END Theory / Evans Relational Framework

A Formal Relational-Graph Model of Emergent Time, Persistent Nodes, Local Capacity, and Testable Wave Dynamics

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Research manuscript v1.0

Scientific status: formalized research programme with a closed scalar/link benchmark and explicit open closure problems. This manuscript does not claim empirical confirmation of a theory of everything.

Abstract

The Node-Reality Matrix Theory (NRMT) began from a compact conceptual proposal: reality is not built on a pre-existing stage but emerges from potential states, their relations, and the persistent patterns produced when those states interact. The later END Theory / Evans Relational Framework corpus developed that intuition into a more disciplined sequence: ordered micro-configurations precede physical time; recurrent patterns provide operational clocks; persistence defines candidate objects; intervention and response define effective adjacency; and a finite local capacity restricts admissible change. This paper consolidates the project uploads into one research manuscript and separates three logically different layers: **(i)** exact mathematics of a declared graph benchmark, **(ii)** proposed bridges to observed physics, and **(iii)** detector-level tests.

On a stabilized weighted graph, a scalar-plus-link action yields a deterministic central-difference update, a positive graph-Laplacian spectrum, an exact half-step invariant in the linear sector, an exact lattice dispersion relation, and a necessary-and-sufficient linear stability bound. A two-mode Hamiltonian replaces the historical frequency-matching delta function with a precise resonance and avoided-crossing model. A local convex projection gives a mathematically auditable form of the capacity idea. The manuscript then states the missing closure obligations for quantum probabilities, chiral matter, operational geometry, gravitation, and cosmology rather than treating them as solved by analogy.

Four public-data routes are specified: GWOSC propagation and detector-lag tests, CERN/ATLAS scalar and contact-interaction templates, XENONnT recoil spectra, and DUNE flavour evolution. The accompanying code package reproduces closed-benchmark integrity checks and constructs a 42-entry constants dependency audit using the 2022 CODATA values. Those calculations classify definitions, measured inputs, algebraic identities, matching relations, and genuine predictions. The result is a usable foundation for independent mathematical review and for future pre-registered tests, while preserving the core idea and removing unsupported numerical claims.

Keywords: relational physics; emergent time; graph dynamics; lattice field theory; node pairing; local capacity; falsifiable model construction.

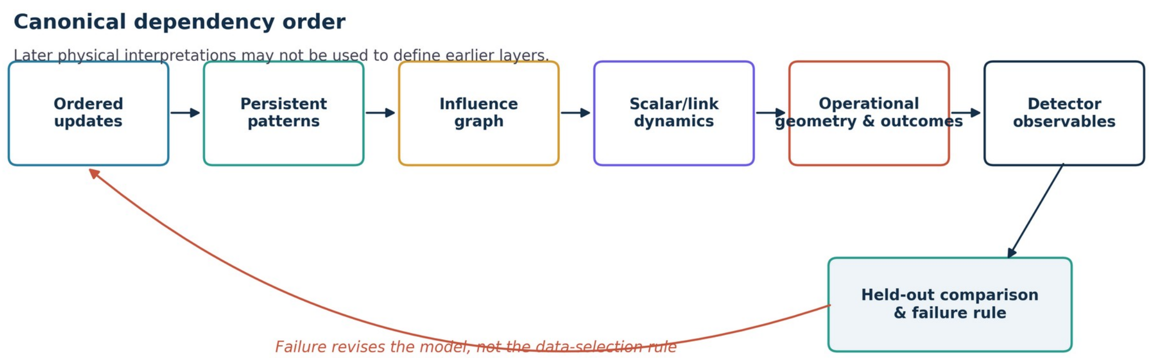
1. Purpose, provenance, and scientific scope

The project archive contains two distinct generations of work. The early NRMT documents state the original intuition in direct language: nodes represent potential states; resonance or pairing produces realized events; space is relational geometry; time is the sequence of interactions; and forces, including gravity, may emerge from collective node behaviour [1]. Later editions under the names END Theory, Evans Node Dialect, and Matrix Node Theory introduced a proof discipline, a canonical graph benchmark, explicit closure labels, and public-data analysis contracts [2], [3], [4].

The purpose of this manuscript is not to add another broad claim. It is to reduce the archive to one dependency-ordered paper that a mathematical physicist, data analyst, or experimental collaborator can audit. The central question is:

Can a fixed relational micro-model, without presupposing physical spacetime, produce persistent excitations, operational clocks and distances, quantum-compatible probabilities, relativistic effective dynamics, and detector-level predictions with fewer independent inputs than the held-out observables it explains?

The project is scientifically valuable before that question is fully answered because part of the mathematics is already closed and reproducible. The scalar graph sector is not a theory of all observed particles, but it is a non-trivial benchmark that can be varied, simulated, and falsified. The uploaded technical reference correctly states that a heat-kernel coefficient does not by itself derive Einstein dynamics, a deterministic update does not by itself derive Born probabilities, and numerical agreement is not a prediction when the target selected the parameters [4]. This paper adopts that rule throughout.



The canonical dependency order. A later physical interpretation is not permitted to define an earlier layer.

1.1 Claim grammar

Every consequential statement is assigned one of the following statuses.

Status	Meaning	Required check
DEFINED	A symbol, convention, state space, or classification has been fixed.	Check consistency and units.
DERIVED	The statement follows from declared assumptions.	Repeat the algebra, proof, variation, or numerical unit test.
PROPOSED	A candidate law or bridge has been selected.	Check internal consistency and derive distinguishing consequences.
CONDITIONAL	A deduction holds only if a named bridge assumption holds.	Test both the mathematics and the bridge assumption.
CLOSURE	A result required for physical completion has not yet been obtained.	Supply the missing map or theorem without importing the target observable.
MEASURED INPUT	A value enters from metrology or experiment.	Trace source, uncertainty, scale, covariance, and unit system.

Status	Meaning	Required check
CALIBRATED	A parameter was fixed from declared calibration data.	Freeze it before opening held-out data.
TESTED	A frozen forward model reaches a held-out observable.	Apply the registered likelihood and failure rule.
FALSIFIED / RETIRED	A frozen statement failed or an equation was found inconsistent.	Preserve the failure in the ledger; do not silently reuse it.

This grammar is not cosmetic. It prevents a chain such as “standard identity → fitted coefficient → reproduced target” from being described as a novel prediction.

1.2 What is retained from the historical NRMT draft

The following conceptual commitments survive the reconstruction:

physical spacetime should not be assumed at the primitive layer;

relational states and links are prior to macroscopic objects;

stable objects may be persistent patterns rather than immutable microscopic pieces;

wave-like and localized behaviour can be different dynamical regimes of the same substrate;

pairing or resonance should have a precise dynamical representation;

time should be read from change, not inserted as an unexplained physical clock;

a local limit on available change may influence propagation and clock rates; and

a theory earns physical status only through a complete observation map and held-out tests.

1.3 Historical equations not retained as fundamental laws

Several equations in early AI-assisted drafts expressed the intended idea but did not yet meet the requirements of a physical law. They are therefore treated as historical notation, not as the master theory.

