

Radian-Transition Spectator Feedback for Predictable Qubit Phase Coherence

Company technical paper, reproducibility specification, IBM hardware screen and patent-counsel work package

EVIDENCE	STATUS
Frozen simulations passed their stated model checks. No IBM hardware result is claimed in this document. Hardware language is conditional on the included notebook returning a preregistered PASS.	

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Document control and legal-use boundary

NO HARDWARE ASSUMPTION MAY BE REPORTED AS FACT

The paper includes exact wording for a future pass because the requested IBM time is short. The values in that wording must come from END_QM_hardware_decision.json. Simulated values may never fill hardware fields.

Item	Value
Document	END Qubit Method long-form technical paper
Author / proposed inventor	Jordan Ryan Evans
Evidence freeze	Simulation seed 20260806; hardware prospective
Primary implementation	END-QM-6, public IBM dynamic circuit
Secondary implementation	Telemetry lexicon; lab-only END-QM/Dressed
License posture	Internal confidential evaluation only; signed terms recommended
Patent status	Unresolved; do not state patent pending until filed

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Abstract

We disclose END-QM, a confidence-gated phase-feedback method that converts two time-ordered, multi-axis sentinel-qubit observations into a bounded virtual-Z update on a separate data qubit. Three sentinels implement a six-sector radian encoder at two boundaries. The two three-bit observations are packed as a transition code; a pilot-only circular lexicon maps supported codes to conditional data phase. Ambiguous/low-support codes fall back to zero, and a reserved pilot-validation subset disables any local map that does not improve estimated coherence. A held-out comparison uses a latency-matched open-loop circuit, wrong-sign, raw-idle and XY4 controls across 12 start-phase windows. In a frozen synthetic common-mode drift model, END-QM achieved a 1.2219× median normalized equatorial-coherence AUC gain over matched open loop; the weakest window was 1.1055× and its simultaneous lower bound 1.0968×. These values are model-specific and not hardware evidence. A 96-circuit pilot and randomized 480-circuit confirmation notebook targets current IBM Runtime dynamic circuits within three requested job caps totaling 540 seconds. The package defines falsification, replication and IP-review boundaries and preserves an earlier richer-telemetry forecast separately.

Keywords: quantum control; dephasing; spectator qubits; dynamic circuits; virtual-Z; circular statistics; phase feedback; radian lexicon; reproducibility.

Contents

1. Executive decision brief
2. Claim taxonomy and non-claims
3. Conceptual provenance from Evans's END, radian lexicon and QDS work
4. Decoherence problem decomposition
5. Rotating-frame model
6. Radian state, frequency change and causal timing
7. Three-sentinel six-sector encoder
8. Circular lexicon and confidence structure
9. Controller and circuit semantics
10. Dressed-state extension
11. Synthetic model design
12. Frozen END-QM-6 results
13. Earlier telemetry-layer evidence
14. Negative controls and causal specificity
15. Robustness and domain of applicability
16. Statistical analysis plan
17. IBM hardware experiment
18. Hardware acceptance and falsification criteria

Contents — continued

19. Security, credentials and reproducibility
20. Patentability, prior art and ownership
21. Disclosure and filing strategy
22. Commercial evaluation pathway
23. Engineering roadmap
24. Limitations and failure interpretation
25. Conclusion
26. Equations and algorithm
27. Frozen numerical tables
28. Hardware submission checklist
29. Conditional result language
30. References
Appendix A. Simulation core
Appendix B. Frozen simulation runner
Appendix C. Unit tests
Appendix D. IBM Colab code

Notation and abbreviations

Symbol / term	Meaning
END-QM	END Qubit Method family
END-QM-6	Six-sector two-frame public-hardware embodiment
QDS / Δ QDS	Qubit dressed-state modulation / detuning-modulated variant
$\varphi, \Delta\varphi$	Wrapped phase and phase transition in radians
$f, \Delta f$	Frequency and frame-to-frame frequency change
c	Six-bit raw transition code
u_c	Bounded correction for code c
R	Circular resultant/concentration
AUC	Area under normalized coherence curve
matched_open	Same sensing/reset/latency as END-QM, with no correction
oracle	Simulation-only use of inaccessible true common phase
MCM	Mid-circuit measurement
VZ	Virtual-Z phase-frame update

1. Executive decision brief

END-QM is presented as a reproducible research asset, not as a guaranteed cure for decoherence. Its narrow claim is that a data qubit can sometimes retain more equatorial coherence when a repeatable, common-mode phase component is observed on separate sentinel qubits and converted into a bounded, independently validated frame correction. The method cannot restore energy lost through T1 relaxation, and it is expected to fail when the dominant noise is local, uncorrelated, too fast for the measurement latency, or too weakly observed.

The company-facing package separates three evidence levels. First, the full telemetry simulation inherited from QDC-RAFS shows what a high-quality external phase estimate could enable under a deliberately slow common-detuning model. Second, END-QM-6 models the public IBM embodiment with binary sentinels, ambiguous states, finite support, wrong-sign and shuffled controls, and a pilot-validation gate. Third, the IBM notebook executes the causal hardware screen. Only the third can support a hardware statement, and even a pass is limited to one device and calibration interval until replicated.

The requested commercial posture is therefore staged. A recipient can audit the mathematics and code without accepting a valuation; execute a short controlled screen; reproduce on additional dates/qubits; then negotiate an evaluation option or license if the result persists. This ordering reduces the chance that an enthusiastic simulation statement is mistaken for hardware evidence or a binding guarantee. It also creates a better record for patent counsel because the claimed combination, implementation details, failure modes and known prior-art pressure are stated before selective hardware reporting.

2. Claim taxonomy and non-claims

The phrase “extends a qubit state” is ambiguous. A method may rotate the observed phase, suppress an ensemble dephasing envelope, improve a task-specific survival probability, reduce coherent error, or change an exponential fit without altering intrinsic materials limits. END-QM therefore uses equatorial coherence and a fixed-grid normalized AUC as its primary observable. It does not rename that metric T2. A T2-style fit may be reported secondarily only with the exact fit window, model, weighting and uncertainty.

A virtual-Z operation is an efficient phase-frame update [1]. By itself it cannot increase the radial length of a Bloch vector; a precomputed virtual-Z schedule can merely rotate measurement phase. END-QM requires information that is correlated with the data-qubit phase error and a correction selected causally from that information. This is why the public embodiment measures separate sentinels and why the latency-matched open-loop circuit is the primary control. If END and matched open loop are indistinguishable, the method has not shown a feedback benefit.

The package makes no promise that IBM will accept a paper, buy a method, provide more time, or endorse the result. Scientific merit is determined by the protocol, data and replication; business interest is a separate decision. It also makes no claim that SMART was copied from Evans's work. The accessible SMART publication predates the supplied 2026 records [3]. Any earlier private provenance would need independently dated evidence and legal review.

- Claimable after simulation only: code executed as specified; frozen model behavior; unit-test results.
- Claimable after a one-device pass: only the named device/date/qubits and preregistered metric.
- Not claimable without replication: general coherence extension, scalability, fault-tolerance benefit, valuation or vendor acceptance.
- Never claim: reversal of T1 loss, universal decoherence solution, theft, patentability or freedom to operate without evidence.

3. Conceptual provenance from Evans's END, radian lexicon and QDS work

Evans's supplied materials repeatedly emphasize phase anchors, frame-to-frame changes, frequency and change in frequency, circular rather than linear phase handling, and an operational lexicon that maps recurrent patterns to corrections. The useful technical kernel is not a broad cosmological interpretation; it is the recognition that a periodic state must be estimated on a circle, that successive frames contain directional information, and that an observed change should actuate a later frame rather than be retroactively applied.

The QDS and Δ QDS materials add a control-system language based on a rotating-frame Hamiltonian with detuning, drive amplitude and drive phase. Their earlier Markovian simulation reported approximately 54.55 microseconds for a baseline fit and 78.63 microseconds for a Δ QDS fit, but the inferred ratio changed substantially with the fit window: a narrow window could yield about 2.12 while a broader 10–120 microsecond window yielded about 1.01. That sensitivity is evidence for stricter metrics, not evidence for a universal factor. END-QM keeps the Hamiltonian as a laboratory embodiment and replaces fit-driven screening with fixed-grid AUC.

The newer END monograph's explicit withdrawal of unsupported empirical claims is adopted as a methodological constraint. Frame count is not physical time unless calibrated; radian labels are coordinates, not mechanisms; pattern matching in unrelated time series does not prove quantum control. The present paper uses the lexicon only where a physical sensor, a causal data path and a falsifiable control exist.

4. Decoherence problem decomposition

For a data qubit, relaxation and dephasing must be separated. T1 processes change population and bound recoverable transverse coherence. Dephasing includes deterministic offsets, slowly wandering frequencies, common-mode fields, local stochastic noise, drive-induced shifts and measurement back-action. A feedback method is valuable only for the component that is both observable soon enough and correlated with the data error. Unobserved local noise remains in the residual channel.

Dynamical decoupling reshapes sensitivity to environmental spectra [2]. It can be highly effective without measuring the environment, but finite pulses, calibration, idle scheduling and hardware-specific noise affect its result [7]. Real-time Hamiltonian estimation and predictive compensation demonstrate that measured correlations can suppress dephasing in suitable systems [4,5]. Spectator-qubit methods likewise establish that separate quantum sensors can reveal shared disturbances [6]. END-QM is positioned within this known control landscape, not outside it.

The design challenge is therefore selective correction. If the controller reacts to low-support or ambiguous sensor outcomes, estimation noise becomes applied phase noise. If the controller learns and evaluates on the same shots, apparent gains can be optimistic. If it is compared only with raw idle, measurement latency can be misread as a feedback effect. END-QM addresses these issues with ambiguity fallback, support gates, a reserved pilot-validation subset and a matched dynamic-circuit control.

5. Rotating-frame model

In the rotating frame and under the usual two-level approximation, a driven qubit may be represented by a Hamiltonian containing detuning along z and a transverse drive in the equatorial plane. The END-QM public embodiment uses only calibrated gates, delays, measurement, reset and virtual-Z conditionals. The laboratory embodiment can expose the continuous controls explicitly.

Let the data phase accumulated during storage be the sum of a common component, an independent component and deterministic start phase. The sentinels measure a noisy function of the common component at two boundaries. The controller receives no privileged access to the true phase. The oracle curve in simulation uses true common phase only to show an unattainable upper bound; it is never part of the implementable result.

A control update changes the data-frame phase but cannot undo amplitude damping. With T_1 envelope $A(t)$, the observed equatorial vector is $A(t)$ times the circular mean of the residual phase. This decomposition makes the limitation visible: even perfect common-phase cancellation approaches the oracle envelope but cannot exceed the T_1 -limited amplitude represented in the model.

6. Radian state, frequency change and causal timing

The physical premise is that the phase advance between two timestamps depends on the integral of instantaneous detuning. When frequency changes, the next phase cannot be inferred from a single instantaneous phase alone. END-QM retains an ordered pair of observations. In the telemetry form this state may include measured frequency f_k and difference Δf_k . In END-QM-6 the ordered binary patterns act as a coarse proxy for the same transition.

Causality imposes an important boundary. A sensor observation at frame k may affect only operations scheduled after the corresponding classical result is available. The IBM circuit therefore keeps the data qubit alive across two sentinel measurements, performs the conditional virtual-Z update after the second measurement, and then leaves a final storage segment before analysis. The matched control preserves the same mid-circuit operations but omits the correction.

The two boundaries are fixed at 0.34 and 0.70 of the requested storage duration. They are not asserted to be globally optimal. They create a measurable transition and leave 30% of the nominal interval after actuation. A future design can optimize boundaries prospectively, but changing them after confirmation results would create a new protocol version.

7. Three-sentinel six-sector encoder

A single binary Ramsey observation supplies too little angular information for a one-shot phase label. END-QM-6 prepares three sentinel qubits with analysis axes separated by 120 degrees. Across the circle, six non-unanimous three-bit patterns dominate ideal sector responses. The unanimous patterns 000 and 111 are treated as ambiguous rather than forced into a sector.

This encoder trades precision for compatibility. It uses no two-qubit gate and therefore does not require connectivity between the data qubit and sentinels. It does require common-mode sensitivity: physical separation, frequency mismatch and local noise can reduce correlation. The notebook records selected qubits and calibration but cannot guarantee spatially common noise. A hardware failure may therefore indicate absent observability rather than an incorrect software implementation.

Two three-bit observations are packed into a raw code $c=s_0+8s_1$. Packing preserves both direction and endpoint without assuming arithmetic inside the dynamic circuit. IBM Runtime hardware supports classical conditions but places restrictions on arithmetic and nested flow [10]. The notebook therefore compiles a finite set of direct equality conditions, each applying a previously frozen angle.

