END THEORY 2.0

The Evans Relational Framework — Technical Reference Edition

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> **SCIENTIFIC SCOPE.** END Theory / Evans Node Dialect / Matrix Node Theory is presented as an independent relational-physics research programme. The scalar-graph benchmark contains checkable mathematics. Bridges to quantum probability, chiral matter, gravitation, cosmology, and unique empirical predictions remain explicitly gated where the required construction is absent.

This edition preserves the strongest existing textbook as its conceptual and mathematical baseline, then adds ninety two-page technical dossiers. Every added statement page has a unique formal object; every working page has a construction, dimensional audit, decisive calculation, adversarial edge case, and failure rule. The expansion does not promote conjectures by repetition.

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Publication and non-promotion rule

A deterministic update does not derive quantum probabilities. A heat-kernel coefficient does not by itself derive Einstein dynamics or Newton's constant. A sign-changing profile does not by itself produce a chiral anomaly-free spectrum. A scalar potential does not by itself derive dark energy. A numerical agreement is not a prediction when the target selected the parameters, labels, branch, truncation, or accepted run.

The claim grammar is **DEFINED**, **DERIVED**, **PROPOSED**, **CONDITIONAL**, **CLOSURE**, **MEASURED INPUT**, **CALIBRATED**, **TESTED**, or **FALSIFIED / RETIRED**. Consequential statements use the strongest label actually earned by the construction.

> **COMPLETION CRITERION.** END becomes a quantitatively competitive physical theory when one immutable microscopic model crosses its closure gates, fixes fewer independent inputs than the number of held-out observables it explains, publishes its failure regions, and survives independent reproduction.

Reading routes

Proof route

Read the exact scalar-graph benchmark before using any continuum or phenomenological bridge. Reproduce incidence factorization, positivity, the discrete variation, normal modes, exact dispersion, stability, the discrete invariant, and finite-spacing gauge covariance.

Construction route

Follow the dependency order from primitive state through capacity, graph evolution, continuum scaling, gauge and matter structure, quantum maps, operational geometry, cosmology, and detector-level tests.

Experimental route

Begin with the universal forward-model contract, then use the GWOSC, CERN, XENONnT, DUNE, DESI, Planck, Euclid, and transient dossiers. Do not call a forecast or template constraint a validation of the full theory.

Adversarial route

Read the failure rule on every working page. A claim advances only after the stated calculation succeeds without changing the frozen parameter and analysis cards.

Dossier map

Part 1. Relational foundations. F01 Primitive update order; F02 Microscopic state domain; F03 Bounded relational locality; F04 Persistent-pattern equivalence; F05 Operational clock construction; F06 Influence-defined adjacency; F07 Graph distance from route cost; F08 Scale-dependent spectral dimension; F09 Observable completion map; F10 Structural identifiability.

Part 2. Capacity as a mathematical constraint. C01 Demand functional; C02 Hard admissible set; C03 Weighted metric projection; C04 KKT active-set system; C05 Nonexpansive convex projection; C06 Residual-capacity current; C07 Explicit reservoir balance; C08 Local proposal limiter; C09 Simultaneous saturation; C10 Capacity-induced dissipation.

Part 3. Exact scalar-graph benchmark. B01 Incidence factorization; B02 Laplacian positivity and kernel; B03 Central-difference action; B04 Discrete Euler-Lagrange update; B05 Generalized normal modes; B06 Exact lattice dispersion; B07 Long-wave correction; B08 Linear stability bound; B09 Exact half-step invariant; B10 Weak nonlinear frequency shift.

Part 4. Graph evolution and continuum control. E01 Graph evolution in the state; E02 Birth and death rules; E03 Local reconnection; E04 Joint refinement trajectory; E05 Continuum extrapolation; E06 Graph-family universality; E07 Universal causal cone; E08 Anisotropy operator basis; E09 Heat-kernel asymptotics; E10 Renormalized parameter card.

Part 5. Gauge structure and matter targets. G01 Abelian link covariance; G02 Non-Abelian plaquette holonomy; G03 Finite-spacing gauge action; G04 Gauge fixing and ghost gate; G05 Lattice Dirac operator; G06 Fermion-doubling obstruction; G07 Ginsparg-Wilson closure route; G08 Domain-wall closure route; G09 Anomaly cancellation ledger; G10 Mass and mixing overlap map.

Part 6. Quantum map and entanglement. Q01 Preparation measure; Q02 Outcome basin map; Q03 Born-frequency gate; Q04 Bell-CHSH statistic; Q05 Measurement-independence audit; Q06 Operational no-signalling; Q07 Contextuality gate; Q08 Decoherence functional; Q09 Detector-record map; Q10 Statistical distinguishability.

Part 7. Operational geometry and gravitation. R01 Radar-time metric data; R02 World-function reconstruction; R03 Tetrad and transport map; R04 Torsion as loop nonclosure; R05 Regge curvature comparison; R06 Induced curvature term; R07 Einstein-dynamics gate; R08 Newtonian limit; R09 Post-Newtonian dossier; R10 Strong-field and wave sector.

Part 8. EQEF cosmology and the dark sector. K01 Coarse residual-capacity field; K02 Leading EQEF action; K03 Homogeneous background; K04 Scalar perturbation system; K05 Dark-sector sound speed; K06 Effective dark-matter map; K07 Early-universe gate; K08 BAO distance map; K09 CMB transfer map; K10 Weak-lensing and shear map.

Part 9. Phenomenology and public-data dossiers. D01 Universal forward-model contract; D02 GWOSC dispersion test; D03 CERN neutral-scalar template; D04 CERN contact-interaction test; D05 XENONnT recoil dossier; D06 DUNE oscillation programme; D07 DESI expansion and growth; D08 Planck CMB likelihood dossier; D09 Euclid imaging dossier; D10 Transient timing test.

Part 10. Closure, inference, and reproducibility. A01 Immutable parameter card; A02 Dependency DAG; A03 Dimensional and circularity audit; A04 Calibration and holdout partition; A05 Likelihood construction; A06

Injection and coverage campaign; A07 Nuisance provenance table; A08 Preregistered stop rules; A09 Theory-wide falsification matrix; A10 Reproducibility archive.

The conceptual textbook that follows the front matter remains the entry route. The dossier volume resumes after it and deepens the parts that require formal closure or executable testing.

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PART I

A universe built from relations

1

Begin without a stage

Newtonian mechanics begins with positions and a time variable; relativity combines space and time into geometry; quantum field theory assigns operators or fields to spacetime points. End Theory asks a prior question: what mathematical structure is sufficient before a physical distance or duration has been established? The proposed answer is an ordered relational history.

Primitive history DEFINED

$$X_0 \prec X_1 \prec X_2 \prec \cdots, \qquad X_n \in \mathcal{X}, \tag{1.1}$$

where X_n is a complete micro-configuration and $\prec$ means "comes before in the update order." The integer n is dimensionless. It orders records; it is not yet a reading in seconds.

The state X_n may contain internal labels, admissible relations, and a rule for local change. It must not quietly contain the metric that the framework later claims to construct. A useful abstract package is

$$\mathcal{P} = (\mathcal{X}, \mathcal{R}, \mathcal{A}, U_p), \tag{1.2}$$

where $\mathcal{R}$ lists direct relations, $\mathcal{A}$ lists admissible changes, and U_p is an update with fixed parameter vector p.

FIRST LOOK

Imagine a flipbook before anyone has drawn a ruler or a clock inside it. The pages have an order. A character can discover duration only by finding something that repeats: a bouncing light pulse, a turning wheel, or a stable oscillation. Page number is order. Clock reading is a property of a repeating pattern.

1.1 Order becomes time through a clock

Let a persistent subsystem K possess a complex clock observable

$$Z_K(X_n) = A_K(X_n) \exp[\mathrm{i}\Theta_K(X_n)]. \tag{1.3}$$

Unwrap the phase so it increases continuously along a stable cycle. The number of completed cycles is

$$N_K(n) = \frac{\operatorname{unwrap} \Theta_K(n) - \Theta_K(0)}{2\pi}. \tag{1.4}$$

After one reference period T_K has been fixed, the clock reads

$$t_K(n) = T_K N_K(n). \tag{1.5}$$

This is the first essential distinction of the framework: n counts updates, while t_K is inferred from dynamics.

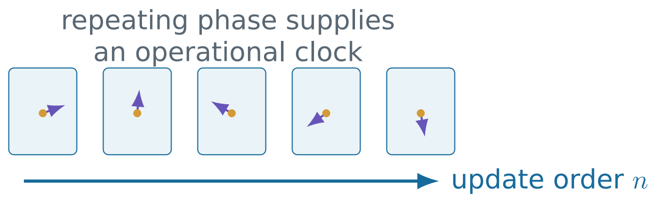


Figure 1.1: The pages advance uniformly in update order, while the internal hand supplies the clock reading.

1.2 When clocks agree

One clock is not yet a universal time coordinate. Suppose two low-load clock types K and L satisfy

$$\frac{t_K(n_2) - t_K(n_1)}{t_L(n_2) - t_L(n_1)} \longrightarrow \gamma_{KL} \tag{1.6}$$

with the same constant γ_{KL} for all sufficiently quiet histories. Then the clocks define compatible duration up to a calibration factor. A relativistic completion must go further: motion and local load must change clock rates universally, not according to the construction details of each clock.

WORK IT OUT

A clock phase advances by 0.40 radians per update. How many updates make one cycle?

$$N = \frac{2\pi}{0.40} = 15.708\ldots$$

Because an update count is an integer, the hand passes one full cycle between updates 15 and 16. If one calibrated cycle is $1\,\mathrm{s}$, then each update corresponds to approximately $1/15.708 = 0.06366\,\mathrm{s}$ in this homogeneous example.

RESEARCH GATE

Construct at least two stable clock patterns from the same update rule. Measure their phase noise, perturbation recovery, and rate ratio. Universal low-energy agreement is the gate from ordered change to physical time.

2

Persistence draws the map

Objects need not be primitive. A flame, a wave crest, and a whirlpool retain recognizable structure while their material constituents change. End Theory uses that familiar idea as a mathematical filter: a candidate object is a pattern that remains equivalent to itself under allowed transport, phase change, or relabeling.

2.1 A test for identity

Choose a coarse-graining map C_R at resolution R, a comparison distance d_R, a group $\mathscr{G}_R$ of allowed transformations, and a tolerance ε_R. A pattern persists for T updates when

$$\inf_{g \in \mathscr{G}_R} d_R(C_R X_{n+T}, gC_R X_n) < \varepsilon_R. \tag{2.1}$$

The corresponding equivalence class is

$$[X]_{R,\varepsilon} = \left\{ Y : \inf_{g \in \mathscr{G}_R} d_R(C_R Y, gC_R X) < \varepsilon_R \right\}. \tag{2.2}$$

The lifetime estimator

$$\tau_R = \inf \{T : d_R(C_R X_{n+T}, \mathscr{G}_R C_R X_n) \geq \varepsilon_R\} \tag{2.3}$$

turns persistence into a number.

FIRST LOOK

You recognize a song even if it is played on another piano. The exact air vibrations differ, but the pattern survives a change of instrument. The transformation group says which differences are allowed; the tolerance says how far the tune may drift before it becomes another tune.

2.2 Influence becomes adjacency

Let a and b label persistent classes. Perturb a by a small admissible change δ and measure the resolved response of b:

$$I_{ab}(n) = \frac{\left\| C_R X_{n+\Delta}^{(a+\delta)} - C_R X_{n+\Delta} \right\|_b}{\|\delta\|}. \tag{2.4}$$

Average over a fixed observation window,

$$w_{ab} = \frac{1}{T} \sum_{n=1}^{T} I_{ab}(n), \tag{2.5}$$

and retain a link when $w_{ab} > w_{\text{min}}$. This constructs an effective weighted graph

$$G_R = (V_R, E_R, W_R). \tag{2.6}$$

The word “local” now has an operational meaning: directly linked patterns respond without an intervening chain.

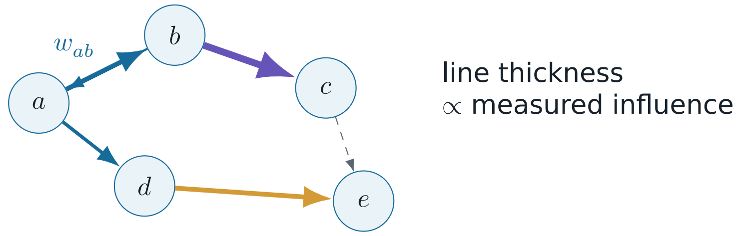


Figure 2.1: The graph is inferred by intervention and response. Direction is permitted until reciprocity is demonstrated.

2.3 Distance and dimension

Assign a positive edge length $\ell_{ab} = \ell_0 f(w_{ab})$ with a declared decreasing function f. The shortest-path distance is

$$d_G(a,b) = \min_{\gamma: a \to b} \sum_{e \in \gamma} \ell_e. \tag{2.7}$$

Dimension can be estimated without first drawing the graph as a cubic lattice. If $\mathsf{L} = D - W$ is its Laplacian, diffusion for parameter σ is generated by

$$K(\sigma) = \exp(-\sigma \mathsf{L}), \qquad P(\sigma) = \frac{1}{|V|} \operatorname{Tr} K(\sigma). \tag{2.8}$$

The scale-dependent spectral dimension is

$$d_s(\sigma) = -2 \frac{\mathrm{d} \ln P(\sigma)}{\mathrm{d} \ln \sigma}. \tag{2.9}$$

For ordinary diffusion in d flat spatial dimensions, $P(\sigma) \propto \sigma^{-d/2}$, so $d_s = d$.

PROOF PATH

Take $P(\sigma) = A\sigma^{-d/2}$. Then $\ln P = \ln A - (d/2) \ln \sigma$. Differentiating with respect to $\ln \sigma$ gives $-d/2$. Multiplying by -2 yields $d_s = d$. No special dimension has been inserted.

DRY NOTE FROM THE FLIGHT DECK

The ratio of a d-ball's volume to its boundary area is $V_d / S_{d-1} = r/d$. It is not unique to three dimensions. A serious dimensional claim therefore comes from dynamical stability and a scale plateau such as Equation (2.9), not from admiring the number three until it confesses.

3

A local capacity for change

The distinctive structural proposal of End Theory is a finite local transition capacity. The constraint is local: each node and its neighbourhood must process a bounded demand during one update. A regional budget is obtained by summing local budgets after the local rule has been applied.

3.1 Demand and residual capacity

For a scalar benchmark, define the local velocity

$$v_i^n = \frac{q_i^{n+1} - q_i^n}{\Delta\tau} \tag{3.1}$$

and an energy-like demand

$$E_i^n = \frac{\mu}{2}(v_i^n)^2 + \frac{\kappa}{4a^2}\sum_{j\sim i} w_{ij}(q_i^n - q_j^n)^2 + V_+(q_i^n), \tag{3.2}$$

where $V_+ \geq 0$ is the portion of the onsite energy assigned to the capacity ledger. With capacity scale $E_\star > 0$,

$$C_i^n = \frac{E_i^n}{E_\star}, \qquad 0 \leq C_i^n \leq 1, \qquad r_i^n = 1 - C_i^n. \tag{3.3}$$

Here r_i^n is residual capacity. It is dimensionless bookkeeping until a coarse-grained physical normalization is supplied.

WORK IT OUT

Take $\mu = 2$, $v = 0.30$, link contribution 0.21, onsite contribution 0.05, and $E_\star = 0.50$ in consistent energy units. Then

$$E = \tfrac{1}{2}(2)(0.30)^2 + 0.21 + 0.05 = 0.35,$$

so $C = 0.35/0.50 = 0.70$ and $r = 0.30$. The calculation is simple on purpose: every later capacity model must reduce to a ledger that a reader can audit term by term.

3.2 From a proposal to an admissible update

Suppose an unconstrained equation proposes $\tilde{q}_i^{n+1}$. Define its proposed increment $\Delta_i = \tilde{q}_i^{n+1} - q_i^n$. A local limiter may be written

$$q_i^{n+1} = q_i^n + \eta_i^n \Delta_i, \qquad 0 \leq \eta_i^n \leq 1, \tag{3.4}$$

with η_i^n chosen from neighbourhood data so that $C_i^n \leq 1$. This is a constructive pattern, not a unique law. A promoted update must satisfy five simultaneous checks:

1. **Closure:** admissible input states map to admissible output states.
2. **Single-valuedness:** every branch and tie-break has one result.
3. **Locality:** the calculation uses a bounded neighbourhood.

4. **Conservation or controlled dissipation:** declared charges obey an exact balance law.
5. **Refinement:** observables converge as the graph and update scale are refined.

The complete capacity-preserving law can be expressed as

$$(X_{n+1}, X_n) = U_p(X_n, X_{n-1}), qquad U_p(\mathcal{A}) \subseteq \mathcal{A}, \tag{3.5}$$

where $\mathcal{A}$ is the capacity-admissible state set.

RESEARCH GATE

Choose a concrete η_i^n. Prove closure by induction, then calculate the discrete energy difference $H^{n+1} - H^n$. A limiter that respects capacity but invents energy has not completed the gate.

Figure 3.1: A visual analogy for local capacity. The overloaded tray changes the robot's local response; the rest of the spacecraft does not need an instantaneous universe-wide inventory. The black hole is off duty and merely provides atmosphere.

4

Patterns can carry physics

In a field theory a particle is associated with an excitation carrying energy, momentum, spin, and charge. In a relational framework the same observable roles must arise from persistent dynamical patterns. The word "particle" is earned by a checklist, not granted to every attractive knot in a simulation.

4.1 The pattern checklist

A candidate pattern P requires:

1. a localized or controlled-support solution;
2. stability under small perturbations;
3. a dispersion relation $E(\mathbf{p})$;
4. conserved or topological labels;
5. a transport rule;
6. scattering or decay amplitudes;
7. a detector-level map.

The first three items can be explored in a scalar benchmark. Charge requires a symmetry and current. Spin requires transformation properties under the emergent rotation or Lorentz group. A familiar mass label can be assigned only after operational energy, momentum, and clock units have been built.

FIRST LOOK

A particle may be more like a travelling stadium wave than a marble. People sit down and stand up; the wave moves around the stadium. The moving pattern is real even though no one person travels with it. The difficult physics begins when the pattern must also carry exact labels, collide predictably, and survive noise.

4.2 A two-node example

Two identical coupled coordinates provide the smallest transparent pattern calculation:

$$\mu\ddot{q}_1 + \mu\Omega_0^2 q_1 + \kappa(q_1 - q_2) = 0, \tag{4.1}$$

$$\mu\ddot{q}_2 + \mu\Omega_0^2 q_2 + \kappa(q_2 - q_1) = 0. \tag{4.2}$$

Introduce normal coordinates

$$q_+ = \frac{q_1 + q_2}{\sqrt{2}}, \qquad q_- = \frac{q_1 - q_2}{\sqrt{2}}. \tag{4.3}$$

Adding and subtracting the equations gives

$$\ddot{q}_+ + \Omega_0^2 q_+ = 0, \tag{4.4}$$

$$\ddot{q}_- + \left(\Omega_0^2 + \frac{2\kappa}{\mu}\right) q_- = 0. \tag{4.5}$$

Thus the collective frequencies are

$$\omega_+ = \Omega_0, \qquad \omega_- = \sqrt{\Omega_0^2 + \frac{2\kappa}{\mu}}. \tag{4.6}$$

The in-phase mode does not stretch the link; the out-of-phase mode does. This is the seed of band structure on a larger graph.

WORK IT OUT

Let $\Omega_0 = 2$, $\kappa = 3$, and $\mu = 6$ in consistent units. Then

$$\omega_+ = 2, \qquad \omega_- = \sqrt{4+1} = \sqrt{5} \approx 2.236.$$

Substitute $q_1 = A\cos(2t)$ and $q_2 = A\cos(2t)$ into the original equations: the link terms cancel. Substitute $q_1 = A\cos(\sqrt{5}t)$ and $q_2 = -q_1$: each link term contributes $2\kappa q_1$, producing the second frequency.

4.3 Wave, bound pattern, and observation

Three levels should remain distinct:

$$\text{micro-state trajectory} \longrightarrow \text{persistent pattern class} \longrightarrow \text{detector record.} \tag{4.7}$$

A wave observation theory specifies every arrow. A detector need not reveal a microscopic node; it responds to coarse conserved quantities and calibrated transfer functions. Chapters 19 and 23 turn this statement into a manual.

5

The logical succession of End Theory

The framework is organized as a dependency chain. Later objects may not be used to define earlier ones. This prevents physical metres, seconds, particle masses, or observed couplings from reappearing as unexplained ingredients in the foundational layer.

5.1 Eight foundational statements

1. **Relational proto-state.** The initial description supplies states, admissible relations, and local changes rather than a prior physical spacetime.
2. **Ordered updates.** Equation (1.1) precedes metric duration.
3. **Operational clocks.** Persistent recurrent patterns define readings by Equation (1.5).
4. **Persistent identity.** Objects are equivalence classes satisfying Equation (2.1).
5. **Emergent adjacency.** A stable influence pattern defines the weighted graph in Equation (2.6).
6. **Local capacity.** Every neighbourhood satisfies Equation (3.3).
7. **Deterministic evolution.** One fixed map of the form (3.5) advances admissible states.
8. **Observable completion.** Preparation, evolution, coarse-graining, and measurement maps determine empirical probabilities and detector records.

5.2 The evidence ladder

Let a model specification be

$$\mathcal{M} = (\mathcal{X}, U_p, B, C_R, \mathcal{O}, \Pi), \tag{5.1}$$

where B denotes boundary/initial conditions, C_R the coarse-graining, $\mathcal{O}$ the observable map, and Π the parameter and calibration policy. A quantity Q has different status depending on how it is obtained:

$$Q \equiv f(\text{definitions}) \qquad \text{identity,} \tag{5.2}$$

$$Q = F_Q(\mathcal{M}) \qquad \text{internal result,} \tag{5.3}$$

$$p \leftarrow D_{\text{cal}} \qquad \text{calibration,} \tag{5.4}$$

$$Q_{\text{test}} = F_Q(\mathcal{M}_{\text{frozen}}) \qquad \text{prediction.} \tag{5.5}$$

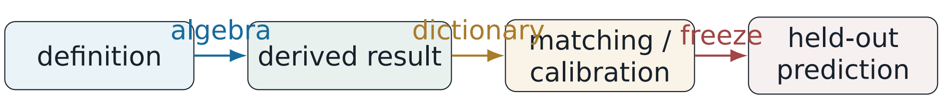


Figure 5.1: A claim advances only by an explicit operation. Reproducing a calibration target is not counted again as a prediction.

5.3 The canonical benchmark

The book now introduces a calculable graph-field benchmark. It supplies an action, an explicit deterministic update, an exact normal-mode spectrum, a stability boundary, and exact finite-spacing Abelian gauge invariance. These results establish the mathematical laboratory in which the broader relational ideas can be tested.

DRY NOTE FROM THE FLIGHT DECK

A grand title does not exempt an equation from arithmetic. In this book, even the universe has to show its working.

PART II

The mathematical engine

6

The small toolkit that carries the theory

The derivations in this book use four recurring operations: finite differences, matrix eigenmodes, variations of an action, and series expansions. This chapter places them in one toolbox so the later equations can be checked rather than merely admired.

6.1 From change to derivative

For a smooth function $q(t)$, Taylor's theorem gives

$$q(t+h) = q + h\dot{q} + \frac{h^2}{2}\ddot{q} + \frac{h^3}{6}q^{(3)} + \frac{h^4}{24}q^{(4)} + \mathcal{O}(h^5), \tag{6.1}$$

$$q(t-h) = q - h\dot{q} + \frac{h^2}{2}\ddot{q} - \frac{h^3}{6}q^{(3)} + \frac{h^4}{24}q^{(4)} + \mathcal{O}(h^5). \tag{6.2}$$

Subtracting gives a centred first derivative,

$$\dot{q}(t) = \frac{q(t+h) - q(t-h)}{2h} + \mathcal{O}(h^2), \tag{6.3}$$

while adding and rearranging gives the centred second derivative,

$$\ddot{q}(t) = \frac{q(t+h) - 2q(t) + q(t-h)}{h^2} + \mathcal{O}(h^2). \tag{6.4}$$

FIRST LOOK

Speed asks, "How much did the position change per tick?" Acceleration asks, "How much did the speed change per tick?" Equation (6.4) compares the present value with both neighbours in time, so straight motion cancels and curvature remains.

6.2 Dimensions before numbers

Write $[Q]$ for the dimensions of Q. An equation is dimensionally admissible only if every term being added has the same dimensions. If

$$\mu\ddot{q} + Kq = 0, \tag{6.5}$$

then $[K/\mu] = T^{-2}$. The angular frequency $\omega = \sqrt{K/\mu}$ has dimensions T^{-1}.

For a monomial

$$Q = \prod_{j=1}^{N} x_j^{a_j}, \tag{6.6}$$

the dimension vector of Q is the matrix product of the input-dimension matrix and the exponent vector $\mathbf{a}$. This linear-algebra view is the quickest way to expose a missing metre, second, kelvin, or charge unit.

WORK IT OUT

Check $\ell_P = \sqrt{\hbar G/c^3}$. Using $[\hbar] = ML^2T^{-1}$, $[G] = L^3M^{-1}T^{-2}$, and $[c] = LT^{-1}$,

$$\left[\frac{\hbar G}{c^3}\right] = \frac{(ML^2T^{-1})(L^3M^{-1}T^{-2})}{L^3T^{-3}} = L^2.$$

The square root therefore has dimension L.

6.3 Eigenmodes

For a real symmetric matrix A, the spectral theorem supplies orthonormal eigenvectors $u^{(r)}$ and real eigenvalues λ_r:

$$Au^{(r)} = \lambda_r u^{(r)}. \tag{6.7}$$

Expand $q = \sum_r Q_r u^{(r)}$. The coupled equation

$$\ddot{q} + Aq = 0 \tag{6.8}$$

becomes independent oscillators

$$\ddot{Q}_r + \lambda_r Q_r = 0. \tag{6.9}$$

The graph Laplacian will play the role of A for spatial coupling.

6.4 Varying an action

Let $S[q] = \int L(q, \dot{q}, t)\,\mathrm{d}t$. Perturb the path, $q \mapsto q + \epsilon\eta$, with η vanishing at the endpoints. To first order,

$$\begin{aligned}
\delta S &= \frac{\mathrm{d}}{\mathrm{d}\epsilon} S[q+\epsilon\eta]\Big|_{\epsilon=0} && (6.10)\\
&= \int \left(\frac{\partial L}{\partial q}\eta + \frac{\partial L}{\partial \dot{q}}\dot{\eta}\right)\mathrm{d}t && (6.11)\\
&= \int \left[\frac{\partial L}{\partial q} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right)\right]\eta\,\mathrm{d}t. && (6.12)
\end{aligned}$$

Because η is arbitrary,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0. \tag{6.13}$$

The discrete calculation in Chapter 8 repeats this logic with sums and neighbouring time slices.