Historical expression	Mathematical issue	Replacement in this manuscript
P(f_i,f_j) = delta(f_i-f_j)	A Dirac delta is a distribution, not a normalized pairing probability; it does not specify dynamics, linewidth, or noise.	Two-mode Hamiltonian, phase-lock energy, lifetime, and pole tests.
S(x,t) = sum_ij N(x_ij,t_ij)	The node interaction term and continuum map were undefined.	Intervention-defined graph, coarse-graining, and controlled continuum target.
F_g = G m_1 m_2 H(f_1,f_2)/r^2	H had no derivation, units, or independent calibration; Newton's law was inserted rather than recovered.	Operational metric and Newton/Einstein recovery are explicit closure obligations.
Compact resultant-reality equation using (hbar c)^2/G, E/psi, and adjustable factors	The output dimension, wavefunction normalization, and fitted parameters were not fixed, allowing target-dependent agreement.	One microscopic action, a unit dictionary, parameter freezing, and held-out likelihoods.

This removal strengthens rather than weakens the core idea. It isolates what is genuinely original and makes the remaining mathematical tasks visible.

2. Primitive relational structure

Status: DEFINED.

The primitive object is an ordered history of complete micro-configurations,

$$X_0 \prec X_1 \prec X_2 \prec \cdots, X_n \in X.$$

The integer n orders records. It is dimensionless and is not yet a physical duration. A compact primitive specification is

$$P=(X,R,A,U_p),$$

where X is the state space, R the admissible direct relations, A the admissible changes, and U_p a single-valued update with parameter vector p. If U_p is second-order in update order, the state can be augmented so that the evolution remains first-order on the enlarged state space.

A canonical stabilized configuration may be written

$$G_n=(V_n,E_n,w_n),\Psi_i^n=\rho_i^n e^{i\theta_i^n}\in C,U_{ij}^n\in G,$$

with $U_{ji}^n=(U_{ij}^n)^{-1}$ and covariant link difference

$$D_{ij}\Psi^n=U_{ij}^n\Psi_j^n-\Psi_i^n.$$

Here G_n is a locally finite weighted graph and U_{ij} parallel-transports an internal state along an oriented link. The minimal closed benchmark fixes $G_n=G$ and first studies real scalar coordinates. The complex and gauge-linked form is a controlled extension, not a claim that the Standard Model group or its matter representations have already been derived.

2.1 Graph relabelling and primitive locality

Node labels carry no physical meaning. A graph relabelling $i\mapsto\pi(i)$ must leave all observable predictions unchanged. Primitive locality means that U_p uses a bounded neighbourhood in the graph at each update. It does not mean that the graph has already been embedded in Euclidean space.

A complete microscopic proposal must therefore state:

the state carried by each node and link;

the graph-generation or graph-update rule;

the neighbourhood used by the update;

every branch and tie-break;

boundary and initial conditions;

conserved quantities or controlled dissipation;

the capacity rule; and

the parameter domain.

3. Operational time and persistent identity

3.1 Time from a recurrent pattern

Status: DEFINED; universality is CLOSURE.

Let a persistent subsystem K possess a complex clock observable

$$Z_K(X_n)=A_K(X_n)e^{i\Theta_K(X_n)}.$$

After unwrapping its phase, the completed cycle count is

$$N_K(n)=\frac{unwrap\,\Theta_K(n)-\Theta_K(0)}{2\pi},$$

and a calibrated clock reading is

$$t_K(n)=T_K N_K(n).$$

This distinguishes update order from measured duration. A physical time coordinate requires more than one clock. Different low-load clock constructions must converge to stable rate ratios, and motion or local load must affect all admissible clocks through the same low-energy law. That clock-universality theorem has not yet been derived from a frozen microscopic model.

3.2 Persistent patterns as candidate objects

Status: DEFINED.

Choose a coarse-graining map C_R, a distance d_R, an allowed transformation group G_R, and tolerance ϵ_R. A pattern persists for T updates when

$$\inf_{g \in G_R} d_R\left(C_R X_{n+T}, g C_R X_n\right) < \epsilon_R .$$

Its lifetime estimator is

$$\tau_R = \inf\left\{T : d_R\left(C_R X_{n+T}, G_R C_R X_n\right) \geq \epsilon_R\right\}.$$

This converts “the same object later” into an algorithm. The resolution, tolerance, allowed transformations, perturbation ensemble, and observation window are all part of the definition. A candidate particle must additionally carry a dispersion relation, conserved or topological labels, scattering amplitudes, and a detector-level map.

3.3 Influence defines adjacency

Status: DEFINED; graph emergence from a microscopic rule is PROPOSED.

Perturb persistent class a by a small admissible change δ and measure the resolved response of class b after Δ updates. A finite-difference influence score is

$$I_{ab}(n) = \frac{\left\| C_R X_{n+\Delta}^{(a+\delta)} - C_R X_{n+\Delta} \right\|_b}{\| \delta \|}.$$

Averaging over a fixed window gives w_{ab}. Retaining links above a registered threshold produces an effective graph $G_R = \left(V_R, E_R, W_R\right)$. Distance can then be defined from positive edge lengths and shortest paths. Scale-dependent spectral dimension can be estimated from the graph heat kernel,

$$K(\sigma) = e^{-\sigma L}, P(\sigma) = \frac{1}{|V|} Tr\, K(\sigma), d_s(\sigma) = -2 \frac{d \ln P}{d \ln \sigma}.$$

A plateau in d_s is an empirical property of an ensemble and scale range; dimension is not chosen merely because the graph is convenient to draw in three dimensions.

4. Local capacity as an admissibility constraint

The distinctive END proposal is that every neighbourhood has a finite capacity for change during one update. This idea becomes scientific only after a non-negative demand, an admissible set, a projection or constrained update, and a conservation audit are specified.

4.1 Scalar capacity ledger

Status: PROPOSED benchmark definition.

For a real scalar coordinate on a weighted graph, define

$$v_i^n = \frac{q_i^{n+1} - q_i^n}{\Delta \tau},$$

and the local energy-like demand

$$E_i^n = \frac{\mu}{2}\left(v_i^n\right)^2 + \frac{\kappa}{4a^2}\sum_{j\sim i} w_{ij}\left(q_i^n - q_j^n\right)^2 + V_{+¿\left(q_i^n\right),¿}$$

where $V_{+¿\geq 0¿}$ is the portion of the onsite potential assigned to the ledger. With capacity scale $E_\star > 0$,

$$C_i^n = \frac{E_i^n}{E_\star} \leq 1, r_i^n = 1 - C_i^n .$$

The residual r_i^n is dimensionless bookkeeping until a coarse physical normalization is established. It must not be renamed dark energy, vacuum energy, or curvature merely because those interpretations are desired.

4.2 Convex capacity projection

Status: DERIVED for the declared candidate projector.