Six resolvable radian sectors from three 120-degree sentinels

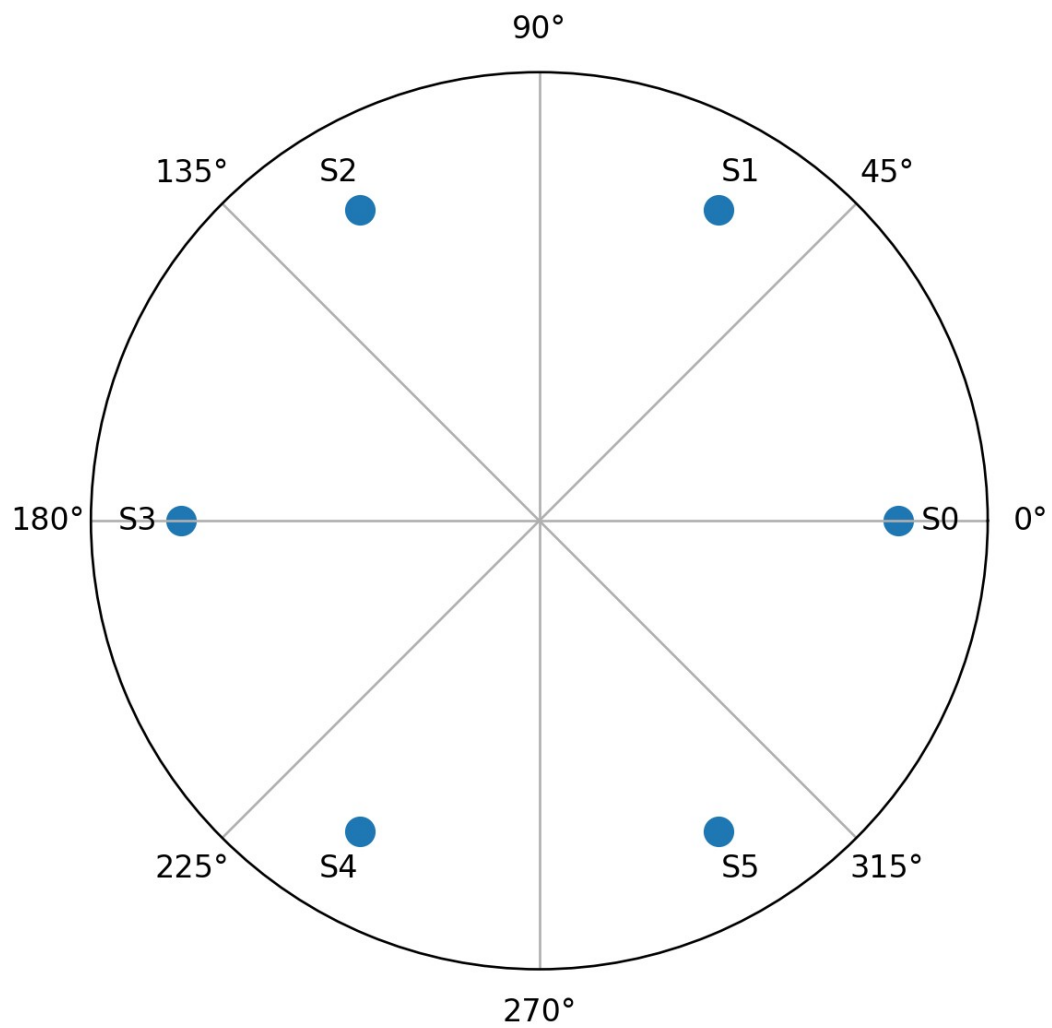


Figure 1. Six-sector encoding; unanimous patterns are fallback states.

8. Circular lexicon and confidence structure

For each start window, duration and transition code, pilot X/Y tomography estimates a conditional complex vector $m_x + i m_y$. Its argument is the predicted data phase, while its magnitude summarizes conditional concentration and readout attenuation. The proposed correction is the negative angle, clipped to $\pm\pi/2$. A code with insufficient X or Y support, an undefined vector, or an ambiguous half-state receives zero.

The local support rule prevents a rare transition from carrying a large phase based on a handful of outcomes. It does not prove that the map generalizes. END-QM therefore reserves 40% of pilot shots. The entire window-duration map is evaluated on those untouched pilot-validation shots; if it does not beat no correction, every angle in that local map is set to zero. This fail-closed gate is applied before confirmation circuit compilation.

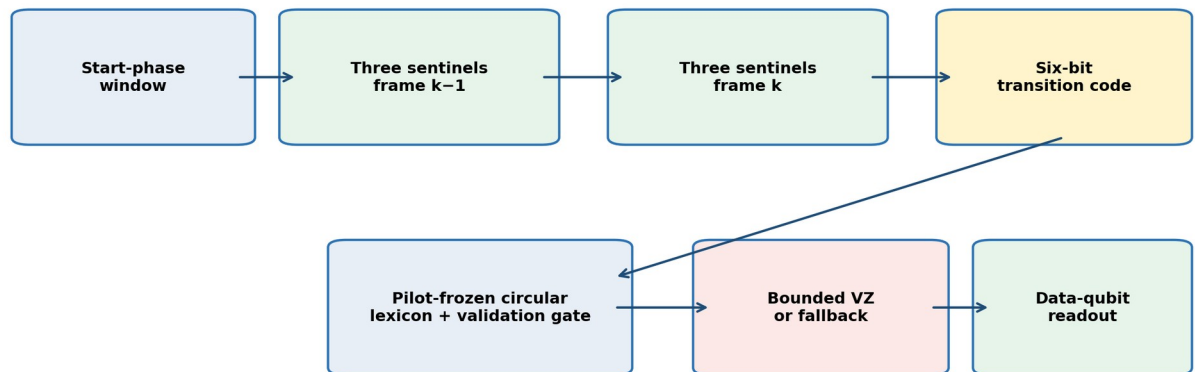
The hardware notebook records both pre-gate and final corrections. An evaluator can therefore distinguish a physically unobserved sector, a low-support sector and an apparently harmful full map. The fallback rate is a result, not an inconvenience. A method that reports gains only after discarding fallback-heavy windows would violate the all-window design.

9. Controller and circuit semantics

The dynamic circuit begins with an equatorial data state and three equatorial sentinels under the selected start-phase window. After the first delay segment, each sentinel is rotated into its analysis axis and measured. The sentinels are reset and re-prepared, then measured again after the second segment. The six classical bits select at most one bounded virtual-Z update on the data qubit. The final delay and X/Y analysis follow.

Mid-circuit measurement is not free. It may introduce hundreds of nanoseconds or more of latency, plus dephasing, reset error and crosstalk. For this reason, ``matched_open`` executes the same sensing and reset sequence with no conditional update. ``wrong_sign`` applies the frozen angle in the opposite direction. A genuine phase-cancellation mechanism should outperform both. Raw idle and XY4 help contextualize the engineering cost but do not replace the matched causal contrast.

The public notebook uses Qiskit 2.5-era circuit control flow and qiskit-ibm-runtime SamplerV2 conventions documented by IBM [9,10]. It does not call Qiskit Pulse because IBM removed public pulse-level control and Qiskit 2.0 removed the pulse module [15]. This boundary avoids presenting code that cannot run on the intended service.



Confirmation circuits compare END-QM with latency-matched open loop, wrong sign, raw idle and XY4.

Figure 2. END-QM-6 causal architecture and matched confirmation structure.

10. Dressed-state extension

The lab-only END-QM/Dressed embodiment combines the conditional radian update with Δ QDS-style modulation of detuning, amplitude and drive phase. The purpose is not to rename SMART or claim continuous driving broadly. It is to test whether a slowly varying dressed basis can reduce sensitivity while the discrete lexicon cancels a residual predictable component. These mechanisms may interact constructively or destructively.

A proper laboratory test must calibrate Rabi rate, detuning amplitude, modulation frequency, phase relation, leakage outside the computational subspace, AC Stark shift, power heating and pulse distortion. It must include continuous-drive-only, feedback-only, combined, wrong-sign and equal-power controls. A combined improvement cannot be assigned to the lexicon unless the feedback-only and wrong-sign contrasts are retained.

The supplied QDS simulation's strong fit-window dependence is a warning. The dressed protocol should use a preregistered time grid and AUC, then optionally fit physical models with residual diagnostics. Any claim that a modulation “extends T2” must report whether the estimate is Ramsey T2*, echo T2, driven coherence, or task-specific survival.

11. Synthetic model design

The END-QM-6 simulation is a mechanism forecast with common static detuning, linear slope, a synchronous sinusoidal component, independent data detuning, phase diffusion, finite sentinel noise and T1 attenuation. It does not simulate a named IBM backend. Parameters were frozen before the final held-out run; 30,000 pilot trajectories and 70,000 independent test trajectories were used for each window-duration cell.

The model gives the controller exactly the type of structure it is designed to exploit: a slow common component measured with high correlation. This makes a positive result plausible by construction, so the model cannot estimate the probability of success on an arbitrary QPU. Its legitimate role is to verify signs, circular wrapping, transition packing, fallback behavior, negative-control ordering, bootstrapping and output generation.

Twelve start-phase windows and five times from zero to 220 microseconds are evaluated. The fixed seed is 20260806. The simulator produces method coherence components, paired covariance, AUC, per-window ratios, simultaneous intervals, robustness sweeps, lexicon entries and an audit sample. Raw held-out phases are retained in compressed form.

12. Frozen END-QM-6 results

The final gated run produced a median normalized coherence-AUC gain of $1.2219\times$ over matched open loop. Its bootstrap 95% interval was $1.2178\text{--}1.2248\times$. The weakest of 12 start-phase windows was $1.1055\times$ and the weakest familywise lower bound was $1.0968\times$. Every window exceeded parity in this synthetic model.

The median fallback-code fraction was 0.6094. That high fraction is intentional: unanimous sensor halves, rare transitions and pilot maps that fail reserved validation are suppressed. The median latent correlation between common data phase and the second sentinel latent phase was 0.9923, which explains why the forecast is favorable and also shows why it cannot be generalized to hardware without measuring observability.

The median AUC values were 96.23 microseconds for END-QM, 78.76 for matched open loop, 75.12 for wrong sign, 75.09 for the shuffled map and 157.01 for the non-implementable common-phase oracle. The gap to the oracle indicates uncorrected independent noise, T1 loss, coarse sectorization and fallback. The control ordering is consistent with causal correction inside this model; it is not evidence about a QPU.

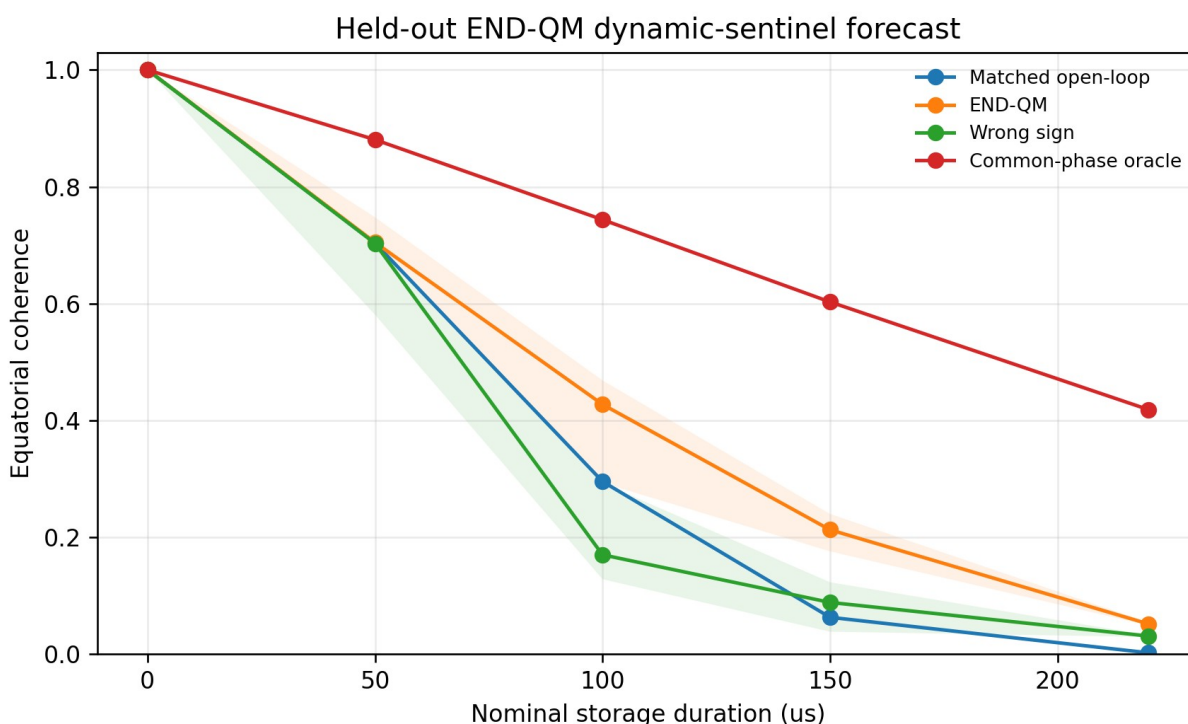


Figure 3. Frozen synthetic END-QM-6 coherence curves. Model result only.

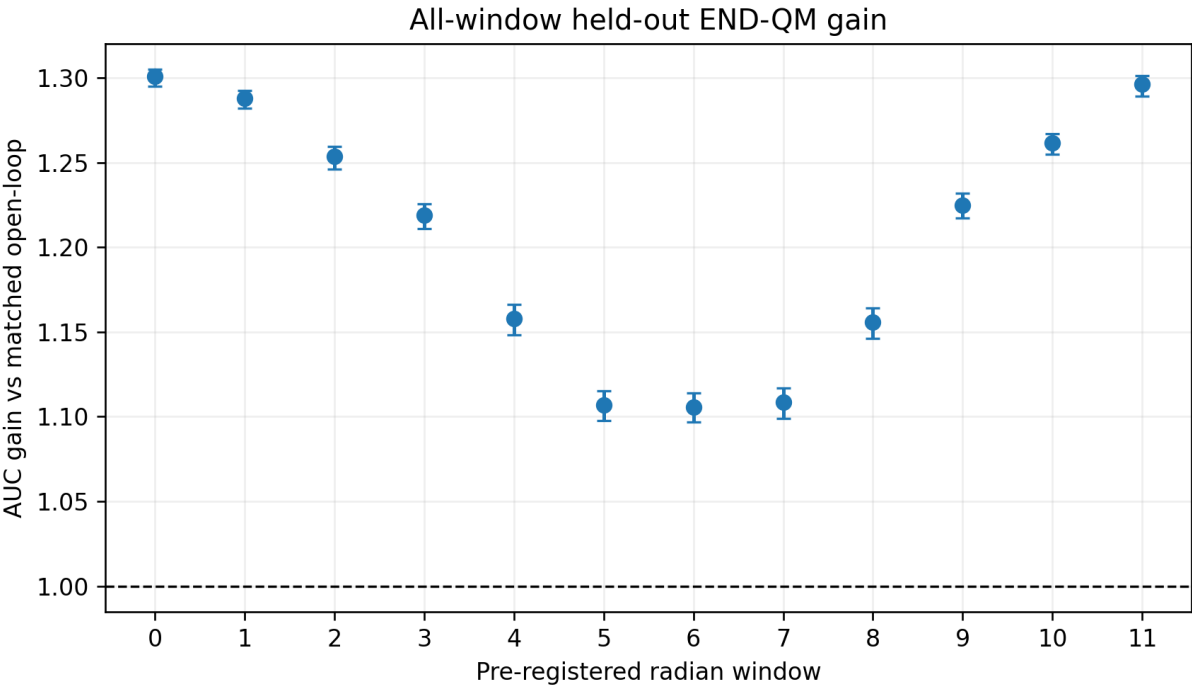


Figure 4. All-window END/matched AUC gains with simultaneous intervals.