7

Graphs, links, and the Laplacian

Let $G = (V, E, W)$ be a finite weighted graph with symmetric nonnegative weights $w_{ij} = w_{ji}$. Its degree matrix and adjacency matrix are

$$D_{ii} = \sum_j w_{ij}, \qquad W_{ij} = w_{ij}. \tag{7.1}$$

The weighted graph Laplacian is

$$\mathsf{L} = D - W, \qquad (\mathsf{L}q)_i = \sum_j w_{ij}(q_i - q_j). \tag{7.2}$$

7.1 Why it behaves like curvature

The quadratic link energy is

$$q^{\mathsf{T}}\mathsf{L}q = \frac{1}{2}\sum_{i,j} w_{ij}(q_i - q_j)^2 \geq 0. \tag{7.3}$$

To verify the identity, expand the square:

$$\frac{1}{2}\sum_{ij} w_{ij}(q_i - q_j)^2 = \frac{1}{2}\sum_{ij} w_{ij}(q_i^2 + q_j^2 - 2q_iq_j) \tag{7.4}$$

$$= \sum_i \left(\sum_j w_{ij}\right) q_i^2 - \sum_{ij} w_{ij}q_iq_j \tag{7.5}$$

$$= q^{\mathsf{T}}(D - W)q. \tag{7.6}$$

Hence all eigenvalues of L are nonnegative. For a connected graph, the constant vector is the unique zero mode.

FIRST LOOK

Put the same number on every node. Every difference $q_i - q_j$ is zero, so the Laplacian sees no bend. Put a bump on one node, and the differences point away from the bump. The Laplacian measures how different a node is from its neighbours.

7.2 A periodic chain

On a one-dimensional chain with spacing a and unit weights,

$$(\mathsf{L}q)_j = 2q_j - q_{j+1} - q_{j-1}. \tag{7.7}$$

For the plane wave $q_j = \exp(\mathrm{i}kja)$,

$$(\mathsf{L}q)_j = \left(2 - \mathrm{e}^{\mathrm{i}ka} - \mathrm{e}^{-\mathrm{i}ka}\right) q_j \tag{7.8}$$

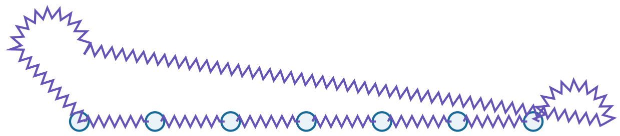
$$= 4\sin^2\left(\frac{ka}{2}\right) q_j. \tag{7.9}$$

Thus

$$\lambda(k) = 4\sin^2\left(\frac{ka}{2}\right). \tag{7.10}$$

In d hypercubic dimensions,

$$\lambda(\mathbf{k}) = 4\sum_{r=1}^{d}\sin^2\left(\frac{k_r a}{2}\right). \tag{7.11}$$



periodic chain: the last node links back to the first

Figure 7.1: A regular benchmark graph. The periodic boundary removes edge effects and makes plane waves exact eigenmodes.

8

The discrete action and update

Fix a graph during a stabilized phase. Let q_i^n be a real coordinate at node i and update n. The benchmark action is

$$\boxed{S_d = \sum_n \Delta\tau \left[\frac{\mu}{2} \sum_i \left(\frac{q_i^{n+1} - q_i^n}{\Delta\tau} \right)^2 - \frac{\kappa}{2a^2} \sum_{\langle ij \rangle} w_{ij} (q_i^n - q_j^n)^2 - \sum_i V(q_i^n) \right].} \tag{8.1}$$

Here $\mu > 0$ is an inertia, $\kappa > 0$ a link stiffness, a a stabilized link scale, and $\Delta\tau$ a benchmark clock step.

8.1 Variation one line at a time

Only the $n-1$ and n kinetic terms contain q_i^n. Their derivative is

$$\frac{\mu}{\Delta\tau} \left[(q_i^n - q_i^{n-1}) - (q_i^{n+1} - q_i^n) \right]. \tag{8.2}$$

The link term contributes

$$-\Delta\tau \frac{\kappa}{a^2} (\mathsf{L} q^n)_i, \tag{8.3}$$

and the onsite term contributes $-\Delta\tau V'(q_i^n)$. Setting the full derivative to zero and dividing by $\Delta\tau$ yields

$$\mu \frac{q_i^{n+1} - 2q_i^n + q_i^{n-1}}{\Delta\tau^2} = -\frac{\kappa}{a^2} (\mathsf{L} q^n)_i - V'(q_i^n). \tag{8.4}$$

Solving for the next state gives the explicit update

$$\boxed{q_i^{n+1} = 2q_i^n - q_i^{n-1} - \frac{\Delta\tau^2}{\mu} \left[\frac{\kappa}{a^2} (\mathsf{L} q^n)_i + V'(q_i^n) \right].} \tag{8.5}$$

PROOF PATH

For a finite graph and differentiable V, every term on the right side of Equation (8.5) is a single real number determined by (q^{n-1}, q^n). Therefore the benchmark recursion is algebraically deterministic. This statement concerns the benchmark update; a capacity projection is a separately declared layer.

8.2 A hand calculation on three nodes

Take a three-node path $1-2-3$ with unit weights, $\mu = \kappa = a = 1$, $\Delta\tau = 0.1$, and $V = 0$. Let

$$q^{n-1} = (0, 0, 0), \qquad q^n = (0, 1, 0). \tag{8.6}$$

The Laplacian is

$$\mathsf{L} = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \qquad \mathsf{L} q^n = (-1, 2, -1). \tag{8.7}$$

Equation (8.5) gives

$$q^{n+1} = 2(0,1,0) - (0,0,0) - 0.01(-1,2,-1) \tag{8.8}$$
$$= (0.01, 1.98, 0.01). \tag{8.9}$$

The peak begins to spread to its neighbours. With zero initial velocity, the chosen pair (q^{n-1}, q^n) actually contains a sudden displacement, so the centre also continues forward before the restoring force reverses it.

8.3 An exact discrete invariant in the linear sector

Let $V(q) = \mu\Omega_0^2 q^2/2$ and define the symmetric positive matrix

$$A = \Omega_0^2 I + \frac{\kappa}{\mu a^2}\mathsf{L}. \tag{8.10}$$

The linear update is

$$q^{n+1} - 2q^n + q^{n-1} + \Delta\tau^2 A q^n = 0. \tag{8.11}$$

It preserves

$$H^{n+1/2} = \frac{1}{2}\left\|\frac{q^{n+1} - q^n}{\Delta\tau}\right\|^2 + \frac{1}{2}(q^{n+1})^{\mathsf{T}} A q^n. \tag{8.12}$$

Subtract $H^{n-1/2}$, factor $q^{n+1} - q^{n-1}$, and insert the update; the result is zero. Positivity of this invariant is tied to the stability condition derived in Chapter 10.

9

Exact lattice waves

Use the harmonic onsite potential $V(q) = \mu\Omega_0^2 q^2/2$ on a periodic d-dimensional cubic lattice. Insert the plane wave

$$q_{\mathbf{j}}^{\,n} = A\exp\left[\mathrm{i}(\mathbf{k}\cdot\mathbf{x}_{\mathbf{j}} - \omega n\Delta\tau)\right] \tag{9.1}$$

into Equation (8.4).

9.1 Temporal factor

The centred second difference contributes

$$q^{n+1} - 2q^n + q^{n-1} = \left(\mathrm{e}^{-\mathrm{i}\omega\Delta\tau} - 2 + \mathrm{e}^{\mathrm{i}\omega\Delta\tau}\right) q^n \tag{9.2}$$

$$= -4\sin^2\left(\frac{\omega\Delta\tau}{2}\right) q^n. \tag{9.3}$$

9.2 Spatial factor

Equation (7.11) supplies the spatial eigenvalue. With

$$c^2 \equiv \frac{\kappa}{\mu}, \tag{9.4}$$

the exact lattice dispersion relation is

$$\boxed{\frac{4}{\Delta\tau^2}\sin^2\left(\frac{\omega\Delta\tau}{2}\right) = \Omega_0^2 + \frac{4c^2}{a^2}\sum_{r=1}^{d}\sin^2\left(\frac{k_r a}{2}\right).} \tag{9.5}$$

This is an exact internal result for the declared linear benchmark.

WORK IT OUT

In one dimension take $a = 1$, $\Delta\tau = 0.5$, $c = 1$, $\Omega_0 = 0$, and $k = \pi/3$. The right side is

$$4\sin^2(\pi/6) = 1.$$

Therefore $16\sin^2(\omega/4) = 1$, so $\sin(\omega/4) = 1/4$ and

$$\omega = 4\arcsin(1/4) \approx 1.01072.$$

The continuum value would be $ck = 1.04720$; the lattice mode is slightly lower because finite spacing bends the band.

9.3 The long-wave expansion

For $|k_r a| \ll 1$ and $|\omega\Delta\tau| \ll 1$,

$$\frac{4}{\Delta\tau^2}\sin^2\left(\frac{\omega\Delta\tau}{2}\right) = \omega^2 - \frac{\Delta\tau^2\omega^4}{12} + \mathcal{O}(\Delta\tau^4\omega^6), \tag{9.6}$$

$$\frac{4}{a^2}\sin^2\left(\frac{k_r a}{2}\right) = k_r^2 - \frac{a^2 k_r^4}{12} + \mathcal{O}(a^4 k_r^6). \tag{9.7}$$

Hence

$$\omega^2 = \Omega_0^2 + c^2|\mathbf{k}|^2 + \frac{\Delta\tau^2\omega^4}{12} - \frac{c^2a^2}{12}\sum_r k_r^4 + \cdots. \tag{9.8}$$

The leading term is relativistic in form; the displayed corrections quantify temporal discretization and lattice anisotropy.

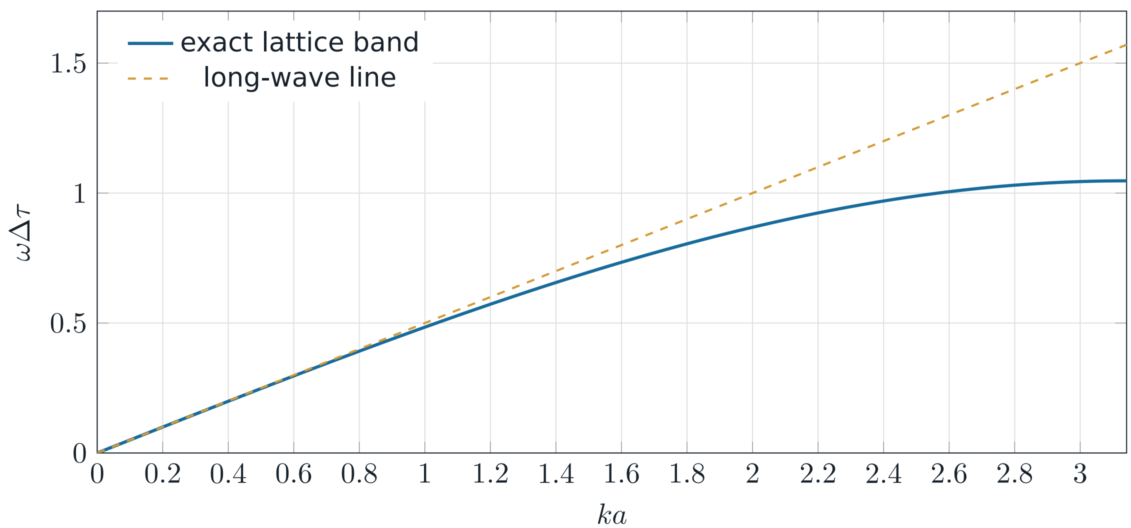


Figure 9.1: A massless one-dimensional example with $c\Delta\tau/a = 0.5$. The exact band agrees with the continuum line near $k = 0$ and bends at short wavelength.

10

Stability, speed, and the mass gap

Define

$$A(\mathbf{k}) = \Omega_0^2 + \frac{4c^2}{a^2}\sum_{r=1}^{d}\sin^2\left(\frac{k_r a}{2}\right). \tag{10.1}$$

Real ω requires the argument of the inverse sine to lie in $[0,1]$:

$$0 \le \frac{\Delta\tau^2}{4}A(\mathbf{k}) \le 1. \tag{10.2}$$

The maximum occurs at a Brillouin-zone corner, where every sine squared is one. Therefore all linear modes are stable precisely when

$$\boxed{\Delta\tau^2\left(\Omega_0^2 + \frac{4dc^2}{a^2}\right) \le 4.} \tag{10.3}$$

PROOF PATH

Each normal coordinate obeys $Q^{n+1} - 2Q^n + Q^{n-1} + \Delta\tau^2 A Q^n = 0$. Its characteristic roots are

$$z_\pm = 1 - \frac{\Delta\tau^2 A}{2} \pm \mathrm{i}\sqrt{\Delta\tau^2 A - \frac{\Delta\tau^4 A^2}{4}}.$$

They lie on the unit circle when $0 \le \Delta\tau^2 A \le 4$. Beyond that interval one root has magnitude greater than one, producing exponential growth.

10.1 Group velocity

Solving Equation (9.5),

$$\omega(\mathbf{k}) = \frac{2}{\Delta\tau}\arcsin\left(\frac{\Delta\tau}{2}\sqrt{A(\mathbf{k})}\right). \tag{10.4}$$

Differentiation gives

$$v_{g,r} = \frac{\partial\omega}{\partial k_r} = \frac{(c^2/a)\sin(k_r a)}{sqrtA(\mathbf{k})\sqrt{1-\Delta\tau^2 A(\mathbf{k})/4}}. \tag{10.5}$$

In the simultaneous long-wave and small-step limit,

$$v_{g,r} \longrightarrow \frac{c^2 k_r}{\sqrt{\Omega_0^2 + c^2|\mathbf{k}|^2}}. \tag{10.6}$$

For $\Omega_0 = 0$, the magnitude approaches c as $\mathbf{k} \to 0$.

10.2 Gap and effective mass

At $\mathbf{k} = 0$,

$$\frac{4}{\Delta\tau^2}\sin^2\left(\frac{\omega_0\Delta\tau}{2}\right) = \Omega_0^2. \tag{10.7}$$

For a small clock step, $\omega_0 \approx \Omega_0$. After an independently established quantum action scale $\hbar$ and physical causal speed c_{phys} have been matched, a relativistic effective mass may be assigned by

$$m_{\text{eff}} c_{\text{phys}}^2 = \hbar\omega_0, \qquad m_{\text{eff}} = \frac{\hbar\omega_0}{c_{\text{phys}}^2}. \tag{10.8}$$

The gap is an internal spectral result. The conversion to kilograms or electronvolts is a physical dictionary.

WORK IT OUT

If a matched mode has $\hbar\omega_0 = 1\,\text{eV}$, then

$$m = \frac{1\,\text{eV}}{c^2} = \frac{1.602176634 \times 10^{-19}\ \text{J}}{(2.99792458 \times 10^8\ \text{m/s})^2} \approx 1.78266 \times 10^{-36}\ \text{kg}.$$

This is a unit conversion, not a new mass prediction.

11

Nonlinearity: clocks that notice amplitude

Take a uniform mode with

$$V(q) = \frac{\mu\Omega_0^2}{2}q^2 + \frac{\lambda}{4}q^4. \tag{11.1}$$

Its continuum-time equation is the Duffing oscillator

$$\ddot{q} + \Omega_0^2 q + \frac{\lambda}{\mu}q^3 = 0. \tag{11.2}$$

11.1 A perturbative frequency shift

Assume $q(t) \approx A\cos(\Omega t)$. The identity

$$\cos^3 x = \frac{3}{4}\cos x + \frac{1}{4}\cos 3x \tag{11.3}$$

shows that the cubic force contains a fundamental term and a third harmonic. Keeping the fundamental balance gives

$$-\Omega^2 A + \Omega_0^2 A + \frac{3\lambda A^3}{4\mu} \approx 0, \tag{11.4}$$

so

$$\Omega^2 \approx \Omega_0^2 + \frac{3\lambda A^2}{4\mu}. \tag{11.5}$$

For $|\lambda|A^2 \ll \mu\Omega_0^2$, expand the square root:

$$\boxed{\Omega(A) \approx \Omega_0\left(1 + \frac{3\lambda A^2}{8\mu\Omega_0^2}\right).} \tag{11.6}$$

Positive λ hardens the oscillator; negative λ softens it within the stable amplitude range.

FIRST LOOK

A small swing and a large swing need not keep exactly the same rhythm when the restoring force is nonlinear. That makes an oscillator a load-sensitive clock. End Theory's broader clock programme asks whether one local capacity mechanism can make many different clocks acquire the same low-energy dilation law.

11.2 The search for localized modes

On a graph, use the stationary ansatz

$$q_i(t) = \phi_i\cos(\Omega t) + \text{higher harmonics}. \tag{11.7}$$

Harmonic balance yields a nonlinear algebraic problem of the schematic form

$$\left[\Omega_0^2 I + \frac{\kappa}{\mu a^2}\mathsf{L} - \Omega^2 I\right]\phi + \frac{3\lambda}{4\mu}\phi^{\circ 3} = 0, \tag{11.8}$$

where $\phi^{\circ 3}$ means componentwise cubing. A continuation algorithm can begin from an anti-continuum limit, increase coupling gradually, and track the solution. Stability follows from the eigenvalues of the linearized Floquet map.

RESEARCH GATE

Search Equation (11.8) without including any known particle mass in the objective. Report dimensionless frequencies, participation ratio

$$P = \frac{(\sum_i |\phi_i|^2)^2}{\sum_i |\phi_i|^4},$$

Floquet multipliers, and persistence under graph perturbations. A small P indicates localization.

12

Phase on nodes, connection on links

Replace the real coordinate by a complex node field

$$\psi_i = \rho_i \mathrm{e}^{\mathrm{i}\theta_i} \tag{12.1}$$

and assign each oriented link a compact transporter

$$U_{ij} = \mathrm{e}^{\mathrm{i}qA_{ij}}, \qquad U_{ji} = U_{ij}^{-1}. \tag{12.2}$$

The covariant link difference is

$$D_{ij}\psi = U_{ij}\psi_j - \psi_i. \tag{12.3}$$

12.1 Exact finite-spacing invariance

Under a local phase change α_i,

$$\psi_i \mapsto \psi_i' = \mathrm{e}^{\mathrm{i}q\alpha_i}\psi_i, \qquad U_{ij} \mapsto U_{ij}' = \mathrm{e}^{\mathrm{i}q\alpha_i}U_{ij}\mathrm{e}^{-\mathrm{i}q\alpha_j}. \tag{12.4}$$

Then

$$D_{ij}'\psi' = U_{ij}'\psi_j' - \psi_i' \tag{12.5}$$

$$= \mathrm{e}^{\mathrm{i}q\alpha_i}U_{ij}\mathrm{e}^{-\mathrm{i}q\alpha_j}\mathrm{e}^{\mathrm{i}q\alpha_j}\psi_j - \mathrm{e}^{\mathrm{i}q\alpha_i}\psi_i \tag{12.6}$$

$$= \mathrm{e}^{\mathrm{i}q\alpha_i}D_{ij}\psi. \tag{12.7}$$

Therefore

$$|D_{ij}'\psi'|^2 = |D_{ij}\psi|^2. \tag{12.8}$$

This invariance is exact at finite spacing.

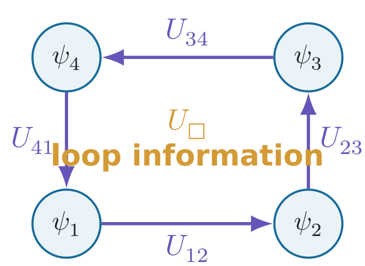


Figure 12.1: A plaquette. Node phases cancel around the closed loop, leaving gauge-invariant holonomy.

12.2 Plaquette curvature

For an oriented plaquette,

$$U_\square = U_{12}U_{23}U_{34}U_{41}. \tag{12.9}$$

All local phase factors cancel under Equation (12.4). A standard compact Abelian lattice action is

$$S_g = -\frac{\beta}{a^4} \sum_{n,\square} \Delta\tau \left(1 - \operatorname{Re} U^n_\square\right). \tag{12.10}$$

If $U_\square = \exp(\mathrm{i} q a^2 F_{\mu\nu} + \mathcal{O}(a^3))$, then

$$1 - \operatorname{Re} U_\square = \frac{1}{2} q^2 a^4 F_{\mu\nu} F^{\mu\nu} + \mathcal{O}(a^6). \tag{12.11}$$

This supplies the continuum gauge-field form after coefficient matching, following the lattice-gauge construction developed by Wilson and successors [33, 34].

12.3 Diagram language for interactions

Feynman diagrams summarize terms in a perturbative expansion; they are not pictures of tiny objects choosing routes. Once a coarse action supplies a vertex $g\phi\bar{\psi}\psi$, its lowest-order bookkeeping diagram is

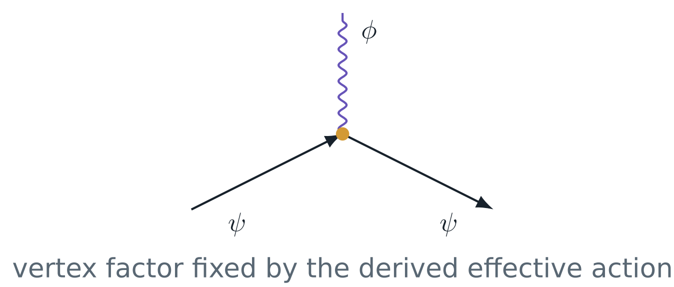


Figure 12.2: Interaction bookkeeping. End Theory reaches this diagram only after a node-pattern calculation supplies the external states and vertex coefficient.

RESEARCH GATE

Extend the link variables to a declared non-Abelian group, construct anomaly-free chiral matter, and calculate the low-energy representation content. The Nielsen–Ninomiya obstruction [39] makes this a precise mathematical design problem.

PART III

Bridges to observed physics

13

From a graph to a continuum field

A continuum limit is a controlled comparison of observables at scales much larger than the link spacing. It is not obtained by replacing a node index with the letter x. The benchmark makes the scaling explicit.

13.1 Field normalization

On a regular d-dimensional phase with cell volume a^d, define

$$q_i(t) = a^{d/2}\phi(\mathbf{x}_i, t). \tag{13.1}$$

Then

$$\sum_i q_i^2 = \sum_i a^d \phi_i^2 \longrightarrow \int \phi^2 \, \mathrm{d}^d x. \tag{13.2}$$

For a nearest-neighbour link in direction r,

$$\frac{q_{i+\hat{r}} - q_i}{a} = a^{d/2}\left(\partial_r \phi + \frac{a}{2}\partial_r^2 \phi + \cdots\right). \tag{13.3}$$

Summing links therefore produces

$$\sum_{\langle ij \rangle} \frac{(q_i - q_j)^2}{a^2} \longrightarrow \int |\nabla \phi|^2 \, \mathrm{d}^d x + \mathcal{O}(a^2). \tag{13.4}$$

The continuum benchmark action is

$$S_{\text{cont}} = \int \mathrm{d}t \, \mathrm{d}^d x \left[\frac{\mu}{2}(\partial_t \phi)^2 - \frac{\kappa}{2}|\nabla \phi|^2 - V_c(\phi)\right], \tag{13.5}$$

where $V_c(\phi) = \lim_{a \to 0} a^{-d} V(a^{d/2}\phi)$ under the chosen scaling.

13.2 Matching the causal cone

Rescale to canonical time kinetic normalization with $\varphi = \sqrt{\mu}\,\phi$. Equation (13.5) becomes

$$S_{\text{cont}} = \int \mathrm{d}t \, \mathrm{d}^d x \left[\frac{1}{2}(\partial_t \varphi)^2 - \frac{c^2}{2}|\nabla \varphi|^2 - \widetilde{V}(\varphi)\right], \qquad c^2 = \frac{\kappa}{\mu}. \tag{13.6}$$

The scalar sector therefore has a low-wave-number causal speed c. A universal relativistic limit requires every light sector to share the same cone and all direction-dependent corrections to lie below observation.

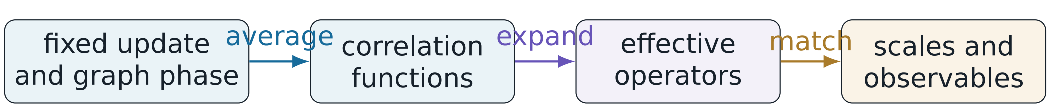


Figure 13.1: The continuum bridge. Coefficients are inferred from correlation functions and response, then matched once to physical units.

13.3 The effective-action target

At long distance, locality and symmetry organize a general action

$$S_{\text{eff}} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_\star^2}{2}R - \frac{1}{4}Z_{ab}F^a_{\mu\nu}F^{b\mu\nu} + \bar{\psi}\mathrm{i}e^\mu{}_a\gamma^a D_\mu\psi + \frac{1}{2}G_{AB}\partial_\mu\phi^A\partial^\mu\phi^B - U(\phi) + \sum_{n>4}\frac{c_n\mathcal{O}_n}{\Lambda^{n-4}}\right]. \tag{13.7}$$

This is a target form, using standard effective-field-theory logic [73]. End Theory becomes distinctive when the microscopic update calculates the field content, symmetries, and coefficients rather than selecting them after observation.

RESEARCH GATE

Calculate two- and four-point correlation functions on several graph refinements. Fit the same operator basis, report coefficient flow with a, and demonstrate a scale window in which irrelevant anisotropies shrink while the desired low-energy coefficients stabilize.

14

Quantum observation as a complete map

Deterministic evolution and quantum prediction address different mathematical objects. A micro-history maps one complete state to the next. An experiment reports frequencies conditioned on a preparation and a setting. A quantum completion must connect these levels explicitly.

14.1 Preparation, setting, and outcome

Let $\lambda \in \Lambda$ denote the complete micro-state relevant to one run. A preparation procedure P induces an ensemble measure

$$\mu_P(\mathrm{d}\lambda). \tag{14.1}$$

An apparatus setting a and response map $A(a, \lambda)$ produce an outcome. The model probability is

$$\Pr(A = x \mid a, P) = \int_\Lambda \mathbf{1}[A(a, \lambda) = x]\, \mu_P(\mathrm{d}\lambda). \tag{14.2}$$

The Born-rule target for a quantum state ρ_P and measurement operators $E_x^{(a)}$ is

$$\Pr(A = x \mid a, P) = \mathrm{Tr}\Big(\rho_P E_x^{(a)}\Big)\,. \tag{14.3}$$

Recovering Equation (14.3) is a calculation of ensemble frequencies; calling the micro-update chaotic does not perform the integral.

FIRST LOOK

A coin can follow ordinary mechanics and still look random when you do not know its exact launch. Quantum theory asks for a much more specific pattern of probabilities, including interference and entanglement. The task is not to say “hidden details exist.” The task is to calculate the exact observed frequency from the hidden details.

14.2 Bell–CHSH as a design constraint

For two separated wings with settings a, a' and b, b', outcomes $A, B \in \{-1, +1\}$ define correlations

$$E(a, b) = \int A(a, \lambda) B(b, \lambda) \rho(\lambda)\, \mathrm{d}\lambda. \tag{14.4}$$

Local response and measurement independence imply the CHSH bound

$$S = |E(a, b) + E(a, b') + E(a', b) - E(a', b')| \le 2. \tag{14.5}$$

Quantum mechanics reaches $2\sqrt{2}$, and loophole-controlled experiments violate the local bound [79, 86, 84]. A deterministic End Theory completion must identify, in equations, the probability structure that changes one of the assumptions used to derive Equation (14.5).

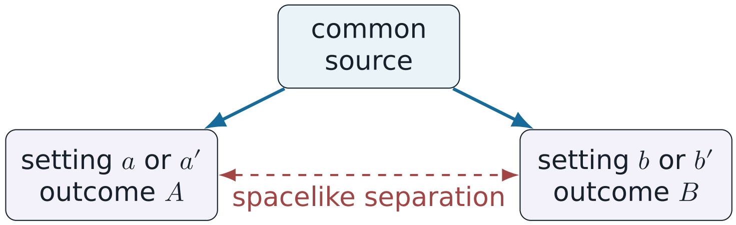


Figure 14.1: Bell-test structure. The preparation distribution, setting dependence, local response, and outcome map must all be declared.