Let $\tilde{v}_i$ be an unconstrained proposed velocity and S_i^n the static part of the local load. The admissible disk is

$$A_i^n = \left\{ v_i : \frac{\mu}{2}\left|v_i\right|^2 \leq E_\star - S_i^n \right\}.$$

The Euclidean projection minimizes

$$\frac{\mu}{2}\left|v_i - \tilde{v}_i\right|^2$$

subject to the capacity inequality. The Karush-Kuhn-Tucker conditions give the radial solution

$$v_i = s_i \tilde{v}_i, s_i = min\left\{1, \sqrt{\frac{2\left(E_\star - S_i^n\right)}{\mu\left|\tilde{v}_i\right|^2}}\right\},$$

with $s_i = 1$ for $\tilde{v}_i = 0$. Projection onto a closed convex set is non-expansive,

$$\| P_A(x) - P_A(y) \| \leq \| x - y \| .$$

This is a rigorous stability property of the projector itself. It does not prove that the full projected nonlinear time-stepping scheme conserves energy, remains reversible, or has a Lorentz-invariant continuum limit. Those properties must be checked for the composed update.

4.3 Residuals required in every constrained simulation

A capacity-limited run should record at least:

the discrete energy residual $E^{n+1} - E^n$;

time-reversal residual after a forward/backward step;

lattice momentum residual where applicable;

the fraction of nodes with $s_i < 1$;

convergence under graph and step refinement; and

correlation between claimed anomalies and the activation of the limiter.

If an effect disappears when the limiter or grid orientation is changed, it is a numerical/model artefact until a continuum scaling law is demonstrated.

5. Closed scalar-graph benchmark

The benchmark supplies the strongest presently closed mathematical sector. It is deliberately modest: a real scalar coordinate on a fixed weighted graph, with a declared onsite potential. It can be solved, simulated, and rejected without invoking particle names or cosmological interpretation.

5.1 Graph Laplacian

Status: DERIVED.

For symmetric non-negative weights $W=(w_{ij})$, let $D_{ii}=\sum_j w_{ij}$ and

$$L=D-W\,,\,(Lq)_i=\sum_j w_{ij}(q_i-q_j).$$

The quadratic form is

$$q^T Lq=\frac{1}{2}\sum_{i,j} w_{ij}(q_i-q_j)^2\geq 0.$$

Thus all Laplacian eigenvalues are non-negative. On a connected graph, the constant vector is the unique zero mode.

5.2 Discrete action and update

Status: DEFINED action; DERIVED update.

On a stabilized graph, take

$$S_d=\sum_n \Delta\tau\left[\frac{\mu}{2}\sum_i\left(\frac{q_i^{n+1}-q_i^n}{\Delta\tau}\right)^2-\frac{\kappa}{2a^2}\sum_{\langle ij\rangle} w_{ij}(q_i^n-q_j^n)^2-\sum_i V(q_i^n)\right).$$

Discrete variation yields

$$\mu\frac{q_i^{n+1}-2q_i^n+q_i^{n-1}}{\Delta\tau^2}=-\frac{\kappa}{a^2}(Lq^n)_i-V'(q_i^n),$$

or explicitly

$$q_i^{n+1}=2q_i^n-q_i^{n-1}-\frac{\Delta\tau^2}{\mu}\left[\frac{\kappa}{a^2}(Lq^n)_i+V'(q_i^n)\right).$$

For a finite graph and a single-valued differentiable V, the recursion is algebraically deterministic. A capacity projection is an additional declared layer and must be audited separately.

5.3 Exact invariant in the linear sector

Let

$$V(q)=\frac{\mu\Omega_0^2}{2}q^2\,,\,A=\Omega_0^2 I+\frac{\kappa}{\mu a^2}L.$$

The linear update

$$q^{n+1}-2q^n+q^{n-1}+\Delta\tau^2 Aq^n=0$$

preserves the half-step bilinear quantity

$$H^{n+1/2} = \frac{1}{2}\left\|\left(\frac{q^{n+1}-q^{n}}{\Delta\tau}\right)^{2}\right. + \frac{1}{2}\left(q^{n+1}\right)^{T} A\, q^{n}.$$

Positivity of this invariant is linked to the stability interval below. The companion code reproduces relative drift of approximately 4.6×10^{-15} in a stable double-precision run, which is an implementation check rather than a physical measurement.

5.4 Exact lattice dispersion

Status: DERIVED.

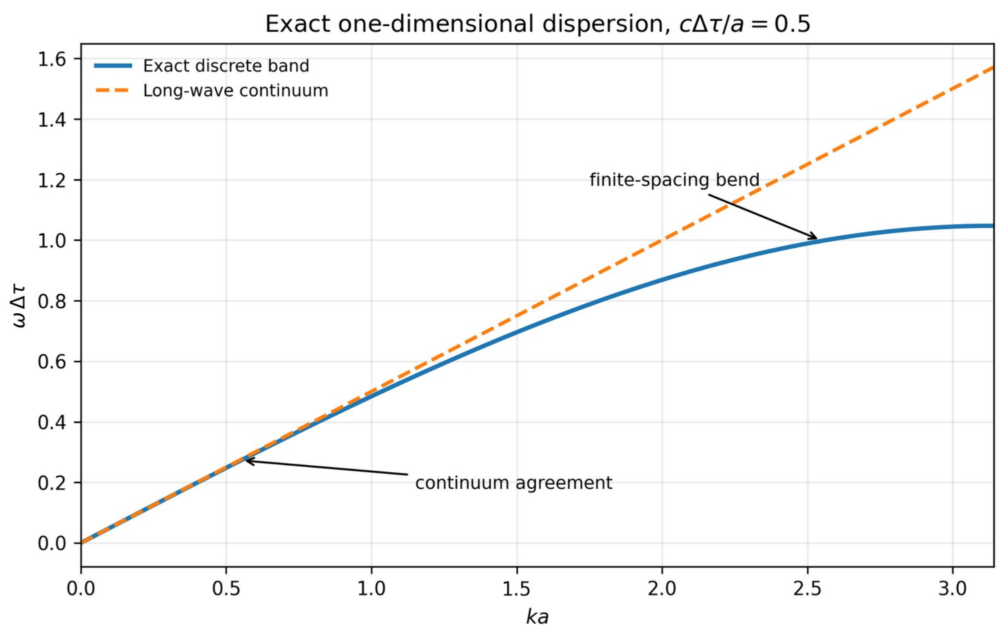
For a periodic d-dimensional cubic lattice and plane wave

$$q_{j,n} = A_0 e^{i\left(k\cdot x_j - \omega n \Delta\tau\right)},$$

with $c^2 = \kappa/\mu$, the exact band is

$$\frac{4}{\Delta\tau^2}\sin^2\left(\frac{\omega\Delta\tau}{2}\right) = \Omega_0^2 + \frac{4c^2}{a^2}\sum_{r=1}^{d}\sin^2\left(\frac{k_r a}{2}\right).$$

This is a genuine output of the declared benchmark. It is not yet a statement about photons or gravitons because a physical clock, ruler, field normalization, source, and detector response have not been matched.



The exact lattice band and its long-wave approximation.

For $|k_r a| \ll 1$ and $|\omega\Delta\tau| \ll 1$,

$$\omega^2 = \Omega_0^2 + c^2|k|^2 + \frac{\Delta\tau^2\omega^4}{12} - \frac{c^2 a^2}{12}\sum_r k_r^4 + \cdots.$$

The leading term has relativistic form. The displayed corrections quantify temporal discretization and spatial anisotropy. A relativistic interpretation additionally requires rotational/Lorentz recovery, universal coupling to admissible clocks, and a detector-level dictionary.