13. Earlier telemetry-layer evidence

The preserved QDC-RAFS v0.1 simulation used a richer telemetry estimate rather than binary dynamic-circuit sentinels. Its held-out median AUC gain over unprotected storage was 2.6717×. The weakest window was 2.6290× and the weakest simultaneous lower bound was 2.5053×. At 160 microseconds the reported coherence was approximately 0.708 versus 0.096 unprotected.

That factor should not be attached to END-QM-6 hardware expectations. The telemetry model assumes a useful external estimate and compares primarily with unprotected storage. Its idealized XY4 AUC was about 224.93 microseconds and its QDC-RAFS AUC about 231.23 microseconds, only roughly 2.8% higher. This is the relevant scale when comparing with established suppression under the same slow-noise model.

The earlier model remains valuable as a software provenance layer and as an upper engineering scenario for systems with high-quality frequency telemetry. It is retained with original checksums and verifier output. No number was retroactively changed to match the new dynamic-sentinel simulation.

14. Negative controls and causal specificity

A wrong-sign correction should worsen or fail to improve coherence if the learned phase direction is meaningful. A shuffled map destroys the association between transition state and correction while preserving the angle distribution. An oracle establishes the best possible cancellation of the modeled common component. These controls answer different questions and should not be collapsed into a single baseline.

In the END-QM-6 forecast, both wrong-sign and shuffled maps fall below matched open loop in median AUC, while the oracle remains far above the implementable method. That ordering is stronger evidence of code correctness than a sole positive ratio. Hardware may not reproduce the ordering; a wrong-sign method that performs equally well or better would undermine the claimed causal direction even if END appeared above raw idle.

The IBM notebook implements wrong sign directly. It does not include shuffled map in the 10-minute confirmation allocation because the five-method, all-window design already contains 480 circuits. A paid or extended run should add a preregistered shuffled-map arm and repeated randomized compilations.

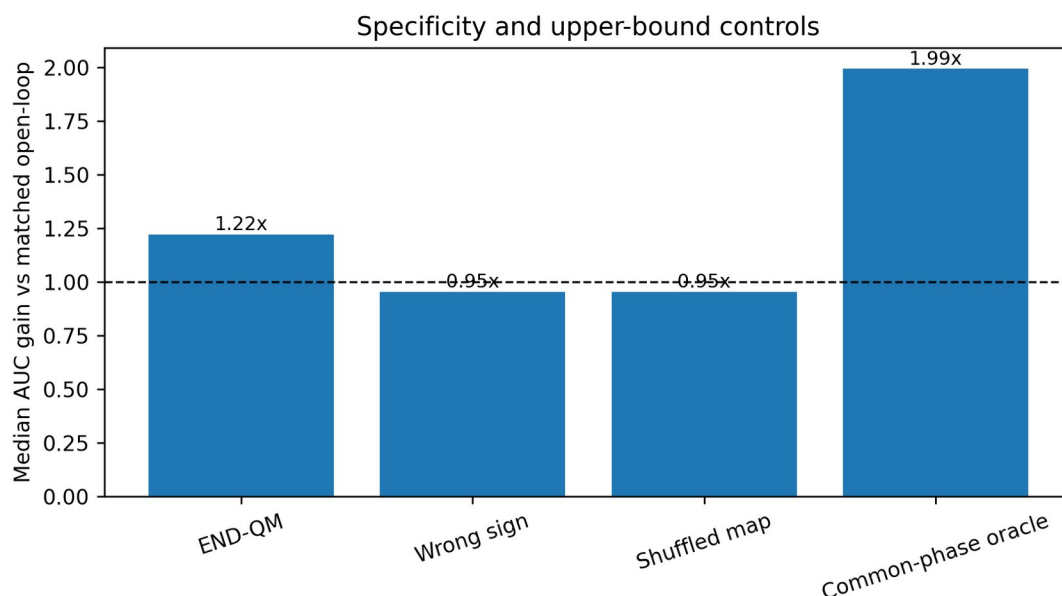


Figure 5. Frozen negative-control ordering and oracle upper bound.

15. Robustness and domain of applicability

The robustness sweep varies common-mode amplitude and sentinel noise. When common structure is absent, the controller should fall back or fail to improve. As common-mode observability grows, the method can cross from parity to benefit. High sensor noise reduces sector reliability and increases the need for support/validation gating. These are domain boundaries, not parameters to tune after hardware results.

The all-window safety gate is conservative but does not make END-QM risk-free. Pilot validation is itself noisy, and repeated opportunities can admit a false-positive map. A production study should use stronger multiplicity control, hierarchical shrinkage, sequential monitoring with error spending, and an independent day-level test. The short screen instead prioritizes transparent falsifiability under a tight QPU budget.

Hardware-specific risks include drift between pilot and confirmation, differing noise at sentinel locations, reset-induced heating, conditional-branch latency, readout bias, compiler changes and phase conventions. Raw circuits, calibration and job IDs are therefore part of the result. A single summary plot is insufficient for technical diligence.

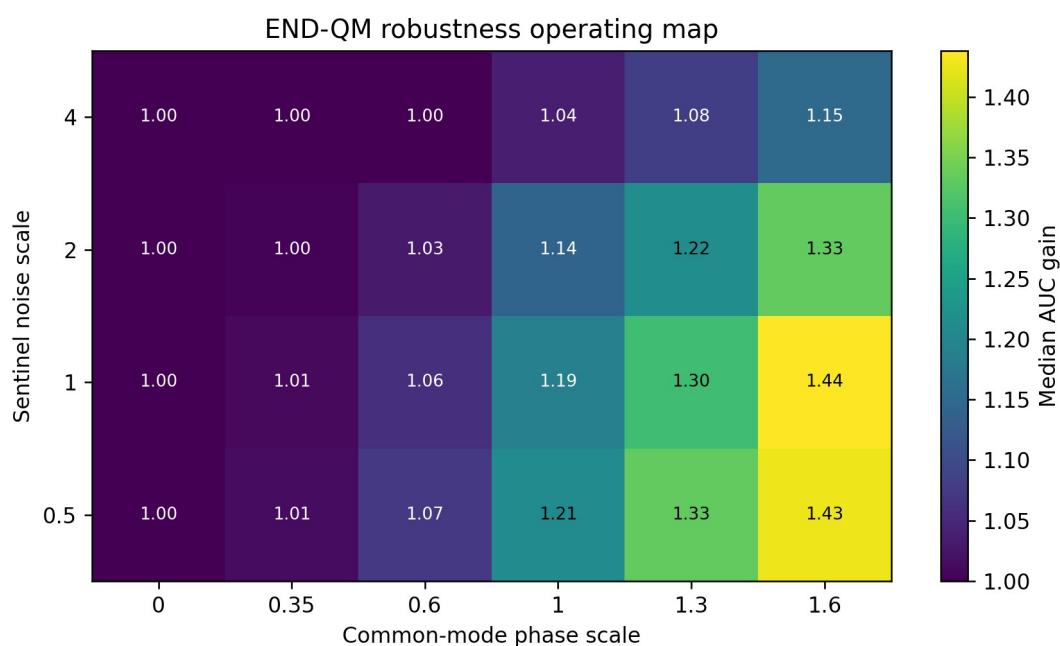


Figure 6. Sensitivity to common-mode amplitude and sentinel noise.

16. Statistical analysis plan

At each method/window/duration, separate X and Y circuits estimate an equatorial vector. The coherence magnitude is normalized by the zero-delay value for the same method and window. AUC is integrated over the adaptively selected but prospectively fixed duration grid. Normalization reduces state-preparation/readout scaling differences, while the matched comparison reduces dynamic-circuit overhead confounding.

The hardware notebook computes a ratio for each of 12 start windows and bootstraps the median by resampling windows. This treats windows as the screening unit; it does not fully model shot-level binomial uncertainty or temporal drift. A subsequent confirmatory analysis should use a hierarchical binomial model or paired bootstrap incorporating shot and circuit layers, report simultaneous intervals for every window, and predefine missing-data behavior.

The threshold of 1.10 is an engineering screening threshold chosen before hardware results. It is not derived from a cost model and does not imply economic materiality. Effect size, interval, control ordering, fallback rate and operational burden should be reported together. P-values without these quantities would not resolve the practical question.

17. IBM hardware experiment

The notebook authenticates with `QiskitRuntimeService(token=key, instance="auto")`, selects an operational QPU, ranks qubits using current T1/T2 and readout error, and chooses four without requiring coupling. Durations are zero plus 0.25, 0.50 and 0.85 of the current data-qubit T2, clipped to 16–220 microseconds. The selected values are printed before submission.

The pilot has 96 circuits at 2,048 shots. Its job requests a 160-second maximum. The 480 held-out circuits are randomized and split into two 240-circuit jobs at 512 shots, each requesting 190 seconds. Requested caps sum to 540 seconds, within a ten-minute target, but the QPU controls actual metering and may reject a plan. IBM documents a rolling Open Plan allowance and job execution limits [11–13].

The code records each job ID immediately before blocking for a result. If Colab disconnects, the job is recovered from the service rather than blindly resubmitted. All raw per-shot sentinel/data records, the pilot split, frozen lexicon, confirmation AUC, decision JSON and plot are downloaded. The executed notebook must be sanitized to remove the API key before sharing.

18. Hardware acceptance and falsification criteria

The screen reports PASS only when the median END/matched AUC ratio is at least 1.10, its paired-window bootstrap lower 95% bound exceeds 1.00, and END exceeds wrong sign. These criteria are conjunctive. A favorable raw-idle comparison cannot rescue failure against the matched control. A favorable subset of windows cannot rescue a failed median or a broad interval.

A PASS permits a narrow sentence naming backend, qubits, calibration date, metric, interval and pending replication. It does not permit the phrases “solves decoherence,” “extends intrinsic T2,” “accepted by IBM,” “worth \$100 billion,” or “proves patent value.” A DO_NOT_CLAIM_PASS result is preserved and used to decide whether observability, timing or physical assumptions failed.

Generalization requires at least three dates and two data qubits, with failed attempts included. A stronger company study should add randomized duplicate circuits, shuffled maps, measurement-error analysis, leakage measurements, T1 controls, neighbor idling, crosstalk monitors and compiler/version locking. Those are not hidden conditions for the first screen; they are the next diligence tier.

19. Security, credentials and reproducibility

The notebook exposes one simple credential line, ``key="..."``, because the user requested immediate copy-paste use. It does not call an account-saving API. This minimizes persistence but does not make a published notebook safe: an executed cell can still retain the literal key. The key must be removed, rotated if exposed, and never included in a company package.

Reproducibility depends on more than code. The package stores frozen configuration, environment versions, complete CSVs, compressed audit samples, figures, unit tests and source checksums. The IBM result must additionally store backend name, physical layout, calibrations, transpiled circuits or QPY artifacts where possible, job IDs, UTC times and Qiskit versions.

The simulator and notebook serve different purposes. Local tests verify pure functions and output contracts. Static notebook verification checks syntax and required current-API tokens but cannot substitute for importing Qiskit and transpiling against a live backend. The package labels this limitation instead of presenting an unexecuted notebook as hardware-validated code.

20. Patentability, prior art and ownership

Patent protection cannot be created by strong wording inside a paper. Protection may arise from a filed patent application, enforceable confidentiality, copyright in expression/code, trade-secret practices, or contracts. Each has different requirements. A confidentiality legend is useful notice but may not bind a recipient without agreement. “Patent pending” should not appear until an application has actually been filed.

The field is crowded. Virtual-Z gates, dynamical decoupling, real-time Hamiltonian estimation, predictive compensation, spectator sensing, SMART modulation and closed-loop frequency feedback predate this package [1–8,16–18]. US11695417B1 is particularly relevant to broad closed-loop Ramsey/frequency claims. Counsel should claim no known component in isolation and should prepare element-by-element charts against patents and non-patent literature.

The proposed point of differentiation is the ordered combination: anchor window; two time-ordered multi-axis sensor observations; transition encoding; circular conditional map; support/ambiguity gate; bounded actuation; reserved pilot-validation gate that disables a local map; and matched-latency causal validation. Whether that combination is novel and non-obvious remains unresolved. The invention disclosure and matrix are work inputs, not legal conclusions.

21. Disclosure and filing strategy

CIPO advises inventors to consider filing before disclosure, and grace-period rules differ internationally [19,20]. A Canadian or US grace period is not a safe substitute for a coordinated foreign filing strategy. Before sending non-public implementation details, record the recipient, purpose, exact files, date and NDA status. Public cloud execution may itself create logs; counsel should advise whether that matters for the chosen strategy.

A provisional US application can establish a filing date but must contain adequate technical disclosure and does not mature without further action [21]. It is not a one-page placeholder for an idea. The present package improves enablement by including circuit structure, maps, thresholds, alternatives and failure cases, but counsel must decide jurisdiction, claim scope and inventorship.

Trade-secret protection requires reasonable secrecy measures [22]. If the core map-construction or calibration process is kept secret, distribute only what is necessary under written controls. Conversely, a hardware reproducibility paper may require disclosure. The commercialization decision must therefore choose deliberately between publication, patent filing, confidential evaluation and retained know-how.