14.3 Operational no-signalling

Whatever completion is chosen, the observable marginal on one wing must not depend on the remote freely varied setting:

$$\sum_y \mathrm{Pr}(x, y \mid a, b, P) = \sum_y \mathrm{Pr}(x, y \mid a, b', P). \tag{14.6}$$

This is an empirical constraint on the outcome distribution. It can coexist with correlations that violate Equation (14.5).

14.4 Localization as a measurable process

A capacity threshold may participate in amplification if a declared functional $T[X; \mathcal{R}]$ over detector region $\mathcal{R}$ reaches a fixed value $T_\star$. A complete model then specifies

$$T < T_\star : \text{distributed response}, \qquad T \geq T_\star : \text{stable outcome channel}, \tag{14.7}$$

and calculates the frequency of channels across the preparation ensemble. Conservation is checked by including the apparatus and environment in the balance law.

RESEARCH GATE

Choose one two-path interference experiment. Define P, λ, the phase-control setting, the detector amplification map, and the ensemble measure. Recover the fringe law and vary a which-path coupling without changing the model parameters.

15

Geometry, gravity, and torsion

Gravity enters only after clocks, rods, propagation, and coarse geometry exist. The bridge has three levels: reconstruct a metric, derive its universal coupling to matter, and calculate the effective action governing its response.

15.1 Metric from operational intervals

Suppose stable signal modes define local travel times and clock patterns define durations. At long wavelength, fit the eikonal relation

$$g^{\mu\nu}(x)k_\mu k_\nu = 0 \tag{15.1}$$

for several independent massless sectors. A common $g^{\mu\nu}$ defines the causal geometry. Massive pattern trajectories should then approach

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}s^2} + \Gamma^\mu_{\nu\rho}\frac{\mathrm{d}x^\nu}{\mathrm{d}s}\frac{\mathrm{d}x^\rho}{\mathrm{d}s} = 0 \tag{15.2}$$

with the same connection, up to controlled spin or finite-size effects.

15.2 Torsion as loop nonclosure

With tetrad one-forms e^a and spin connection $\omega^a_{\ b}$, torsion is

$$T^a = \mathrm{d}e^a + \omega^a_{\ b} \wedge e^b. \tag{15.3}$$

For two small displacement vectors u and v, the leading failure of a transported parallelogram to close is

$$\Delta x^\mu = T^\mu_{\nu\rho}u^\nu v^\rho + \mathcal{O}(|u|^2|v|, |u||v|^2). \tag{15.4}$$

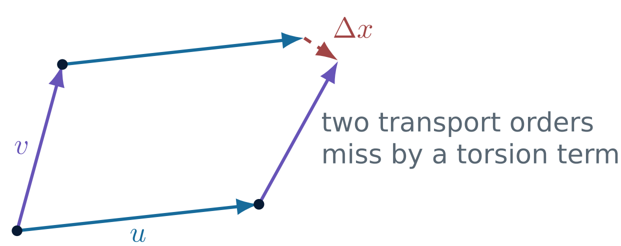


Figure 15.1: A geometric interpretation of torsion. A node model must derive the transport rule before assigning the mismatch.

The phrase “progression imbalance” can be given a disciplined meaning by calculating a coarse transport nonclosure and identifying it with Equation (15.3). In Riemann–Cartan gravity, spin can source torsion [94]; the effective coefficients must be obtained from the coarse node response.

15.3 Newtonian and Einstein targets

In the weak static limit, write

$$g_{00} \approx -\left(1 + \frac{2\Phi_N}{c^2}\right). \tag{15.5}$$

The required long-distance equation is

$$\nabla^2 \Phi_N = 4\pi G \rho. \tag{15.6}$$

At the covariant level, the Einstein–Hilbert term is

$$S_{\text{EH}} = \frac{c^3}{16\pi G} \int R\sqrt{-g}\, \mathrm{d}^4 x, \tag{15.7}$$

with additional torsion terms if the independent connection persists. The bridge calculation has a clear output: derive Equation (15.7), determine G from microscopic parameters, recover Equation (15.6), and test universal free fall.

WORK IT OUT

For a spherical mass M, solve $\nabla^2\Phi = 0$ outside the source. Spherical symmetry gives

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 \frac{\mathrm{d}\Phi}{\mathrm{d}r}\right) = 0.$$

Integrate twice: $r^2\Phi' = C_1$, so $\Phi = -C_1/r + C_2$. Matching to Equation (15.6) fixes $C_1 = GM$ and a zero at infinity fixes $C_2 = 0$.

16

Residual capacity and the EQEF sector

EQEF is used here in one controlled sense: it is the coarse description of expressed and deferred local transition capacity. At the discrete level the primary quantity is the residual ledger $r_i^n = 1 - C_i^n$.

16.1 A causal balance law

Let $J_{ij}^n = -J_{ji}^n$ be residual-capacity transport along a link and s_i^n a local conversion term. A candidate balance equation is

$$r_i^{n+1} - r_i^n + \sum_{j \sim i} J_{ij}^n = s_i^n. \tag{16.1}$$

Summing over a finite region $\mathcal{R}$ cancels internal link currents:

$$\sum_{i \in \mathcal{R}} (r_i^{n+1} - r_i^n) + \sum_{\substack{i \in \mathcal{R} \\ j \notin \mathcal{R}}} J_{ij}^n = \sum_{i \in \mathcal{R}} s_i^n. \tag{16.2}$$

This is the desired causal structure: change in a region equals boundary flux plus local conversion.

16.2 Coarse field

Choose a normalized kernel $K_R(x, i)$ and background $\bar{r}$. Define

$$\chi_R(x) = Z_R \sum_i K_R(x, i)(r_i - \bar{r}). \tag{16.3}$$

A conservative scalar effective action may take the form

$$S_\chi = \int \sqrt{-g}\, \mathrm{d}^4x \left[-\frac{Z_\chi}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) - \frac{\xi}{2} \chi^2 R \right]. \tag{16.4}$$

The microscopic calculation must determine the sign of Z_χ, the potential, the coupling ξ, and the source map. Positivity $Z_\chi > 0$ supplies the ordinary second-order kinetic sector.

16.3 Homogeneous cosmology target

For a spatially flat Friedmann–Lemaître–Robertson–Walker metric and homogeneous $\chi(t)$, minimal coupling gives

$$\rho_\chi = \frac{Z_\chi}{2} \dot{\chi}^2 + U(\chi), \tag{16.5}$$

$$p_\chi = \frac{Z_\chi}{2} \dot{\chi}^2 - U(\chi), \tag{16.6}$$

$$\ddot{\chi} + 3H\dot{\chi} + Z_\chi^{-1} U'(\chi) = 0, \tag{16.7}$$

with Friedmann equation

$$H^2 = \frac{8\pi G}{3}\left(\rho_{\text{known}} + \rho_\chi\right) \tag{16.8}$$

after the gravitational bridge has been established. Perturbations $\delta\chi$ then determine structure and microwave-background consequences. The model parameters are frozen before those spectra are opened.

FIRST LOOK

Residual capacity is like unused room in a local schedule. A region can store it, pass it to neighbours, or convert it through a declared rule. A cosmic field is justified only when many local ledgers average into one smooth variable with its own measured response.

17

Constants, units, and honest dependencies

Constants enter a theory in different ways. Some define units. Some are measured independent scales. Some are algebraic combinations. A small number may eventually be genuine outputs of a microscopic spectrum. Clear classification prevents a familiar identity from being counted as a new prediction.

17.1 The four-level ladder

Level	Operation	Example
Definition	Fixes a unit or convention	$c = 299792458\,\mathrm{m/s}$ in SI
Identity	Recombines established quantities	$E = mc^2$, $\hbar = h/(2\pi)$
Matching	Equates a model coefficient to a measured scale	$c_{\text{model}} a/\Delta\tau = c_{\text{SI}}$
Prediction	Calculates a held-out dimensionless or dimensional observable after matching	A new mass ratio from two independently identified pattern gaps

Table 17.1: A constant's numerical agreement has meaning only after its operation is classified.

17.2 Planck combinations, fully worked

Using $\hbar$, c, G, and k_B, the standard Planck combinations are

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616255e - 35\,\mathrm{m}, \tag{17.1}$$

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \approx 5.391247e - 44\,\mathrm{s}, \tag{17.2}$$

$$m_P = \sqrt{\frac{\hbar c}{G}} \approx 2.176434e - 8\,\mathrm{kg}, \tag{17.3}$$

$$E_P = m_P c^2 \approx 1.95608e9\,\mathrm{J}, \tag{17.4}$$

$$T_P = \frac{E_P}{k_B} \approx 1.41678e32\,\mathrm{K}. \tag{17.5}$$

These are dependency calculations: G, $\hbar$, c, and k_B are inputs.

The area of a sphere of radius ℓ_P is

$$A_{P,\text{sphere}} = 4\pi\ell_P^2 \approx 3.283e - 69\,\mathrm{m}^2. \tag{17.6}$$

Planck power is

$$P_P = \frac{E_P}{t_P} = \frac{c^5}{G} \approx 3.628e52\,\mathrm{W}. \tag{17.7}$$

17.3 Blackbody radiation

Planck's spectral energy density per angular frequency is

$$u_\omega(T) = \frac{\hbar\omega^3}{\pi^2 c^3} \frac{1}{\exp(\hbar\omega/k_B T) - 1}. \tag{17.8}$$

Integrate with $x = \hbar\omega/(k_B T)$:

$$\begin{aligned} u(T) &= \int_0^\infty u_\omega \, \mathrm{d}\omega & (17.9) \\ &= \frac{(k_B T)^4}{\pi^2 \hbar^3 c^3} \int_0^\infty \frac{x^3}{\mathrm{e}^x - 1} \, \mathrm{d}x & (17.10) \\ &= \frac{\pi^2 k_B^4}{15 \hbar^3 c^3} T^4, & (17.11) \end{aligned}$$

because the integral is $\pi^4/15$. Isotropic radiation carries outward flux $F = cu/4$, so

$$F = \sigma T^4, \qquad \boxed{\sigma = \frac{\pi^2 k_B^4}{60 \hbar^3 c^2}} \approx 5.670374419e - 8\,\mathrm{W.m^{-2}.K^{-4}}. \tag{17.12}$$

This derives the Stefan–Boltzmann constant from the quantum radiation law and SI defining constants; it is not a separate End Theory output.

17.4 Electromagnetic coupling

In SI,

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}. \tag{17.13}$$

Today e, h, and c are exact SI defining constants, while ε_0 is experimentally related through the fine-structure constant. A lattice theory predicts α only if its bare charge content and renormalization flow determine the low-energy dimensionless coupling without using the target value.

17.5 Strong coupling: a corrected one-loop example

At one loop,

$$\alpha_s(Q) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)}, \qquad \beta_0 = 11 - \frac{2n_f}{3}. \tag{17.14}$$

For $n_f = 5$, $Q = 91.1876\,\mathrm{GeV}$, and $\Lambda = 0.213\,\mathrm{GeV}$,

$$\beta_0 = \frac{23}{3}, \qquad \ln(Q^2/\Lambda^2) \approx 12.12, \tag{17.15}$$

so the one-loop value is approximately 0.135, not 0.118. Higher-loop running and scheme-dependent threshold matching change the relation between Λ_{QCD} and $\alpha_s(M_Z)$. Reversing Equation (17.14) at one loop to match 0.118 gives an illustrative $\Lambda \approx 0.088\,\mathrm{GeV}$. Precision work uses the declared loop order and scheme [117].

17.6 A natural-unit cross-section example

Suppose a contact estimate is

$$\sigma \sim \frac{g^2}{4\pi m^2}. \tag{17.16}$$

With $g = 10^{-2}$ and $m = 100\,\mathrm{GeV}$,

$$\sigma \approx 7.96 \times 10^{-10}\ \mathrm{GeV}^{-2}. \tag{17.17}$$

Using $1\ \mathrm{GeV}^{-2} = 0.389379 \times 10^{-27}\ \mathrm{cm}^2$,

$$\sigma \approx 3.10 \times 10^{-37}\ \mathrm{cm}^2. \tag{17.18}$$

The conversion line is essential; a result written directly in square centimetres without it cannot be audited.

17.7 A solar-mass black hole

For $M = M_\odot \approx 1.98847e30\,\mathrm{kg}$,

$$r_s = \frac{2GM}{c^2} \approx 2.953\,\mathrm{km}. \tag{17.19}$$

The Bekenstein–Hawking entropy is

$$S_{\mathrm{BH}} = \frac{k_B A}{4\ell_P^2} = \frac{k_B \pi r_s^2}{\ell_P^2} \approx 1.45 \times 10^{54}\ \mathrm{J/K}, \tag{17.20}$$

or $S_{\mathrm{BH}}/k_B \approx 1.05 \times 10^{77}$ dimensionless information units. The units distinguish those two quoted magnitudes.

RESEARCH GATE

For every proposed constant, print an input ledger: formula, dimensions, numerical inputs, covariance, scale and scheme, calibration/hold-out assignment, and code checksum. The next strong End Theory result should be a dimensionless ratio or scaling exponent calculated before its comparison value is opened.

18

Complex adaptive systems as a later layer

The relational language can also describe biological or artificial networks, but this layer begins after the physical theory has supplied time, energy, matter, and measurement. It is therefore kept separate from the foundational action.

18.1 Memory as persistent state

Let $z_i(t) = r_i(t)\mathrm{e}^{\mathrm{i}\theta_i(t)}$ represent coarse activity in subsystem i. A memory candidate is a metastable attractor $\mathcal{A}_m$ whose return probability after a standardized perturbation is

$$R_m(\Delta) = \Pr[z(t+\Delta) \in \mathcal{A}_m \mid z(t) \in \mathcal{A}_m,\ \text{probe}]\,. \tag{18.1}$$

This turns "persistent configuration" into an experimental statistic.

18.2 Phase coordination

A standard synchronization order parameter is

$$\mathcal{R}(t)\mathrm{e}^{\mathrm{i}\Psi(t)} = \frac{1}{N}\sum_{j=1}^{N} \mathrm{e}^{\mathrm{i}\theta_j(t)}, \qquad 0 \le \mathcal{R} \le 1. \tag{18.2}$$

Large $\mathcal{R}$ indicates phase alignment, but alignment alone is not awareness. A testable integrative index may combine coordination, causal influence, and adaptive prediction:

$$\mathcal{I}_T = \frac{1}{T}\int_{t_0}^{t_0+T} \mathcal{R}(t)\,\mathcal{C}(t)\,\mathcal{P}(t)\,\mathrm{d}t, \tag{18.3}$$

where each factor is normalized and independently operationalized. Equation (18.3) is a phenomenological research definition, not a fundamental law.

FIRST LOOK

Many musicians can play the same beat, yet a good orchestra also listens, predicts, corrects mistakes, and remembers the score. Synchrony measures only the shared beat. A serious theory of awareness must measure the additional causal and adaptive structure.

18.3 A clean research boundary

Claims about attention, temporal perception, memory, and self-maintaining feedback can be studied through perturbation recovery, information flow, phase coordination, and behavioural report. They enter End Theory as applications of a mature relational dynamics, not as evidence for the fundamental node model.

PART IV

Wave observation and verification manual

19

Wave observation theory

A wave equation becomes experimental only after a source, propagation law, sensor response, sampling rule, and likelihood are connected. This chapter defines that chain in a form that can be used for lattice simulations, laboratory waves, gravitational signals, or collider distributions.

19.1 The observation chain

Let $s(t, \mathbf{x}; \vartheta)$ be a source with parameters ϑ. A Green function G propagates it to a field

$$\phi(t, \mathbf{x}) = \int G(t - t', \mathbf{x}, \mathbf{x}'; p) s(t', \mathbf{x}'; \vartheta)\, \mathrm{d}t'\, \mathrm{d}^d x'. \tag{19.1}$$

A detector with response kernel R_a produces channel a:

$$y_a(t) = \int R_a(t - t', \mathbf{x}') \phi(t', \mathbf{x}')\, \mathrm{d}t'\, \mathrm{d}^d x' + n_a(t), \tag{19.2}$$

where n_a is noise. Sampling at times t_m gives data vector $\mathbf{y}$.

Figure 19.1: A complete observation map. Each block has parameters, calibration data, uncertainty, and a validation test.

For Gaussian noise with covariance C and model mean $\mathbf{m}(p, \vartheta)$,

$$-2 \ln \mathcal{L} = (\mathbf{y} - \mathbf{m})^\mathsf{T} C^{-1} (\mathbf{y} - \mathbf{m}) + \ln \det C + \text{constant}. \tag{19.3}$$

Non-Gaussian counting or threshold data use the corresponding likelihood rather than forcing a least-squares form.

19.2 Reading dispersion from phase

For two sensors separated by L along propagation, the cross spectrum is

$$C_{12}(\omega) = \tilde{y}_1^*(\omega) \tilde{y}_2(\omega). \tag{19.4}$$

Its phase $\Delta\varphi(\omega) = \arg C_{12}$ estimates

$$k(\omega) = \frac{\text{unwrap}\, \Delta\varphi(\omega)}{L}, \tag{19.5}$$

after accounting for the response-phase difference of the sensors. Comparing $k(\omega)$ with Equation (9.5) tests the lattice band without fitting a particle identity.

WORK IT OUT

Two probes are $L = 3\,\text{m}$ apart. At $f = 20\,\text{Hz}$ the calibrated cross-phase is 1.50 rad. Then $k = 1.50/3 = 0.50\,\text{rad.m}^{-1}$ and $\omega = 2\pi f \approx 125.66\,\text{rad.s}^{-1}$. The phase velocity is

$$v_p = \omega/k \approx 251.3\,\text{m/s}.$$

Repeat across frequency; a single point cannot determine dispersion.

19.3 Sampling and aliases

With sampling interval Δt_s, the Nyquist frequency is

$$f_N = \frac{1}{2\Delta t_s}. \tag{19.6}$$

A frequency $f > f_N$ aliases to a lower observed frequency. Spatial sampling on a regular graph has the parallel limit $|k_r| \leq \pi/a$. The Brillouin-zone edge in the exact dispersion relation is therefore not a decorative mathematical boundary; it is the shortest distinct wavelength represented by the lattice.

19.4 A wave-result ledger

Every reported wave result records:

1. source waveform and nuisance parameters;
2. propagation equation and boundary conditions;
3. sensor transfer function and calibration covariance;
4. sample rate, window, filtering, and missing-data policy;
5. estimator and uncertainty;
6. baseline model;
7. held-out frequency or geometry range;
8. pass/fail rule fixed before opening the hold-out.

20

The reproducible lattice laboratory

The first executable project verifies the benchmark before attaching it to known particles. Use periodic chains, square/cubic lattices, and approximately isotropic irregular graphs. The program should reproduce the exact spectrum, stability boundary, gauge invariants, and refinement behaviour.

20.1 Minimal scalar update

```
inputs: graph W, mu, kappa, a, dt, Omega0, nonlinear dW(q)
state:  q_prev, q_now

repeat for n = 0 ... N-1:
    lap = (degree(W) - W) @ q_now
    force = (kappa/a^2) * lap + mu*Omega0^2*q_now + dW(q_now)
    q_next = 2*q_now - q_prev - (dt^2/mu) * force
    record observables(q_prev, q_now, q_next)
    q_prev, q_now = q_now, q_next
```

The implementation test for one eigenmode u_r with Laplacian eigenvalue λ_r is

$$\omega_r^{\text{exact}} = \frac{2}{\Delta\tau} \arcsin\left[\frac{\Delta\tau}{2}\sqrt{\Omega_0^2 + \frac{\kappa\lambda_r}{\mu a^2}}\right]. \tag{20.1}$$

20.2 Initial data for a pure mode

Choose amplitude A and phase φ:

$$q^0 = A u_r \cos\varphi, \tag{20.2}$$
$$q^1 = A u_r \cos(\varphi - \omega_r \Delta\tau). \tag{20.3}$$

Project the trajectory back onto the mode,

$$Q_r^n = (u_r)^{\mathsf{T}} q^n, \tag{20.4}$$

and fit its phase. The relative frequency residual is

$$\epsilon_\omega = \frac{\hat{\omega}_r - \omega_r^{\text{exact}}}{\omega_r^{\text{exact}}}. \tag{20.5}$$

20.3 Gauge-invariance unit test

Generate random node phases α_i. Transform ψ and U by Equation (12.4). Compute

$$\epsilon_D = \frac{\left|\sum_{\langle ij\rangle} |D'_{ij}\psi'|^2 - \sum_{\langle ij\rangle} |D_{ij}\psi|^2\right|}{\max\left(1, \sum_{\langle ij\rangle} |D_{ij}\psi|^2\right)}. \tag{20.6}$$

Double-precision code should place ϵ_D near its accumulated floating-point tolerance.

20.4 Refinement study

Let an observable have leading error $O(a^p)$. For spacings a, $a/2$, and $a/4$, the differences satisfy

$$\frac{O(a) - O(a/2)}{O(a/2) - O(a/4)} \longrightarrow 2^p. \tag{20.7}$$

Estimate

$$\hat{p} = \log_2 \left| \frac{O(a) - O(a/2)}{O(a/2) - O(a/4)} \right|. \tag{20.8}$$

This separates a continuum trend from a single attractive plot.

Test	Procedure	Pass record
Normal modes	Initialize each Laplacian eigenvector	Frequency residual vs. precision and duration
Stability edge	Sweep $\Delta\tau$ across Equation (10.3)	Growth begins at the analytic boundary within scan resolution
Invariant	Track Equation (8.12)	Bounded roundoff-scale drift in the linear stable sector
Gauge transform	Random local phase rotations	ϵ_D and plaquette differences at floating-point tolerance
Refinement	Repeat fixed physical initial data	Estimated order $\hat{p}$ and extrapolated observable
Irregular graphs	Compare isotropy estimators	Directional residual distribution and spectral dimension

Table 20.1: Core verification suite. Thresholds are recorded numerically in the project's parameter card.

DRY NOTE FROM THE FLIGHT DECK

"The plot looked stable" is not a unit test. It is a plot wearing a lab coat.

21

CERN-facing interaction templates

The microscopic framework currently supplies exact benchmark structure rather than a unique collider mass. CERN analysis can still be prepared through parameterized templates, with every coefficient labeled as matching input until a node-pattern calculation fixes it.

21.1 Neutral scalar template

For a neutral scalar pattern s, a low-energy interaction basis may include

$$\mathcal{L}_s = \frac{1}{2}(\partial s)^2 - \frac{1}{2}m_s^2 s^2 - \frac{\lambda_{sH}}{2}s^2 H^\dagger H - \sum_f y_f^{(s)} s\bar{f}f - \frac{c_g}{4\Lambda} sG^a_{\mu\nu}G^{a\mu\nu} - \frac{c_\gamma}{4\Lambda} sF_{\mu\nu}F^{\mu\nu}. \tag{21.1}$$

A microscopic calculation must identify the pattern, normalize its state, and compute m_s, λ_{sH}, $y_f^{(s)}$, c_g, and c_γ.

21.2 Contact correction template

High-energy deviations can be organized by dimension-six operators such as

$$\mathcal{L}_{\text{contact}} = \sum_i \frac{C_i}{\Lambda^2}\mathcal{O}_i. \tag{21.2}$$

The event-level validity region $\hat{s} < \Lambda^2_{\text{valid}}$ is applied before the fit. Limits then constrain coefficient combinations; they become End Theory tests only after the micro-to-operator map is frozen.

21.3 Binned likelihood

For observed counts n_b and expected counts

$$\nu_b(\theta, \eta) = s_b(\theta, \eta) + b_b(\eta), \tag{21.3}$$

use

$$\mathcal{L}(\theta, \eta) = \prod_b \frac{\nu_b^{n_b} \mathrm{e}^{-\nu_b}}{n_b!} \prod_k \pi_k(\eta_k), \tag{21.4}$$

where η are nuisance parameters with constraint terms π_k. Correlated systematic effects are shared across bins and channels rather than added independently in quadrature when a covariance is available.

21.4 Open-data entry point

CERN's ATLAS 13 TeV educational release provides 10 fb^{-1} of 2016 proton–proton data with matched simulated samples, including four-lepton and diphoton collections [120, 121]. It is suitable for learning the analysis chain and reproducing public examples. A research-grade claim uses the collaboration's current analysis objects, calibrations, and likelihood information.

RESEARCH GATE

Build the likelihood with a background-only model and one template coefficient. Validate it on a known injected signal. Freeze event selection, binning, nuisance correlations, and the validity cut before opening the held-out distribution.

22

Evidence without retrofitting

Let the full parameter vector be split into structural parameters p_s, calibration parameters p_c, and nuisance parameters η. A clean protocol assigns data once:

$$D = D_{\text{design}} \cup D_{\text{cal}} \cup D_{\text{test}}, \qquad D_{\text{cal}} \cap D_{\text{test}} = \emptyset. \tag{22.1}$$

22.1 Freeze before comparison

The model card contains

$$\mathcal{C}_{\text{model}} = \{\text{equations, graph rule, } p_s, p_c, \text{ priors, code version, seeds, units, data IDs}\}. \tag{22.2}$$

Hash the card and code. The prediction function is then

$$\hat{\mathbf{y}}_{\text{test}} = F(\mathcal{C}_{\text{model}}, D_{\text{cal}}; X_{\text{test}}), \tag{22.3}$$

evaluated without changing $\mathcal{C}_{\text{model}}$.

22.2 Uncertainty propagation

For outputs $\mathbf{y} = f(\mathbf{x})$ and input covariance V_x, the linearized covariance is

$$V_y = J V_x J^{\mathsf{T}}, \qquad J_{ij} = \frac{\partial f_i}{\partial x_j}. \tag{22.4}$$

For one multiplicative quantity $Q = \prod_j x_j^{a_j}$,

$$\frac{u^2(Q)}{Q^2} = \sum_{ij} a_i a_j \frac{\text{Cov}(x_i, x_j)}{x_i x_j}. \tag{22.5}$$

Ignoring covariance can either inflate or shrink the final uncertainty.

22.3 Model comparison

Report absolute fit and improvement over a baseline. For nested models, a profile likelihood statistic is

$$q = -2 \ln \frac{\mathcal{L}(\theta_0, \hat{\hat{\eta}})}{\mathcal{L}(\hat{\theta}, \hat{\eta})}. \tag{22.6}$$

Also report parameter count, prior volume where relevant, residual structure, robustness to declared alternatives, and negative results.

WORK IT OUT

Suppose $Q = x^2/y$, with $x = 3.0 \pm 0.1$, $y = 2.0 \pm 0.2$, and zero covariance. Then $Q = 4.5$ and

$$\frac{u^2(Q)}{Q^2} = 4\left(\frac{0.1}{3.0}\right)^2 + \left(\frac{0.2}{2.0}\right)^2 = 0.01444.$$

Thus $u(Q) = 4.5\sqrt{0.01444} \approx 0.54$, so $Q = 4.50 \pm 0.54$ at one standard uncertainty.

23

The falsification ladder

The research programme advances through gates in dependency order. Passing a later-looking comparison does not repair an earlier undefined bridge.