5.5 Necessary and sufficient linear stability

Real mode frequencies require

$$0 \leq \frac{\Delta\tau^2}{4}\left[\Omega_0^2 + \frac{4c^2}{a^2}\sum_r \sin^2\left(\frac{k_r a}{2}\right)\right] \leq 1.$$

The maximum occurs at a Brillouin-zone corner, giving the global bound

$$\Delta\tau^2\left(\Omega_0^2 + \frac{4dc^2}{a^2}\right) \leq 4.$$

This condition is necessary and sufficient for all normal-mode roots to lie on the unit circle in the linear hypercubic benchmark.

5.6 Group velocity and mass-gap dictionary

The exact branch is

$$\omega(k) = \frac{2}{\Delta\tau}\arcsin\left[\frac{\Delta\tau}{2}\sqrt{A(k)}\right].$$

At $k=0$, the gap is ω_0. Only after a physical action scale $\hbar_{eff}$ and causal speed c_{phys} have been independently matched may one define

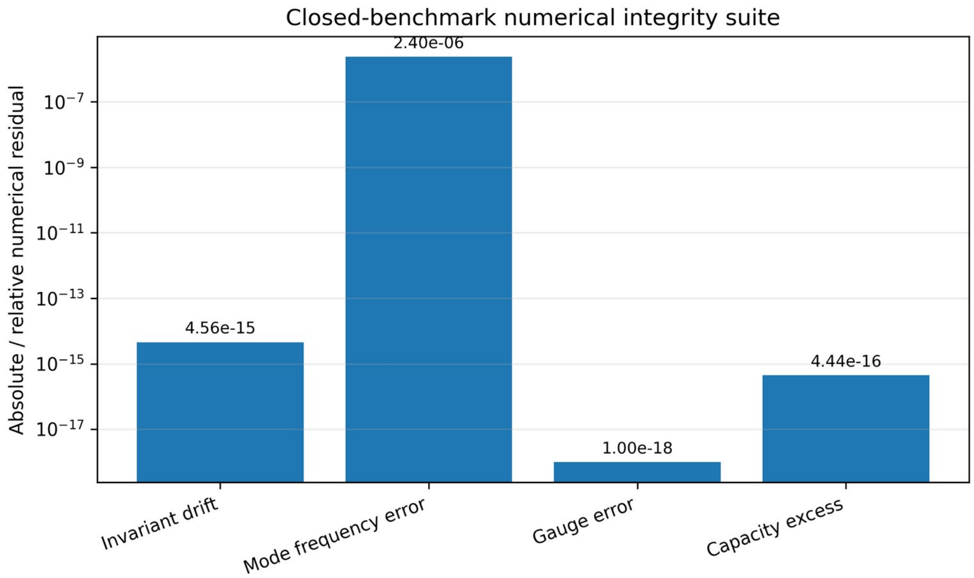
$$m_{eff} c_{phys}^2 = \hbar_{eff}\,\omega_0.$$

This is a dictionary. It is not a particle-mass prediction until the gap follows from a frozen pattern calculation and the unit matching was fixed on separate data.

5.7 Numerical integrity suite

The supplied toolkit runs normal-mode, invariant, gauge, stability, and capacity tests with deterministic seeds. The reference execution produced:

Check	Result	Interpretation
Half-step invariant relative range	4.56 x 10^-15	Roundoff-scale drift in the stable linear sector.
Maximum fitted mode-frequency relative error	2.40 x 10^-6	Phase-estimator accuracy against exact dispersion.
Analytic stability margin	3.8351	Positive; chosen run is inside the stable region.
U(1) covariant-link energy relative error	< 10^-18	Finite transformation unit test.
Maximum post-projection capacity ratio	1 + 4.4 x 10^-16	Capacity bound satisfied to floating-point tolerance.



Numerical residuals from the closed-benchmark integrity suite.

These checks validate code against mathematics already derived. They do not validate a bridge to nature.

6. A precise model of node pairing

The early phrase “nodes pair when their frequencies match” becomes useful when it is translated into coupled-mode dynamics, finite linewidth, binding energy, lifetime, and an observable pole.

6.1 Two-mode Hamiltonian

Status: DEFINED model; eigenvalues DERIVED.

The minimal one-excitation Hamiltonian is

$$H = \epsilon_1 a_1^\dagger a_1 + \epsilon_2 a_2^\dagger a_2 - J\left(a_1^\dagger a_2 + a_2^\dagger a_1\right).$$

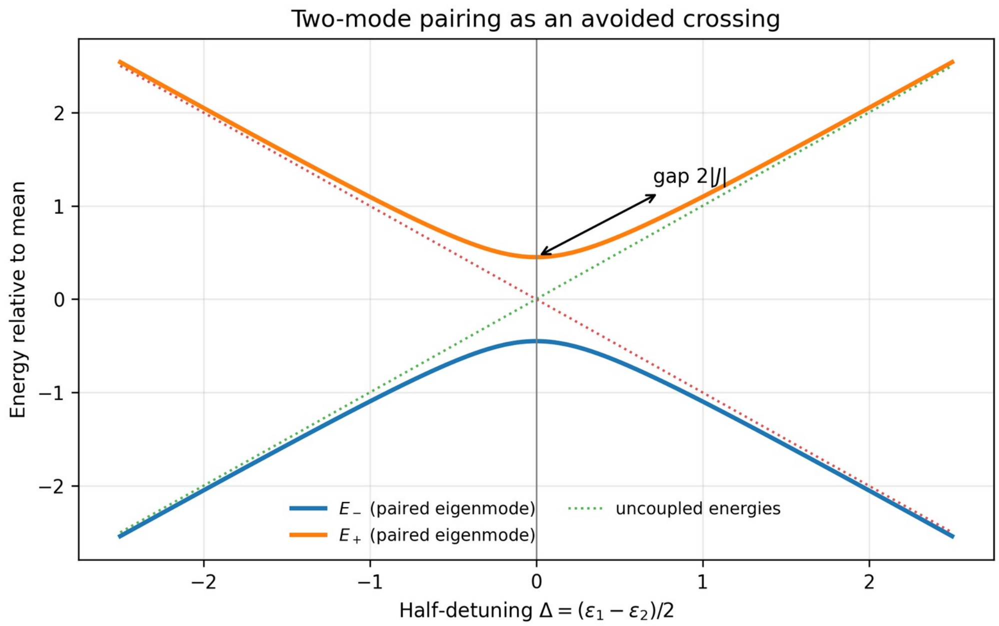
In the one-excitation sector,

$$H_1 = \begin{pmatrix} \epsilon_1 & -J \\ -J & \epsilon_2 \end{pmatrix}.$$

With $\acute{\epsilon} = \left(\epsilon_1 + \epsilon_2\right)/2$ and $\Delta = \left(\epsilon_1 - \epsilon_2\right)/2$,

$$E_\pm = \acute{\epsilon} \pm \sqrt{\Delta^2 + J^2}.$$

At resonance, the level splitting is $2|J|$. This is an avoided crossing rather than a singular yes/no delta function.