22. Commercial evaluation pathway

A technical buyer is unlikely to value an unreplicated simulation at a large figure. A credible path begins with measurable burden and benefit: extra qubits, mid-circuit latency, shots, calibration frequency, compilation complexity and improvement relative to the buyer's best existing suppression. END-QM consumes three sentinels per data qubit in its simplest form, which is expensive for scaling and must be justified by a meaningful effect.

The first commercial milestone is not a sale; it is an independently reproduced evaluation under signed terms. The second is a scoped integration study showing repeatability, resource cost and failure detection. The third is a negotiated option, license, sponsored research agreement or assignment with clear fields of use, milestones, improvements, publication rights, indemnities and background-IP definitions.

No valuation is supplied in this paper. Value depends on effect size, reproducibility, compatibility with error-correction stacks, freedom to operate, alternative solutions and buyer-specific economics. The strongest negotiating asset is a clean, timestamped record of an enabled method and independent data —not an unsupported savings estimate.

23. Engineering roadmap

Phase 1 is the short IBM screen. Phase 2 repeats without changing thresholds on three dates and two data qubits, adds shuffled maps and stores transpiled artifacts. Phase 3 estimates common-mode spatial correlation and tests whether one sentinel bank can serve multiple data qubits. Phase 4 compares END-QM with optimized DD sequences under equal wall-clock time and compiler constraints.

A laboratory branch should test analog END-QM/Dressed only after gate-level observability is established. Its experiments should prospectively sweep modulation rate and amplitude, quantify leakage and heating, and compare equal-power SMART-like controls. A theory branch should derive a noise-transfer function that includes sensor measurement and feedback latency rather than relying only on Monte Carlo forecasts.

A product branch would require online calibration, drift detection, automatic map rollback, audit logs and integration with scheduling. Maps should expire when calibration changes or observability drops. The fallback behavior that appears conservative in simulation becomes a core safety feature in operation.

24. Limitations and failure interpretation

The simulation uses a high common-phase correlation and therefore demonstrates a conditional mechanism rather than a realistic backend forecast. It omits many microscopic effects, including leakage, non-Markovian local noise, readout crosstalk, reset heating, branch-duration variation and correlated gate errors. It treats the selected time grid and T1 envelope simply. Hardware can fail despite correct code.

A positive one-device result can also be misleading. Calibration drift between pilot and confirmation might favor one randomized block; readout asymmetry can bias reconstructed coherence; the virtual-Z convention may interact with analysis phases; and dynamic-circuit scheduling may differ by branch. These risks motivate raw-shot release, randomization, matched controls and replication.

A negative result does not prove that every END concept is false. It can show that this encoder, QPU, timing, resource allocation or common-mode assumption is inadequate. But the preregistered embodiment must still be called failed. Redesign is legitimate only as a new version with a new prospective test.

25. Conclusion

END-QM converts Evans's frame-to-frame radian idea into a bounded and falsifiable quantum-control protocol. Its substantive contribution is not a promise of perfect coherence; it is an experimentally auditable path from two physical sensor frames through a circular transition lexicon to a later data-qubit actuation, with conservative fallback and a matched causal control.

The frozen END-QM-6 simulation passed its model acceptance checks with a 1.222× median AUC ratio and a 1.106× weakest window. The favorable model assumes measurable common-mode drift, so hardware remains the decision point. The included notebook is designed to make that decision quickly while preserving failure evidence.

For company and patent review, the package is intentionally exact about what exists and what does not. It provides enablement, source, tests, outputs, prior-art pressure and conditional result language. It does not manufacture an IBM endorsement, hardware result, valuation or legal monopoly. Those boundaries make a later positive result more credible and a later filing more useful to counsel.

26. Equations and exact algorithm

$$H(t) = -\Delta(t)\sigma_z/2 + \Omega(t)[\cos \varphi_d(t)\sigma_x + \sin \varphi_d(t)\sigma_y]/2 \quad (1)$$

$$\varphi_k = \text{wrap}[\varphi_0 + 2\pi \int_0^{t_k} \delta f(t) dt] \quad (2)$$

$$c = s_{\{k-1\}} + 8 s_k, \quad s_k \in \{0, \dots, 7\} \quad (3)$$

$$\mu_c = \text{atan2}(m_{\{y,c\}}, m_{\{x,c\}}), \quad u_c = \text{clip}(-\mu_c, -\pi/2, +\pi/2) \quad (4)$$

$$C(t) = \sqrt{[\langle X \rangle^2 + \langle Y \rangle^2]}, \quad \text{AUC} = \int C(t)/C(0) dt \quad (5)$$

$$G_w = \text{AUC}_{\text{END},w} / \text{AUC}_{\text{matched},w} \quad (6)$$

Algorithm 1 — pilot and confirmation

- Freeze windows, boundary fractions, durations, code-support threshold, correction bound, pilot split and pass rule.
- Collect pilot X/Y shots with two sentinel epochs and no data correction.
- Use pilot-fit shots to estimate conditional circular phase for each window-duration-code cell.
- Set correction to zero for ambiguous/unsupported cells; clip every other angle.
- Evaluate each complete local map on untouched pilot-validation shots; disable it unless gain > 1.
- Compile and randomize held-out END, matched-open, wrong-sign, raw-idle and XY4 circuits.
- Calculate normalized equatorial AUC and all-window ratios; issue PASS only under the frozen conjunctive rule.
- Archive raw shots, maps, calibrations, job IDs, versions and failures.

27. Frozen numerical tables

Window	AUC gain	FWER 95% low	FWER 95% high
0	1.3007	1.2949	1.3050
1	1.2878	1.2819	1.2924
2	1.2534	1.2462	1.2594
3	1.2188	1.2111	1.2255
4	1.1579	1.1481	1.1661
5	1.1067	1.0976	1.1151
6	1.1055	1.0968	1.1140
7	1.1086	1.0990	1.1170
8	1.1556	1.1463	1.1640
9	1.2249	1.2171	1.2317
10	1.2613	1.2548	1.2669
11	1.2960	1.2891	1.3014

Table 1. Frozen END-QM-6 held-out synthetic ratios. FWER intervals are simultaneous model-based intervals. They do not quantify backend uncertainty.

Method	Median AUC (μ s)
end_qm	96.2255
matched_open	78.7557
oracle_common_phase	157.0142
shuffled_map	75.0863
wrong_sign	75.1235

27.1 Robustness grid

Common scale	Sensor scale	Median correlation	Median gain	Min gain
0.00	0.50	—	1.000	1.000
0.00	1.00	—	1.000	1.000
0.00	2.00	—	1.000	1.000
0.00	4.00	—	1.000	1.000
0.35	0.50	0.992	1.014	1.000
0.35	1.00	0.992	1.011	1.000
0.35	2.00	0.992	1.004	1.000
0.35	4.00	0.992	1.000	1.000
0.60	0.50	0.992	1.073	1.000
0.60	1.00	0.992	1.061	1.000
0.60	2.00	0.992	1.032	1.000
0.60	4.00	0.992	1.000	1.000
1.00	0.50	0.992	1.212	1.111
1.00	1.00	0.992	1.191	1.083
1.00	2.00	0.992	1.141	1.023
1.00	4.00	0.992	1.036	1.000

INTERPRETATION

The favorable frozen setting corresponds to a highly observable common phase. The zero-common rows define a required failure domain. Hardware must measure its own location on this observability continuum.

28. Hardware submission checklist

- ☐ Installation cell completed with Qiskit≈2.5 and qiskit-ibm-runtime≈0.47.
- ☐ API key pasted only in key= and removed before sharing.
- ☐ Backend, four physical qubits, calibration values and durations captured.
- ☐ First transpiled pilot circuit visually inspected.
- ☐ Pilot job ID copied before waiting.
- ☐ Pilot raw CSV and frozen lexicon downloaded.
- ☐ Two confirmation job IDs copied before waiting.
- ☐ Confirmation raw CSV, AUC CSV, decision JSON and plot downloaded.
- ☐ No simulation value inserted into a hardware statement.
- ☐ Failures, missing circuits and backend recalibrations reported.
- ☐ Executed notebook sanitized; key rotated if exposed.
- ☐ Counsel consulted before public disclosure or “patent pending” language.

29. Conditional result language

USE	ONLY	AFTER	JSON	STATUS	=	PASS
On [backend], calibration timestamp [timestamp], the preregistered END-QM-6 screen produced a median normalized equatorial-coherence AUC gain of [value] over the latency-matched dynamic-circuit control, with a paired-window bootstrap 95% interval of [low, high], and outperformed the wrong-sign control. This is a one-device screening result; replication across at least three calibration dates and two data qubits is pending.						

USE	AFTER	ANY	NON-PASS
The preregistered END-QM-6 hardware screen did not meet its decision threshold. The result is retained as a falsification or redesign input; no hardware coherence-extension claim is made.			

A company cover note should attach raw data and identify the exact notebook commit/checksum. It should not imply that IBM reviewed or accepted the method merely because an IBM QPU executed circuits.

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Appendix A. Simulation core — listing 1 of 9

```

0001 """Causal END-QM two-frame radian-transition model.
0002
0003 This module models only a correctable phase-noise channel. It does not model
0004 hardware as a whole and it does not claim to remove T1 relaxation, leakage,
0005 readout error, or unobserved bath entanglement. Frequencies are in hertz,
0006 times are in seconds, and phases are in radians.
0007 """
0008
0009 from __future__ import annotations
0010
0011 from dataclasses import asdict, dataclass, replace
0012 from typing import Any
0013
0014 import numpy as np
0015
0016
0017 TAU = 2.0 * np.pi
0018 SENSOR_AXES_RAD = np.array([0.0, TAU / 3.0, 2.0 * TAU / 3.0])
0019
0020
0021 def wrap_angle(value: np.ndarray | float) -> np.ndarray | float:
0022     """Wrap an angle to [-pi, pi)."""
0023
0024     return (np.asarray(value) + np.pi) % TAU - np.pi
0025
0026
0027 def raw_transition_code(first_pattern: np.ndarray, second_pattern: np.ndarray) -> np.ndarray:
0028     """Pack two three-bit sentinel patterns into one six-bit integer."""
0029
0030     return np.asarray(first_pattern, dtype=np.int16) + 8 * np.asarray(second_pattern,
dtype=np.int16)
0031
0032
0033 def pattern_is_ambiguous(pattern: int) -> bool:
0034     """All-equal three-sensor outcomes have no preferred hexagonal sector."""
0035
0036     return int(pattern) in (0, 7)
0037
0038
0039 def transition_is_ambiguous(code: int) -> bool:
0040     """Return True when either three-bit frame has no sector direction."""
0041
0042     return pattern_is_ambiguous(code & 7) or pattern_is_ambiguous((code >> 3) & 7)

```

Appendix A. Simulation core — listing 2 of 9

```

0043
0044
0045 def sector_vector(pattern: int) -> complex:
0046     """Return the signed three-axis vector for a sentinel pattern."""
0047
0048     signs = np.array([1.0 if ((pattern >> j) & 1) == 0 else -1.0 for j in range(3)])
0049     return complex(np.sum(signs * np.exp(1j * SENSOR_AXES_RAD)))
0050
0051
0052 def pattern_sector(pattern: int) -> int | None:
0053     """Map a non-ambiguous pattern to one of six radian sectors."""
0054
0055     vector = sector_vector(pattern)
0056     if abs(vector) < 1e-12:
0057         return None
0058     return int(np.round((np.angle(vector) % TAU) / (TAU / 6.0))) % 6
0059
0060
0061 @dataclass(frozen=True)
0062 class ENDQMConfig:
0063     """Frozen synthetic model and confidence-gate parameters."""
0064
0065     seed: int = 20260806
0066     windows: int = 12
0067     times_us: tuple[float, ...] = (0.0, 50.0, 100.0, 150.0, 220.0)
0068     first_boundary_fraction: float = 0.34
0069     second_boundary_fraction: float = 0.70
0070     t1_us: float = 260.0
0071     common_static_sigma_hz: float = 2_080.0
0072     common_slope_sigma_hz_per_s: float = 11_700_000.0
0073     common_sync_sigma_hz: float = 780.0
0074     common_sync_period_us: float = 100.0
0075     data_independent_static_sigma_hz: float = 650.0
0076     data_diffusion_rad_at_220us: float = 0.30
0077     sentinel_independent_sigma_hz: float = 400.0
0078     sentinel_readout_phase_sigma_rad: float = 0.11
0079     concentration_floor: float = 0.10
0080     support_shrinkage: float = 35.0
0081     minimum_cell_support: int = 15
0082     pilot_fit_fraction: float = 0.60
0083     pilot_validation_minimum_gain: float = 1.0
0084     maximum_correction_rad: float = np.pi / 2.0

```

Appendix A. Simulation core — listing 3 of 9

```

0085     n_train: int = 30_000
0086     n_test: int = 70_000
0087     n_bootstrap: int = 6_000
0088
0089     def to_dict(self) -> dict[str, Any]:
0090         return asdict(self)
0091
0092
0093     @dataclass(frozen=True)
0094     class LexiconEntry:
0095         """One raw transition-code entry in the circular residual lexicon."""
0096
0097         mean_phase_rad: float
0098         resultant_length: float
0099         support: int
0100         confidence: float
0101         correction_rad: float
0102         fallback: bool
0103
0104
0105     def _integrated_common_phase(
0106         time_s: float,
0107         static_hz: np.ndarray,
0108         slope_hz_per_s: np.ndarray,
0109         sync_hz: np.ndarray,
0110         sync_phase: np.ndarray,
0111         sync_period_s: float,
0112     ) -> np.ndarray:
0113         """Integrate a static + linear + sinusoidal detuning trajectory."""
0114
0115         omega = TAU / sync_period_s
0116         sync_integral = sync_hz / omega * (
0117             np.cos(sync_phase) - np.cos(omega * time_s + sync_phase)
0118         )
0119         return TAU * (
0120             static_hz * time_s
0121             + 0.5 * slope_hz_per_s * time_s * time_s
0122             + sync_integral
0123         )
0124
0125
0126     def generate_shots(

