
   b. $-3 + \sqrt{-1}$
   c. $-5 - 2i$
   d. $\frac{1}{4+3i}$
   e. $\frac{(1+i)^2}{3-i}$
   f. $\frac{(1+i)(2+i)}{3+i}$
   g. $(6+5i)^2$
4. Find the modulus of
   a. $-i$
   b. $2 + \sqrt{-3}$
   c. $(3i-1)^2$
5. Find the modulus of $\left(\frac{1+i}{1-i} - \frac{1-i}{1+i}\right)$
6. $If\ (x+iy)^3 = (u+iv)$
   $then\ show\ that\ \left(\frac{u}{x} + \frac{v}{y}\right) = 4(x^2 - y^2)$
7. $If\ (a+ib) = \sqrt{\frac{1+i}{1-i}}$
   $then\ show\ that\ (a^2+b^2) = 1$

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8. **Simplify :**

   a. $(-2+\sqrt{-3})(-3+\sqrt{-3})$

   b. $(2-3i)^2$

   c. $(3+4i)^2$

   d. $(1+2i)^3$

   e. $(1-3i)^{-3}$

   f. $(3-2i)^{-1}$

   g. $(1-i)^2(1+i)-(3-4i)^2$

   h. $(1+i)^3-(1-i)^3$

9. **Reduce following to the standard form :**

   a. $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$

   b. $\frac{i}{1+i}$

   c. $\frac{1}{3+4i}$

   d. $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

   e. $\frac{(1+i)(3+i)}{3-i}-\frac{(1-i)(3-i)}{3+i}$

   f. $\frac{(2+3i)^2}{2-i}$

   g. $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

   h. $\frac{3-4i}{(4-2i)(1+i)}$

   i. $\frac{3+2i}{2-3i}+\frac{3-2i}{2+3i}$

   j. $\frac{1-i}{1+i}$

10. **Represent the complex number into polar form :**

    a. $(-1+i\sqrt{3})$

    b. $-\frac{16}{1+i\sqrt{3}}$

    c. $-1-i$

    d. $-1-\sqrt{3}i$

    e. $1-i$

    f. $1+i$

    g. $\sqrt{3}+i$

    h. 3

    i. i

    j. -3

    k. $2i$

    l. $1+i\sqrt{3}$

    m. $\frac{5-i}{2-3i}$

    n. $\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$

11. **If $z=(\sqrt{2}-\sqrt{-3})$, find Re(z), $Im(z)$, $\bar{z}$, $|z|$.**

12. **If $z_1=(1-i)$ and $z_2=(-2+4i)$, find $Im\left(\frac{z_1 z_2}{\overline{z_1}}\right)$**

13. Show that $\frac{\sqrt{7}+i\sqrt{3}}{\sqrt{7}-i\sqrt{3}}+\frac{\sqrt{7}-i\sqrt{3}}{\sqrt{7}+i\sqrt{3}}$ is real.

14. Find real values of θ for which $\frac{3+2isin\theta}{1-2isin\theta}$ is purely real.

15. $If\ (x+iy)^{\frac{1}{3}}=a+ib$, then show that $\left(\frac{x}{a}+\frac{y}{b}\right)=4(a^2-b^2)$

16. $If\ (x+iy)=\frac{a+ib}{a-ib}$, prove that $x^2+y^2=1$

17. Separate $\frac{3+\sqrt{-1}}{2-\sqrt{-1}}$ into real and imaginary parts, and find its modulus.

18. Find the least positive integral value of m for which $\left(\frac{1+i}{1-i}\right)^m=1$

19. Find the value of x and y , if

a. $2+(x+iy)=(3-i)$

b. $x+4iy=ix+y+3$

c. $\frac{x-1}{3+i}+\frac{y-1}{3-i}=i$

d. $(x+iy)(2-3i)=(4+i)$

20. Find the value of x and y , if

$(1+i)y^2+(6+i)=(2+i)x$

21. Solve

a. $3x^2+8ix+3=0$

b. $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

c. $x^2+\frac{x}{\sqrt{2}}+1=0$

d. $2x^2+1=0$

e. $3x^2+5=7x$

f. $2x^2-\sqrt{3}x+1=0$

g. $2x^2+3ix+2=0$

22. Find the square root of i.

23. Find the square root of $-i$.

24. Find the square root of $-15-8i$.

25. Find the square root of $7-24i$.

26. Find the cube root of unity.