Coupling replaces an exact-frequency delta function with a measurable avoided crossing.

6.2 Phase locking

For complex amplitudes $\Psi_r = \sqrt{n_r} e^{i\theta_r}$, the hopping energy is

$$-J\left(\Psi_1^{*}\Psi_2 + \Psi_2^{*}\Psi_1\right) = -2J\sqrt{n_1 n_2}\cos\varphi\,,\varphi = \theta_2 - \theta_1.$$

For $J > 0$, the minimum is at $\varphi = 0$; for $J < 0$, it is at $\varphi = \pi$. A noisy or open system replaces perfect locking with a finite distribution governed by damping, drive, temperature, and detuning.

6.3 Binding and particle criteria

Define the binding energy

$$B = \left(E_1 + E_2\right) - E_{12}.$$

$B > 0$ is necessary for an energetically bound pattern, but it is insufficient for a particle. A particle candidate must also demonstrate localization or controlled support, perturbative stability, a dispersion relation, conserved or topological labels, scattering states, a continuum limit, and a detector map.

For interpolating operator O, a stable one-particle state appears as a pole in the two-point function,

$$G\left(p^2\right) = \int_0^\infty ds\,\frac{\rho(s)}{p^2 - s + i0}\,,\rho(s) \supset Z\,\delta\left(s - m^2\right),$$

so that

$$G\left(p^2\right) \sim \frac{Z}{p^2 - m^2 + i0}.$$

This supplies a field-theoretic test for whether a persistent node pattern behaves as a particle.

7. Finite-spacing internal symmetry

7.1 U(1) link covariance

Status: DERIVED for the declared lattice implementation.

Let

$$\Psi_i \mapsto e^{iq\alpha_i}\Psi_i, U_{ij} \mapsto e^{iq\alpha_i} U_{ij} e^{-iq\alpha_j}.$$

Then

$$D_{ij}\Psi = U_{ij}\Psi_j - \Psi_i \mapsto e^{iq\alpha_i} D_{ij}\Psi,$$

and $\left|D_{ij}\Psi\right|^2$ is exactly invariant at finite spacing. For a smooth Abelian field,

$$U_\mu(x) = e^{iqaA_\mu(x)},$$

and the plaquette satisfies

$$U_{\mu\nu}(x) = e^{iqa^2F_{\mu\nu}(x) + O(a^3)},$$

so

$$1 - \Re U_{\mu\nu} = \frac{q^2a^4}{2} F_{\mu\nu}^2 + O(a^6).$$

This shows how a chosen compact gauge structure may be implemented. It does not derive why nature selects $SU(3)\times SU(2)\times U(1)$, which representations occur, or how chirality and anomaly cancellation emerge. Those are separate closure gates constrained by lattice no-go results [5], [6], [7], [8].

7.2 Candidate scalar-plus-link master action

Status: PROPOSED microscopic candidate; scalar/link consequences are DERIVED after the action is assumed.

A consolidated candidate on a stabilized graph is

$$S_\star[\Psi, U] = \sum_n \Delta\tau [\text{¿} \frac{\mu}{2}\sum_i \left|\frac{\Psi_i^{n+1} - \Psi_i^n}{\Delta\tau}\right|^2 - \frac{\kappa}{2a^2}\sum_{\langle ij\rangle} w_{ij}\left|D_{ij}\Psi^n\right|^2 \text{¿} - \sum_i V\left(\left|\Psi_i^n\right|^2\right) - \frac{\beta}{a^4}\sum_p \left(1 - \frac{1}{d_R}\Re\, tr\, U_p^n\right)].$$

An effective onsite expansion may be written

$$V\left(|\Psi|^2\right) = V_0 + rac\, m_0^2 2|\Psi|^2 + rac\,\lambda_4 4|\Psi|^4 + rac\,\lambda_6 6 M_\star^2 |\Psi|^6 + \cdots.$$

The coefficients are model parameters unless a deeper update kernel fixes them. m_0 is a spectral coefficient, not automatically the mass of a known particle.

8. Continuum and quantum-field-theory programme

8.1 Controlled continuum recovery

Status: CONDITIONAL.

A continuum field may be defined from coarse graph modes only when normalization, isotropy, and convergence are demonstrated. For a regular graph, the long-wave action has the schematic form

$$S_{eff} = \int dt\, d^d x \left[Z_t |\partial_t \psi|^2 - Z_s |D_i \psi|^2 - V_{eff}(|\psi|^2) - \frac{1}{4 g_R^2} F_{\mu\nu} F^{\mu\nu} + \sum_n \frac{c_n}{\Lambda^{n-d-1}} O_n \right).$$

The coefficient map must be obtained by correlation functions, response, or controlled blocking. Refinement should demonstrate an error law rather than a visually smooth plot. Discrete variational methods supply a standard mathematical comparison for the scalar update [9].

8.2 Quantum probability closure

Status: CLOSURE.

A deterministic update does not by itself produce the Born rule. A complete operational model must define preparation P, settings x, y, hidden or effective state λ, response functions, outcome probabilities, and the distribution over initial conditions. It must reproduce interference, contextuality, Bell/CHSH violations, and operational no-signalling with one frozen specification [10], [11].

The target quantum map can be written

$$P(o \vee x, \rho) = Tr\left(\rho E_o^{(x)}\right),$$

but writing the target does not derive it. Any proposed deterministic completion must state which Bell assumption is altered and demonstrate that the resulting model does not permit controllable superluminal signalling.

8.3 Matter and chirality closure

Status: CLOSURE.

A physical unification requires stable fermionic excitations, the observed chiral representations, anomaly cancellation, mass generation, mixing, and a renormalized coupling flow. The exact scalar/link benchmark provides none of these automatically. A viable extension must confront fermion doubling and demonstrate the Standard Model spectrum or a falsifiable alternative without inserting the answer.

9. Geometry, gravitation, and cosmology

The relational premise is compatible with established programmes in which geometry is reconstructed from discrete relations, including Regge calculus and causal-set approaches [12], [13]. Compatibility of motivation, however, is not a derivation.

9.1 Operational metric target

Status: CLOSURE.

An effective metric should be inferred from physical clock readings, signal travel times, and transport around loops. Locally, measured intervals would have to fit a universal quadratic form,

$$d\tau^2 = -\frac{1}{c_{phys}^2} g_{\mu\nu} dx^\mu dx^\nu,$$

with the same $g_{\mu\nu}$ for all admissible low-energy clocks and probes. The graph must supply an embedding-independent construction of the tangent structure or an operational substitute.

9.2 Newtonian and Einstein limits

Status: CLOSURE.

A completed geometric sector must first recover the Newtonian limit,

$$\nabla^2 \Phi = 4\pi G \rho.$$

It must also recover the Einstein field equation,

$$G_{\mu\nu}+\Lambda g_{\mu\nu}=\frac{8\pi G}{c^4}T_{\mu\nu}.$$

These limits require the equivalence principle, gravitational redshift, lensing, post-Newtonian parameters, gravitational-wave polarizations and speed, black-hole exterior solutions, and conservation through the Bianchi identity. Adding an undefined node tensor to Einstein's equation is not sufficient; the tensor, its divergence, and its coupling must follow from a variational or coarse-graining construction.