```

Appendix A. Simulation core — listing 4 of 9

```

0127     rng: np.random.Generator,
0128     n_shots: int,
0129     time_s: float,
0130     start_phase_rad: float,
0131     cfg: ENDQMConfig,
0132     *,
0133     common_scale: float = 1.0,
0134     sensor_phase_scale: float = 1.0,
0135 ) -> dict[str, np.ndarray]:
0136     """Generate data phase and two measured three-sentinel frames.
0137
0138     The same latent detuning drives the data qubit and both sentinel frames.
0139     Independent data and sentinel terms make transfer imperfect. The model is
0140     intentionally favorable to a common-mode tracker; robustness sweeps expose
0141     where that assumption fails.
0142     """
0143
0144     static = rng.normal(0.0, cfg.common_static_sigma_hz * common_scale, n_shots)
0145     slope = rng.normal(0.0, cfg.common_slope_sigma_hz_per_s * common_scale, n_shots)
0146     sync = rng.normal(0.0, cfg.common_sync_sigma_hz * common_scale, n_shots)
0147     sync_phase = rng.uniform(0.0, TAU, n_shots)
0148     period_s = cfg.common_sync_period_us * 1e-6
0149
0150     def common_at(fraction: float) -> np.ndarray:
0151         return _integrated_common_phase(
0152             time_s * fraction, static, slope, sync, sync_phase, period_s
0153         )
0154
0155     common_total = common_at(1.0)
0156     diffusion_sigma = cfg.data_diffusion_rad_at_220us * np.sqrt(
0157         max(time_s, 0.0) / 220e-6
0158     )
0159     data_phase = (
0160         common_total
0161         + TAU
0162         * rng.normal(0.0, cfg.data_independent_static_sigma_hz, n_shots)
0163         * time_s
0164         + rng.normal(0.0, diffusion_sigma, n_shots)
0165         + start_phase_rad
0166     )
0167
0168     patterns: list[np.ndarray] = []

```

Appendix A. Simulation core — listing 5 of 9

```

0169     sentinel_latent: list[np.ndarray] = []
0170     for fraction in (cfg.first_boundary_fraction, cfg.second_boundary_fraction):
0171         boundary_s = time_s * fraction
0172         common_phase = common_at(fraction)
0173         phase = (
0174             common_phase[:, None]
0175             + start_phase_rad
0176             + TAU
0177             * rng.normal(
0178                 0.0,
0179                 cfg.sentinel_independent_sigma_hz * sensor_phase_scale,
0180                 size=(n_shots, 3),
0181             )
0182             * boundary_s
0183             + rng.normal(
0184                 0.0,
0185                 cfg.sentinel_readout_phase_sigma_rad * sensor_phase_scale,
0186                 size=(n_shots, 3),
0187             )
0188         )
0189         probability_zero = (1.0 + np.cos(phase - SENSOR_AXES_RAD[None, :])) / 2.0
0190         bits = (rng.random((n_shots, 3)) > probability_zero).astype(np.int8)
0191         patterns.append(bits[:, 0] + 2 * bits[:, 1] + 4 * bits[:, 2])
0192         sentinel_latent.append(common_phase)
0193
0194     code = raw_transition_code(patterns[0], patterns[1])
0195     return {
0196         "data_phase_rad": data_phase,
0197         "transition_code": code,
0198         "first_pattern": patterns[0],
0199         "second_pattern": patterns[1],
0200         "common_total_phase_rad": common_total,
0201         "second_sentinel_latent_phase_rad": sentinel_latent[1],
0202     }
0203
0204
0205 def fit_transition_lexicon(
0206     data_phase_rad: np.ndarray,
0207     transition_code: np.ndarray,
0208     cfg: ENDQMConfig,
0209     *,
0210     zero_correction: bool = False,

```

Appendix A. Simulation core — listing 6 of 9

```

0211 ) -> tuple[np.ndarray, list[LexiconEntry]]:
0212     """Fit a training-only 64-code circular lexicon with automatic fallback."""
0213
0214     corrections = np.zeros(64, dtype=float)
0215     entries: list[LexiconEntry] = []
0216     for code in range(64):
0217         values = data_phase_rad[transition_code == code]
0218         support = int(values.size)
0219         if support:
0220             circular_mean = np.mean(np.exp(1j * values))
0221             mu = float(np.angle(circular_mean))
0222             resultant = float(abs(circular_mean))
0223         else:
0224             mu = 0.0
0225             resultant = 0.0
0226         concentration = np.clip(
0227             (resultant - cfg.concentration_floor)
0228             / max(1.0 - cfg.concentration_floor, 1e-12),
0229             0.0,
0230             1.0,
0231         )
0232         support_factor = np.sqrt(support / (support + cfg.support_shrinkage)) if support else 0.0
0233         confidence = float(concentration * support_factor)
0234         fallback = (
0235             zero_correction
0236             or support < cfg.minimum_cell_support
0237             or transition_is_ambiguous(code)
0238         )
0239         correction = 0.0 if fallback else float(
0240             np.clip(-confidence * mu, -cfg.maximum_correction_rad, cfg.maximum_correction_rad)
0241         )
0242         corrections[code] = correction
0243         entries.append(
0244             LexiconEntry(
0245                 mean_phase_rad=mu,
0246                 resultant_length=resultant,
0247                 support=support,
0248                 confidence=confidence,
0249                 correction_rad=correction,
0250                 fallback=fallback,
0251             )
0252     )

```

Appendix A. Simulation core — listing 7 of 9

```

0253     return corrections, entries
0254
0255
0256 def _coherence_stats(phase_rad: np.ndarray, t1_envelope: float) -> dict[str, float]:
0257     values = np.column_stack((np.cos(phase_rad), np.sin(phase_rad)))
0258     mean = values.mean(axis=0)
0259     cov = np.cov(values, rowvar=False, ddof=1) / max(len(values), 1)
0260     return {
0261         "x": float(mean[0] * t1_envelope),
0262         "y": float(mean[1] * t1_envelope),
0263         "coherence": float(np.hypot(*mean) * t1_envelope),
0264         "cov_xx": float(cov[0, 0] * t1_envelope**2),
0265         "cov_xy": float(cov[0, 1] * t1_envelope**2),
0266         "cov_yy": float(cov[1, 1] * t1_envelope**2),
0267         "shots": int(len(values)),
0268     }
0269
0270
0271 def simulate_window(
0272     rng: np.random.Generator,
0273     time_us: float,
0274     window_index: int,
0275     cfg: ENDQMConfig,
0276     *,
0277     common_scale: float = 1.0,
0278     sensor_phase_scale: float = 1.0,
0279     n_train: int | None = None,
0280     n_test: int | None = None,
0281 ) -> dict[str, Any]:
0282     """Fit on pilot shots and evaluate independent held-out shots."""
0283
0284     time_s = time_us * 1e-6
0285     start_phase = TAU * window_index / cfg.windows
0286     train = generate_shots(
0287         rng,
0288         n_train or cfg.n_train,
0289         time_s,
0290         start_phase,
0291         cfg,
0292         common_scale=common_scale,
0293         sensor_phase_scale=sensor_phase_scale,
0294     )

```

Appendix A. Simulation core — listing 8 of 9

```

0295     split = int(len(train["data_phase_rad"]) * cfg.pilot_fit_fraction)
0296     corrections, entries = fit_transition_lexicon(
0297         train["data_phase_rad"][:split],
0298         train["transition_code"][:split],
0299         cfg,
0300         zero_correction=(time_us == 0.0),
0301     )
0302     validation_phase = train["data_phase_rad"][split:]
0303     validation_code = train["transition_code"][split:]
0304     validation_open = abs(np.mean(np.exp(1j * validation_phase)))
0305     validation_end = abs(
0306         np.mean(np.exp(1j * (validation_phase + corrections[validation_code])))
0307     )
0308     validation_gain = float(validation_end / max(validation_open, 1e-12))
0309     pilot_gate_pass = bool(
0310         time_us > 0.0 and validation_gain > cfg.pilot_validation_minimum_gain
0311     )
0312     if not pilot_gate_pass:
0313         corrections = np.zeros_like(corrections)
0314         entries = [
0315             replace(entry, correction_rad=0.0, fallback=True) for entry in entries
0316         ]
0317     test = generate_shots(
0318         rng,
0319         n_test or cfg.n_test,
0320         time_s,
0321         start_phase,
0322         cfg,
0323         common_scale=common_scale,
0324         sensor_phase_scale=sensor_phase_scale,
0325     )
0326     code = test["transition_code"]
0327     data = test["data_phase_rad"]
0328     shuffled = np.roll(corrections, 8)
0329     t1_envelope = float(np.exp(-time_us / (2.0 * cfg.t1_us)))
0330     methods = {
0331         "matched_open": data,
0332         "end_qm": data + corrections[code],
0333         "wrong_sign": data - corrections[code],
0334         "shuffled_map": data + shuffled[code],
0335         "oracle_common_phase": data - test["common_total_phase_rad"],
0336     }

```


Appendix A. Simulation core — listing 9 of 9

```

0337     stats = {name: _coherence_stats(phase, t1_envelope) for name, phase in methods.items()}
0338     base_phase = methods["matched_open"]
0339     base_vectors = np.column_stack((np.cos(base_phase), np.sin(base_phase))) * t1_envelope
0340     for name, phase in methods.items():
0341         method_vectors = np.column_stack((np.cos(phase), np.sin(phase))) * t1_envelope
0342         joint = np.column_stack((method_vectors, base_vectors))
0343         joint_cov = np.cov(joint, rowvar=False, ddof=1) / max(len(joint), 1)
0344         for row in range(4):
0345             for col in range(4):
0346                 stats[name][f"paired_cov_{row}{col}"] = float(joint_cov[row, col])
0347         stats[name]["identical_to_matched"] = bool(
0348             np.array_equal(phase, base_phase)
0349         )
0350     true_phase = test["common_total_phase_rad"]
0351     sentinel_phase = test["second_sentinel_latent_phase_rad"]
0352     if np.std(true_phase) < 1e-15 or np.std(sentinel_phase) < 1e-15:
0353         corr = float("nan")
0354     else:
0355         corr = float(np.corrcoef(true_phase, sentinel_phase)[0, 1])
0356     fallback_fraction = float(np.mean([entry.fallback for entry in entries]))
0357     return {
0358         "time_us": float(time_us),
0359         "window": int(window_index),
0360         "start_phase_rad": float(start_phase),
0361         "methods": stats,
0362         "corrections_rad": corrections,
0363         "lexicon_entries": entries,
0364         "latent_phase_correlation": corr,
0365         "fallback_code_fraction": fallback_fraction,
0366         "pilot_validation_gain": validation_gain,
0367         "pilot_gate_pass": pilot_gate_pass,
0368         "audit_sample": {
0369             "data_phase_rad": data[:512],
0370             "transition_code": code[:512],
0371         },
0372     }

```

Appendix B. Frozen simulation runner — listing 1 of 10

```

0001 #!/usr/bin/env python3
0002 """Run the frozen END-QM synthetic validation and robustness suite."""
0003
0004 from __future__ import annotations
0005
0006 import argparse
0007 import csv
0008 import gzip
0009 import json
0010 import platform
0011 import sys
0012 from dataclasses import asdict
0013 from pathlib import Path
0014
0015 import matplotlib.pyplot as plt
0016 import numpy as np
0017 import pandas as pd
0018 from scipy.stats import norm
0019
0020 from endqm.core import ENDQMConfig, simulate_window
0021
0022
0023 METHOD_LABELS = {
0024     "matched_open": "Matched open-loop",
0025     "end_qm": "END-QM",
0026     "wrong_sign": "Wrong sign",
0027     "shuffled_map": "Shuffled map",
0028     "oracle_common_phase": "Common-phase oracle",
0029 }
0030
0031
0032 def write_json(path: Path, value: object) -> None:
0033     path.parent.mkdir(parents=True, exist_ok=True)
0034     path.write_text(json.dumps(value, indent=2), encoding="utf-8")
0035
0036
0037 def run_full(cfg: ENDQMConfig, out_dir: Path) -> tuple[pd.DataFrame, pd.DataFrame, dict]:
0038     rng = np.random.default_rng(cfg.seed)
0039     rows: list[dict] = []
0040     lexicons: list[dict] = []
0041     audit_rows: list[dict] = []
0042     for window in range(cfg.windows):

```

Appendix B. Frozen simulation runner — listing 2 of 10

```

0043     for time_us in cfg.times_us:
0044         result = simulate_window(rng, time_us, window, cfg)
0045         for method, stats in result["methods"].items():
0046             rows.append(
0047                 {
0048                     "window": window,
0049                     "start_phase_rad": result["start_phase_rad"],
0050                     "time_us": time_us,
0051                     "method": method,
0052                     **stats,
0053                     "latent_phase_correlation": result["latent_phase_correlation"],
0054                     "fallback_code_fraction": result["fallback_code_fraction"],
0055                     "pilot_validation_gain": result["pilot_validation_gain"],
0056                     "pilot_gate_pass": result["pilot_gate_pass"],
0057                 }
0058             )
0059         lexicons.append(
0060             {
0061                 "window": window,
0062                 "time_us": time_us,
0063                 "entries": [asdict(entry) for entry in result["lexicon_entries"]],
0064             }
0065         )
0066         audit = result["audit_sample"]
0067         for phase, code in zip(audit["data_phase_rad"], audit["transition_code"]):
0068             audit_rows.append(
0069                 {
0070                     "window": window,
0071                     "time_us": time_us,
0072                     "data_phase_rad": float(phase),
0073                     "transition_code": int(code),
0074                 }
0075             )
0076
0077     curves = pd.DataFrame(rows)
0078     curves.to_csv(out_dir / "dynamic_coherence_stats.csv", index=False)
0079     pd.DataFrame(audit_rows).to_csv(
0080         out_dir / "dynamic_audit_phase_samples.csv.gz", index=False, compression="gzip"
0081     )
0082     with gzip.open(out_dir / "dynamic_lexicon_entries.json.gz", "wt", encoding="utf-8") as
handle:
0083         json.dump(lexicons, handle)
0084