Gate	Object	Required calculation	Decision record
1	Benchmark dynamics	Exact update, spectrum, invariant, stability, gauge test	Reproduction logs and tolerances
2	Capacity completion	Closure, locality, deterministic tie-breaks, conservation	Proof plus saturation stress tests
3	Emergent geometry	Persistence, graph construction, dimension plateau, isotropy, clocks	Scale window and failure regions
4	Pattern spectrum	Localized solutions, stability, dispersion, scattering	Blind dimensionless catalogue
5	Known fields	Gauge group, chirality, representations, anomaly cancellation	Derived low-energy operator content
6	Quantum map	Preparation ensemble, Born frequencies, Bell assumption, no-signalling	Preregistered interference and Bell simulations
7	Gravity/EQEF	Common metric, Newtonian/Einstein limit, torsion and capacity sources	Solar-system, wave, and cosmology likelihoods
8	Prediction	Frozen calibration and held-out observable	External comparison with uncertainty

23.1 Decision logic for a new idea

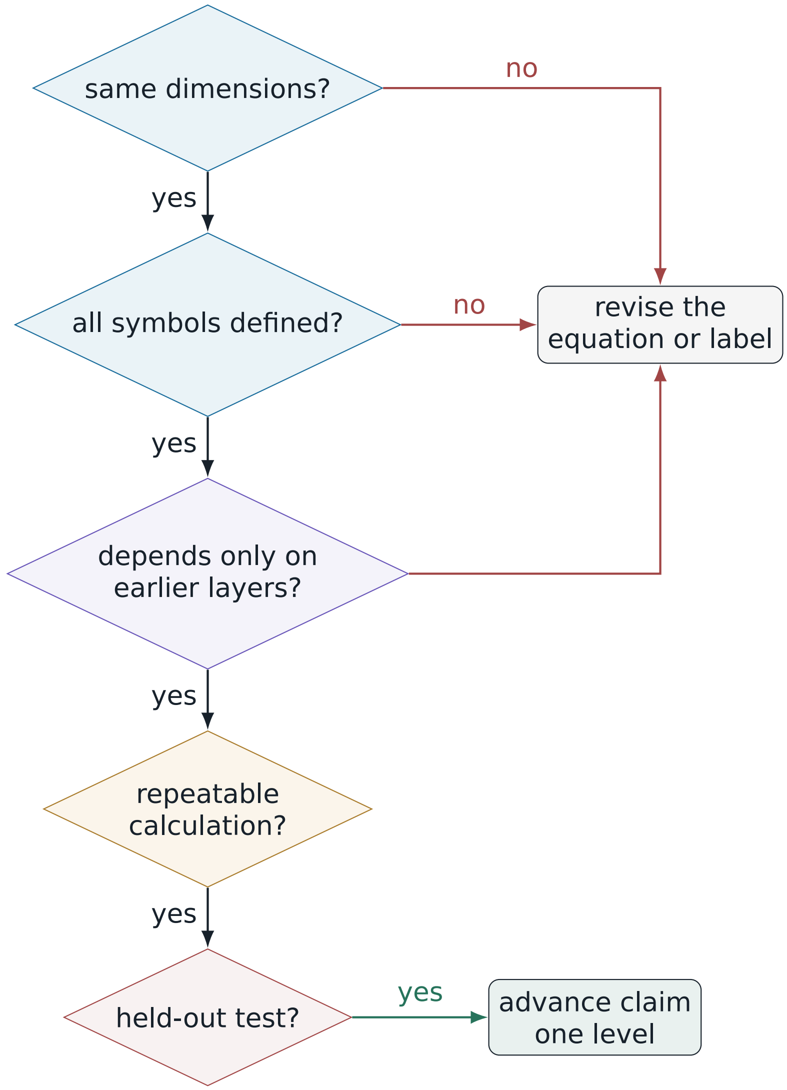


Figure 23.1: A compact author decision tool. The label changes only after the corresponding operation has been completed.

23.2 The completion criterion

End Theory becomes a quantitatively competitive physical theory when one immutable micro-model crosses the gates with fewer calibrated inputs than independent held-out results, publishes its negative regions, and remains stable under independent reproduction. Until then, the scientifically useful object is the ordered framework, exact benchmark, and explicit programme recorded in this textbook.

DRY NOTE FROM THE FLIGHT DECK

Changing the finish line after the race is excellent exercise and poor inference.

PART V

The End Theory concept atlas

How to use the atlas

The atlas is ordered by dependency. Each topic occupies a compact statement page followed by a working page. The first page tells the reader what the object means; the second records inputs, algebra, dimensions, and the calculation that advances it.

Relational proto-state

DEFINED

Compact equation

$$\mathcal{P} = (\mathcal{X}, \mathcal{R}, \mathcal{A}, U_p), \quad X_n \notin \mathcal{M}_{\text{spacetime}}$$

FIRST LOOK

Begin with possible states, direct relations, legal local changes, and an update. Physical distance and duration are later observables.

Dependency: None; this is the root definition.

F01 Working page: Relational proto-state

Derivation or construction

Choose a finite alphabet Σ and a relational hypergraph H_n. One explicit prototype is $X_n = (H_n, s_n)$ with $s_n : V(H_n) \to \Sigma$. Locality means $[U_p(X)]_i$ depends on a bounded relational neighbourhood.

Dimensions and normalization

The primitive labels are dimensionless unless the model explicitly assigns a scale. Metres and seconds enter only through derived rods and clocks.

Decisive calculation

Run the update without a coordinate embedding. If the rule requires Euclidean distance to decide which primitive elements interact, the pre-geometric claim has been replaced by a background geometry.

READER CHECK

Name every object needed to compute X_1 from X_0. Circle any object whose definition already assumes position, time, energy, or particle identity.

Ordered updates before time

DEFINED

Compact equation

$$X_{n+1} = U_p(X_n), \qquad n \in \mathbb{Z}$$

FIRST LOOK

The index orders snapshots. A clock reading is built from a stable repeating process inside the snapshots.

Dependency: F01: relational proto-state and update.

F02 Working page: Ordered updates before time

Derivation or construction

Iterate one fixed map to form the history $\Gamma(X_0) = \{X_0, U_p X_0, U_p^2 X_0, ...\}$. Causal order means effects at step $n+1$ depend only on the declared past neighbourhood.

Dimensions and normalization

n is dimensionless. A conversion $t = n\Delta t$ is a homogeneous matching rule unless Δt is generated by clock dynamics.

Decisive calculation

Verify that histories are single-valued and that changing the update schedule does not change observables unless the schedule itself is part of the state.

READER CHECK

A process repeats every 12 updates. If one period is calibrated as $0.75\,\mathrm{s}$, compute the homogeneous duration per update: $0.75/12 = 0.0625\,\mathrm{s}$. Label the operation as matching.

Operational clock map

PROPOSED

Compact equation

$$N_K(n) = \frac{\operatorname{unwrap}\Theta_K(n)}{2\pi}, \qquad t_K = T_K N_K$$

FIRST LOOK

A clock is a robust orbit whose phase can be counted. Time is what the orbit reads, not the name of the update counter.

Dependency: F02 and a persistent recurrent subsystem.

F03 Working page: Operational clock map

Derivation or construction

Select a complex observable $Z_K = A_K e^{i\Theta_K}$. Unwrap its phase and count crossings of 2π. Linearize one period to obtain a monodromy matrix; transverse multipliers inside the unit circle indicate recovery from small disturbances.

Dimensions and normalization

N_K and Θ_K are dimensionless. T_K carries time and must be calibrated once or related to another predicted scale.

Decisive calculation

Build two clock types. In a common low-load regime, test whether their rate ratio approaches a constant and whether imposed load changes both rates by the same factor.

READER CHECK

If Θ_K advances by $\pi/8$ per update, show that one cycle takes 16 updates.

Persistence across frames

DEFINED

Compact equation

$$\inf_{g \in \mathcal{G}_R} d_R(C_R X_{n+T}, g C_R X_n) < \varepsilon_R$$

FIRST LOOK

A pattern keeps its identity when coarse features stay close after an allowed motion, phase change, or relabeling.

Dependency: F01–F03 and a declared resolution.

F04 Working page: Persistence across frames

Derivation or construction

Fix C_R, d_R, $\mathcal{G}_R$, and ε_R before searching. Define the equivalence tube $[X]_{R,\varepsilon}$ and record first-exit time τ_R. Compare τ_R with the internal cycle period.

Dimensions and normalization

d_R is dimensionless or normalized by a fixed scale. T counts updates until an operational clock is applied.

Decisive calculation

Perturb pattern candidates and measure the lifetime distribution across resolutions. Robust classes show a plateau rather than appearing only at one tuned tolerance.

READER CHECK

Explain why a translated wave packet can remain the same pattern even though its node-by-node vector has changed.

Emergent weighted graph

PROPOSED

Compact equation

$$w_{ab} = \frac{1}{T} \sum_{n=1}^{T} I_{ab}(n), \qquad G_R = (V_R, E_R, W_R)$$

FIRST LOOK

Tap one persistent pattern and measure which other patterns respond. The response network draws effective neighbourhood.

Dependency: F04 persistent classes and an intervention rule.

F05 Working page: Emergent weighted graph

Derivation or construction

Perturb class a by standardized δ, measure resolved response in b, normalize by $\|\delta\|$, and average over a fixed window. Retain a link when $w_{ab} > w_{\min}$. Keep direction until reciprocity is demonstrated.

Dimensions and normalization

I_{ab} and w_{ab} are dimensionless response ratios. Physical link length requires a separate scale and function $\ell(w)$.

Decisive calculation

Repeat with different admissible probes. A useful graph has a stable link backbone and predicts response to probes not used in its construction.

READER CHECK

Given $w_{ab} = 0.8$, $w_{bc} = 0.3$, and threshold 0.5, draw the retained graph.

Graph distance and dimension

DERIVED

Compact equation

$$d_G(a,b) = \min_{\gamma:a\to b} \sum_{e\in\gamma} \ell_e, \qquad d_s = -2\frac{\mathrm{d}\ln P}{\mathrm{d}\ln\sigma}$$

FIRST LOOK

Distance comes from route cost; dimension comes from how diffusion spreads and returns.

Dependency: F05, a positive edge-length map, and graph diffusion.

F06 Working page: Graph distance and dimension

Derivation or construction

Set $\ell_{ab} = \ell_0 f(w_{ab})$ with fixed monotone f. Construct $\mathsf{L} = D - W$ and $K(\sigma) = \mathrm{e}^{-\sigma \mathsf{L}}$. Average the return probability and differentiate its log slope.

Dimensions and normalization

ℓ_0 supplies length. P and d_s are dimensionless; σ is normalized to the Laplacian convention.

Decisive calculation

Look for a broad d_s plateau shared by diffusion, volume growth, and wave propagation. Quantify anisotropy across directions and graph ensembles.

READER CHECK

If $P(\sigma) = 3\sigma^{-3/2}$, calculate $d_s = 3$.

Finite local transition capacity

PROPOSED

Compact equation

$$C_i = \frac{D_i}{D_{\star,i}} \leq 1, \qquad r_i = 1 - C_i$$

FIRST LOOK

Every local junction has a finite change budget. Busy regions alter nearby evolution through the same local rule.

Dependency: F01 locality and a nonnegative demand functional.

F07 Working page: Finite local transition capacity

Derivation or construction

Construct D_i from the same variables advanced by the update: amplitude change, phase change, link strain, or rewiring demand. Normalize by a fixed local capacity $D_{\star,i}$.

Dimensions and normalization

The numerator and denominator have identical dimensions, so C_i and r_i are dimensionless. Terms inside D_i must be mutually commensurate.

Decisive calculation

Stress the lattice at increasing load. Record onset, conservation residuals, propagation of saturation, and dependence on boundary size.

READER CHECK

For $D_i = 0.72D_\star$, compute C_i and r_i.

Admissible deterministic update

PROPOSED

Compact equation

$$(X_{n+1}, X_n) = U_p(X_n, X_{n-1}), \qquad U_p(\mathcal{A}) \subseteq \mathcal{A}$$

FIRST LOOK

A complete rule says which legal move happens next and keeps every local demand within capacity.

Dependency: F07 admissible set plus fixed parameters and boundary conditions.

F08 Working page: Admissible deterministic update

Derivation or construction

Prove single-valuedness, then closure by induction on n. Derive exact conserved charges or a declared Lyapunov inequality. Treat graph-change branches and ties explicitly.

Dimensions and normalization

U_p maps like-dimensioned states. Each component of p has a unit or dimensionless normalization.

Decisive calculation

Adversarially sample states near every capacity boundary. Verify closure, locality radius, conservation, stability, and reproducibility across execution order.

READER CHECK

Write one example of an ambiguous tie and a deterministic tie-break that depends only on local state.

Discrete graph action

DEFINED

Compact equation

$$S_d = \sum_n \Delta\tau \left[\frac{\mu}{2} \left\| \frac{q^{n+1} - q^n}{\Delta\tau} \right\|^2 - \frac{\kappa}{2a^2} q^{n\mathsf{T}} \mathsf{L} q^n - \sum_i V(q_i^n) \right]$$

FIRST LOOK

The action balances change from one update to the next against link strain and onsite energy.

Dependency: A fixed weighted graph phase and scalar node coordinates.

D01 Working page: Discrete graph action

Derivation or construction

Insert Equation (7.3) to show that the link term equals the sum of squared neighbour differences. The discrete time sum approximates an integral when the clock map and refinement limit exist.

Dimensions and normalization

The bracket has energy dimensions: $[\mu q^2/\Delta\tau^2] = [\kappa q^2/a^2] = [V]$. Multiplication by $\Delta\tau$ gives action.

Decisive calculation

Evaluate the action on a constant configuration, one-node bump, and plane wave. Compare signs and scaling with amplitude.

READER CHECK

Why does the constant mode have zero link energy on a connected graph?

D02

BENCHMARK

Discrete Euler–Lagrange update

DERIVED

Compact equation

$$\mu \frac{q_i^{n+1} - 2q_i^n + q_i^{n-1}}{\Delta\tau^2} = -\frac{\kappa}{a^2}(\mathsf{L}q^n)_i - V'(q_i^n)$$

FIRST LOOK

The next value is determined by the previous two values and the force at the present value.

Dependency: D01 and differentiable onsite potential.

D02 Working page: Discrete Euler–Lagrange update

Derivation or construction

Differentiate the $n-1$ and n kinetic terms with respect to q_i^n. Add the link derivative $-(\kappa/a^2)(\mathsf{L}q^n)_i$ and onsite derivative $-V'(q_i^n)$, then solve for q_i^{n+1}.

Dimensions and normalization

Both sides have force-like dimensions $[\mu q/T^2]$.

Decisive calculation

Compare analytic gradients of S_d with finite-difference gradients on random small graphs.

READER CHECK

For a free isolated node, show that the update gives $q^{n+1} - q^n = q^n - q^{n-1}$: constant discrete velocity.

D03

BENCHMARK

Weighted graph Laplacian

DERIVED

Compact equation

$$(\mathsf{L}q)_i = \sum_j w_{ij}(q_i - q_j), \qquad q^{\mathsf{T}}\mathsf{L}q = \frac{1}{2}\sum_{ij} w_{ij}(q_i - q_j)^2$$

FIRST LOOK

The Laplacian compares each value with a weighted neighbour average and produces the restoring link force.

Dependency: Symmetric nonnegative weights.

D03 Working page: Weighted graph Laplacian

Derivation or construction

Expand the quadratic form. Symmetry makes the two q_i^2 sums equal, leaving $q^{\mathsf{T}}(D - W)q$. Positivity follows term by term.

Dimensions and normalization

w_{ij} is dimensionless in the benchmark; κ/a^2 carries the physical stiffness normalization.

Decisive calculation

Diagonalize L. Require nonnegative eigenvalues and one zero eigenvalue per connected component.

READER CHECK

Compute the Laplacian matrix of a three-node path and verify its row sums are zero.

Deterministic recursion theorem

DERIVED

Compact equation

$$q^{n+1} = 2q^n - q^{n-1} - \frac{\Delta\tau^2}{\mu}\left(\frac{\kappa}{a^2}\mathsf{L}q^n + V'(q^n)\right)$$

FIRST LOOK

Once two consecutive states are known, ordinary arithmetic gives exactly one next benchmark state.

Dependency: D02, finite graph, $\mu > 0$, and single-valued V'.

D04 Working page: Deterministic recursion theorem

Derivation or construction

The graph Laplacian is a finite matrix. Every vector operation on the right is single-valued. Division by positive μ is defined. Therefore the recursion gives a unique real vector.

Dimensions and normalization

The coefficient $\Delta\tau^2/\mu$ converts force to coordinate increment.

Decisive calculation

Run two independent implementations from byte-identical inputs and compare every output bit or declared tolerance.

READER CHECK

List which inputs must be stored to reproduce update 100 without ambiguity.

Discrete energy ledger

DERIVED

Compact equation

$$H^{n+1/2} = \frac{1}{2}\left\|\frac{q^{n+1} - q^{n}}{\Delta\tau}\right\|^2 + \frac{1}{2}(q^{n+1})^{\mathsf{T}} A q^{n}$$

FIRST LOOK

The linear central update carries an exact step-to-step balance that can be checked numerically.

Dependency: Linear harmonic benchmark with symmetric A.

D05 Working page: Discrete energy ledger

Derivation or construction

Subtract consecutive half-step expressions. Factor $q^{n+1} - q^{n-1}$ and use $q^{n+1} - 2q^n + q^{n-1} = -\Delta\tau^2 A q^n$. Symmetry cancels the remaining terms.

Dimensions and normalization

As written, H is normalized per unit inertia. Multiply by μ for benchmark energy dimensions.

Decisive calculation

Track $H^{n+1/2}$ over long stable runs. Report maximum relative drift and scaling with floating-point precision.

READER CHECK

Verify the invariant for one oscillator over the first two updates by direct substitution.

Local proposal limiter

PROPOSED

Compact equation

$$q_i^{n+1} = q_i^n + \eta_i(\tilde{q}_i^{n+1} - q_i^n), \qquad 0 \le \eta_i \le 1$$

FIRST LOOK

When an unconstrained move asks too much of a node, a local factor shortens that move.

Dependency: D02 proposal and F07 demand.

D06 Working page: Local proposal limiter

Derivation or construction

Compute $\tilde{q}^{n+1}$ from the benchmark. Solve the local inequality $C_i(\eta_i) \le 1$ for the largest admissible factor, including coupled neighbour constraints by a deterministic local iteration.

Dimensions and normalization

η_i is dimensionless. The demand inequality carries the units.

Decisive calculation

Map capacity violations, energy residuals, and propagation speed as the load crosses threshold.

READER CHECK

If kinetic demand alone scales as η^2 and the proposal has $C = 1.44$, find $\eta = 1/\sqrt{1.44} = 5/6$.

Capacity projection

PROPOSED

Compact equation

$$X_{n+1} = \operatorname*{argmin}_{Y \in \mathcal{A}_{\text{local}}} \left\| Y - \widetilde{X}_{n+1} \right\|_M^2$$

FIRST LOOK

Choose the closest admissible local state according to a fixed metric, with a fixed rule for equal minima.

Dependency: D06, a convex or otherwise well-specified admissible set, and metric M.

D07 Working page: Capacity projection

Derivation or construction

Formulate the local constrained optimization, include conserved linear charges as equality constraints, and prove existence and uniqueness under the chosen geometry. If the set is not convex, specify deterministic branch selection.

Dimensions and normalization

The metric M normalizes unlike coordinates so the squared distance is dimensionless or has one declared unit.

Decisive calculation

Compare the projected change with an independently integrated constrained dynamics. Check closure and charge balance at every saturated node.

READER CHECK

Project the scalar proposal $y = 1.3$ onto interval $[-1, 1]$ under Euclidean distance. The answer is 1.

Continuum scaling

DERIVED

Compact equation

$$q_i = a^{d/2}\phi(x_i), \qquad \sum_i q_i^2 \to \int \phi^2 \, \mathrm{d}^d x$$

FIRST LOOK

The field is normalized so a sum over nodes becomes an integral over space.

Dependency: D01 and a regular graph phase with cell volume a^d.

D08 Working page: Continuum scaling

Derivation or construction

Insert the normalization, Taylor-expand neighbour differences, and replace $\sum_i a^d$ by $\int \mathrm{d}^d x$. Keep the leading a^2 correction to measure convergence.

Dimensions and normalization

a carries length after metric matching. The field dimension follows from the canonical continuum kinetic term.

Decisive calculation

Repeat one fixed physical wave packet at $a, a/2, a/4$ and estimate order by Equation (20.8).

READER CHECK

Show that $\sum_i q_i^2$ is unchanged in the continuum normalization when the number of cells doubles in each dimension and a halves.

Plane-wave eigenmodes

DERIVED

Compact equation

$$q_{\mathbf{j}}^{,n} = A\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x}_{\mathbf{j}}-\omega n\Delta\tau)}$$

FIRST LOOK

A plane wave repeats the same phase pattern across a periodic regular lattice.

Dependency: D02 on a translation-invariant graph.

S01 Working page: Plane-wave eigenmodes

Derivation or construction

Shift the lattice index by one cell; the mode acquires factor $e^{ik_r a}$. Translation invariance makes it a simultaneous eigenvector of all shift operators and hence of the Laplacian.

Dimensions and normalization

k has inverse-length dimensions after a is matched; ω has inverse-time dimensions after $\Delta\tau$ is matched.

Decisive calculation

Initialize one exact mode and measure leakage into all other discrete Fourier components.

READER CHECK

For a chain of N periodic nodes, list allowed $k_m = 2\pi m/(Na)$.

Exact lattice dispersion

DERIVED

Compact equation

$$\frac{4}{\Delta\tau^2}\sin^2\frac{\omega\Delta\tau}{2} = \Omega_0^2 + \frac{4c^2}{a^2}\sum_r \sin^2\frac{k_r a}{2}$$

FIRST LOOK

The equation tells every represented wavelength exactly how fast its phase advances per update.

Dependency: S01, harmonic onsite potential, and cubic periodic lattice.

S02 Working page: Exact lattice dispersion

Derivation or construction

Evaluate the temporal second difference and spatial Laplacian eigenvalue separately; cancel the nonzero plane-wave amplitude.

Dimensions and normalization

Every term has dimensions T^{-2}. The sine arguments are dimensionless.

Decisive calculation

Compare fitted mode frequencies with the analytic inverse-sine expression over the whole Brillouin zone.

READER CHECK

Set $k = 0$. Show that the frequency is the lattice gap determined by Ω_0.

Long-wavelength expansion

DERIVED

Compact equation

$$\omega^2 = \Omega_0^2 + c^2k^2 + \frac{\Delta\tau^2\omega^4}{12} - \frac{c^2a^2}{12}\sum_r k_r^4 + \cdots$$

FIRST LOOK

Long waves forget much of the lattice, while the first corrections remember spacing and direction.

Dependency: S02 and small $|k_r a|$, $|\omega\Delta\tau|$.

S03 Working page: Long-wavelength expansion

Derivation or construction

Use $\sin^2(x/2) = x^2/4 - x^4/48 + \mathcal{O}(x^6)$ on both sides of the exact dispersion relation.

Dimensions and normalization

Each correction has dimensions T^{-2}. Directional $\sum k_r^4$ differs from rotational $(k^2)^2$ and measures anisotropy.

Decisive calculation

Fit simulated residuals to $a^2 \sum k_r^4$ and $\Delta\tau^2\omega^4$ on refinements.

READER CHECK

Compare waves with $\mathbf{k} = (k, 0, 0)$ and $(k/\sqrt{3}, k/\sqrt{3}, k/\sqrt{3})$; calculate which has larger $\sum k_r^4$.

Group velocity and causal cone

DERIVED

Compact equation

$$v_{g,r} = \frac{(c^2/a)\,\sin(k_r a)}{\sqrt{A(\mathbf{k})}\sqrt{1 - \Delta\tau^2 A(\mathbf{k})/4}}$$

FIRST LOOK

A packet moves at the slope of the frequency band, not generally at the phase speed of one crest.

Dependency: S02 within the stable domain.

S04 Working page: Group velocity and causal cone

Derivation or construction

Solve for $\omega(\mathbf{k})$ with an inverse sine and differentiate. The chain rule supplies $\partial A/\partial k_r = (2c^2/a)\sin(k_r a)$.

Dimensions and normalization

$\partial\omega/\partial k$ has dimensions length/time.

Decisive calculation

Launch narrow-band packets and compare measured envelope speed with the analytic derivative. Test direction dependence.

READER CHECK

For massless long waves, insert $A \approx c^2k^2$ and show $|\mathbf{v}_g| \to c$.

Courant stability domain

DERIVED

Compact equation

$$\Delta\tau^2\left(\Omega_0^2 + \frac{4dc^2}{a^2}\right) \leq 4$$

FIRST LOOK

The time step must be short enough to resolve the fastest lattice mode.

Dependency: S02 and all normal modes of the linear benchmark.

S05 Working page: Courant stability domain

Derivation or construction

Require the inverse-sine argument to be at most one. Maximize the spatial sine sum at a Brillouin-zone corner, where it equals d.

Dimensions and normalization

The bracket has T^{-2}, so the product is dimensionless.

Decisive calculation

Sweep across the boundary at fixed graph size. Fit the growth exponent of the corner mode.

READER CHECK

For $d = 1$, $a = c = 1$, and $\Omega_0 = 0$, find $\Delta\tau \leq 1$.

Mass gap

DERIVED

Compact equation

$$m_{\mathrm{eff}} = \frac{\hbar\omega_0}{c_{\mathrm{phys}}^2}$$

FIRST LOOK

A nonzero lowest frequency costs energy even when the pattern has no spatial momentum.

Dependency: S02 at $\mathbf{k} = 0$, plus matched $\hbar$ and physical causal speed.

S06 Working page: Mass gap

Derivation or construction

Find ω_0 from the exact lattice gap. In the relativistic low-energy dictionary set rest energy $E_0 = \hbar\omega_0 = mc^2$.

Dimensions and normalization

$\hbar\omega_0$ is energy; division by speed squared gives mass.

Decisive calculation

Identify multiple stable patterns without target masses. Compare their dimensionless gap ratios on refinements.

READER CHECK

If $\omega_a = 3\omega_b$, show that $m_a/m_b = 3$ after the common dictionary cancels.

Nonlinear frequency shift

DERIVED

Compact equation

$$\Omega(A) \approx \Omega_0 \left(1 + \frac{3\lambda A^2}{8\mu\Omega_0^2}\right)$$

FIRST LOOK

A nonlinear oscillator's clock rate can depend on how strongly it is excited.

Dependency: Duffing equation and weak nonlinearity.

S07 Working page: Nonlinear frequency shift

Derivation or construction

Insert $q = A\cos\Omega t$, use $\cos^3 x = (3\cos x + \cos 3x)/4$, balance the fundamental, and expand the square root.

Dimensions and normalization

$\lambda A^2/(\mu\Omega_0^2)$ is dimensionless.

Decisive calculation

Measure Ω versus A^2 at small amplitude. Compare the fitted slope with $3\lambda/(8\mu\Omega_0)$.

READER CHECK

For positive λ, decide whether frequency increases or decreases with amplitude.

Localized nonlinear modes

PROPOSED

Compact equation

$$\left[\Omega_0^2 I + \frac{\kappa}{\mu a^2}\mathsf{L} - \Omega^2 I\right]\phi + \frac{3\lambda}{4\mu}\phi^{\circ 3} = 0$$

FIRST LOOK

Nonlinearity can balance dispersion and hold a vibrating pattern near a small set of nodes.

Dependency: S07 on a graph, harmonic balance, and a localization search.

S08 Working page: Localized nonlinear modes

Derivation or construction

Use an anti-continuum seed or constrained Newton solve. Continue coupling, compute participation ratio, and linearize one period for Floquet multipliers.