9.3 Residual capacity and cosmology

Status: PROPOSED research direction.

The residual $r_i^n=1-C_i^n$ may be coarse-grained into a field only after a local balance law is specified. A generic conservative form is

$$\frac{r_i^{n+1}-r_i^n}{\Delta\tau}+\sum_j J_{ij}^n=S_i^n,$$

with antisymmetric exchange $J_{ij}=-J_{ji}$ and a declared source term. Homogeneous cosmology would then require a continuum stress-energy tensor and equation of state derived from the micro-dynamics. Neither accelerated expansion nor the observed cosmological constant follows from the existence of unused local capacity alone.

10. From theory to data: the universal forward model

A wave or particle equation becomes experimental only when it is connected to a source, propagation law, detector response, sampling rule, nuisance model, and likelihood.

Let $s(t,x;\vartheta)$ be a source. A propagator G produces

$$\phi(t,x)=\int G(t-t',x,x';p)\,s(t',x';\vartheta)\,dt'\,d^dx'.$$

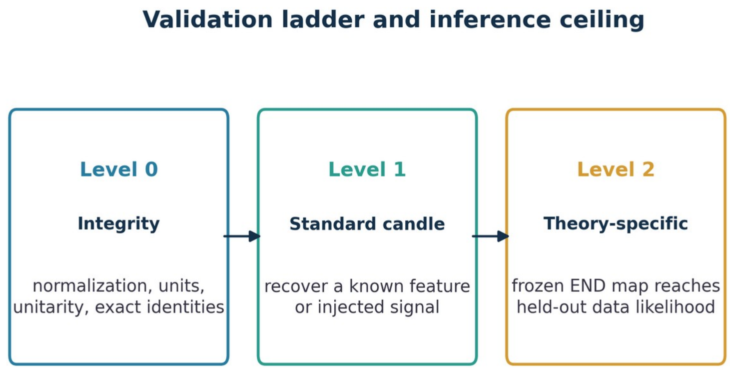
Detector channel a has response kernel R_a,

$$y_a(t)=\int R_a(t-t',x')\,\phi(t',x')\,dt'\,d^dx'+n_a(t).$$

For Gaussian data vector y with mean m and covariance C,

$$-2\ln L=(y-m)^T C^{-1}(y-m)+\ln\det C+\text{constant}.$$

Counting and threshold data require their proper Poisson or collaboration likelihoods. Every arrow in the chain must have units, calibration data, uncertainties, and a synthetic injection/recovery test.



The uploaded public-data companion establishes Levels 0-1; a complete frozen micro-to-observable map is still required for Level 2.

Three levels of validation. The existing public-data work reaches integrity and standard-candle checks, not a full theory-specific likelihood.

The uploaded validation companion correctly concludes that its four analyses validate code paths and detector-response logic, not END coefficients [14]. The Level 3 stress suite further labels the GW coefficient as not identifiable, the ATLAS contact coefficient as lacking an event-rate map, the XENON point as a boundary comparison, and the DUNE result as forecast/integrity only [15].

11. GWOSC validation programme

11.1 Standard-candle timing test

The official GW150914 release provides calibrated Hanford and Livingston strain and documents an inter-site delay of about 6.9 ms for the displayed waveforms [16], [17]. The project companion recovered a 7.324 ms lag with a transparent band-limited cross-correlation [14]. That is a useful pipeline test because it checks data loading, conditioning, sign convention, sampling, and lag recovery.

It does **not** measure propagation from the source because the two detectors are separated by Earth-scale distance and see the same astrophysical waveform. A cosmological dispersion coefficient requires a source model and a propagation kernel over the source distance.

11.2 Modified-dispersion bridge

Status: PROPOSED phenomenological bridge.

A commonly testable deformation is

$$E^2 = p^2 c^2 + A_\alpha p^\alpha c^\alpha ,$$

where A_α has units of energy$^{2-\alpha}$. For a small correction,

$$\frac{v_g}{c} \simeq 1 + \frac{\alpha - 1}{2} A_\alpha p^{\alpha-2} c^{\alpha-2} .$$

A source at redshift z then requires a convention-specific phase or time-delay integral, source-frame frequency evolution, cosmology, waveform family, calibration uncertainty, and population hierarchy. The mapping from microscopic graph parameters $(a, \Delta\tau, c, \dots)$ to A_α must be derived and frozen before events are selected.

A valid programme is:

reproduce a general-relativistic baseline on official GWOSC data;

inject known phase coefficients into off-source or simulated data;

verify unbiased recovery and coverage;

freeze waveform alternatives, calibration treatment, priors, and event partition;

calculate the END coefficient from the microscopic model, including units;

apply a hierarchical held-out comparison; and

retire the bridge if detectors, waveform families, or event classes disagree beyond the registered tolerance.

The toolkit includes the detector-lag standard candle. It intentionally refuses to interpret that lag as A_{α}.

12. CERN / ATLAS validation programme

12.1 Neutral scalar template

Status: CONDITIONAL.

A persistent scalar pattern may be represented at low energy by

$$L_s = ¿ \frac{1}{2}(\partial s)^2 - r a c 12 m_s^2 s^2 - \lambda_{sH} s^2 H^{\dagger} H - \sum_f y_f^{(s)} s \acute{f} f$$
$$- \frac{c_g}{4\Lambda} s G_{\mu\nu}^a G^{a\mu\nu} - \frac{c_\gamma}{4\Lambda} s F_{\mu\nu} F^{\mu\nu}.$$

This becomes an END prediction only when m_s, width, and couplings follow from a frozen node-pattern calculation. Scanning them freely produces limits on a template family, not confirmation of the framework.

12.2 Contact interaction

A dimension-six example is

$$L_{EFT} \supset \frac{C}{\Lambda^2} (\acute{q} \gamma^{\mu} q)(\acute{\ell} \gamma_{\mu} \ell).$$

A collider prediction requires operator flavour structure, interference with the Standard Model, running, parton distributions, event generation, detector selection, nuisance covariance, and an EFT validity rule. Accepted events with partonic energy above the registered cutoff cannot be used to claim a valid EFT constraint.

12.3 Educational four-lepton standard candle

CERN Open Data record 15005 provides an educational ATLAS Run-2 sample and explicitly warns that it is not suitable for scientific publication [18]. The project companion used the four data ROOT files, selected tight isolated four-lepton events, reconstructed

$$m_{4\ell}^2 = \left(\sum_i E_i\right)^2 - \left|\sum_i p_i\right)^2,$$

and fitted a fixed 125 GeV Gaussian plus an exponential background [14]. This is a valid reconstruction and likelihood exercise. It is not a discovery analysis and does not test the historical value $C/\Lambda^2 = 0.015\, TeV^{-2}$ because no mapping to signal yield was supplied.

For binned counts,

$$L(\mu,\theta)=\prod_b Pois\left(n_b \mid \mu s_b(\theta)+b_b(\theta)\right)\prod_k \pi_k(\theta_k).$$

Profile-likelihood intervals and look-elsewhere corrections should follow established methods [19], [20]. The toolkit includes a transparent educational fitter and optional ROOT reader, while preserving the publication limitation.