```

Appendix B. Frozen simulation runner — listing 3 of 10

```

0085     auc_rows: list[dict] = []
0086     for (window, method), group in curves.groupby(["window", "method"]):
0087         group = group.sort_values("time_us")
0088         auc_rows.append(
0089             {
0090                 "window": int(window),
0091                 "method": str(method),
0092                 "auc_us": float(np.trapezoid(group.coherence, group.time_us)),
0093             }
0094         )
0095     auc = pd.DataFrame(auc_rows)
0096     auc.to_csv(out_dir / "dynamic_auc_by_window.csv", index=False)
0097
0098     # Conditional parametric bootstrap of held-out shot means. The pilot map is
0099     # held frozen, matching the intended pilot/confirmation hardware split.
0100     bootstrap_rng = np.random.default_rng(cfg.seed + 11)
0101     boot_auc: dict[tuple[int, str], np.ndarray] = {}
0102     boot_gain: dict[tuple[int, str], np.ndarray] = {}
0103     for (window, method), group in curves.groupby(["window", "method"]):
0104         group = group.sort_values("time_us")
0105         coherence_draws = []
0106         matched_draws = []
0107         for row in group.itertuples(index=False):
0108             if bool(row.identical_to_matched):
0109                 cov = np.array([[row.cov_xx, row.cov_xy], [row.cov_xy, row.cov_yy]])
0110                 cov += np.eye(2) * 1e-15
0111                 draws = bootstrap_rng.multivariate_normal(
0112                     np.array([row.x, row.y]), cov, size=cfg.n_bootstrap
0113                 )
0114                 coherence = np.clip(np.hypot(draws[:, 0], draws[:, 1]), 0.0, 1.0)
0115                 coherence_draws.append(coherence)
0116                 matched_draws.append(coherence)
0117             else:
0118                 covariance = np.array(
0119                     [
0120                         [getattr(row, f"paired_cov_{r}{c}") for c in range(4)]
0121                         for r in range(4)
0122                     ]
0123                 )
0124                 covariance += np.eye(4) * 1e-15
0125                 base_row = curves[
0126                     (curves.window == row.window)

```

Appendix B. Frozen simulation runner — listing 4 of 10

```

0127         & (curves.time_us == row.time_us)
0128         & (curves.method == "matched_open")
0129     ].iloc[0]
0130     mean = np.array([row.x, row.y, base_row.x, base_row.y])
0131     draws = bootstrap_rng.multivariate_normal(
0132         mean, covariance, size=cfg.n_bootstrap
0133     )
0134     coherence_draws.append(
0135         np.clip(np.hypot(draws[:, 0], draws[:, 1]), 0.0, 1.0)
0136     )
0137     matched_draws.append(
0138         np.clip(np.hypot(draws[:, 2], draws[:, 3]), 0.0, 1.0)
0139     )
0140     boot_auc[(int(window), str(method))] = np.trapezoid(
0141         np.vstack(coherence_draws), group.time_us.to_numpy(), axis=0
0142     )
0143     paired_base_auc = np.trapezoid(
0144         np.vstack(matched_draws), group.time_us.to_numpy(), axis=0
0145     )
0146     boot_gain[(int(window), str(method))] = boot_auc[(int(window), str(method))] / np.clip(
0147         paired_base_auc, 1e-12, None
0148     )
0149
0150     point = auc.pivot(index="window", columns="method", values="auc_us")
0151     gain_rows: list[dict] = []
0152     family_tail = 0.05 / (2.0 * cfg.windows)
0153     for window in range(cfg.windows):
0154         for method in ["end_qm", "wrong_sign", "shuffled_map", "oracle_common_phase"]:
0155             samples = boot_gain[(window, method)]
0156             gain_rows.append(
0157                 {
0158                     "window": window,
0159                     "method": method,
0160                     "auc_gain_vs_matched": float(
0161                         point.loc[window, method] / point.loc[window, "matched_open"]
0162                     ),
0163                     "fwer95_low": float(np.quantile(samples, family_tail)),
0164                     "fwer95_high": float(np.quantile(samples, 1.0 - family_tail)),
0165                 }
0166             )
0167     gains = pd.DataFrame(gain_rows)
0168     gains.to_csv(out_dir / "dynamic_gain_by_window.csv", index=False)

```

Appendix B. Frozen simulation runner — listing 5 of 10

```

0169
0170     end_gain = gains[gains.method == "end_qm"]
0171     med_boot = np.median(
0172         np.vstack(
0173             [
0174                 boot_gain[(window, "end_qm")]
0175                 for window in range(cfg.windows)
0176             ]
0177         ),
0178         axis=0,
0179     )
0180     summary = {
0181         "status": "synthetic_held_out_forecast_only",
0182         "protocol": "END-QM two-frame six-sector spectator feedback",
0183         "seed": cfg.seed,
0184         "windows": cfg.windows,
0185         "times_us": list(cfg.times_us),
0186         "n_train_per_window_time": cfg.n_train,
0187         "n_test_per_window_time": cfg.n_test,
0188         "median_auc_gain_vs_matched": float(end_gain.auc_gain_vs_matched.median()),
0189         "median_auc_gain_ci95": [
0190             float(np.quantile(med_boot, 0.025)),
0191             float(np.quantile(med_boot, 0.975)),
0192         ],
0193         "minimum_window_auc_gain": float(end_gain.auc_gain_vs_matched.min()),
0194         "minimum_window_fwer95_low": float(end_gain.fwer95_low.min()),
0195         "all_windows_point_gain_gt_1": bool((end_gain.auc_gain_vs_matched > 1.0).all()),
0196         "all_windows_fwer95_low_gt_1": bool((end_gain.fwer95_low > 1.0).all()),
0197         "all_windows_point_gain_gte_1": bool((end_gain.auc_gain_vs_matched >= 1.0).all()),
0198         "all_windows_fwer95_low_gte_1": bool((end_gain.fwer95_low >= 1.0).all()),
0199         "median_latent_phase_correlation": float(curves.latent_phase_correlation.median()),
0200         "median_fallback_code_fraction": float(curves.fallback_code_fraction.median()),
0201         "method_median_auc_us": {
0202             method: float(values.median())
0203             for method, values in auc.groupby("method").auc_us
0204         },
0205         "claim_boundary": (
0206             "This is a synthetic forecast conditioned on common-mode measurable phase drift. "
0207             "It is not an IBM hardware result and does not establish intrinsic T2 extension."
0208         ),
0209     }
0210     write_json(out_dir / "dynamic_summary.json", summary)

```

Appendix B. Frozen simulation runner — listing 6 of 10

```

0211     return curves, gains, summary
0212
0213
0214 def run_robustness(cfg: ENDQMConfig, out_dir: Path) -> pd.DataFrame:
0215     rows: list[dict] = []
0216     common_scales = [0.0, 0.35, 0.60, 1.0, 1.30, 1.60]
0217     sensor_scales = [0.5, 1.0, 2.0, 4.0]
0218     for i, common_scale in enumerate(common_scales):
0219         for j, sensor_scale in enumerate(sensor_scales):
0220             rng = np.random.default_rng(cfg.seed + 1000 + i * 100 + j)
0221             auc_by_method: dict[str, list[float]] = {"matched_open": [], "end_qm": []}
0222             correlations: list[float] = []
0223             for window in range(cfg.windows):
0224                 curves = {"matched_open": [], "end_qm": []}
0225                 for time_us in cfg.times_us:
0226                     result = simulate_window(
0227                         rng,
0228                         time_us,
0229                         window,
0230                         cfg,
0231                         common_scale=common_scale,
0232                         sensor_phase_scale=sensor_scale,
0233                         n_train=5_000,
0234                         n_test=12_000,
0235                     )
0236                     correlations.append(result["latent_phase_correlation"])
0237                     for method in curves:
0238                         curves[method].append(result["methods"][method]["coherence"])
0239             for method in curves:
0240                 auc_by_method[method].append(
0241                     float(np.trapezoid(curves[method], cfg.times_us))
0242                 )
0243             gains = np.asarray(auc_by_method["end_qm"]) / np.clip(
0244                 np.asarray(auc_by_method["matched_open"]), 1e-12, None
0245             )
0246             finite_correlations = [value for value in correlations if np.isfinite(value)]
0247             rows.append(
0248                 {
0249                     "common_scale": common_scale,
0250                     "sensor_noise_scale": sensor_scale,
0251                     "median_latent_phase_correlation": (
0252                         float(np.median(finite_correlations))

```

Appendix B. Frozen simulation runner — listing 7 of 10

```

0253             if finite_correlations
0254                 else None
0255             ),
0256             "median_auc_gain": float(np.median(gains)),
0257             "minimum_window_auc_gain": float(np.min(gains)),
0258             "all_windows_gain_gt_1": bool(np.all(gains > 1.0)),
0259         }
0260     )
0261     result = pd.DataFrame(rows)
0262     result.to_csv(out_dir / "dynamic_robustness_sweep.csv", index=False)
0263     return result
0264
0265
0266 def make_figures(
0267     curves: pd.DataFrame,
0268     gains: pd.DataFrame,
0269     robustness: pd.DataFrame,
0270     out_dir: Path,
0271 ) -> None:
0272     fig_dir = out_dir / "figures"
0273     fig_dir.mkdir(parents=True, exist_ok=True)
0274     plt.rcParams.update({"font.size": 10, "axes.titlesize": 12, "axes.labelsize": 10})
0275
0276     fig, ax = plt.subplots(figsize=(7.3, 4.5))
0277     for method in ["matched_open", "end_qm", "wrong_sign", "oracle_common_phase"]:
0278         group = curves[curves.method == method].groupby("time_us").coherence
0279         mean = group.median()
0280         low = group.quantile(0.10)
0281         high = group.quantile(0.90)
0282         ax.plot(mean.index, mean.values, marker="o", label=METHOD_LABELS[method])
0283         ax.fill_between(mean.index, low.values, high.values, alpha=0.10)
0284     ax.set_xlabel="Nominal storage duration (us)", ylabel="Equatorial coherence", ylim=(0, 1.03))
0285     ax.set_title("Held-out END-QM dynamic-sentinel forecast")
0286     ax.grid(alpha=0.25)
0287     ax.legend(frameon=False, fontsize=8)
0288     fig.tight_layout()
0289     fig.savefig(fig_dir / "fig_dynamic_coherence.png", dpi=240)
0290     plt.close(fig)
0291
0292     end = gains[gains.method == "end_qm"].sort_values("window")
0293     fig, ax = plt.subplots(figsize=(7.3, 4.3))
0294     yerr = np.vstack(

```


Appendix B. Frozen simulation runner — listing 8 of 10

```

0295         [
0296             end.auc_gain_vs_matched - end.fwer95_low,
0297             end.fwer95_high - end.auc_gain_vs_matched,
0298         ]
0299     )
0300     ax.errorbar(end.window, end.auc_gain_vs_matched, yerr=yerr, fmt="o", capsize=3)
0301     ax.axhline(1.0, color="black", linewidth=1, linestyle="--")
0302     ax.set(
0303         xlabel="Pre-registered radian window",
0304         ylabel="AUC gain vs matched open-loop",
0305         title="All-window held-out END-QM gain",
0306         xticks=range(12),
0307     )
0308     ax.grid(alpha=0.2)
0309     fig.tight_layout()
0310     fig.savefig(fig_dir / "fig_dynamic_window_gain.png", dpi=240)
0311     plt.close(fig)
0312
0313     auc_median = (
0314         gains.groupby("method").auc_gain_vs_matched.median().reindex(
0315             ["end_qm", "wrong_sign", "shuffled_map", "oracle_common_phase"]
0316         )
0317     )
0318     fig, ax = plt.subplots(figsize=(7.3, 4.2))
0319     bars = ax.bar(
0320         [METHOD_LABELS[index] for index in auc_median.index], auc_median.values
0321     )
0322     ax.axhline(1.0, color="black", linewidth=1, linestyle="--")
0323     ax.set(ylabel="Median AUC gain vs matched open-loop", title="Specificity and upper-bound controls")
0324     ax.tick_params(axis="x", rotation=18)
0325     for bar, value in zip(bars, auc_median.values):
0326         ax.text(bar.get_x() + bar.get_width() / 2, value + 0.02, f"{value:.2f}x", ha="center",
0327             fontsize=9)
0328     fig.tight_layout()
0329     fig.savefig(fig_dir / "fig_dynamic_controls.png", dpi=240)
0330     plt.close(fig)
0331
0332     pivot = robustness.pivot(
0333         index="sensor_noise_scale", columns="common_scale", values="median_auc_gain"
0334     )
0335     fig, ax = plt.subplots(figsize=(7.3, 4.4))
0336     image = ax.imshow(pivot.values, aspect="auto", origin="lower", cmap="viridis")
0337     ax.set_xticks(range(len(pivot.columns)), [f"{v:g}" for v in pivot.columns])