Dimensions and normalization

Normalize ϕ by a fixed norm; report frequencies and amplitudes in dimensionless benchmark units before physical matching.

Decisive calculation

Perturb each candidate, vary graph size and boundary conditions, and publish the region where it persists.

READER CHECK

For a vector supported equally on m nodes and zero elsewhere, show that its participation ratio is m.

Complex node field

DEFINED

Compact equation

$$\psi_i = \rho_i \mathrm{e}^{\mathrm{i}\theta_i}, \qquad \theta_i \sim \theta_i + 2\pi$$

FIRST LOOK

Amplitude says how much; phase says where the local cycle sits.

Dependency: D01 extended from a real coordinate to a complex coordinate.

G01 Working page: Complex node field

Derivation or construction

Write real and imaginary parts or polar variables. Require observables to be insensitive to the chosen 2π representative. Treat $\rho = 0$ separately because phase is undefined there.

Dimensions and normalization

θ is dimensionless. The normalization and mass dimension of ρ follow the chosen action.

Decisive calculation

Evolve a global phase-rotated initial state and compare all invariant observables.

READER CHECK

Show that $|\rho e^{i(\theta+2\pi)}|^2 = \rho^2$.

Covariant link difference

DEFINED

Compact equation

$$D_{ij}\psi = U_{ij}\psi_j - \psi_i, \qquad U_{ji} = U_{ij}^{-1}$$

FIRST LOOK

Before comparing phases at different nodes, transport one phase along the link.

Dependency: G01 and compact link transporter.

G02 Working page: Covariant link difference

Derivation or construction

Assign $U_{ij} \in U(1)$ to each orientation. The difference reduces to $\psi_j - \psi_i$ in gauge $U_{ij} = 1$. Reverse orientation by group inverse.

Dimensions and normalization

U_{ij} and the link exponent are dimensionless. $D_{ij}\psi$ has the same dimensions as ψ.

Decisive calculation

Verify covariance under thousands of random local phase transformations.

READER CHECK

If $U_{ij} = \mathrm{e}^{\mathrm{i}\pi/2}$ and $\psi_j = 1$, $\psi_i = \mathrm{i}$, calculate $D_{ij}\psi = 0$.

Local gauge transformation

DERIVED

Compact equation

$$\psi'_i = \mathrm{e}^{\mathrm{i}q\alpha_i}\psi_i, \qquad U'_{ij} = \mathrm{e}^{\mathrm{i}q\alpha_i}U_{ij}\mathrm{e}^{-\mathrm{i}q\alpha_j}$$

FIRST LOOK

Each node may choose its own phase reference; link variables compensate so physics stays unchanged.

Dependency: G02.

G03 Working page: Local gauge transformation

Derivation or construction

Substitute the transformed objects into $D'_{ij}\psi'$ and factor out $\mathrm{e}^{\mathrm{i}q\alpha_i}$. Absolute squares and closed loops are invariant.

Dimensions and normalization

$q\alpha_i$ is dimensionless. The split between q and A_{ij} depends on charge normalization.

Decisive calculation

Randomize α_i and compare action terms, currents, and loop products at floating-point tolerance.

READER CHECK

Perform the three-line cancellation leading to Equation (12.8).

Plaquette holonomy

DERIVED

Compact equation

$$U_{\square} = \prod_{(ij)\in\partial\square} U_{ij}$$

FIRST LOOK

Transport around a closed elementary loop. The leftover phase records gauge curvature.

Dependency: G03 and an oriented plaquette.

G04 Working page: Plaquette holonomy

Derivation or construction

Multiply the transformed links in order. Every node factor appears once with positive and once with negative exponent, so they cancel. Reverse the loop to obtain $U_{\square}^{-1}$.

Dimensions and normalization

$U_{\square}$ is dimensionless. In a smooth limit its phase is $qa^2F_{\mu\nu} + \cdots$.

Decisive calculation

Compare loop products before and after gauge transformation; then test the area scaling of small smooth fields.

READER CHECK

Write the cancellation explicitly for a four-link square.

Gauge-field action

PROPOSED

Compact equation

$$S_g = -\frac{\beta}{a^4} \sum_{n,\square} \Delta\tau \left(1 - \mathrm{Re}\, U_\square^n\right)$$

FIRST LOOK

The action penalizes loop phase mismatch while preserving exact local gauge symmetry.

Dependency: G04 and a declared lattice normalization.

G05 Working page: Gauge-field action

Derivation or construction

Use $1 - \cos \Phi_\square$ as the compact local curvature energy. Expand for small $\Phi_\square$ to recover a quadratic field-strength term.

Dimensions and normalization

β/a^4 and the spacetime cell measure must combine to action dimensions; conventions differ between Euclidean and real-time formulations.

Decisive calculation

Unit-test gauge invariance and compare small-field dispersion with the intended continuum gauge sector.

READER CHECK

Expand $1 - \cos x = x^2/2 + \mathcal{O}(x^4)$.

Continuum gauge coupling

PROPOSED

Compact equation

$$1 - \mathrm{Re}\, U_{\square} \approx \frac{1}{2} q^2 a^4 F_{\mu\nu} F^{\mu\nu}$$

FIRST LOOK

A measured low-energy coupling comes from the normalization and renormalization of the lattice link sector.

Dependency: G05, matter content, regulator, and continuum limit.

G06 Working page: Continuum gauge coupling

Derivation or construction

Match the expanded plaquette action to $-F^2/(4g^2)$ in a declared convention. Compute correlation functions and run the coupling to the comparison scale.

Dimensions and normalization

g is dimensionless in four spacetime dimensions; bare and renormalized couplings carry scale and scheme labels.

Decisive calculation

Predict coupling flow at a held-out scale after fixing the bare model on calibration observables.

READER CHECK

Explain why inserting the measured α into the bare charge and recovering it later is calibration.

Charge representations

PROPOSED

Compact equation

$$\psi_i \mapsto R(g_i)\psi_i, \qquad U_{ij} \mapsto R(g_i)U_{ij}R(g_j)^{-1}$$

FIRST LOOK

Different pattern types may transform in different representations of the same local symmetry.

Dependency: G03 generalized to a declared group.

G07 Working page: Charge representations

Derivation or construction

Identify stable pattern multiplets and compute how link transport acts on their internal state space. The generators and representation weights define charges.

Dimensions and normalization

Charge labels are dimensionless representation data; physical coupling strength belongs to the action normalization.

Decisive calculation

Derive conserved lattice currents and verify charge addition in scattering states.

READER CHECK

For $U(1)$, show that charges q_1 and q_2 make a product field transform with charge $q_1 + q_2$.

Chiral fermion hurdle

TEST

Compact equation

$$\{\gamma_5, D\} = 0 \quad \text{and} \quad \text{locality, translation symmetry, no doubling}$$

FIRST LOOK

Left- and right-handed matter cannot be placed naively on a regular lattice without extra states or a refined construction.

Dependency: G07 plus a fermionic node operator.

G08 Working page: Chiral fermion hurdle

Derivation or construction

State which hypothesis of the Nielsen–Ninomiya theorem is modified by the construction: operator relation, locality notion, lattice regularity, boundary/domain-wall structure, or another explicit mechanism.

Dimensions and normalization

Fermion fields have mass dimension $3/2$ in four-dimensional continuum normalization.

Decisive calculation

Compute the full low-energy spectrum, chirality, anomaly coefficients, and unwanted mirror modes on refinements.

READER CHECK

State the difference between a Dirac mass coupling left and right components and a purely chiral gauge representation.

Conservative amplitude–phase action

PROPOSED

Compact equation

$$\mathcal{L} = \tfrac{1}{2}(\partial\Phi)^2 + \tfrac{1}{2}f_\theta^2(\Phi)(\partial\theta)^2 - U(\Phi) - \tfrac{\eta}{2}(\Box\Phi)^2$$

FIRST LOOK

Amplitude and phase can share one effective field action, with every sign and derivative order displayed.

Dependency: G01 continuum field plus a stated effective regime.

L01 Working page: Conservative amplitude–phase action

Derivation or construction

Write $\Psi = \Phi e^{i\theta}$. The ordinary kinetic identity $|\partial\Psi|^2 = (\partial\Phi)^2 + \Phi^2(\partial\theta)^2$ motivates $f_\theta(\Phi) = \Phi$ in the simplest case. Higher derivatives are kept as separate effective operators.

Dimensions and normalization

In four dimensions $[\Phi] = [f_\theta] = M$, $[U] = M^4$, and $[\eta] = M^{-2}$ in natural units.

Decisive calculation

Derive the quadratic propagator around each background. Check pole signs, gradient stability, and the scale below which the derivative expansion is valid.

READER CHECK

Expand $|\partial(\Phi e^{i\theta})|^2$ and show the cross terms cancel.

Link phase mismatch

DERIVED

Compact equation

$$|U_{ij}\psi_j - \psi_i|^2 = \rho_i^2 + \rho_j^2 - 2\rho_i\rho_j\cos(\theta_j - \theta_i + qA_{ij})$$

FIRST LOOK

A link stores energy when neighbouring transported phases fail to align.

Dependency: G01–G03.

L02 Working page: Link phase mismatch

Derivation or construction

Insert polar forms and multiply by the complex conjugate. The real part of $\mathrm{e}^{\mathrm{i}(\theta_j-\theta_i+qA_{ij})}$ produces the cosine.

Dimensions and normalization

The squared mismatch has the square of the field amplitude's units. The cosine argument is dimensionless.

Decisive calculation

Compare the expanded small-mismatch form with a phase-stiffness action and measure nonlinear phase slips.

READER CHECK

Set equal amplitudes ρ and small mismatch δ. Use $1-\cos\delta \approx \delta^2/2$.

Positivity and derivative order

TEST

Compact equation

$$K_{AB}\dot{\varphi}^A\dot{\varphi}^B > 0, \qquad \det K \neq 0$$

FIRST LOOK

The kinetic matrix must assign positive energy to ordinary small disturbances in its domain of use.

Dependency: L01 expanded about a specified background.

L03 Working page: Positivity and derivative order

Derivation or construction

Compute the Hessian of the Lagrangian with respect to time derivatives. Diagonalize K_{AB}. Treat higher-time-derivative terms through their declared effective cutoff or a degenerate constrained construction.

Dimensions and normalization

Eigenvalues of K share the normalization needed to make kinetic energy density. Rescale fields only with a recorded Jacobian.

Decisive calculation

Scan backgrounds and momenta below the cutoff for ghost, gradient, and tachyonic instabilities.

READER CHECK

For $K = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, calculate eigenvalues 1 and 3, both positive.

Residual capacity

DEFINED

Compact equation

$$r_i^n = 1 - C_i^n, \qquad 0 \le r_i^n \le 1$$

FIRST LOOK

Residual capacity is the unused fraction of one node's local change budget.

Dependency: F07 local demand.

C01 Working page: Residual capacity

Derivation or construction

Normalize the nonnegative demand by a fixed capacity. Subtract from one. Keep the ledger local; regional residual is a sum over a declared finite region.

Dimensions and normalization

C_i and r_i are dimensionless. A continuum field normalization is a later construction.

Decisive calculation

Record the distribution and correlation length of r_i as load changes.

READER CHECK

If three nodes have $C = (0.2, 0.7, 1.0)$, compute $r = (0.8, 0.3, 0)$.

Causal capacity balance

PROPOSED

Compact equation

$$r_i^{n+1} - r_i^n + \sum_j J_{ij}^n = s_i^n, \qquad J_{ij} = -J_{ji}$$

FIRST LOOK

Residual capacity may move through links or change through a declared local source.

Dependency: C01 and an oriented link-current rule.

C02 Working page: Causal capacity balance

Derivation or construction

Sum the local equations over a region. Antisymmetric internal currents cancel, leaving boundary flux and local sources as in Equation (16.2).

Dimensions and normalization

Every term is residual fraction per update. Multiply by a local capacity and divide by clock step for physical power-like units.

Decisive calculation

Inject a localized source and measure finite-speed spreading, boundary balance, and dependence on update order.

READER CHECK

Show explicitly that $J_{12} + J_{21} = 0$ cancels inside a two-node region.

Coarse EQEF field

PROPOSED

Compact equation

$$\chi_R(x) = Z_R \sum_i K_R(x,i)(r_i - \bar{r})$$

FIRST LOOK

Average local residual fluctuations into a smooth field at resolution R.

Dependency: C01–C02 and a normalized coarse kernel.

C03 Working page: Coarse EQEF field

Derivation or construction

Choose $\sum_i K_R(x,i) = 1$. Subtract the background and select Z_R so the two-point function has a stable continuum normalization.

Dimensions and normalization

r_i is dimensionless; Z_R supplies the continuum field dimension. State it before writing the action.

Decisive calculation

Measure $\langle \chi(x)\chi(y) \rangle$, dispersion, and response to a known capacity source over refinements.

READER CHECK

If a kernel averages four equal weights and residual deviations are $(.2, .1, -.1, -.2)$, show $\chi/Z_R = 0$.

Response and source map

PROPOSED

Compact equation

$$\frac{\delta\langle\chi(x)\rangle}{\delta s(y)} = G_{\chi s}(x, y)$$

FIRST LOOK

A field earns physical meaning through how it responds to a controlled source.

Dependency: C03 plus an intervention that changes local demand.

C04 Working page: Response and source map

Derivation or construction

Perturb $s(y)$ by a small impulse. Average repeated trajectories or spatially equivalent trials. The linear response kernel is the functional derivative of the mean field.

Dimensions and normalization

$G_{\chi s}$ carries the field units divided by source units. Fourier poles define propagation and damping scales.

Decisive calculation

Compare the measured response kernel with the one derived from Equation (16.4).

READER CHECK

For a linear oscillator response $G(\omega) = 1/(m^2 - \omega^2)$, locate its poles.

Torsion from loop nonclosure

PROPOSED

Compact equation

$$\Delta x^{\mu} = T^{\mu}_{\nu\rho} u^{\nu} v^{\rho} + \cdots$$

FIRST LOOK

If two transport orders end at different coarse positions, their small-loop mismatch defines a torsion candidate.

Dependency: F05–F06 geometry and a derived transport law.

C05 Working page: Torsion from loop nonclosure

Derivation or construction

Transport a localized probe around both orders of a small loop. Convert endpoint pattern difference through the coarse position map. Divide by oriented area and take the refinement limit.

Dimensions and normalization

$T^{\mu}_{\nu\rho}$ has inverse-length dimensions in a coordinate basis. The loop mismatch has length.

Decisive calculation

Check tensor transformation, orientation reversal, universality across probes, and scaling with loop area.

READER CHECK

Reverse u and v in Equation (15.4); show the leading mismatch changes sign because torsion is antisymmetric in the last two indices.

Tetrad and connection bridge

PROPOSED

Compact equation

$$T^a = \mathrm{d}e^a + \omega^a{}_b \wedge e^b, \qquad R^a{}_b = \mathrm{d}\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b$$

FIRST LOOK

A tetrad converts local frame readings into spacetime intervals; a connection compares frames at neighbouring points.

Dependency: C05 plus clocks, rods, and orientation observables.

C06 Working page: Tetrad and connection bridge

Derivation or construction

Build local orthonormal response modes, define $e^a{}_\mu$ from their calibrated intervals, and infer ω from transport. Compute torsion and curvature as exterior field strengths.

Dimensions and normalization

$e^a{}_\mu$ depends on coordinate conventions; ω and torsion carry inverse-length scale after physical matching.

Decisive calculation

Reconstruct the same geometry with independent clock and probe species. Quantify non-universal residuals.

READER CHECK

Use $g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$ to show that a local Lorentz rotation leaves g invariant.

Emergent metric

PROPOSED

Compact equation

$$g^{\mu\nu}(x)k_{\mu}k_{\nu} = 0$$

FIRST LOOK

The common wavefront relation of several light sectors defines the effective causal geometry.

Dependency: F03 clocks, F06 distance, and continuum wave modes.

R01 Working page: Emergent metric

Derivation or construction

Fit local dispersion surfaces to a quadratic form in covectors. Normalize with one clock/rod convention and compare the resulting metric across sectors.

Dimensions and normalization

k_μ has inverse-coordinate units; $g^{\mu\nu}$ carries the corresponding conversion so the eikonal equation is homogeneous.

Decisive calculation

Test one common metric against held-out wave directions, polarizations, frequencies, and matter species.

READER CHECK

For flat space, insert $g^{\mu\nu} = \mathrm{diag}(-1/c^2, 1, 1, 1)$ and derive $\omega^2 = c^2k^2$.

Equivalence-principle target

TEST

Compact equation

$$\frac{m_g}{m_i} = 1 + \eta_{\mathrm{comp}}, \qquad \eta_{\mathrm{comp}} \to 0$$

FIRST LOOK

Different stable patterns should fall alike when size, spin, and tidal corrections are controlled.

Dependency: R01 and pattern inertial/gravitational response maps.

R02 Working page: Equivalence-principle target

Derivation or construction

Define inertial mass from acceleration under a calibrated non-gravitational force and gravitational charge from response to the emergent potential. Compare ratios across compositions.

Dimensions and normalization

η_{comp} is dimensionless and is the clean universality diagnostic.

Decisive calculation

Simulate or measure differential acceleration with one frozen graph/capacity model and publish the maximum composition residual.

READER CHECK

If two accelerations are a_1 and a_2, write the Eötvös ratio $2|a_1 - a_2|/(|a_1 + a_2|)$.

Newtonian limit

PROPOSED

Compact equation

$$\nabla^2 \Phi_N = 4\pi G \rho, \qquad g_{00} \approx -(1 + 2\Phi_N / c^2)$$

FIRST LOOK

A weak, slowly varying source should produce the familiar inverse-distance potential.

Dependency: R01–R02 and a coarse source density.

R03 Working page: Newtonian limit

Derivation or construction

Linearize the derived geometric equations about a homogeneous background. Identify the scalar potential in g_{00} and solve the static response to localized density.

Dimensions and normalization

G has dimensions $L^3M^{-1}T^{-2}$. The source and potential units must reproduce $\nabla^2\Phi$.

Decisive calculation

Fit the same G to multiple source shapes and distances, then test a held-out geometry and composition.

READER CHECK

Integrate the spherical vacuum equation and recover $\Phi = -GM/r$.

Einstein-sector matching

PROPOSED

Compact equation

$$S_g = \frac{c^3}{16\pi G}\int R\sqrt{-g}\,\mathrm{d}^4x + S_{\mathrm{corr}}$$

FIRST LOOK

The long-distance geometry needs one universal curvature response plus calculable corrections.

Dependency: C06 and R01–R03.

R04 Working page: Einstein-sector matching

Derivation or construction

Integrate out short graph modes or match long-wave response functions. Identify the coefficient of R, list all symmetry-allowed corrections, and compute their suppression scales.

Dimensions and normalization

c^3/G times $R\,\mathrm{d}^4x$ has action dimensions in SI. Never mix this coefficient with a natural-unit expression without conversion.

Decisive calculation

Recover post-Newtonian observables, wave polarizations and speed, and the same matter coupling from one coefficient set.

READER CHECK

Vary $\sqrt{-g}R$ formally and state the resulting Einstein tensor term $G_{\mu\nu}$.

Homogeneous cosmology

PROPOSED

Compact equation

$$H^2 = \frac{8\pi G}{3}\rho_{\text{total}} - \frac{k}{a_{\text{cos}}^2} + \frac{\Lambda}{3}$$

FIRST LOOK

Average the derived fields on large scales and calculate how the cosmic scale factor evolves.

Dependency: R04 plus derived matter/EQEF stress energy and initial conditions.

R05 Working page: Homogeneous cosmology

Derivation or construction

Insert a homogeneous isotropic ansatz into the effective equations. Derive the continuity equations from covariant conservation rather than choosing density histories independently.

Dimensions and normalization

H has inverse-time units. Each term in the Friedmann equation has T^{-2}. Distinguish lattice spacing a from cosmological scale factor $a_{\cos}$.

Decisive calculation

Fit calibration parameters to a declared subset, then predict held-out distances, expansion rates, and age relations with covariance.

READER CHECK

For constant equation of state $p = w\rho$, derive $\rho \propto a_{\cos}^{-3(1+w)}$.

Cosmological perturbations

TEST

Compact equation

$$\delta\ddot{X}_k + A_k(t)\delta\dot{X}_k + B_k(t)\delta X_k = S_k(t)$$

FIRST LOOK

Small ripples around the cosmic background seed observable structure and radiation patterns.

Dependency: R05 and the complete quadratic effective action.

R06 Working page: Cosmological perturbations

Derivation or construction

Expand every field to background plus perturbation, construct gauge-invariant variables, and derive the coupled mode equations and initial-state prescription.

Dimensions and normalization

Power spectra carry the field normalization; quote the dimensionless spectrum $\Delta^2(k) = k^3 P(k)/(2\pi^2)$ where appropriate.

Decisive calculation

Calculate microwave-background and matter spectra with the same frozen background parameters; compare full covariance, not selected peaks.

READER CHECK

If $P(k) \propto k^{n_s - 4}$ for curvature conventions, compute the scaling of $\Delta^2(k)$.

Ensemble map

PROPOSED

Compact equation

$$P \longmapsto \mu_P(\mathrm{d}\lambda)$$

FIRST LOOK

A repeatable preparation selects a distribution over complete micro-states, even when each state evolves deterministically.

Dependency: F08 and a laboratory preparation protocol.

Q01 Working page: Ensemble map

Derivation or construction

Specify all controlled variables, uncontrolled environmental variables, and how frequencies over repeated preparations define the measure. State whether settings are statistically independent of λ.

Dimensions and normalization

μ_P is normalized: $\int \mu_P = 1$. Densities inherit inverse units of their coordinates.

Decisive calculation

Generate blind repeated preparations and compare predicted sufficient statistics with empirical distributions.

READER CHECK

For a discrete ensemble with weights $(0.2, 0.3, 0.5)$, verify normalization and calculate the probability of the last two states.

Born-rule target

TEST

Compact equation

$$\Pr(x|a,P) = \int \mathbf{1}[A(a,\lambda) = x]\mu_P(\,\mathrm{d}\lambda) = \mathrm{Tr}(\rho_P E_x^{(a)})$$

FIRST LOOK

The micro-model must reproduce the squared-amplitude outcome frequencies for every declared preparation and measurement.

Dependency: Q01 and a detector outcome map.

Q02 Working page: Born-rule target

Derivation or construction

Derive basin measures or amplification-channel frequencies from μ_P. Compare them with the operator probability for phase, amplitude, and entangled preparations.

Dimensions and normalization

Probabilities are dimensionless and sum to one. Detector efficiencies enter once through the response map.

Decisive calculation

Use held-out measurement bases and multi-outcome positive-operator-valued measurements, not only the basis used to tune the ensemble.

READER CHECK

For state amplitudes $(\sqrt{1/3}, \sqrt{2/3})$, calculate Born probabilities $(1/3, 2/3)$.

Bell–CHSH assumptions

TEST

Compact equation

$$S = |E(a,b) + E(a,b') + E(a',b) - E(a',b')|$$

FIRST LOOK

Entangled correlations diagnose the combined assumptions of local response, setting independence, and ordinary probability structure.

Dependency: Q01–Q02 and spacelike separated response maps.

Q03 Working page: Bell–CHSH assumptions

Derivation or construction

Write the joint distribution and identify the factorization or conditional-independence step used to derive $S \leq 2$. A completion states precisely which mathematical step changes.

Dimensions and normalization

S is dimensionless. Setting and outcome conventions must be identical across theory and data.

Decisive calculation

Reproduce the full correlation curve, marginal distributions, setting statistics, and loophole controls with one parameter set.

READER CHECK

Insert the singlet correlation $E(a, b) = -\cos(a - b)$ at optimal angles and obtain $2\sqrt{2}$.

Operational no-signalling

TEST

Compact equation

$$\sum_y P(x,y|a,b) = \sum_y P(x,y|a,b')$$

FIRST LOOK

Remote setting choices may change correlations but not the local outcome frequency available for signalling.

Dependency: Q03 joint outcome distribution.

Q04 Working page: Operational no-signalling

Derivation or construction

Marginalize the joint distribution over the remote outcome. Prove equality for all permitted settings and preparations, including detector imperfections modeled symmetrically.

Dimensions and normalization

Both sides are probabilities. Compare with uncertainty intervals rather than exact finite-sample equality.

Decisive calculation

Run randomized remote settings and test local marginal invariance with a preregistered statistic.

READER CHECK

Given a 2×2 joint table, sum each row and show how to compare the marginal across two remote settings.

Non-circular derivation protocol

DEFINED

Compact equation

$$D = D_{\text{design}} \cup D_{\text{cal}} \cup D_{\text{test}}, \qquad D_{\text{cal}} \cap D_{\text{test}} = \emptyset$$

FIRST LOOK

Use one set of data to choose the model, another to set permitted scales, and a sealed set to test it.

Dependency: The complete model specification and data provenance.

E01 Working page: Non-circular derivation protocol

Derivation or construction

Record which datum selected every equation, prior, parameter, cut, and unit. Freeze the model card and hash before evaluating the test set.

Dimensions and normalization

Unit definitions are not empirical predictions; matched dimensional scales are calibration inputs.

Decisive calculation

Audit the full dependency graph for any path from the held-out target back into model selection or calibration.

READER CHECK

Classify $m = E/c^2$ after inserting measured E and defined c: it is an identity/conversion.

Dimensionless predictions

TEST

Compact equation

$$R = \frac{\Omega_a}{\Omega_b} = \frac{m_a}{m_b}$$

FIRST LOOK

Ratios cancel a common clock or energy scale and are strong early targets for a new model.

Dependency: Two independently identified stable patterns and a shared physical dictionary.

E02 Working page: Dimensionless predictions

Derivation or construction

Compute both gaps from the frozen micro-model, propagate finite-size and refinement uncertainty, and take the ratio before comparing with particle data.

Dimensions and normalization

R is dimensionless. Common multiplicative scale uncertainty cancels, while correlated numerical errors require covariance.

Decisive calculation

Blind the physical labels during the pattern search, then compare the ranked ratio catalogue with held-out targets using a declared multiplicity correction.

READER CHECK

If gaps are 2.40 ± 0.03 and 1.20 ± 0.02 with zero covariance, calculate $R = 2.00$ and propagate its uncertainty.

Uncertainty and covariance

DERIVED

Compact equation

$$V_y = J V_x J^{\mathsf{T}}, \qquad J_{ij} = \partial f_i / \partial x_j$$

FIRST LOOK

Uncertainty follows the slope of the calculation, and shared inputs make outputs move together.

Dependency: A differentiable output map and input covariance.

E03 Working page: Uncertainty and covariance

Derivation or construction

Linearize $f(x)$ about the central inputs: $\delta y \approx J\delta x$. Take the expectation of $\delta y \delta y^{\mathsf{T}}$. Use Monte Carlo or higher-order methods when nonlinearity is material.

Dimensions and normalization

Covariance entries carry products of output units. Correlation coefficients are dimensionless.

Decisive calculation

Validate propagated intervals with synthetic draws from the full input distribution and measure coverage.

READER CHECK

For $y = x^2$, show $u(y) \approx 2|x|u(x)$ at first order.

E04

EVIDENCE

Frozen falsification protocol

TEST

Compact equation

$$\hat{y}_{\text{test}} = F(\mathcal{C}_{\text{frozen}}; X_{\text{test}}), \qquad \mathcal{D} = \text{pass or fail rule}$$

FIRST LOOK

A prediction is made by an immutable calculation and judged by a rule written before the answer is seen.