13. XENONnT validation programme

Status: TEST framework; END particle bridge is CLOSURE.

For elastic dark-matter-like nuclear recoil,

$$\frac{dR}{dE_R}=\frac{\rho_\chi}{m_\chi m_T}\int_{v>v_{min}} d^3v\, v\, f(v)\frac{d\sigma_T}{dE_R}.$$

A detector prediction must then include nuclear response, charge and light yields, threshold, efficiencies, migration, backgrounds, and the collaboration likelihood or official recast interface. XENONnT has released data products for its low-energy ionization search [21], [22]. Reproducing a published boundary is a strong pipeline check but not a prediction if the mass and cross section were selected from that boundary.

An END-specific test requires:

a stable pattern with a pole-defined mass;

a derived mediator and coupling structure;

a nucleon or electron scattering amplitude;

a recoil spectrum with units;

a frozen astrophysical and detector nuisance policy; and

a region required independently by another held-out prediction.

The historical $m_\chi=8$ GeV and $\sigma_{SI}=3.28\times10^{-46}$ cm❑^{2} card should therefore remain labelled a phenomenological benchmark until the microscopic chain is supplied.

14. DUNE validation programme

Status: CONDITIONAL forecast.

Three-flavour propagation in matter with a proposed END contribution is

$$i\frac{d\nu}{dx}=\left[\frac{1}{2E}U\,diag\left(0,\Delta m_{21}^2,\Delta m_{31}^2\right)U^\dagger+diag\left(V_{CC},0,0\right)+H_{END\nu}\right)\nu.$$

The evolution operator $S(L)=e^{-iHL}$ must be unitary for Hermitian H. The uploaded companion verified this numerical integrity to approximately 2.4×10^{-15} and produced a release-based appearance forecast [14]. DUNE's public simulation configurations are appropriate for reproducing TDR-like sensitivities before adding a new Hamiltonian [23].

A valid END forecast must specify the full Hermitian matrix structure of $H_{END\nu}$, its energy and density dependence, neutrino/antineutrino transformation, and its derivation from the microscopic model. It must then propagate through fluxes, cross sections, migration, efficiencies, near-detector constraints, matter-density uncertainty, and correlated systematics. Pseudo-data and sensitivity curves must remain labelled forecasts.

15. Constants, units, and honest dependencies

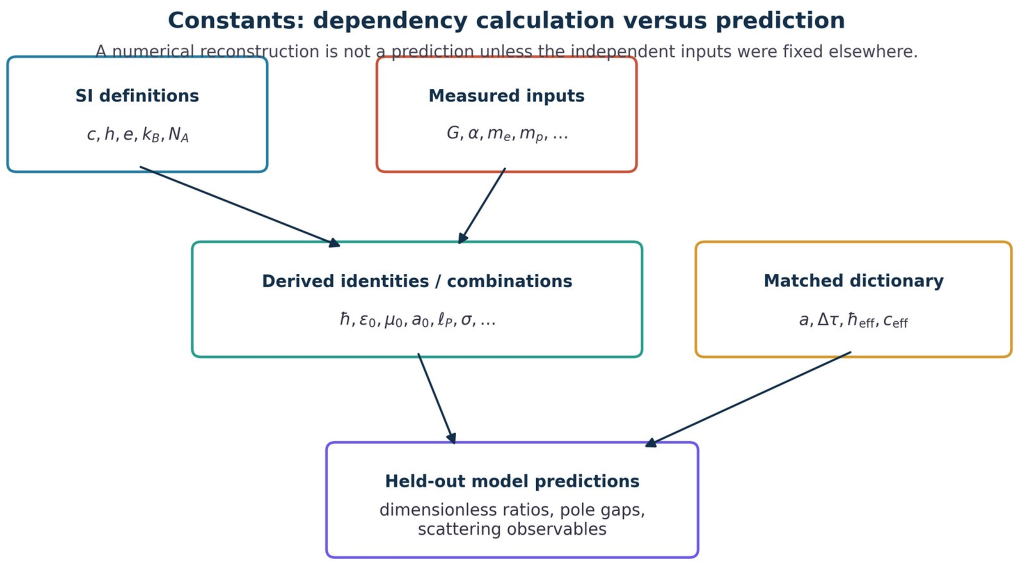
The request to "calculate all constants" can be made rigorous by distinguishing four operations:

Definition: fixes a unit or convention, such as the exact SI value of c.

Identity: recombines established quantities, such as $\hbar = h/(2\pi)$.

Matching: equates a model coefficient to a measured scale, such as $c_{model} a/\Delta\tau = c_{SI}$.

Prediction: calculates a held-out quantity after matching fewer independent inputs elsewhere.



A constants calculation must identify which numbers are definitions, measurements, identities, matches, or predictions.

The toolkit uses the 2022 CODATA central values [24]. Its 42-entry ledger contains five SI definitions, four measured inputs, and 33 derived identities. Selected entries are shown below.

Quantity	Value used or calculated	Status	Dependency
c	299 792 458 m s^-1	DEFINED	SI definition
h	6.62607015 x 10^-34 J s	DEFINED	SI definition
e	1.602176634 x 10^-19 C	DEFINED	SI definition
k_B	1.380649 x 10^-23 J K^-1	DEFINED	SI definition
G	6.67430 x 10^-11 m^3 kg^-1 s^-2	MEASURED INPUT	CODATA
alpha	7.2973525643 x 10^-3	MEASURED INPUT	CODATA
hbar	1.0545718176 x 10^-34 J s	DERIVED IDENTITY	h/(2 pi)
epsilon_0	8.8541878188 x 10^-12 F m^-1	DERIVED IDENTITY	1/(mu_0 c^2)
a_0	5.2917721054 x 10^-11 m	DERIVED IDENTITY	hbar/(m_e c alpha)
l_P	1.6162550 x 10^-35 m	DERIVED IDENTITY	sqrt(hbar G/c^3)
t_P	5.3912464 x 10^-44 s	DERIVED IDENTITY	sqrt(hbar G/c^5)
m_P	2.1764340 x 10^-8 kg	DERIVED IDENTITY	sqrt(hbar c/G)
E_P	1.956082 x 10^9 J	DERIVED IDENTITY	m_P c^2
sigma	5.670374419 x 10^-8 W m^-2 K^-4	DERIVED IDENTITY	2 pi^5 k_B^4/(15 h^3 c^2)

The Planck combinations are

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}}, t_P = \sqrt{\frac{\hbar G}{c^5}}, m_P = \sqrt{\frac{\hbar c}{G}},$$

$$E_P = m_P c^2, T_P = \frac{E_P}{k_B}, P_P = \frac{c^5}{G}.$$

These are exact dependency calculations given the inputs. They do not derive G, α, or the particle spectrum from END Theory.

15.1 What would count as an END prediction of a constant

A genuine constant prediction would require:

one frozen microscopic action or update;

a dimensionless internal spectrum or ratio independent of units;

a unit dictionary matched on a separate minimal set;

renormalization and scale dependence where relevant;

uncertainty from finite size, discretization, fitting, and matching;

a value not used to select the model branch or parameters; and

comparison to a held-out metrological or experimental quantity.