```

Appendix B. Frozen simulation runner — listing 9 of 10

```

0337     ax.set_yticks(range(len(pivot.index)), [f"{v:g}" for v in pivot.index])
0338     ax.set_xlabel="Common-mode phase scale", ylabel="Sentinel noise scale", title="END-QM
robustness operating map")
0339     for row in range(pivot.shape[0]):
0340         for col in range(pivot.shape[1]):
0341             value = pivot.iloc[row, col]
0342             ax.text(col, row, f"{value:.2f}", ha="center", va="center", color="white" if value <
1.2 else "black", fontsize=8)
0343     fig.colorbar(image, ax=ax, label="Median AUC gain")
0344     fig.tight_layout()
0345     fig.savefig(fig_dir / "fig_dynamic_robustness.png", dpi=240)
0346     plt.close(fig)
0347
0348     # Exact six-sector geometry used by the three-sentinel code.
0349     angles = np.arange(6) * 2 * np.pi / 6
0350     fig, ax = plt.subplots(figsize=(5.6, 5.3), subplot_kw={"projection": "polar"})
0351     ax.scatter(angles, np.ones(6), s=80)
0352     for sector, angle in enumerate(angles):
0353         ax.text(angle, 1.12, f"S{sector}", ha="center", va="center")
0354     ax.set_rticks([])
0355     ax.set_ylim(0, 1.25)
0356     ax.set_title("Six resolvable radian sectors from three 120-degree sentinels", pad=20)
0357     fig.tight_layout()
0358     fig.savefig(fig_dir / "fig_six_sector_code.png", dpi=240)
0359     plt.close(fig)
0360
0361 def main() -> None:
0362     parser = argparse.ArgumentParser()
0363     parser.add_argument("--output-dir", type=Path, required=True)
0364     args = parser.parse_args()
0365     args.output_dir.mkdir(parents=True, exist_ok=True)
0366     cfg = ENDQMConfig()
0367     write_json(args.output_dir / "frozen_dynamic_config.json", cfg.to_dict())
0368     curves, gains, summary = run_full(cfg, args.output_dir)
0369     robustness = run_robustness(cfg, args.output_dir)
0370     make_figures(curves, gains, robustness, args.output_dir)
0371     write_json(
0372         args.output_dir / "environment.json",
0373         {
0374             "python": sys.version,
0375             "platform": platform.platform(),
0376             "numpy": np.__version__,
0377             "pandas": pd.__version__,

```

Appendix B. Frozen simulation runner — listing 10 of 10

```
0379         "scipy": sys.modules["scipy"].__version__,
0380         "matplotlib": sys.modules["matplotlib"].__version__,
0381     },
0382 )
0383 print(json.dumps(summary, indent=2))
0384
0385
0386 if __name__ == "__main__":
0387     main()
```

Appendix C. Unit tests — listing 1 of 2

```

0001 import unittest
0002
0003 import numpy as np
0004
0005 from endqm.core import (
0006     ENDQMConfig,
0007     fit_transition_lexicon,
0008     pattern_sector,
0009     raw_transition_code,
0010     simulate_window,
0011     transition_is_ambiguous,
0012     wrap_angle,
0013 )
0014
0015
0016 class CoreTests(unittest.TestCase):
0017     def test_wrap_range(self):
0018         values = wrap_angle(np.linspace(-20.0, 20.0, 1001))
0019         self.assertTrue(np.all(values >= -np.pi))
0020         self.assertTrue(np.all(values < np.pi))
0021
0022     def test_six_non_ambiguous_patterns_map_to_six_sectors(self):
0023         sectors = [pattern_sector(pattern) for pattern in range(8)]
0024         self.assertIsNone(sectors[0])
0025         self.assertIsNone(sectors[7])
0026         self.assertEqual(set(s for s in sectors if s is not None), set(range(6)))
0027
0028     def test_raw_code_packing(self):
0029         first = np.array([0, 1, 7])
0030         second = np.array([7, 2, 0])
0031         np.testing.assert_array_equal(raw_transition_code(first, second), [56, 17, 7])
0032
0033     def test_ambiguous_frames_force_fallback(self):
0034         for code in range(64):
0035             expected = ((code & 7) in (0, 7)) or (((code >> 3) & 7) in (0, 7))
0036             self.assertEqual(transition_is_ambiguous(code), expected)
0037
0038     def test_lexicon_is_bounded_and_support_gated(self):
0039         cfg = ENDQMConfig(minimum_cell_support=15)
0040         phase = np.full(20, 1.2)
0041         code = np.full(20, 9)
0042         corrections, entries = fit_transition_lexicon(phase, code, cfg)

```

Appendix C. Unit tests — listing 2 of 2

```

0043         self.assertLessEqual(abs(corrections[9]), np.pi)
0044         self.assertFalse(entries[9].fallback)
0045         self.assertTrue(entries[10].fallback)
0046
0047     def test_frozen_specificity_direction(self):
0048         cfg = ENDQMConfig(n_train=5000, n_test=15000)
0049         result = simulate_window(np.random.default_rng(1234), 100.0, 4, cfg)
0050         self.assertGreater(
0051             result["methods"]["end_qm"]["coherence"],
0052             result["methods"]["matched_open"]["coherence"],
0053         )
0054         self.assertGreater(
0055             result["methods"]["end_qm"]["coherence"],
0056             result["methods"]["wrong_sign"]["coherence"],
0057         )
0058
0059
0060 if __name__ == "__main__":
0061     unittest.main()
0062

```

Appendix D. IBM 10-minute Colab code — listing 1 of 10

```

0001 # --- Notebook code cell 2 ---
0002 %pip -q install "qiskit~=2.5.0" "qiskit-ibm-runtime~=0.47.0" pandas matplotlib scipy
0003
0004 # --- Notebook code cell 4 ---
0005 key="PASTE_YOUR_IBM_CLOUD_API_KEY_HERE"
0006
0007 if key == "PASTE_YOUR_IBM_CLOUD_API_KEY_HERE" or len(key) < 20:
0008     raise ValueError("Paste your IBM Cloud API key into key= before continuing.")
0009
0010 # --- Notebook code cell 5 ---
0011 from __future__ import annotations
0012
0013 import json, math, random, sys, time
0014 from dataclasses import dataclass
0015 from pathlib import Path
0016
0017 import matplotlib.pyplot as plt
0018 import numpy as np
0019 import pandas as pd
0020 from scipy.stats import bootstrap
0021
0022 from qiskit import ClassicalRegister, QuantumCircuit, QuantumRegister, transpile
0023 from qiskit.transpiler.preset_passmanagers import generate_preset_pass_manager
0024 from qiskit_ibm_runtime import QiskitRuntimeService, SamplerV2 as Sampler
0025
0026 SEED = 20260806
0027 random.seed(SEED)
0028 np.random.seed(SEED)
0029
0030 service = QiskitRuntimeService(token=key, instance="auto")
0031 print("Authenticated. The API key is held only in notebook memory.")
0032
0033 # --- Notebook code cell 7 ---
0034 BACKEND_NAME = None # Optional: set to a specific QPU name, otherwise least-busy is used.
0035
0036 if BACKEND_NAME:
0037     backend = service.backend(BACKEND_NAME)
0038 else:
0039     backend = service.least_busy(operational=True, simulator=False, min_num_qubits=4)
0040
0041 props = backend.properties()
0042 rows = []

```

Appendix D. IBM 10-minute Colab code — listing 2 of 10

```

0043 for q in range(backend.num_qubits):
0044     try:
0045         t1 = float(props.t1(q))
0046         t2 = float(props.t2(q))
0047         readout_error = float(props.readout_error(q))
0048         score = min(t2, 2.0 * t1) / max(0.01, readout_error)
0049         rows.append({"qubit": q, "t1_s": t1, "t2_s": t2,
0050                    "readout_error": readout_error, "score": score})
0051     except Exception:
0052         pass
0053
0054 calibration = pd.DataFrame(rows).sort_values("score", ascending=False)
0055 if len(calibration) < 4:
0056     raise RuntimeError("Could not find four qubits with complete calibration data.")
0057
0058 physical_qubits = calibration.head(4).qubit.astype(int).tolist()
0059 data_physical, *sentinel_physical = physical_qubits
0060 data_t2_us = float(calibration.iloc[0].t2_s * 1e6)
0061 durations_us = sorted(set([0.0] + [
0062     round(float(np.clip(frac * data_t2_us, 16.0, 220.0))), 1)
0063     for frac in (0.25, 0.50, 0.85)
0064 ])))
0065 while len(durations_us) < 4:
0066     durations_us = sorted(set(durations_us + [float(16 * len(durations_us))]))
0067
0068 print("Backend:", backend.name)
0069 print("Physical layout [data, sentinel 0, 1, 2]:", physical_qubits)
0070 print("Data T2 from current calibration (us):", round(data_t2_us, 2))
0071 print("Test durations (us):", durations_us)
0072 display(calibration.head(10))
0073
0074 # --- Notebook code cell 9 ---
0075 WINDOWS = np.linspace(0.0, 2*np.pi, 12, endpoint=False)
0076 SENTINEL_AXES = (0.0, 2*np.pi/3, 4*np.pi/3)
0077 BOUNDARIES = (0.34, 0.70)
0078 PILOT_SHOTS = 2048
0079 CONFIRM_SHOTS = 512
0080 MAX_CORRECTION_RAD = np.pi/2
0081 MIN_CODE_SUPPORT = 32
0082 METHODS = ("end_qm", "matched_open", "wrong_sign", "raw_idle", "xy4")
0083 AMBIGUOUS_HALF_CODES = {0b000, 0b111}
0084 PILOT_JOB_CAP_S = 160

```

Appendix D. IBM 10-minute Colab code — listing 3 of 10

```

0085 CONFIRM_JOB_CAP_S = 190 # two confirmation jobs: 160 + 190 + 190 = 540 seconds
0086
0087 def wrap(x):
0088     return (np.asarray(x) + np.pi) % (2*np.pi) - np.pi
0089
0090 def code_is_ambiguous(code_value):
0091     return ((code_value & 0b111) in AMBIGUOUS_HALF_CODES or
0092            (((code_value >> 3) & 0b111) in AMBIGUOUS_HALF_CODES))
0093
0094 def add_data_analysis(qc, q, axis):
0095     if axis == "x":
0096         qc.h(q)
0097     elif axis == "y":
0098         qc.sdg(q); qc.h(q)
0099     else:
0100         raise ValueError(axis)
0101
0102 def add_sentinel_analysis(qc, q, axis):
0103     qc.rz(-axis, q); qc.h(q)
0104
0105 def base_registers():
0106     q = QuantumRegister(4, "q")
0107     sense = ClassicalRegister(6, "sense")
0108     readout = ClassicalRegister(1, "readout")
0109     return q, sense, readout, QuantumCircuit(q, sense, readout)
0110
0111 def build_dynamic_circuit(window, duration_us, axis, correction_map=None, sign=1.0):
0112     q, sense, readout, qc = base_registers()
0113     data = q[0]; sentinels = [q[1], q[2], q[3]]
0114     qc.h(data); qc.rz(float(window), data)
0115     for s in sentinels:
0116         qc.h(s); qc.rz(float(window), s)
0117
0118     cuts = [BOUNDARIES[0]*duration_us,
0119            (BOUNDARIES[1]-BOUNDARIES[0])*duration_us,
0120            (1-BOUNDARIES[1])*duration_us]
0121     for qb in q:
0122         qc.delay(max(cuts[0], 0.0), qb, unit="us")
0123     qc.barrier()
0124     for i, (s, a) in enumerate(zip(sentinels, SENTINEL_AXES)):
0125         add_sentinel_analysis(qc, s, a)
0126     qc.measure(s, sense[i])

```


Appendix D. IBM 10-minute Colab code — listing 4 of 10

```

0127     qc.barrier()
0128     for s in sentinels:
0129         qc.reset(s); qc.h(s); qc.rz(float(window), s)
0130
0131     for qb in q:
0132         qc.delay(max(cuts[1], 0.0), qb, unit="us")
0133     qc.barrier()
0134     for i, (s, a) in enumerate(zip(sentinels, SENTINEL_AXES)):
0135         add_sentinel_analysis(qc, s, a)
0136         qc.measure(s, sense[i+3])
0137     qc.barrier()
0138
0139     if correction_map:
0140         for raw_code in range(64):
0141             angle = float(sign * correction_map.get(raw_code, 0.0))
0142             if abs(angle) > 1e-12:
0143                 with qc.if_test((sense, raw_code)):
0144                     qc.rz(angle, data)
0145
0146     for qb in q:
0147         qc.delay(max(cuts[2], 0.0), qb, unit="us")
0148     qc.rz(float(-window), data)
0149     add_data_analysis(qc, data, axis)
0150     qc.measure(data, readout[0])
0151     return qc
0152
0153 def build_reference_circuit(window, duration_us, axis, method):
0154     q, sense, readout, qc = base_registers()
0155     data = q[0]
0156     qc.h(data); qc.rz(float(window), data)
0157     if method == "xy4" and duration_us > 0:
0158         for pulse in ("x", "y", "x", "y"):
0159             qc.delay(duration_us/5, data, unit="us")
0160             getattr(qc, pulse)(data)
0161             qc.delay(duration_us/5, data, unit="us")
0162     else:
0163         qc.delay(duration_us, data, unit="us")
0164     qc.rz(float(-window), data)
0165     add_data_analysis(qc, data, axis)
0166     qc.measure(data, readout[0])
0167     return qc
0168