Dependency: E01–E03 and a baseline model.

E04 Working page: Frozen falsification protocol

Derivation or construction

Archive the model card, software environment, data identifiers, checksums, selection, likelihood, and decision threshold. Evaluate once; later revisions receive a new version and a new hold-out.

Dimensions and normalization

The statistic and threshold have declared normalization and sampling distribution.

Decisive calculation

Independent reproduction obtains the same result from the archive and applies the same decision rule.

READER CHECK

Write a one-sentence failure rule for the exact dispersion test that includes a numerical tolerance and wave-number range.

Atlas concordance

Reader goal	Primary codes	Follow next
Understand the ontology	F01–F08	D01–D08 for an executable benchmark
Reproduce exact mathematics	D01–D05, S01–S07, G01–G05	E03 for numerical uncertainty
Develop the capacity mechanism	F07–F08, D06–D07, C01–C04	C05–C06 and R01
Search for particle-like patterns	S06–S08, G07	E01–E04
Build quantum completion	Q01–Q04	Detector maps in Chapter 19
Build gravity/cosmology	C05–C06, R01–R06	Frozen evidence protocol E01–E04
Prepare an empirical test	E01–E04	Chapters 19 and 23

A

Notation and unit discipline

Symbol	Meaning	Discipline
n	micro-update index	Dimensionless order label, not automatically physical time
t_K	clock time	Reading of persistent clock pattern K
X_n	complete micro-state	Contains every variable required by the update
$G = (V, E, W)$	weighted graph	Primitive only in the benchmark; emergent in the full ontology
a	regular-phase link spacing	Length after metric matching; numerical grid scale before it
$\Delta\tau$	benchmark clock step	Time after a clock map; otherwise update parameter
q_i^n	real node coordinate	Normalized so the action terms have common units
ψ_i	complex node field	Amplitude and compact phase
U_{ij}	link transporter	Dimensionless group element; inverse under orientation reversal
L	weighted graph Laplacian	Positive semidefinite for symmetric nonnegative weights
μ	node inertia	$[\mu q^2/T^2] =$ energy
κ	link stiffness parameter	κ/μ sets squared wave speed in the benchmark convention
Ω_0	onsite angular-frequency scale	Inverse time after clock matching
$E_\star$	local capacity scale	Same energy units as local demand
C_i, r_i	demand and residual fractions	Dimensionless and local
χ	coarse EQEF candidate	Normalization and field dimension derived from coarse response
$e^a{}_\mu$	tetrad	Converts coordinate increments to local frame components
$\omega^a{}_b$	spin connection	Compares local frames under transport
$T^a, R^a{}_b$	torsion and curvature	Field strengths of tetrad/connection geometry

A.1 SI and natural units

In SI, c and h have exact defined numerical values and $\hbar = h/(2\pi)$. In natural units one sets $\hbar = c = 1$ as a unit convention. A natural-unit equation is converted only after restoring powers that reproduce the desired SI dimensions.

For example, a mass m written in electronvolts in natural units means the rest energy mc^2 in SI. A cross section in GeV^{-2} requires the conversion

$$1\ \mathrm{GeV}^{-2} = 0.389379 \times 10^{-27}\ \mathrm{cm}^2. \tag{A.1}$$

DRY NOTE FROM THE FLIGHT DECK

Setting $c = 1$ makes the algebra shorter. It does not make a metre equal to a second in the equipment cupboard.

B

Equation audit workbook

Use this six-line audit for every proposed equation.

1. **Types:** What mathematical kind is each symbol: scalar, vector, operator, probability measure, graph, tensor, or group element?
2. **Dimensions:** Do all added terms share dimensions? Are every logarithm, exponential, sine, and phase argument dimensionless?
3. **Dependencies:** Does the equation use only objects constructed earlier in the hierarchy?
4. **Status:** Is it a definition, derived result, matching relation, measured input, or held-out prediction?
5. **Limits:** What happens for zero coupling, constant field, long wavelength, small amplitude, and fine resolution?
6. **Test:** Which calculation or observation changes confidence in the equation?

B.1 Derivative audit

For a derivative $\partial y/\partial x$, check

$$\left[\frac{\partial y}{\partial x}\right] = \frac{[y]}{[x]}. \tag{B.1}$$

For a functional derivative,

$$\delta S = \int \frac{\delta S}{\delta\phi(x)}\delta\phi(x)\,\mathrm{d}^d x, \tag{B.2}$$

so the measure and field variation determine the units of $\delta S/\delta\phi$.

B.2 Matrix audit

Before diagonalizing, verify symmetry or Hermiticity in the correct inner product. For a generalized eigenproblem

$$Ku = \omega^2 Mu, \tag{B.3}$$

the mass matrix M defines the inner product. Normalize modes by $u_r^{\mathsf{T}} M u_s = \delta_{rs}$.

B.3 Probability audit

Verify positivity and normalization:

$$P(x) \geq 0, \qquad \sum_x P(x) = 1 \quad \text{or} \quad \int p(x)\,\mathrm{d}x = 1. \tag{B.4}$$

Conditioning variables must be printed. The expressions $P(x)$, $P(x|a)$, and $P(x|a,P)$ are different claims.

C

Problems

The problems progress from direct substitution to research-level design. Answers follow in the next appendix.

Foundations

P1. A clock advances 0.25 rad per update. Calculate updates per cycle.

P2. A persistent pattern has internal period 20 updates and lifetime 2,000 updates. Calculate the number of survived cycles.

P3. If $P(\sigma) = 5\sigma^{-2}$, calculate spectral dimension.

P4. On a graph with edge lengths $\ell_{ab} = 1$, $\ell_{bc} = 2$, and $\ell_{ac} = 4$, find the shortest distance from a to c.

P5. Local demands are $(0.4, 0.9, 1.0)$ of capacity. Calculate all residuals and the regional residual sum.

Graph dynamics

P6. Construct D, W, and L for a two-node graph of weight w.

P7. Find both Laplacian eigenvalues of that graph.

P8. For an isolated free node, solve the discrete update from $q^0 = 1$, $q^1 = 1.2$ for q^2, q^3.

P9. On a periodic chain, evaluate $\lambda(k)$ at $ka = 0, \pi/2, \pi$.

P10. For $d = 2$, $a = c = 1$, and $\Omega_0 = 0$, find the largest stable $\Delta\tau$.

Waves and nonlinearity

P11. With $a = c = 1$, $\Delta\tau = 0.5$, $\Omega_0 = 0$, and $k = \pi/2$, calculate ω from the exact one-dimensional dispersion relation.

P12. Expand $4\sin^2(ka/2)/a^2$ through order a^2.

P13. For $\Omega_0 = 3$, $\mu = 2$, $\lambda = 0.4$, and $A = 0.5$, estimate the Duffing frequency.

P14. A normalized mode is equal on four nodes and zero elsewhere. Calculate its participation ratio.

P15. Two matched gaps are $\omega_a = 5$ and $\omega_b = 2$. Calculate the mass ratio.

Gauge structure

P16. Show $D_{ij}\psi$ transforms with the phase at node i.

P17. For link phases $(0.2, 0.3, -0.1, -0.4)$ rad around a plaquette, calculate the plaquette phase.

P18. Expand $1 - \cos\Phi$ through fourth order.

P19. If two $U(1)$ fields have charges 2 and -1, determine the charge of their product.

P20. Explain in two sentences why a Feynman diagram requires a derived vertex coefficient.

Constants and evidence

P21. Show dimensionally that c^5/G is power.

P22. Compute the spherical area in units of ℓ_P^2 for radius $2\ell_P$.

P23. Use Equation (17.18) to find how the cross section changes if g doubles.

P24. If $y = x^3$, derive the linearized relative uncertainty $u(y)/|y|$ in terms of $u(x)/|x|$.

P25. A model uses a measured mass to choose a frequency scale and then returns that same mass. Classify the result.

P26. Write a three-way split of a 1,000-event dataset with 200 design events, 300 calibration events, and the remainder held out.

P27. For independent residuals $r = (1, -2, 1)$ with unit variances, calculate $\chi^2 = r^\mathsf{T} r$.

P28. A measured phase difference is 2.4 rad across 6 m. Find k.

P29. Sampling occurs every 0.002 s. Find the Nyquist frequency.

P30. State one numerical failure rule for a gauge-invariance unit test.

Research design

P31. Design a blind search for localized modes that avoids particle-mass tuning.

P32. Name the four maps needed to turn a deterministic micro-state into quantum outcome frequencies.

P33. State the CHSH local bound and the quantum maximum.

P34. Give one operational no-signalling equality.

P35. List the outputs required before a capacity field can be used in a cosmological likelihood.

D

Solutions

S1. $2\pi/0.25 = 8\pi \approx 25.13$ updates; a full crossing occurs between updates 25 and 26.
S2. $2000/20 = 100$ cycles.
S3. $\ln P = \ln 5 - 2\ln\sigma$, so $d_s = -2(-2) = 4$.
S4. The path through b costs $1+2=3$, less than direct cost 4; hence $d_G(a,c)=3$.
S5. $r = (0.6, 0.1, 0)$ and the regional sum is 0.7.
S6. $D = \mathrm{diag}(w, w)$, $W = \left(\begin{smallmatrix} 0 & w \\ w & 0 \end{smallmatrix}\right)$, and $\mathsf{L} = \left(\begin{smallmatrix} w & -w \\ -w & w \end{smallmatrix}\right)$.
S7. The constant vector has eigenvalue 0; the antisymmetric vector has eigenvalue $2w$.
S8. Free motion has constant increment 0.2: $q^2 = 1.4$, $q^3 = 1.6$.
S9. Equation (7.10) gives 0, 2, and 4.
S10. Equation (10.3) gives $\Delta\tau^2(8) \le 4$, so $\Delta\tau \le 1/\sqrt{2}$.
S11. The right side is $4\sin^2(\pi/4) = 2$. Thus $16\sin^2(\omega/4) = 2$, giving $\omega = 4\arcsin(1/\sqrt{8}) \approx 1.445$.
S12. $4\sin^2(ka/2)/a^2 = k^2 - a^2k^4/12 + \mathcal{O}(a^4k^6)$.
S13. The correction is $3\lambda A^2/(8\mu\Omega_0^2) = 0.0020833$, so $\Omega \approx 3.00625$.
S14. Equal support on four nodes gives $P = 4$.
S15. $m_a/m_b = \omega_a/\omega_b = 2.5$.
S16. Direct substitution gives $D'_{ij}\psi' = \mathrm{e}^{\mathrm{i}q\alpha_i}D_{ij}\psi$.
S17. The oriented phase sum is $0.2 + 0.3 - 0.1 - 0.4 = 0$ rad, so $U_\square = 1$.
S18. $1 - \cos\Phi = \Phi^2/2 - \Phi^4/24 + \mathcal{O}(\Phi^6)$.
S19. Charges add: $2 + (-1) = 1$.
S20. The diagram is a term in an amplitude expansion. Its line types and vertex factor come from a normalized effective action, so without the coefficient it supplies no rate.
S21. $[c^5/G] = (L^5T^{-5})/(L^3M^{-1}T^{-2}) = ML^2T^{-3}$, the dimension of energy per time.
S22. $A = 4\pi(2\ell_P)^2 = 16\pi\ell_P^2$.
S23. $\sigma \propto g^2$, so doubling g multiplies σ by four.
S24. $u(y) \approx |3x^2|u(x)$, hence $u(y)/|y| \approx 3u(x)/|x|$.
S25. Calibration followed by identity/conversion, not a held-out prediction.
S26. $D_{\text{design}} = 200$, $D_{\text{cal}} = 300$, and $D_{\text{test}} = 500$ events, with disjoint identifiers.
S27. $\chi^2 = 1^2 + (-2)^2 + 1^2 = 6$.
S28. $k = 2.4/6 = 0.4\,\mathrm{rad.m}^{-1}$.
S29. $f_N = 1/(2\times 0.002) = 250\,\mathrm{Hz}$.
S30. Example: fail if the relative change in the gauge-invariant link action exceeds 10^{-12} for any of 10,000 seeded random transformations.
S31. Fix the graph family, parameters, objective, localization statistic, continuation method, and stability threshold before viewing particle data; publish every stable dimensionless gap with multiplicity accounted for.
S32. Preparation $P \mapsto \mu_P$, deterministic evolution U, coarse state/observable map, and apparatus outcome response $A(a,\lambda)$.
S33. Local CHSH bound 2; quantum Tsirelson maximum $2\sqrt{2}$.
S34. $\sum_y P(x,y|a,b) = \sum_y P(x,y|a,b')$ for every x, a, b, b'.
S35. A derived coarse field normalization; action coefficients; source and response map; background solution; perturbation equations; initial conditions; calibration policy; and covariance-aware observable likelihood.

E

Equation and claim ledger

Equation	Object	Status	Primary dependency
(1.1)	Ordered history	DEFINED	Relational state space
(1.5)	Operational time	PROPOSED	Stable recurrent pattern
(2.1)	Pattern identity	DEFINED	Coarse map, tolerance, symmetry
(3.3)	Local capacity	PROPOSED	Nonnegative local demand
(8.1)	Scalar graph action	DEFINED	Fixed benchmark graph
(8.4)	Benchmark field equation	DERIVED	Variation of the action
(9.5)	Exact lattice spectrum	DERIVED	Plane-wave eigenmodes
(10.3)	Linear stability domain	DERIVED	Maximum lattice eigenvalue
(11.6)	Nonlinear frequency shift	DERIVED	Weak Duffing approximation
(12.8)	Link-action invariance	DERIVED	Local gauge transformation
(13.7)	Continuum operator target	PROPOSED	Coarse matching
(14.3)	Quantum probability target	MEASURED INPUT	Established quantum formalism
(14.5)	Bell–CHSH bound	TEST	Local measurement-independent model
(15.3)	Torsion geometry	DEFINED	Tetrad and connection
(16.1)	EQEF balance candidate	PROPOSED	Residual capacity current
(17.12)	Stefan–Boltzmann constant	DERIVED	Quantum radiation law and SI inputs
(21.4)	Collider counting likelihood	DEFINED	Poisson model and nuisance constraints

F

Author revision protocol

This protocol lets new ideas enter the book without breaking the dependency chain.

F.1 One-page proposal card

For every revision, complete:

1. **Name and version.**
2. **Layer:** foundation, benchmark, spectrum, gauge, capacity, quantum, gravity, cosmology, or evidence.
3. **Replaced object:** exact equation, definition, or map.
4. **New object:** full types, domains, units, parameters, boundary conditions, and tie-breaks.
5. **Reason:** inconsistency repaired or new explanatory role.
6. **Dependencies affected:** atlas codes and equation numbers.
7. **Checks:** algebra, dimensions, limits, symmetry, stability, conservation, code tests.
8. **Empirical policy:** design data, calibration data, held-out data, likelihood, and failure rule.

F.2 Promotion rules

- A metaphor becomes a DEFINED object when its mathematical type and measurement rule are explicit.
- A proposed equation becomes DERIVED only when it follows from earlier displayed assumptions.
- A model coefficient becomes matched when the calibration datum and uncertainty are recorded.
- A result becomes a prediction only after the relevant card is frozen and the comparison target is held out.

F.3 Consistency sweep

After a foundational change, rerun in order: notation/types, dimensions, deterministic closure, conservation, linear spectrum, stability, gauge tests, continuum scaling, pattern search, observable mapping, covariance, and held-out likelihood. Increment the edition number and retain the prior edition as provenance.

G

Glossary

Admissible set
States or state pairs satisfying every local constraint.

Benchmark
A deliberately limited model in which exact calculations can be performed.

Brillouin zone
The distinct wave-vector domain represented by a periodic lattice.

Calibration
Use of data to select a parameter, scale, or dictionary.

Capacity
The fixed maximum of a declared local transition demand.

Coarse-graining
A map that retains resolved large-scale information while averaging microscopic detail.

Covariance
A measure of how uncertainties in two quantities move together.

Dispersion relation
The relation between frequency and wave vector.

Effective field theory
An operator expansion organized by symmetry and scale.

Emergence
Construction of a stable higher-level description from lower-level dynamics.

EQEF
In this book, the ledger and possible coarse field of residual local transition capacity.

Gauge invariance
Independence of physical observables from local internal reference choices.

Graph Laplacian
The operator comparing each node value with its weighted neighbours.

Group velocity
The derivative $\partial\omega/\partial k$ governing a narrow packet's envelope motion.

Held-out test
Data not used to choose equations, cuts, priors, parameters, or units.

Holonomy
Net transporter obtained around a closed loop.

Identity
A relation true by definition or algebraic rearrangement.

Matching
Equating coefficients in two descriptions at a declared scale and convention.

Node
A relational degree of freedom in the benchmark or a persistent class in the emergent ontology.

No-signalling
Independence of a local observable marginal from a remote setting choice.

Operational clock
A robust recurrent pattern used to label duration.

Pattern
A persistent equivalence class under allowed transport, phase change, or relabeling.

Plaquette
An elementary oriented loop used to encode lattice curvature.

Prediction
A held-out observable calculated after the model and calibration are frozen.

Residual capacity
The fraction $1 - C_i$ remaining at a node after local demand is evaluated.

Spectral dimension
Scale-dependent dimension inferred from diffusion return probability.

Torsion
Geometric field strength describing infinitesimal transport nonclosure.

Update order
The primitive succession label n, distinguished from physical clock time.

H

Annotated bibliography and intellectual lineage

End Theory sits beside several established traditions: relational accounts of space and time, discrete dynamics, lattice gauge theory, nonlinear waves, effective field theory, quantum foundations, gravitation with and without torsion, cosmology, metrology, statistical inference, and complex-systems science. The references below credit the people and collaborations whose work defines those neighbouring subjects. Inclusion records relevance; it does not imply endorsement of End Theory.

Bibliography

Classical mechanics, space, time, and relativity

[1] I. Newton, *Philosophiæ Naturalis Principia Mathematica* (1687). The classical synthesis of motion and universal gravitation.

[2] G. W. Leibniz and S. Clarke, *The Leibniz–Clarke Correspondence* (1715–1716). A foundational exchange over relational and absolute accounts of space and time.

[3] J.-L. Lagrange, *Mécanique analytique* (1788). Systematic variational mechanics.

[4] W. R. Hamilton, "On a general method in dynamics," *Philosophical Transactions of the Royal Society* **124**, 247–308 (1834).

[5] E. Mach, *The Science of Mechanics* (1883). Influential critique of absolute space and inertia.

[6] A. Einstein, "On the electrodynamics of moving bodies," *Annalen der Physik* **17**, 891–921 (1905).

[7] H. Minkowski, "Space and time," address to the 80th Assembly of German Natural Scientists and Physicians (1908).

[8] A. Einstein, "The foundation of the general theory of relativity," *Annalen der Physik* **49**, 769–822 (1916).

[9] E. Noether, "Invariant variation problems," *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen*, 235–257 (1918). The symmetry–conservation theorem.

[10] H. Weyl, "Gravitation and electricity," *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 465–480 (1918). Early gauge-geometric unification.

[11] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, 1923).

[12] J. A. Wheeler, "Information, physics, quantum: the search for links," in *Complexity, Entropy, and the Physics of Information* (Addison-Wesley, 1990). The "it from bit" programme.

[13] J. Barbour, *The End of Time* (Oxford University Press, 1999). A modern timeless/relational perspective.

[14] C. Rovelli, *Quantum Gravity* (Cambridge University Press, 2004). Relational observables and loop quantum gravity.

Discrete geometry, graphs, and emergent spacetime

[15] T. Regge, "General relativity without coordinates," *Nuovo Cimento* **19**, 558–571 (1961), doi:10.1007/BF02733251.

[16] R. Penrose, "Angular momentum: an approach to combinatorial space-time," in *Quantum Theory and Beyond* (Cambridge University Press, 1971). Spin-network origins.

[17] L. Bombelli, J. Lee, D. Meyer, and R. D. Sorkin, "Space-time as a causal set," *Physical Review Letters* **59**, 521–524 (1987), doi:10.1103/PhysRevLett.59.521.

[18] G. Brightwell and R. Gregory, "Structure of random discrete spacetime," *Physical Review Letters* **66**, 260–263 (1991).

[19] A. Ashtekar, “New variables for classical and quantum gravity,” *Physical Review Letters* **57**, 2244–2247 (1986).

[20] C. Rovelli and L. Smolin, “Discreteness of area and volume in quantum gravity,” *Nuclear Physics B* **442**, 593–622 (1995).

[21] J. Ambjørn, J. Jurkiewicz, and R. Loll, “Dynamically triangulating Lorentzian quantum gravity,” *Nuclear Physics B* **610**, 347–382 (2001).

[22] S. Carlip, “Dimension and dimensional reduction in quantum gravity,” *Classical and Quantum Gravity* **34**, 193001 (2017).

[23] F. R. K. Chung, *Spectral Graph Theory* (American Mathematical Society, 1997).

[24] B. Mohar, “The Laplacian spectrum of graphs,” in *Graph Theory, Combinatorics, and Applications* (Wiley, 1991).

[25] M. Kac, “Can one hear the shape of a drum?” *American Mathematical Monthly* **73**, 1–23 (1966). Spectra as geometric probes.

[26] R. R. Coifman and S. Lafon, “Diffusion maps,” *Applied and Computational Harmonic Analysis* **21**, 5–30 (2006).

[27] M. Belkin and P. Niyogi, “Laplacian eigenmaps for dimensionality reduction and data representation,” *Neural Computation* **15**, 1373–1396 (2003).

[28] E. H. Lieb and D. W. Robinson, “The finite group velocity of quantum spin systems,” *Communications in Mathematical Physics* **28**, 251–257 (1972), doi:10.1007/BF01645779.

Numerical dynamics and lattice field theory

[29] R. Courant, K. Friedrichs, and H. Lewy, “On the partial difference equations of mathematical physics,” *Mathematische Annalen* **100**, 32–74 (1928). The CFL stability condition.

[30] J. von Neumann and R. D. Richtmyer, “A method for the numerical calculation of hydrodynamic shocks,” *Journal of Applied Physics* **21**, 232–237 (1950).

[31] C. Størmer, “Sur les trajectoires des corpuscules electrises,” *Archives des Sciences Physiques et Naturelles* **24**, 5–18 (1907). Early centred integration associated with the Størmer method.

[32] L. Verlet, “Computer experiments on classical fluids. I. Thermodynamical properties of Lennard-Jones molecules,” *Physical Review* **159**, 98–103 (1967).

[33] K. G. Wilson, “Confinement of quarks,” *Physical Review D* **10**, 2445–2459 (1974), doi:10.1103/PhysRevD.10.2445.

[34] J. Kogut and L. Susskind, “Hamiltonian formulation of Wilson’s lattice gauge theories,” *Physical Review D* **11**, 395–408 (1975).

[35] J. B. Kogut, “An introduction to lattice gauge theory and spin systems,” *Reviews of Modern Physics* **51**, 659–713 (1979).

[36] M. Creutz, *Quarks, Gluons and Lattices* (Cambridge University Press, 1983).

[37] H. J. Rothe, *Lattice Gauge Theories: An Introduction*, 4th ed. (World Scientific, 2012).

[38] I. Montvay and G. Münster, *Quantum Fields on a Lattice* (Cambridge University Press, 1994).

[39] H. B. Nielsen and M. Ninomiya, "A no-go theorem for regularizing chiral fermions," *Physics Letters B* **105**, 219–223 (1981), doi:10.1016/0370-2693(81)91026-1.

[40] P. H. Ginsparg and K. G. Wilson, "A remnant of chiral symmetry on the lattice," *Physical Review D* **25**, 2649–2657 (1982).

[41] D. B. Kaplan, "A method for simulating chiral fermions on the lattice," *Physics Letters B* **288**, 342–347 (1992).

[42] H. Neuberger, "Exactly massless quarks on the lattice," *Physics Letters B* **417**, 141–144 (1998).

Nonlinear waves, solitons, and localization

[43] G. Duffing, *Erzwungene Schwingungen bei veränderlicher Eigenfrequenz* (Vieweg, 1918).

[44] E. Fermi, J. Pasta, S. Ulam, and M. Tsingou, "Studies of nonlinear problems," Los Alamos report LA-1940 (1955).

[45] D. J. Korteweg and G. de Vries, "On the change of form of long waves advancing in a rectangular canal," *Philosophical Magazine* **39**, 422–443 (1895).

[46] N. J. Zabusky and M. D. Kruskal, "Interaction of solitons in a collisionless plasma and the recurrence of initial states," *Physical Review Letters* **15**, 240–243 (1965).

[47] A. C. Scott, F. Y. F. Chu, and D. W. McLaughlin, "The soliton: a new concept in applied science," *Proceedings of the IEEE* **61**, 1443–1483 (1973).

[48] S. Aubry, "Breathers in nonlinear lattices: existence, linear stability and quantization," *Physica D* **103**, 201–250 (1997).

[49] S. Flach and C. R. Willis, "Discrete breathers," *Physics Reports* **295**, 181–264 (1998).

[50] R. S. MacKay and S. Aubry, "Proof of existence of breathers for time-reversible or Hamiltonian networks of weakly coupled oscillators," *Nonlinearity* **7**, 1623–1643 (1994).

[51] P. W. Anderson, "Absence of diffusion in certain random lattices," *Physical Review* **109**, 1492–1505 (1958).

Quantum theory, gauge fields, and effective theories

[52] M. Planck, "On the law of distribution of energy in the normal spectrum," *Annalen der Physik* **4**, 553–563 (1901).

[53] M. Born, "Zur Quantenmechanik der Stoßvorgänge," *Zeitschrift für Physik* **37**, 863–867 (1926). Statistical interpretation of the wavefunction.

[54] W. Heisenberg, "Quantum-theoretical reinterpretation of kinematic and mechanical relations," *Zeitschrift für Physik* **33**, 879–893 (1925).

[55] E. Schrödinger, "Quantization as an eigenvalue problem," *Annalen der Physik* **79**, 361–376 (1926).

[56] P. A. M. Dirac, "The quantum theory of the electron," *Proceedings of the Royal Society A* **117**, 610–624 (1928).

[57] S. Tomonaga, "On a relativistically invariant formulation of the quantum theory of wave fields," *Progress of Theoretical Physics* **1**, 27–42 (1946).

[58] J. Schwinger, "Quantum electrodynamics. I. A covariant formulation," *Physical Review* **74**, 1439–1461 (1948).

[59] R. P. Feynman, "Space-time approach to quantum electrodynamics," *Physical Review* **76**, 769–789 (1949).

[60] F. J. Dyson, "The radiation theories of Tomonaga, Schwinger, and Feynman," *Physical Review* **75**, 486–502 (1949).

[61] C. N. Yang and R. L. Mills, "Conservation of isotopic spin and isotopic gauge invariance," *Physical Review* **96**, 191–195 (1954).

[62] Y. Nambu and G. Jona-Lasinio, "Dynamical model of elementary particles based on an analogy with superconductivity," *Physical Review* **122**, 345–358 (1961).

[63] J. Goldstone, "Field theories with superconductor solutions," *Nuovo Cimento* **19**, 154–164 (1961).

[64] P. W. Higgs, "Broken symmetries and the masses of gauge bosons," *Physical Review Letters* **13**, 508–509 (1964).

[65] F. Englert and R. Brout, "Broken symmetry and the mass of gauge vector mesons," *Physical Review Letters* **13**, 321–323 (1964).

[66] S. L. Glashow, "Partial-symmetries of weak interactions," *Nuclear Physics* **22**, 579–588 (1961).