A promising first target is a dimensionless ratio, such as the ratio of two independently identified pole gaps or a coupling ratio. Dimensionless predictions avoid disguising an arbitrary unit calibration as physical explanation.

16. Parameter identification and the frozen card

A model is identifiable only when different parameter choices do not produce indistinguishable detector predictions over the tested data. Before opening held-out data, the following must be frozen:

microscopic state space, graph family, and update rule;

boundary/initial-condition ensemble;

capacity scale and projection;

coarse-graining and pattern classifier;

physical clock, ruler, action, and field-normalization dictionary;

experiment-specific coefficient map and units;

source and detector models;

nuisance priors and covariance;

calibration/holdout split;

failure, stopping, and multiplicity rules; and

software and input hashes.

The historical Level 3 benchmark card is preserved in the code archive because it documents the project's progress. It is not promoted to a microscopic derivation. In particular, the absence of a unit for A_α makes the GW bridge non-identifiable; the CERN coefficient lacks a generated event-rate map; the XENON point lies on a released exclusion boundary; and the DUNE coefficient was used only as a propagation perturbation [15].

17. Falsification matrix and completion criteria

Claim	Minimum decisive calculation	Failure rule
Persistent patterns behave as particles	Stable localized solution, pole, scattering, refinement, detector map	No isolated pole, unstable under refinement, or detector signature indistinguishable from lattice artefact
Capacity produces universal clock slowing	Two or more distinct clocks under varied local load and motion	Clock response depends on construction after calibration
Relativistic continuum emerges	Isotropic cone, Lorentz-sensitive correlators, controlled a, Delta-tau -> 0 scaling	Preferred-frame or anisotropy residual does not vanish within registered law
Quantum probabilities emerge	One preparation-setting-outcome model reproduces interference, CHSH, contextuality, no-signalling	Any test requires a different hidden distribution or response rule
Gravity emerges	Operational metric, equivalence principle, Newtonian and post-Newtonian limits, wave sector	Universal coupling or conservation fails; G is inserted rather than derived or matched once
Standard Model matter emerges	Chiral anomaly-free spectrum and renormalized couplings	Spectrum is inserted by hand or violates no-go or anomaly constraints
GW dispersion is predicted	Frozen micro-to-A_alpha map with units and hierarchical waveform likelihood	Coefficient changes by event, waveform family, or calibration choice
CERN signal is predicted	Fixed mass, width, couplings, generated signal, detector likelihood, EFT validity	Parameters are scanned after viewing signal region or accepted events violate cutoff
Dark-sector recoil is predicted	Derived mass and amplitude with official detector likelihood	Point was selected from the published limit or required region is excluded
Constants are predicted	Independent dimensionless spectrum and held-out metrology	Target value enters calibration, branch selection, or unit definition

The completion criterion is therefore:

END Theory becomes a quantitatively competitive physical theory when one immutable microscopic model crosses its quantum, matter, geometry, and observation closure gates; fixes fewer independent inputs than the number of held-out observables it explains; publishes its failure regions; and survives independent reproduction.

This criterion is intentionally stronger than “the idea resembles known physics” and more useful than claiming completion before the bridges exist.

18. Reproducible research package

The companion archive contains:

end_theory.core: graph Laplacian, exact update, invariant, dispersion, stability, capacity projection, U(1) unit test, and pairing spectrum;

end_theory.constants: 2022 CODATA parser and 42-entry dependency ledger;

end_theory.statistics: binned Poisson likelihood and educational scalar-template fit;

end_theory.gwosc: official-HDF5 reader and transparent GW150914 detector-lag recovery;

end_theory.atlas: four-lepton kinematics and optional ROOT reader;

end_theory.xenon: recoil kinematic/shape helpers, explicitly not a detector likelihood;

end_theory.neutrino: three-flavour matter Hamiltonian and unitary propagation;

blank and historical parameter cards;

deterministic unit tests and saved result ledgers.

The core checks can be run with:

```
python -m pip install -e .[test]
python scripts/run_core_validation.py
python scripts/calculate_constants.py
pytest -q
```

The generated results are machine-readable JSON/CSV. The code is deliberately modular so that a future microscopic-to-observable map can be inserted without rewriting the data-selection and likelihood layers.

19. Discussion

The strongest part of the project is not the historical compact TOE equation. It is the relational sequence that motivated it. The idea that operational spacetime and stable objects may emerge from patterns of influence is scientifically recognizable and can be expressed without invoking mystical or undefined terms. The capacity proposal adds a distinctive constraint that can be tested in simulations. The exact lattice sector provides a real mathematical backbone: positivity, normal modes, discrete variation, stability, dispersion, finite-spacing gauge covariance, and a precise pairing model.

The principal weakness of the earlier material was not the intuition but the collapse of several logical steps into one formula. Standard constants were combined, adjustable factors were tuned, and agreement was sometimes described as validation even when the target selected the inputs. The latest uploaded technical and validation editions already correct much of that problem. This paper makes the correction central: a candidate ontology, a closed benchmark, a proposed bridge, and a detector test are different scientific objects.

The most efficient next theoretical objective is not to calculate a long list of dimensional constants. It is to freeze one microscopic model and derive one new dimensionless observable. Good candidates include:

a ratio of two persistent-pattern gaps;

a universal load-dependent clock-rate coefficient shared by distinct clock constructions;

a controlled coefficient of the leading lattice-dispersion correction;

a scattering-length or phase-shift ratio from two independently identified patterns; or

a graph-spectral exponent in a scale range not used for calibration.

A successful result at that level would determine whether the core idea has predictive leverage. A failure would also be valuable because the falsification matrix identifies which layer must be revised.

20. Conclusion

This manuscript preserves Jordan Ryan Evans's central proposal while replacing unsupported AI-generated equations with a dependency-ordered research programme. Reality is represented first as ordered relational change. Persistent recurrent patterns define candidate objects and clocks. Intervention defines effective adjacency. A finite local capacity restricts admissible change. On a fixed graph, the resulting scalar/link benchmark has exact, reproducible mathematics. Node pairing is represented by coupled modes and tested by binding, lifetime, scattering, and pole criteria.

The framework is not yet a completed theory of everything. Quantum probabilities, chiral matter, universal operational geometry, Einstein gravity, cosmology, and unique experiment-specific coefficients remain closure problems. The accompanying validation suite and constants audit are designed to make that boundary productive: they show exactly what can already be reproduced, what is only a template, what must be frozen, and what result would count as a genuine prediction.

The project can therefore be presented honestly and strongly as a **relational unified-physics research programme with a closed mathematical benchmark and a defined route to falsification**. That is a defensible foundation for expert review and further development.

Acknowledgements and disclosure

The conceptual framework and original node-reality intuition are attributed to Jordan Ryan Evans. The project corpus was developed through extensive human-AI interaction. AI tools assisted consolidation, equation transcription, editorial restructuring, and generation of the companion code; they are not treated as scientific authorities or authors. The named author remains responsible for checking the mathematics, provenance, and claims before submission. Independent review is required.

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