```

Appendix D. IBM 10-minute Colab code — listing 5 of 10

```

0169 def manifest_key(window_index, duration_index, axis, method):
0170     return f"w{window_index:02d}_t{duration_index:02d}_{axis}_{method}"
0171
0172 # --- Notebook code cell 11 ---
0173 pilot_manifest = []
0174 pilot_circuits = []
0175 for wi, window in enumerate(WINDOWS):
0176     for ti, duration in enumerate(durations_us):
0177         for axis in ("x", "y"):
0178             pilot_manifest.append({"key": manifest_key(wi, ti, axis, "pilot"),
0179                                   "window_index": wi, "duration_index": ti,
0180                                   "window_rad": float(window), "duration_us": float(duration),
0181                                   "axis": axis, "method": "pilot"})
0182             pilot_circuits.append(build_dynamic_circuit(window, duration, axis))
0183
0184 assert len(pilot_circuits) == 96
0185 pass_manager = generate_preset_pass_manager(
0186     backend=backend, optimization_level=1, initial_layout=physical_qubits,
0187     seed_transpiler=SEED,
0188 )
0189 pilot_isa = pass_manager.run(pilot_circuits)
0190 print("Pilot circuits:", len(pilot_isa), "shots each:", PILOT_SHOTS)
0191 print("First pilot circuit depth:", pilot_isa[0].depth())
0192 display(pilot_isa[0].draw("mpl", idle_wires=False, fold=80))
0193
0194 # --- Notebook code cell 12 ---
0195 pilot_sampler = Sampler(mode=backend)
0196 pilot_sampler.options.max_execution_time = PILOT_JOB_CAP_S
0197 pilot_job = pilot_sampler.run(pilot_isa, shots=PILOT_SHOTS)
0198 PILOT_JOB_ID = pilot_job.job_id()
0199 print("PILOT JOB ID - SAVE THIS NOW:", PILOT_JOB_ID)
0200 pilot_result = pilot_job.result()
0201 print("Pilot complete.")
0202
0203 # --- Notebook code cell 14 ---
0204 def extract_pub_shots(pub_result, meta):
0205     sense_strings = pub_result.data.sense.get_bitstrings()
0206     readout_strings = pub_result.data.readout.get_bitstrings()
0207     if len(sense_strings) != len(readout_strings):
0208         raise RuntimeError("Sense/readout shot count mismatch")
0209     rows = []
0210     for shot_index, (sense_bits, readout_bits) in enumerate(zip(sense_strings, readout_strings)):

```

Appendix D. IBM 10-minute Colab code — listing 6 of 10

```

0211         rows.append(**meta, "shot_index": shot_index,
0212                       "transition_code": int(sense_bits.replace(" ", ""), 2),
0213                       "data_bit": int(readout_bits.replace(" ", ""), 2))
0214     return rows
0215
0216     pilot_rows = []
0217     for pub, meta in zip(pilot_result, pilot_manifest):
0218         pilot_rows.extend(extract_pub_shots(pub, meta))
0219     pilot_df = pd.DataFrame(pilot_rows)
0220     pilot_df["split"] = np.where(pilot_df.shot_index < int(0.60*PILOT_SHOTS), "fit", "validation")
0221     pilot_df.to_csv("END_QM_pilot_raw.csv", index=False)
0222     print("Pilot shot records:", len(pilot_df))
0223
0224     def conditional_vectors(frame, split):
0225         subset = frame[frame.split == split]
0226         table = {}
0227         for (wi, ti, code_value), group in subset.groupby(["window_index", "duration_index",
0228 "transition_code"]):
0229             item = {"n_x": 0, "n_y": 0, "mx": np.nan, "my": np.nan}
0230             for axis in ("x", "y"):
0231                 axis_group = group[group.axis == axis]
0232                 if len(axis_group):
0233                     item[f"n_{axis}"] = len(axis_group)
0234                     item[f"m_{axis}"] = float(np.mean(1 - 2*axis_group.data_bit.to_numpy()))
0235             table[(int(wi), int(ti), int(code_value))] = item
0236     return table
0237
0238     fit_vectors = conditional_vectors(pilot_df, "fit")
0239     validation_vectors = conditional_vectors(pilot_df, "validation")
0240     correction_maps = {}
0241     lexicon_rows = []
0242
0243     for wi in range(len(WINDOWS)):
0244         for ti in range(len(durations_us)):
0245             cmap = {}
0246             for raw_code in range(64):
0247                 item = fit_vectors.get((wi, ti, raw_code), {})
0248                 nx, ny = item.get("n_x", 0), item.get("n_y", 0)
0249                 mx, my = item.get("mx", np.nan), item.get("my", np.nan)
0250                 supported = min(nx, ny) >= MIN_CODE_SUPPORT and np.isfinite(mx) and np.isfinite(my)
0251                 fallback = code_is_ambiguous(raw_code) or not supported or ti == 0
0252                 angle = 0.0 if fallback else float(np.clip(-math.atan2(my, mx), -MAX_CORRECTION_RAD,
0253 MAX_CORRECTION_RAD))
0254             cmap[raw_code] = angle

```

Appendix D. IBM 10-minute Colab code — listing 7 of 10

```

0253         lexicon_rows.append({"window_index": wi, "duration_index": ti,
0254                               "transition_code": raw_code, "n_x": nx, "n_y": ny,
0255                               "mx": mx, "my": my, "correction_rad_pre_gate": angle,
0256                               "fallback": fallback})
0257
0258     def aggregate_coherence(vectors, correction):
0259         total = 0.0 + 0.0j; weight = 0.0
0260         for raw_code in range(64):
0261             item = vectors.get((wi, ti, raw_code), {})
0262             if not (np.isfinite(item.get("mx", np.nan)) and np.isfinite(item.get("my",
0263                                     np.nan))):
0264                 continue
0265             n = min(item.get("n_x", 0), item.get("n_y", 0))
0266             total += n * complex(item["mx"], item["my"]) * np.exp(1j*correction.get(raw_code,
0267                                     0.0))
0268             weight += n
0269         return abs(total / weight) if weight else 0.0
0270
0271     base = aggregate_coherence(validation_vectors, {})
0272     corrected = aggregate_coherence(validation_vectors, cmap)
0273     validation_gain = corrected / max(base, 1e-12)
0274     gate_pass = bool(ti > 0 and validation_gain > 1.0)
0275     if not gate_pass:
0276         cmap = {raw_code: 0.0 for raw_code in range(64)}
0277     correction_maps[(wi, ti)] = cmap
0278     for row in lexicon_rows:
0279         if row["window_index"] == wi and row["duration_index"] == ti:
0280             row["pilot_validation_gain"] = validation_gain
0281             row["pilot_gate_pass"] = gate_pass
0282             row["correction_rad"] = cmap[row["transition_code"]]
0283
0284     lexicon_df = pd.DataFrame(lexicon_rows)
0285     lexicon_df.to_csv("END_QM_frozen_lexicon.csv", index=False)
0286     print("Enabled maps:", int(lexicon_df.groupby(["window_index",
0287         "duration_index"]).pilot_gate_pass.first().sum()),
0288           "of", len(WINDOWS)*len(durations_us))
0289     display(lexicon_df.groupby(["window_index", "duration_index"])[["pilot_validation_gain",
0290         "pilot_gate_pass"]].first())
0291
0292     # --- Notebook code cell 16 ---
0293     confirm_items = []
0294     for wi, window in enumerate(WINDOWS):
0295         for ti, duration in enumerate(durations_us):
0296             for axis in ("x", "y"):
0297                 for method in METHODS:
0298                     if method == "end_qm":

```

Appendix D. IBM 10-minute Colab code — listing 8 of 10

```

0295         qc = build_dynamic_circuit(window, duration, axis, correction_maps[(wi, ti)],
+1.0)
0296     elif method == "wrong_sign":
0297         qc = build_dynamic_circuit(window, duration, axis, correction_maps[(wi, ti)],
-1.0)
0298     elif method == "matched_open":
0299         qc = build_dynamic_circuit(window, duration, axis, None)
0300     else:
0301         qc = build_reference_circuit(window, duration, axis, method)
0302     confirm_items.append((manifest_key(wi, ti, axis, method),
0303                          {"key": manifest_key(wi, ti, axis, method),
0304                           "window_index": wi, "duration_index": ti,
0305                           "window_rad": float(window), "duration_us":
float(duration),
0306                           "axis": axis, "method": method}, qc))
0307
0308 random.Random(SEED).shuffle(confirm_items)
0309 assert len(confirm_items) == 480
0310 confirm_manifest = [item[1] for item in confirm_items]
0311 confirm_isa = pass_manager.run([item[2] for item in confirm_items])
0312 chunks = [(confirm_isa[i:i+240], confirm_manifest[i:i+240]) for i in range(0, 480, 240)]
0313 print("Confirmation circuits:", len(confirm_isa), "in chunks:", [len(c[0]) for c in chunks])
0314 print("Total circuit executions planned:", len(pilot_isa)*PILOT_SHOTS +
len(confirm_isa)*CONFIRM_SHOTS)
0315
0316 # --- Notebook code cell 17 ---
0317 CONFIRM_JOB_IDS = []
0318 confirm_results = []
0319 for chunk_index, (circuits, manifest) in enumerate(chunks):
0320     sampler = Sampler(mode=backend)
0321     sampler.options.max_execution_time = CONFIRM_JOB_CAP_S
0322     job = sampler.run(circuits, shots=CONFIRM_SHOTS)
0323     job_id = job.job_id()
0324     CONFIRM_JOB_IDS.append(job_id)
0325     print(f"CONFIRM JOB {chunk_index+1} ID - SAVE THIS NOW:", job_id)
0326     confirm_results.append((job.result(), manifest))
0327 print("All confirmation jobs complete.")
0328
0329 # --- Notebook code cell 19 ---
0330 confirm_rows = []
0331 for chunk_result, manifest in confirm_results:
0332     for pub, meta in zip(chunk_result, manifest):
0333         readout_strings = pub.data.readout.get_bitstrings()
0334         # Raw-idle and XY4 controls intentionally leave the sentinel register
0335         # unmeasured. Some Runtime result versions omit such empty registers.
0336         try:

```

Appendix D. IBM 10-minute Colab code — listing 9 of 10

```

0337         sense_strings = pub.data.sense.get_bitstrings()
0338     except (AttributeError, KeyError):
0339         sense_strings = ["000000"] * len(readout_strings)
0340     for shot_index, readout_bits in enumerate(readout_strings):
0341         sense_bits = sense_strings[shot_index] if shot_index < len(sense_strings) else
"000000"
0342         confirm_rows.append({"**meta", "shot_index": shot_index,
0343                             "transition_code": int(sense_bits.replace(" ", ""), 2),
0344                             "data_bit": int(readout_bits.replace(" ", ""), 2)})
0345 confirm_df = pd.DataFrame(confirm_rows)
0346 confirm_df.to_csv("END_QM_confirmation_raw.csv", index=False)
0347
0348 means = (confirm_df.assign(z=1-2*confirm_df.data_bit)
0349          .groupby(["window_index", "duration_index", "duration_us", "method", "axis"])
0350          .z.agg(["mean", "count"]).reset_index())
0351 pivot = means.pivot_table(index=["window_index", "duration_index", "duration_us", "method"],
0352                            columns="axis", values="mean").reset_index()
0353 pivot["coherence"] = np.hypot(pivot["x"], pivot["y"])
0354
0355 auc_rows = []
0356 for (wi, method), group in pivot.groupby(["window_index", "method"]):
0357     group = group.sort_values("duration_us")
0358     normalized = group.coherence.to_numpy() / max(group.coherence.iloc[0], 1e-12)
0359     auc_rows.append({"window_index": int(wi), "method": method,
0360                    "auc_us": float(np.trapezoid(normalized, group.duration_us.to_numpy()))})
0361 auc = pd.DataFrame(auc_rows)
0362 auc_pivot = auc.pivot(index="window_index", columns="method", values="auc_us")
0363 window_gain = (auc_pivot.end_qm / auc_pivot.matched_open).rename("gain").reset_index()
0364
0365 rng = np.random.default_rng(SEED)
0366 boot = []
0367 values = window_gain.gain.to_numpy()
0368 for _ in range(20000):
0369     boot.append(float(np.median(rng.choice(values, size=len(values), replace=True))))
0370 median_gain = float(np.median(values))
0371 ci_low, ci_high = np.quantile(boot, [0.025, 0.975])
0372 wrong_ratio = float(np.median(auc_pivot.end_qm / auc_pivot.wrong_sign))
0373 screen_pass = bool(median_gain >= 1.10 and ci_low > 1.00 and wrong_ratio > 1.00)
0374
0375 decision = {
0376     "status": "PASS" if screen_pass else "DO_NOT_CLAIM_PASS",
0377     "backend": backend.name,
0378     "physical_qubits": physical_qubits,

```

Appendix D. IBM 10-minute Colab code — listing 10 of 10

```

0379     "pilot_job_id": PILOT_JOB_ID,
0380     "confirmation_job_ids": CONFIRM_JOB_IDS,
0381     "median_auc_gain_vs_matched": median_gain,
0382     "paired_window_bootstrap_ci95": [float(ci_low), float(ci_high)],
0383     "median_end_vs_wrong_sign": wrong_ratio,
0384     "screening_threshold": "median>=1.10; lower95>1.00; END>wrong-sign",
0385     "claim_boundary": "One-device hardware screen only; repeat on >=3 dates and >=2 data qubits
before a general hardware claim."
0386 }
0387 Path("END_QM_hardware_decision.json").write_text(json.dumps(decision, indent=2))
0388 auc.to_csv("END_QM_hardware_auc.csv", index=False)
0389 print(json.dumps(decision, indent=2))
0390
0391 fig, axes = plt.subplots(1, 2, figsize=(12, 4.3))
0392 for method, group in pivot.groupby("method"):
0393     curve = group.groupby("duration_us").coherence.median()
0394     axes[0].plot(curve.index, curve.values, marker="o", label=method)
0395 axes[0].set(xlabel="Delay (us)", ylabel="Median equatorial coherence", title="Held-out hardware
curves")
0396 axes[0].legend(fontsize=8)
0397 axes[1].bar(window_gain.window_index, window_gain.gain, color="#2E74B5")
0398 axes[1].axhline(1.0, color="black", lw=1); axes[1].axhline(1.10, color="#C45A35", ls="--")
0399 axes[1].set(xlabel="Start-phase window", ylabel="END-QM / matched AUC", title="All-window gain")
0400 fig.tight_layout(); fig.savefig("END_QM_hardware_results.png", dpi=220)
0401 plt.show()
0402
0403 # --- Notebook code cell 21 ---
0404 from google.colab import files
0405 for filename in [
0406     "END_QM_pilot_raw.csv", "END_QM_frozen_lexicon.csv",
0407     "END_QM_confirmation_raw.csv", "END_QM_hardware_auc.csv",
0408     "END_QM_hardware_decision.json", "END_QM_hardware_results.png",
0409 ]:
0410     if Path(filename).exists():
0411         files.download(filename)
0412

```