[67] S. Weinberg, "A model of leptons," *Physical Review Letters* **19**, 1264–1266 (1967).

[68] A. Salam, "Weak and electromagnetic interactions," in *Elementary Particle Theory* (Almqvist and Wiksell, 1968).

[69] D. J. Gross and F. Wilczek, "Ultraviolet behavior of non-Abelian gauge theories," *Physical Review Letters* **30**, 1343–1346 (1973).

[70] H. D. Politzer, "Reliable perturbative results for strong interactions?" *Physical Review Letters* **30**, 1346–1349 (1973).

[71] G. 't Hooft, "Renormalizable Lagrangians for massive Yang–Mills fields," *Nuclear Physics B* **35**, 167–188 (1971).

[72] K. G. Wilson, "Renormalization group and critical phenomena. I," *Physical Review B* **4**, 3174–3183 (1971).

[73] S. Weinberg, "Phenomenological Lagrangians," *Physica A* **96**, 327–340 (1979), doi:10.1016/0378-4371(79)90223-1.

[74] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, 1995).

[75] A. Zee, *Quantum Field Theory in a Nutshell*, 2nd ed. (Princeton University Press, 2010).

[76] R. P. Woodard, "The theorem of Ostrogradsky," *Scholarpedia* **10**, 32243 (2015), doi:10.4249/scholarpedia.32243.

Quantum foundations and measurement

[77] D. Bohm, "A suggested interpretation of the quantum theory in terms of hidden variables," *Physical Review* **85**, 166–193 (1952).

[78] H. Everett III, "Relative state formulation of quantum mechanics," *Reviews of Modern Physics* **29**, 454–462 (1957).

[79] J. S. Bell, "On the Einstein Podolsky Rosen paradox," *Physics* **1**, 195–200 (1964), doi:10.1103/PhysicsPhysiqueFizika.1.195.

[80] S. Kochen and E. P. Specker, "The problem of hidden variables in quantum mechanics," *Journal of Mathematics and Mechanics* **17**, 59–87 (1967).

[81] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, "Proposed experiment to test local hidden-variable theories," *Physical Review Letters* **23**, 880–884 (1969).

[82] A. M. Gleason, "Measures on the closed subspaces of a Hilbert space," *Journal of Mathematics and Mechanics* **6**, 885–893 (1957).

[83] B. S. Cirel'son, "Quantum generalizations of Bell's inequality," *Letters in Mathematical Physics* **4**, 93–100 (1980).

[84] A. Aspect, J. Dalibard, and G. Roger, "Experimental test of Bell's inequalities using time-varying analyzers," *Physical Review Letters* **49**, 1804–1807 (1982).

[85] W. H. Zurek, "Decoherence, einselection, and the quantum origins of the classical," *Reviews of Modern Physics* **75**, 715–775 (2003).

[86] B. Hensen *et al.*, "Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres," *Nature* **526**, 682–686 (2015), doi:10.1038/nature15759.

[87] M. Giustina *et al.*, "Significant-loophole-free test of Bell's theorem with entangled photons," *Physical Review Letters* **115**, 250401 (2015).

[88] L. K. Shalm *et al.*, "Strong loophole-free test of local realism," *Physical Review Letters* **115**, 250402 (2015).

[89] H. M. Wiseman, "The two Bell's theorems of John Bell," *Journal of Physics A* **47**, 424001 (2014).

Gravitation, torsion, black holes, and holography

[90] D. Hilbert, "The foundations of physics," *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen*, 395–407 (1915).

[91] E. Cartan, "On a generalization of the notion of Riemann curvature and spaces with torsion," *Comptes Rendus de l'Académie des Sciences* **174**, 593–595 (1922).

[92] D. W. Sciama, "The physical structure of general relativity," *Reviews of Modern Physics* **36**, 463–469 (1964).

[93] T. W. B. Kibble, "Lorentz invariance and the gravitational field," *Journal of Mathematical Physics* **2**, 212–221 (1961).

[94] F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, "General relativity with spin and torsion," *Reviews of Modern Physics* **48**, 393–416 (1976), doi:10.1103/RevModPhys.48.393.

[95] R. M. Wald, *General Relativity* (University of Chicago Press, 1984).

[96] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, 1973).

[97] S. M. Carroll, *Spacetime and Geometry* (Addison-Wesley, 2004).

[98] K. Schwarzschild, "On the gravitational field of a mass point according to Einstein's theory," *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 189–196 (1916).

[99] R. Penrose, "Gravitational collapse and space-time singularities," *Physical Review Letters* **14**, 57–59 (1965).

[100] J. D. Bekenstein, "Black holes and entropy," *Physical Review D* **7**, 2333–2346 (1973).

[101] S. W. Hawking, "Particle creation by black holes," *Communications in Mathematical Physics* **43**, 199–220 (1975).

[102] G. 't Hooft, "Dimensional reduction in quantum gravity," arXiv:gr-qc/9310026 (1993).

[103] L. Susskind, "The world as a hologram," *Journal of Mathematical Physics* **36**, 6377–6396 (1995).

[104] J. Maldacena, "The large-N limit of superconformal field theories and supergravity," *Advances in Theoretical and Mathematical Physics* **2**, 231–252 (1998).

Cosmology and gravitational waves

[105] A. Friedmann, "On the curvature of space," *Zeitschrift für Physik* **10**, 377–386 (1922).

[106] G. Lemaître, "A homogeneous universe of constant mass and increasing radius," *Annales de la Société Scientifique de Bruxelles* **47**, 49–59 (1927).

[107] E. Hubble, "A relation between distance and radial velocity among extra-galactic nebulae," *Proceedings of the National Academy of Sciences* **15**, 168–173 (1929).

[108] H. P. Robertson, "Kinematics and world-structure," *Astrophysical Journal* **82**, 284–301 (1935).

[109] A. G. Walker, "On Milne's theory of world-structure," *Proceedings of the London Mathematical Society* **42**, 90–127 (1937).

[110] A. G. Riess *et al.*, "Observational evidence from supernovae for an accelerating universe and a cosmological constant," *Astronomical Journal* **116**, 1009–1038 (1998).

[111] S. Perlmutter *et al.*, "Measurements of Ω and Λ from 42 high-redshift supernovae," *Astrophysical Journal* **517**, 565–586 (1999).

[112] Planck Collaboration, "Planck 2018 results. VI. Cosmological parameters," *Astronomy & Astrophysics* **641**, A6 (2020).

[113] B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration), "Observation of gravitational waves from a binary black hole merger," *Physical Review Letters* **116**, 061102 (2016).

[114] B. P. Abbott *et al.*, "Multi-messenger observations of a binary neutron star merger," *Astrophysical Journal Letters* **848**, L12 (2017).

[115] S. Weinberg, *Cosmology* (Oxford University Press, 2008).

[116] P. J. E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, 1993).

Particle data, collider measurements, and open data

[117] F. Takahashi *et al.* (Particle Data Group), "Review of Particle Physics," *International Journal of Modern Physics A* **41**, 2630011 (2026).

[118] ATLAS Collaboration, "Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC," *Physics Letters B* **716**, 1–29 (2012).

[119] CMS Collaboration, "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC," *Physics Letters B* **716**, 30–61 (2012).

[120] ATLAS Collaboration, "ATLAS 13 TeV samples collection: at least four leptons," CERN Open Data record 15005 (2020), doi:10.7483/OPENDATA.ATLAS.2Y1T.TLGL.

[121] ATLAS Collaboration, "ATLAS 13 TeV samples collection: gamma–gamma," CERN Open Data record 15006 (2020).

[122] CERN, *CERN Open Data Portal*, opendata.cern.ch. Public datasets, software environments, and educational resources.

[123] E. Maguire, L. Heinrich, and G. Watt, "HEPData: a repository for high energy physics data," *Journal of Physics: Conference Series* **898**, 102006 (2017).

[124] G. Cowan, K. Cranmer, E. Gross, and O. Vitells, "Asymptotic formulae for likelihood-based tests of new physics," *European Physical Journal C* **71**, 1554 (2011).

Constants, thermodynamics, and metrology

[125] L. Boltzmann, "On the relation between the second law of the mechanical theory of heat and probability calculations," *Wiener Berichte* **76**, 373–435 (1877).

[126] J. Stefan, "On the relation between thermal radiation and temperature," *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften* **79**, 391–428 (1879).

[127] W. Wien, "A new relation of the radiation of black bodies to the second law of thermodynamics," *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 55–62 (1893).

[128] N. Bohr, "On the constitution of atoms and molecules," *Philosophical Magazine* **26**, 1–25 (1913).

[129] P. J. Mohr, D. B. Newell, B. N. Taylor, and E. Tiesinga, "CODATA recommended values of the fundamental physical constants: 2022," *Journal of Physical and Chemical Reference Data* **54**, 033105 (2025), doi:10.1063/5.0279860.

[130] Bureau International des Poids et Mesures, *The International System of Units (SI Brochure)*, 9th ed., version 4.01 (2025), doi:10.59161/AUEZ1291.

[131] National Institute of Standards and Technology, *Fundamental Physical Constants: 2022 CODATA Adjustment*, Standard Reference Database 121.

Inference, reproducibility, and model criticism

[132] T. Bayes, "An essay towards solving a problem in the doctrine of chances," *Philosophical Transactions of the Royal Society* **53**, 370–418 (1763).

[133] R. A. Fisher, "On the mathematical foundations of theoretical statistics," *Philosophical Transactions of the Royal Society A* **222**, 309–368 (1922).

[134] J. Neyman and E. S. Pearson, "On the problem of the most efficient tests of statistical hypotheses," *Philosophical Transactions of the Royal Society A* **231**, 289–337 (1933).

[135] S. S. Wilks, "The large-sample distribution of the likelihood ratio," *Annals of Mathematical Statistics* **9**, 60–62 (1938).

[136] H. Akaike, "A new look at the statistical model identification," *IEEE Transactions on Automatic Control* **19**, 716–723 (1974).

[137] G. E. P. Box, "Science and statistics," *Journal of the American Statistical Association* **71**, 791–799 (1976).

[138] R. D. Peng, "Reproducible research in computational science," *Science* **334**, 1226–1227 (2011).

[139] M. D. Wilkinson *et al.*, "The FAIR guiding principles for scientific data management and stewardship," *Scientific Data* **3**, 160018 (2016).

[140] B. A. Nosek *et al.*, "Promoting an open research culture," *Science* **348**, 1422–1425 (2015).

Information, feedback, synchronization, and adaptive systems

[141] C. E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal* **27**, 379–423 and 623–656 (1948).

[142] N. Wiener, *Cybernetics: Or Control and Communication in the Animal and the Machine* (MIT Press, 1948).

[143] W. R. Ashby, *An Introduction to Cybernetics* (Chapman & Hall, 1956).

[144] A. M. Turing, "The chemical basis of morphogenesis," *Philosophical Transactions of the Royal Society B* **237**, 37–72 (1952).

[145] I. Prigogine, *Self-Organization in Nonequilibrium Systems* (Wiley, 1977).

[146] Y. Kuramoto, "Self-entrainment of a population of coupled nonlinear oscillators," in *International Symposium on Mathematical Problems in Theoretical Physics* (Springer, 1975).

[147] J. J. Hopfield, "Neural networks and physical systems with emergent collective computational abilities," *Proceedings of the National Academy of Sciences* **79**, 2554–2558 (1982).

[148] G. Tononi, "An information integration theory of consciousness," *BMC Neuroscience* **5**, 42 (2004).

[149] K. Friston, "The free-energy principle: a unified brain theory?" *Nature Reviews Neuroscience* **11**, 127–138 (2010).

[150] A. K. Seth and T. Bayne, "Theories of consciousness," *Nature Reviews Neuroscience* **23**, 439–452 (2022). A comparative review of operational proposals.

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Closing perspective

End Theory begins with a modest object: an ordered local change. Its ambition lies in the succession of constructions that follow. Repeated change must make clocks. Persistence must make identity. Stable influence must make neighbourhood. Long waves must make fields. Invariant loops must make gauge information. Local capacity must make a distinctive nonlinear response. A detector map must finally turn all of that mathematics into a number that nature can accept or reject.

The framework earns strength one explicit bridge at a time. The reader now has the equations, worked checks, status labels, computational tests, and research gates needed to follow those bridges in order.

Part 1. Relational foundations

This part contains 10 canonical dossiers. Equations have one home, dependencies are explicit, and later uses add consequences rather than repeat definitions.

F01 — Primitive update order

DEFINED

Canonical statement

$n \in \mathbb{Z}, H = \left(X^0, X^1, \ldots \right)$

> **FIRST LOOK.** The integer orders states but is not yet a clock reading.

Dependency. root state specification.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Operational-time and discrete-dynamics literature; Noether (1918); Marsden and West (2001).

F01 working page — Primitive update order

Construction or derivation

Specify the admissible state set and one update relation before importing metres, seconds, particles, or background coordinates.

Dimensions and normalization

Primitive labels are dimensionless until a separately constructed rod, clock, or detector calibration supplies physical units.

Decisive calculation

Construct two recurrent subsystems and test whether their long-run rate ratio is stable under changes of initial phase.

Adversarial edge case

A schedule-dependent update can create different histories from the same state.

> **FAILURE RULE.** Retire the pre-geometric claim if computing the next state requires an undeclared spacetime distance.

Atlas code F01. Return to Part 1: Relational foundations.

F02 — Microscopic state domain

DEFINED

Canonical statement

$$X^n = \left(G^n, q^n, p^n, U^n, r^n, \zeta^n\right) \in X$$

FIRST LOOK. A state is complete only when every quantity needed by the update is stored in it.

Dependency. F01.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Reproducible-computation standards and discrete state-space methods.

F02 working page — Microscopic state domain

Construction or derivation

Assign a domain, orientation convention, boundary rule, and precision policy to every component; delete symbols unused by the frozen update.

Dimensions and normalization

Primitive labels are dimensionless until a separately constructed rod, clock, or detector calibration supplies physical units.

Decisive calculation

Serialize one state, reload it in an independent implementation, and reproduce the next update within a published tolerance.

Adversarial edge case

An unrecorded random seed or implicit-solver cache is hidden state.

> **FAILURE RULE.** A result is not reproducible if the next state depends on information absent from the parameter and state cards.

Atlas code F02. Return to Part 1: Relational foundations.

F03 — Bounded relational locality

PROPOSED

Canonical statement

$$\left[U_\theta(X)\right]_i = f_i\left(X|_{N_R(i;G)}\right)$$

> **FIRST LOOK.** Locality is a property of the executable dependency radius, not of the prose used to describe it.

Dependency. F02 and a graph metric.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Bell (1964); Lieb-Robinson bounds as a comparison class.

F03 working page — Bounded relational locality

Construction or derivation

Trace every read operation performed while updating node i, including constraint solvers and graph rewiring, and bound its radius R.

Dimensions and normalization

Primitive labels are dimensionless until a separately constructed rod, clock, or detector calibration supplies physical units.

Decisive calculation

Flip one input outside the declared neighborhood and verify bitwise invariance of the local output.

Adversarial edge case

A globally normalized demand or matrix inversion violates bounded update locality unless implemented through local substeps.

> **FAILURE RULE.** Withdraw the local-causality label when a remote state changes an output in fewer than the declared transport steps.

Atlas code F03. Return to Part 1: Relational foundations.

F04 — Persistent-pattern equivalence

DEFINED

Canonical statement

$$P_n \sim_R P_m \Leftrightarrow d_R(\Pi_R P_n, \Pi_R P_m) \le \varepsilon_R$$

> **FIRST LOOK.** Objects are equivalence classes of patterns that survive controlled perturbation and coarse-graining.

Dependency. F02 and a coarse map.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Dynamical-systems stability and soliton literature.

F04 working page — Persistent-pattern equivalence

Construction or derivation

Freeze the feature map, symmetry group, metric, and tolerance before searching for long-lived candidates.

Dimensions and normalization

Primitive labels are dimensionless until a separately constructed rod, clock, or detector calibration supplies physical units.

Decisive calculation

Measure first-exit-time distributions after phase shifts, translations, collisions, and graph refinement.

Adversarial edge case

A candidate may be stable only because the tolerance exceeds the separation between distinct states.

> **FAILURE RULE.** Reject particle language if no lifetime plateau or reproducible scattering label survives a reasonable tolerance interval.

Atlas code F04. Return to Part 1: Relational foundations.

F05 — Operational clock construction

PROPOSED

Canonical statement

$$N_C(n)=\frac{unwrap\,\vartheta_C(X^n)}{2\pi},\,t_C=T_C N_C$$

> **FIRST LOOK.** A clock is a robust phase-bearing subsystem; its reading is not the update index.

Dependency. F04 recurrent pattern.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Metrology and nonlinear-oscillator literature.

F05 working page — Operational clock construction

Construction or derivation

Choose a complex or angular observable, locate a stable cycle, compute the monodromy spectrum, and define phase crossings.

Dimensions and normalization

Primitive labels are dimensionless until a separately constructed rod, clock, or detector calibration supplies physical units.

Decisive calculation

Build inequivalent clocks and test constancy of rate ratios over load, amplitude, graph size, and boundary conditions.

Adversarial edge case

Amplitude-dependent frequency can make two clocks disagree even in the same update history.

> **FAILURE RULE.** Do not assign physical duration until at least two clocks exhibit a shared stable scaling regime.

Atlas code F05. Return to Part 1: Relational foundations.

F06 — Influence-defined adjacency

PROPOSED

Canonical statement

$$w_{ab} = T^{-1} \sum_{n=1}^{T} I_{ab}(n), G_R = (V_R, E_R, W_R)$$

FIRST LOOK. Effective neighbors are inferred from reproducible response, not drawn in advance.

Dependency. F04 and an intervention protocol.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Spectral graph theory and causal-intervention methodology.

F06 working page — Influence-defined adjacency

Construction or derivation

Apply standardized local interventions, estimate directed responses with uncertainty, and threshold using a preregistered false-link rate.

Dimensions and normalization

Primitive labels are dimensionless until a separately constructed rod, clock, or detector calibration supplies physical units.

Decisive calculation

Predict responses to held-out probes and compare directed, reciprocal, and symmetrized graph models.

Adversarial edge case

Common drivers can create apparent links without direct influence.

> **FAILURE RULE.** Retire an inferred edge when its response fails replication or disappears under intervention controls.

Atlas code F06. Return to Part 1: Relational foundations.

F07 — Graph distance from route cost

CONDITIONAL

Canonical statement

$$d_G(a,b) = \min_{\gamma : a \to b} \sum_{e \in \gamma} \ell_e$$

FIRST LOOK. Distance becomes meaningful after a stable positive cost is assigned to each operational link.

Dependency. F06 and positive edge costs.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Metric geometry and network-distance literature.

F07 working page — Graph distance from route cost

Construction or derivation

Define the cost from exchange time, attenuation, or another reproducible response and prove positivity and path composition.

Dimensions and normalization

Primitive labels are dimensionless until a separately constructed rod, clock, or detector calibration supplies physical units.

Decisive calculation

Compare route distance with independent radar timing and require stable ratios across graph refinement.

Adversarial edge case

Zero or negative costs destroy ordinary metric properties; directed costs produce a quasi-metric.

> **FAILURE RULE.** Do not call d_G physical distance if its ordering changes arbitrarily under admissible probe choices.

Atlas code F07. Return to Part 1: Relational foundations.

F08 — Scale-dependent spectral dimension

DERIVED

Canonical statement

$$d_s(\sigma) = -2\frac{d\ln P(\sigma)}{d\ln\sigma}$$

FIRST LOOK. Dimension is diagnosed by how a diffusion process returns across scale.

Dependency. F06 and graph diffusion.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Ambjorn, Jurkiewicz, and Loll (2005); Vassilevich (2003).

F08 working page — Scale-dependent spectral dimension

Construction or derivation

Construct the heat kernel from the declared Laplacian, average diagonal return probability, smooth only by a frozen rule, and differentiate its log slope.

Dimensions and normalization

Primitive labels are dimensionless until a separately constructed rod, clock, or detector calibration supplies physical units.

Decisive calculation

Seek a plateau shared by graph ensembles, boundary treatments, and at least one independent dimension estimator.

Adversarial edge case

Finite graphs force late-time return saturation and can mimic a falling dimension.

> **FAILURE RULE.** A single point or narrow shoulder is not evidence for an emergent dimension.

Atlas code F08. Return to Part 1: Relational foundations.

F09 — Observable completion map

DEFINED

Canonical statement

$O = D \circ C \circ E \circ P$

> **FIRST LOOK.** A physical claim ends in a detector-level record, not in a microscopic picture.

Dependency. F02-F08.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Statistical decision theory and detector-response methodology.

F09 working page — Observable completion map

Construction or derivation

Specify preparation, evolution, coarse-graining, and detector maps with all stochastic and nuisance variables exposed.

Dimensions and normalization

Primitive labels are dimensionless until a separately constructed rod, clock, or detector calibration supplies physical units.

Decisive calculation

Generate synthetic microstates and verify that two independent pipelines produce the same observable distribution.

Adversarial edge case

A many-to-one coarse map can erase the feature claimed to distinguish the theory.

> **FAILURE RULE.** A claim remains CLOSURE until the complete composition reaches a measurable statistic.

Atlas code F09. Return to Part 1: Relational foundations.

F10 — Structural identifiability

TEST

Canonical statement

$$rank\left(\partial O_a / \partial \theta_b\right) = dim\,\theta$$

FIRST LOOK. Parameters are learnable only when observables respond in linearly independent directions.

Dependency. F09 and a parameter card.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Fisher-information and structural-identifiability literature.

F10 working page — Structural identifiability

Construction or derivation

Evaluate the sensitivity or Fisher matrix over the allowed domain and identify exact or approximate null combinations.

Dimensions and normalization

Primitive labels are dimensionless until a separately constructed rod, clock, or detector calibration supplies physical units.

Decisive calculation

Profile each parameter and verify that held-out observables break the same degeneracies predicted by sensitivity analysis.

Adversarial edge case

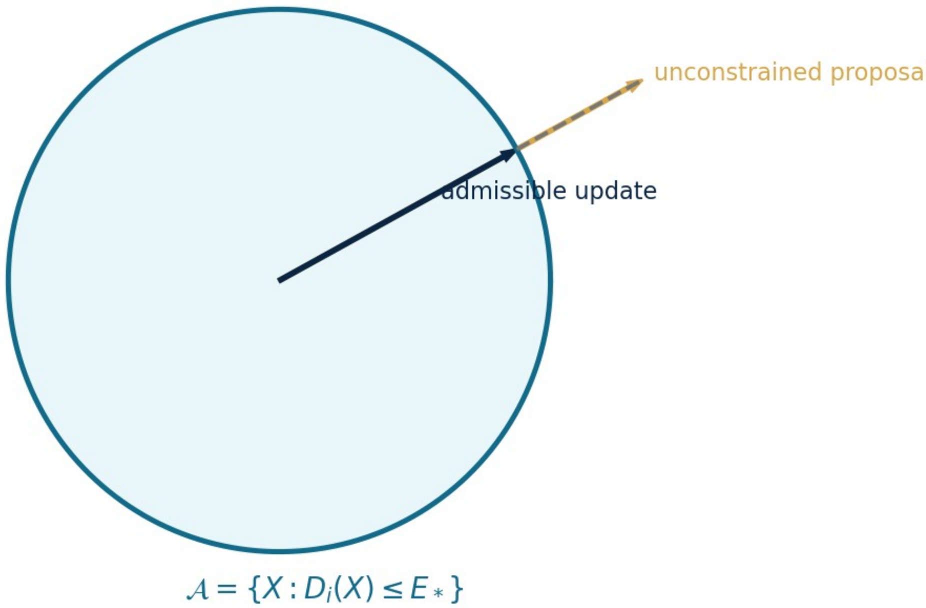
Discrete label changes can imitate continuous parameter shifts.

> **FAILURE RULE.** Do not call a fitted microscopic parameter measured when only a degenerate combination is constrained.

Atlas code F10. *Return to Part 1: Relational foundations.*

Part 2. Capacity as a mathematical constraint

This part contains 10 canonical dossiers. Equations have one home, dependencies are explicit, and later uses add consequences rather than repeat definitions.



Technical orientation figure for Part 2

C01 — Demand functional

PROPOSED

Canonical statement

$D_i(X;\theta) \geq 0, C_i = D_i / E_*$

FIRST LOOK. Capacity begins with a computable nonnegative demand, not a metaphor.

Dependency. F02.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Convex analysis and constrained-mechanics literature.

C01 working page — Demand functional

Construction or derivation

Choose commensurate local contributions from field increments, link strain, rewiring, or reservoir exchange and freeze their coefficients.

Dimensions and normalization

Every term in a capacity comparison must be normalized to the same demand scale; ratios and active-set tests are dimensionless.

Decisive calculation

Evaluate demand on vacuum, uniform motion, a single-node impulse, and a saturated collision.

Adversarial edge case

Cancellation inside a signed demand can hide arbitrarily large local activity.

> **FAILURE RULE.** Reject the capacity law if negative demand or unit mismatch occurs in any admissible state.

Atlas code C01. Return to Part 2: Capacity as a mathematical constraint.

C02 — Hard admissible set

DEFINED

Canonical statement

$$A = \{ X : C_i(X) \leq 1 \; \forall i \}$$

FIRST LOOK. A hard capacity is a state-space boundary that every accepted update must respect.

Dependency. C01.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Rockafellar (1970); Karush-Kuhn-Tucker theory.

C02 working page — Hard admissible set

Construction or derivation

Prove that the set is nonempty and characterize closure, convexity, and connected components for the selected demand.

Dimensions and normalization

Every term in a capacity comparison must be normalized to the same demand scale; ratios and active-set tests are dimensionless.

Decisive calculation

Sample boundary neighborhoods and verify that the proposed update maps admissible inputs back into the set.

Adversarial edge case

Nonconvex components can make a nearest-state prescription multivalued.

> **FAILURE RULE.** One observed capacity violation falsifies closure of the stated hard-constraint update.

Atlas code C02. Return to Part 2: Capacity as a mathematical constraint.

C03 — Weighted metric projection

PROPOSED

Canonical statement

$$X^{n+1} = \arg\min_{X \in A} 1/2 \parallel X - Y^{n+1} \parallel_W^2$$

FIRST LOOK. Projection selects the closest legal state according to one frozen geometry.

Dependency. C02 and positive-definite W.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Convex projection and numerical-optimization literature.

C03 working page — Weighted metric projection

Construction or derivation

Define W, solve the constrained problem, and publish deterministic tolerances and tie-breaking when uniqueness is absent.

Dimensions and normalization

Every term in a capacity comparison must be normalized to the same demand scale; ratios and active-set tests are dimensionless.

Decisive calculation

Compare analytic optimality residuals with an independent optimizer on adversarial near-boundary states.

Adversarial edge case

A fitted metric can encode the desired phenomenology directly.

> **FAILURE RULE.** Projection is not a physical law until W and the admissible set are fixed independently of target data.

Atlas code C03. Return to Part 2: Capacity as a mathematical constraint.

C04 — KKT active-set system

DERIVED

Canonical statement

$$W(X-Y)+\sum_i \lambda_i \nabla g_i = 0, \lambda_i g_i = 0$$

> **FIRST LOOK.** Active multipliers identify which local boundaries redirect a proposal.

Dependency. C03 and a constraint qualification.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Karush (1939); Kuhn and Tucker (1951).

C04 working page — KKT active-set system

Construction or derivation

Derive stationarity, primal feasibility, dual feasibility, and complementarity; state the qualification guaranteeing necessity.

Dimensions and normalization

Every term in a capacity comparison must be normalized to the same demand scale; ratios and active-set tests are dimensionless.

Decisive calculation

Solve small cases symbolically and verify all residuals before trusting a large numerical active set.

Adversarial edge case

Dependent gradients make multipliers nonunique even when the projected state is unique.

> **FAILURE RULE.** Do not interpret multipliers as forces when constraint scaling or the projection metric changes their values.

Atlas code C04. Return to Part 2: Capacity as a mathematical constraint.

C05 — Nonexpansive convex projection

DERIVED

Canonical statement

$$\| P_A x - P_A y \|_W \leq \| x - y \|_W$$

> **FIRST LOOK.** Convex metric projection cannot amplify separation in its own norm.

Dependency. C03 with closed convex A.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Hilbert-space projection theorems.

C05 working page — Nonexpansive convex projection

Construction or derivation

Use the two projection variational inequalities, add them, and apply Cauchy-Schwarz.

Dimensions and normalization

Every term in a capacity comparison must be normalized to the same demand scale; ratios and active-set tests are dimensionless.

Decisive calculation

Draw random pairs, project them independently, and measure the maximum Lipschitz ratio.

Adversarial edge case

Nonconvex admissible sets can produce discontinuous jumps between components.

> **FAILURE RULE.** Nonexpansiveness does not imply energy conservation, symplecticity, or causal transport.

Atlas code C05. Return to Part 2: Capacity as a mathematical constraint.

C06 — Residual-capacity current

PROPOSED

Canonical statement

$$r_i^{n+1} - r_i^n + \Delta n \sum_j J_{ij}^{n+1/2} = \Delta n\, s_i^{n+1/2}$$

> **FIRST LOOK.** Residual capacity becomes a field only when it obeys an explicit local balance.

Dependency. C01 and oriented links.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Discrete continuity equations and finite-volume methods.

C06 working page — Residual-capacity current

Construction or derivation

Define antisymmetric link currents, local sources, boundary flux, and coupling to every reservoir variable.

Dimensions and normalization

Every term in a capacity comparison must be normalized to the same demand scale; ratios and active-set tests are dimensionless.

Decisive calculation

Sum over a closed graph and verify that internal currents cancel to machine precision.

Adversarial edge case

A symmetric current convention double-counts transport and breaks global balance.

> **FAILURE RULE.** Do not call unused capacity conserved when the summed residual changes without a declared source.

Atlas code C06. Return to Part 2: Capacity as a mathematical constraint.

C07 — Explicit reservoir balance

PROPOSED

Canonical statement

$\Delta E_{field} + \Delta E_{res} + \Phi_{\partial G} = 0$

> **FIRST LOOK.** Apparent loss must enter a stored reservoir or a declared boundary flux.

Dependency. C06 and an energy map.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Discrete mechanics and conservation-law numerics.

C07 working page — Explicit reservoir balance

Construction or derivation

Add reservoir variables to the state, give them local updates, and derive the discrete balance by summing node and link equations.

Dimensions and normalization

Every term in a capacity comparison must be normalized to the same demand scale; ratios and active-set tests are dimensionless.

Decisive calculation

Run closed, periodic, and open boundaries and verify the appropriate global ledger.

Adversarial edge case

A hidden thermostat can stabilize simulations while silently violating isolated dynamics.

> **FAILURE RULE.** Unexplained secular drift retires exact-conservation language and must be labeled controlled or uncontrolled dissipation.

Atlas code C07. Return to Part 2: Capacity as a mathematical constraint.

C08 — Local proposal limiter

PROPOSED

Canonical statement

$$q_i^{n+1} = q_i^n + \eta_i \left(\tilde{q}_i^{n+1} - q_i^n \right), 0 \le \eta_i \le 1$$

FIRST LOOK. A limiter shortens a proposed move without changing its local direction.

Dependency. C01-C02.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Flux-limiter and projected-dynamics comparisons.

C08 working page — Local proposal limiter

Construction or derivation

Solve the local capacity inequality for the maximal admissible factor and specify iteration order when neighboring constraints couple.

Dimensions and normalization

Every term in a capacity comparison must be normalized to the same demand scale; ratios and active-set tests are dimensionless.

Decisive calculation

Map limiter activation, conservation residuals, and front speed as load crosses the threshold.

Adversarial edge case

Asynchronous updates can make the result depend on node enumeration.

> **FAILURE RULE.** Reject determinism if different legal evaluation orders produce observably different final states.

Atlas code C08. Return to Part 2: Capacity as a mathematical constraint.

C09 — Simultaneous saturation

CLOSURE

Canonical statement

$I(X)=\{i : g_i(X)=0\}$

> **FIRST LOOK.** Multiple active nodes turn capacity handling into a coupled geometric problem.

Dependency. C04.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. Complementarity problems and active-set methods.

C09 working page — Simultaneous saturation

Construction or derivation

Enumerate compatible active sets, test gradient rank, and define a local algorithm that reaches the same solution without a global oracle.

Dimensions and normalization

Every term in a capacity comparison must be normalized to the same demand scale; ratios and active-set tests are dimensionless.

Decisive calculation

Construct symmetric collisions with several saturated nodes and perturb them to expose branch instability.

Adversarial edge case

Incompatible constraints can leave the admissible set locally empty.

> **FAILURE RULE.** The theory is incomplete until every reachable active-set pattern has a defined next state or a declared termination event.

Atlas code C09. Return to Part 2: Capacity as a mathematical constraint.

C10 — Capacity-induced dissipation

CONDITIONAL

Canonical statement

$\Delta H^n = H^{n+1/2} - H^{n-1/2} \leq 0$

> **FIRST LOOK.** A capacity correction may dissipate a benchmark invariant, but the sign must be proved or measured.

Dependency. C03 or C08 plus an energy.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Lyapunov methods and geometric numerical integration.

C10 working page — Capacity-induced dissipation

Construction or derivation

Insert the corrected update into the discrete energy difference and isolate projection work, reservoir exchange, and truncation error.

Dimensions and normalization

Every term in a capacity comparison must be normalized to the same demand scale; ratios and active-set tests are dimensionless.

Decisive calculation

Sweep step size and precision to distinguish physical dissipation from numerical drift.

Adversarial edge case

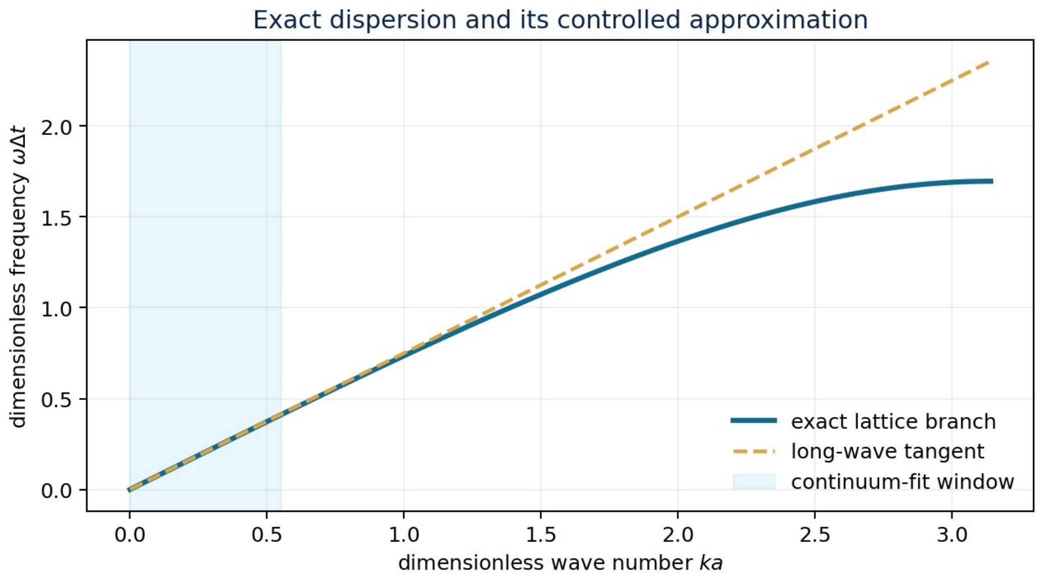
Some projections inject energy when the energy metric differs from W.

> **FAILURE RULE.** Do not infer an arrow of time unless a monotone quantity is established beyond selected examples.

Atlas code C10. Return to Part 2: Capacity as a mathematical constraint.

Part 3. Exact scalar-graph benchmark

This part contains 10 canonical dossiers. Equations have one home, dependencies are explicit, and later uses add consequences rather than repeat definitions.



Technical orientation figure for Part 3

B01 — Incidence factorization

DERIVED

Canonical statement

$$L = B W B^T$$

> **FIRST LOOK.** Orientation enters the incidence matrix but cancels from the Laplacian.

Dependency. fixed undirected weighted graph.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Chung, Spectral Graph Theory (1997).

B01 working page — Incidence factorization

Construction or derivation

Assign one orientation per edge, form B and diagonal W, multiply, and recover degree minus adjacency.

Dimensions and normalization

The benchmark keeps graph weights dimensionless while mass, spacing, time step, stiffness, and fields carry the displayed physical dimensions.

Decisive calculation

Reverse random edge orientations and verify that L is unchanged.

Adversarial edge case

Negative weights destroy the immediate positivity argument.

> **FAILURE RULE.** The factorization is a graph identity, not evidence that nature selects this graph.

Atlas code B01. Return to Part 3: Exact scalar-graph benchmark.

B02 — Laplacian positivity and kernel

DERIVED

Canonical statement

$$x^T L x = \sum_{(i,j)} w_{ij} (x_i - x_j)^2 \geq 0$$

FIRST LOOK. Link strain is nonnegative and constant on each connected component.

Dependency. B01 with nonnegative weights.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Chung (1997); standard spectral-graph results.

B02 working page — Laplacian positivity and kernel

Construction or derivation

Expand the quadratic form and show that every edge contributes one square; characterize equality by path connectivity.

Dimensions and normalization

The benchmark keeps graph weights dimensionless while mass, spacing, time step, stiffness, and fields carry the displayed physical dimensions.

Decisive calculation

Count zero eigenvalues and compare with the number of connected components.

Adversarial edge case

Nearly disconnected graphs produce small eigenvalues that can be confused with exact zero modes.

> **FAILURE RULE.** A negative eigenvalue falsifies symmetry, nonnegative weights, or implementation correctness.

Atlas code B02. Return to Part 3: Exact scalar-graph benchmark.

B03 — Central-difference action

DEFINED

Canonical statement

$$S_d = \sum_n \Delta t \left[1/2 \left(\delta_t q^n \right)^T M \left(\delta_t q^n \right) - 1/2 \left(q^n \right)^T K q^n - V \left(q^n \right) \right)$$

FIRST LOOK. The benchmark balances temporal change, link strain, mass, and onsite interaction.

Dependency. B01 and a fixed-graph interval.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Marsden and West (2001).

B03 working page — Central-difference action

Construction or derivation

State endpoint conditions, time placement of each term, mass-matrix positivity, and whether the graph is fixed during variation.

Dimensions and normalization

The benchmark keeps graph weights dimensionless while mass, spacing, time step, stiffness, and fields carry the displayed physical dimensions.

Decisive calculation

Evaluate the action for a constant configuration, one-node impulse, and one normal mode.

Adversarial edge case

Topology change inside the variation interval creates omitted derivatives of B, W, and the vertex set.

> **FAILURE RULE.** All derived benchmark equations are restricted to the declared fixed-graph phase.

Atlas code B03. Return to Part 3: Exact scalar-graph benchmark.

B04 — Discrete Euler-Lagrange update

DERIVED

Canonical statement

$$M\,\frac{q^{n+1}-2\,q^{n}+q^{n-1}}{\Delta t^{2}}+K\,q^{n}+\nabla V\left(q^{n}\right)=0$$

FIRST LOOK. Two neighboring kinetic cells and the present potential determine the update.

Dependency. B03.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Discrete variational mechanics.

B04 working page — Discrete Euler-Lagrange update

Construction or derivation

Vary q^n with fixed endpoints, collect the n-1 and n kinetic contributions, and solve for q^{n+1}.

Dimensions and normalization

The benchmark keeps graph weights dimensionless while mass, spacing, time step, stiffness, and fields carry the displayed physical dimensions.

Decisive calculation

Compare the analytic action gradient with centered finite differences on random small graphs.

Adversarial edge case

A singular mass matrix leaves some accelerations undetermined and introduces constraints.

> **FAILURE RULE.** Call the recursion deterministic only after every nonlinear branch and constraint is single-valued.

Atlas code B04. *Return to Part 3: Exact scalar-graph benchmark.*

B05 — Generalized normal modes

DERIVED

Canonical statement

$$K\,\phi_\alpha = \Omega_\alpha^2 M\,\phi_\alpha,\ \phi_\alpha^T M\,\phi_\beta = \delta_{\alpha\beta}$$

> **FIRST LOOK.** Positive mass geometry diagonalizes linear benchmark motion into independent modes.

Dependency. B04 linear sector.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Generalized symmetric eigenvalue theory.

B05 working page — Generalized normal modes

Construction or derivation

Whiten with M^{-1/2}, diagonalize the symmetric operator, and transform back to M-orthonormal eigenvectors.

Dimensions and normalization

The benchmark keeps graph weights dimensionless while mass, spacing, time step, stiffness, and fields carry the displayed physical dimensions.

Decisive calculation

Reconstruct random trajectories from modal amplitudes and compare with direct matrix evolution.

Adversarial edge case

Degenerate eigenspaces permit basis rotations that can confuse mode identity across parameter scans.

> **FAILURE RULE.** Observables must be invariant under basis changes inside an exactly degenerate subspace.

Atlas code B05. Return to Part 3: Exact scalar-graph benchmark.

B06 — Exact lattice dispersion

DERIVED

Canonical statement

$$\frac{4\mu}{\Delta t^2}\sin^2\frac{\omega\Delta t}{2}=m_0^2+\frac{4\kappa}{a^2}\sum_r \sin^2\frac{k_r a}{2}$$

FIRST LOOK. Finite spacing predicts a trigonometric dispersion law, not an exact continuum cone.

Dependency. B04 on a periodic hypercubic graph.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Standard lattice-wave analysis.

B06 working page — Exact lattice dispersion

Construction or derivation

Insert a plane wave into the update and simplify temporal and spatial second differences.

Dimensions and normalization

The benchmark keeps graph weights dimensionless while mass, spacing, time step, stiffness, and fields carry the displayed physical dimensions.

Decisive calculation

Estimate omega from simulated phase advance and compare every Fourier mode with the closed form.

Adversarial edge case

Near the Nyquist edge, branch structure and vanishing group velocity invalidate a linear approximation.

> **FAILURE RULE.** Do not apply this scalar result to photons, gravitons, or matter without separate sector derivations.

Atlas code B06. Return to Part 3: Exact scalar-graph benchmark.

B07 — Long-wave correction

DERIVED

Canonical statement

$$\mu\omega^2 = m_0^2 + \kappa|k)^2 - \frac{\kappa a^2}{12}\sum_r k_r^4 + \frac{\mu\Delta t^2}{12}\omega^4 + \cdots$$

FIRST LOOK. Leading spacing effects expose anisotropy and temporal-discretization error.

Dependency. B06.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Effective-operator and lattice-error analysis.

B07 working page — Long-wave correction

Construction or derivation

Expand each sine squared through sixth order and keep a declared joint scaling of a and Delta t.

Dimensions and normalization

The benchmark keeps graph weights dimensionless while mass, spacing, time step, stiffness, and fields carry the displayed physical dimensions.

Decisive calculation

Fit simulated residuals against a^2 and Delta t^2 and recover the analytic coefficients.

Adversarial edge case

Choosing only axial momenta can hide anisotropic fourth-order corrections.

> **FAILURE RULE.** A continuum claim must include off-axis modes and a residual panel, not only the lowest momentum.

Atlas code B07. Return to Part 3: Exact scalar-graph benchmark.

B08 — Linear stability bound

DERIVED

Canonical statement

$$0 \le \Delta t^2 \lambda_\alpha \left(M^{-1} K \right) \le 4$$

FIRST LOOK. Central differences remain oscillatory only inside a mode-by-mode step-size interval.

Dependency. B04 linear sector.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Von Neumann stability analysis.

B08 working page — Linear stability bound

Construction or derivation

Insert an amplification factor, solve the quadratic characteristic equation, and require both roots on the unit circle.

Dimensions and normalization

The benchmark keeps graph weights dimensionless while mass, spacing, time step, stiffness, and fields carry the displayed physical dimensions.

Decisive calculation

Cross the boundary in controlled increments and compare exponential growth with the analytic root.

Adversarial edge case

Roundoff can excite the unstable highest mode even when initial data omit it.

> **FAILURE RULE.** All numerical evidence must publish the stability margin and maximum constraint residual.

Atlas code B08. Return to Part 3: Exact scalar-graph benchmark.

B09 — Exact half-step invariant

DERIVED

Canonical statement

$$H^{n+1/2}=1/2 \parallel \delta_t q^n \parallel_M^2 + 1/2\left(q^{n+1}\right)^T K q^n$$

FIRST LOOK. The linear recursion conserves a discrete invariant distinct from continuum energy.

Dependency. B04 linear symmetric sector.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Geometric numerical integration.

B09 working page — Exact half-step invariant

Construction or derivation

Subtract consecutive half-step expressions, factor the central update, and use symmetry of M and K.

Dimensions and normalization

The benchmark keeps graph weights dimensionless while mass, spacing, time step, stiffness, and fields carry the displayed physical dimensions.

Decisive calculation

Track relative drift over long runs and verify precision rather than step-size scaling.

Adversarial edge case

The bilinear potential term need not be positive outside the stability regime.

> **FAILURE RULE.** Do not advertise conservation after adding nonlinear limiters unless a replacement ledger is derived.

Atlas code B09. Return to Part 3: Exact scalar-graph benchmark.

B10 — Weak nonlinear frequency shift

DERIVED

Canonical statement

$$\omega(A)=\omega_0\left[1+\frac{3\gamma A^2}{8\omega_0^2}+O\left(A^4\right)\right)$$

FIRST LOOK. A quartic interaction makes clock rate and wave frequency amplitude dependent.

Dependency. single Duffing mode.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Nonlinear oscillation theory.

B10 working page — Weak nonlinear frequency shift

Construction or derivation

Apply a Poincare-Lindstedt rescaling, remove resonant first-order forcing, and solve for the frequency correction.

Dimensions and normalization

The benchmark keeps graph weights dimensionless while mass, spacing, time step, stiffness, and fields carry the displayed physical dimensions.

Decisive calculation

Fit frequency versus A squared at small amplitude and verify the predicted slope.

Adversarial edge case

Mode coupling on a graph invalidates the isolated-mode approximation near resonances.

> **FAILURE RULE.** A frequency shift alone neither proves localized particles nor specifies their stability.

Atlas code B10. Return to Part 3: Exact scalar-graph benchmark.

Part 4. Graph evolution and continuum control

This part contains 10 canonical dossiers. Equations have one home, dependencies are explicit, and later uses add consequences rather than repeat definitions.

E01 — Graph evolution in the state

DEFINED

Canonical statement

$$G^{n+1} = \Gamma_\theta\left(G^n, q^n, p^n, r^n\right)$$

FIRST LOOK. Topology change is dynamics only when its rule is explicit and state-complete.

Dependency. F02.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Dynamic-graph and graph-rewriting literature.

E01 working page — Graph evolution in the state

Construction or derivation

Specify legal vertex and edge operations, locality radius, weight update, tie-breaks, and conservation bookkeeping.

Dimensions and normalization

Regulator powers, renormalization conditions, and graph-family comparisons must be expressed through fixed dimensionless physical ratios.

Decisive calculation

Replay stored states and verify identical topology changes across implementations.

Adversarial edge case

Relabeling-sensitive rules encode an unphysical node ordering.

> **FAILURE RULE.** Reject a graph law whose output changes under an allowed vertex permutation.

Atlas code E01. Return to Part 4: Graph evolution and continuum control.

E02 — Birth and death rules

PROPOSED

Canonical statement

$\Delta|V) = N_{birth} - N_{death}$

FIRST LOOK. Changing vertex number requires rules for inherited and discarded quantities.

Dependency. E01.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Conservative remeshing and graph-rewriting methods.

E02 working page — Birth and death rules

Construction or derivation

Define trigger surfaces, newborn-state initialization, link creation, removal transfer, and simultaneous-event ordering.

Dimensions and normalization

Regulator powers, renormalization conditions, and graph-family comparisons must be expressed through fixed dimensionless physical ratios.

Decisive calculation

Test isolated birth, isolated death, and coincident events with permutation-related inputs.

Adversarial edge case

Deleting a loaded node can erase energy or charge.

> **FAILURE RULE.** No topology-changing run is admissible until removed content has a declared destination.

Atlas code E02. Return to Part 4: Graph evolution and continuum control.

E03 — Local reconnection

PROPOSED

Canonical statement

$$(i,j),(k,\ell) \mapsto (i,k),(j,\ell)$$

> **FIRST LOOK.** Rewiring may alter emergent geometry while preserving selected local totals.

Dependency. E01 and a local motif.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Network dynamics and local Pachner-move comparisons.

E03 working page — Local reconnection

Construction or derivation

Enumerate candidate reconnections inside a bounded motif and select with a permutation-invariant local score.

Dimensions and normalization

Regulator powers, renormalization conditions, and graph-family comparisons must be expressed through fixed dimensionless physical ratios.

Decisive calculation

Measure component count, degree distribution, spectral gap, and conserved quantities before and after each event.

Adversarial edge case

Greedy reconnection can generate hidden long-range links in the emergent metric.

> **FAILURE RULE.** A proposed local rule fails if its physical influence radius grows without bound under refinement.

Atlas code E03. Return to Part 4: Graph evolution and continuum control.

E04 — Joint refinement trajectory

DEFINED

Canonical statement

$a \to 0, \Delta t \to 0, L \to \infty, \theta \to \theta_R$

> **FIRST LOOK.** A continuum limit is a trajectory through several regulators, not one fine simulation.

Dependency. B06 and a finite-volume sequence.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Wilson (1974); renormalization-group literature.

E04 working page — Joint refinement trajectory

Construction or derivation

Choose graph family, boundary scaling, temporal scaling, renormalization conditions, and order or joint rate of limits.

Dimensions and normalization

Regulator powers, renormalization conditions, and graph-family comparisons must be expressed through fixed dimensionless physical ratios.

Decisive calculation

Use at least three resolutions and fit regulator powers with correlated uncertainty.

Adversarial edge case

Taking volume and spacing limits in different orders can yield different phases.

> **FAILURE RULE.** The continuum claim is conditional until path independence or a physically selected path is shown.

Atlas code E04. Return to Part 4: Graph evolution and continuum control.

E05 — Continuum extrapolation

TEST

Canonical statement

$$O(a,\Delta t,L)=O_0+c_a a^p+c_t \Delta t^q+c_L e^{-L/\xi}+\cdots$$

FIRST LOOK. The limiting observable and the discretization residual are inferred together.

Dependency. E04.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Lattice continuum-extrapolation practice.

E05 working page — Continuum extrapolation

Construction or derivation

Fit all resolutions with covariance, compare justified correction orders, and reserve points for extrapolation validation.

Dimensions and normalization

Regulator powers, renormalization conditions, and graph-family comparisons must be expressed through fixed dimensionless physical ratios.

Decisive calculation

Predict a withheld finer resolution and require residuals consistent with the fitted truncation model.

Adversarial edge case

A flexible correction series can absorb nonconvergence.

> **FAILURE RULE.** Reject the limit if O_0 changes beyond tolerance under justified fit windows or graph families.

Atlas code E05. Return to Part 4: Graph evolution and continuum control.

E06 — Graph-family universality

TEST

Canonical statement

$$\lim_{a \to 0} O_{G_1} = \lim_{a \to 0} O_{G_2}$$

> **FIRST LOOK.** Universal physics must not depend on an arbitrary microscopic regulator family.

Dependency. E04-E05.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Universality and random-lattice literature.

E06 working page — Graph-family universality

Construction or derivation

Match renormalized conditions on inequivalent graph sequences and compare remaining dimensionless observables.

Dimensions and normalization

Regulator powers, renormalization conditions, and graph-family comparisons must be expressed through fixed dimensionless physical ratios.

Decisive calculation

Use regular, irregular, and mildly disordered families while preserving coarse dimension and target symmetries.

Adversarial edge case

Matching one observable by construction does not test the rest of the spectrum.

> **FAILURE RULE.** Persistent graph-family dependence is a regulator artifact or an extra physical parameter that must be declared.

Atlas code E06. Return to Part 4: Graph evolution and continuum control.

E07 — Universal causal cone

CLOSURE

Canonical statement

$$\lim_{|k|\to 0} \omega_A(k)/|k| = c_* \; \forall A$$

> **FIRST LOOK.** Relativity requires all propagating low-energy sectors to share one protected limiting speed.

Dependency. sector-specific dispersions.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. Lorentz-symmetry and effective-field-theory tests.

E07 working page — Universal causal cone

Construction or derivation

Derive each kinetic operator, diagonalize mixing, and identify the symmetry controlling speed equality and corrections.

Dimensions and normalization

Regulator powers, renormalization conditions, and graph-family comparisons must be expressed through fixed dimensionless physical ratios.

Decisive calculation

Compare scalar, gauge, fermion, and geometric branches under common renormalization conditions.

Adversarial edge case

Independent parameter fits can force numerical equality without a universal mechanism.

> **FAILURE RULE.** Any stable species-dependent low-energy speed outside empirical bounds falsifies this bridge.

Atlas code E07. Return to Part 4: Graph evolution and continuum control.

E08 — Anisotropy operator basis

DERIVED

Canonical statement

$$\delta L = a^2 \sum_i c_i O_i^{(6)}$$

FIRST LOOK. Orientation breaking appears as higher-dimension operators in the effective theory.

Dependency. B07 and continuum symmetries.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Symanzik effective-action methodology.

E08 working page — Anisotropy operator basis

Construction or derivation

Classify operators allowed by the discrete symmetry and remove redundancies by integration by parts and equations of motion.

Dimensions and normalization

Regulator powers, renormalization conditions, and graph-family comparisons must be expressed through fixed dimensionless physical ratios.

Decisive calculation

Fit angular dependence at fixed $|k|$ and compare with the operator basis rather than an arbitrary polynomial.

Adversarial edge case

Irregular graphs can introduce additional tensor structures.

> **FAILURE RULE.** Do not quote one Lorentz-violation coefficient until the complete symmetry-allowed basis is considered.

Atlas code E08. Return to Part 4: Graph evolution and continuum control.

E09 — Heat-kernel asymptotics

CONDITIONAL

Canonical statement

$$Tr\, e^{-s\Delta} \sim (4\pi s)^{-d/2} \sum_{n\geq 0} s^n A_{2n}$$

> **FIRST LOOK.** Spectral asymptotics organize local geometric invariants under precise hypotheses.

Dependency. a Laplace-type smooth limit.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Vassilevich (2003).

E09 working page — Heat-kernel asymptotics

Construction or derivation

Establish a manifold-like limit, operator type, boundary conditions, and asymptotic window before identifying coefficients.

Dimensions and normalization

Regulator powers, renormalization conditions, and graph-family comparisons must be expressed through fixed dimensionless physical ratios.

Decisive calculation

Recover known coefficients on benchmark manifolds and compare graph estimators under refinement.

Adversarial edge case

Finite graphs at large s and rough graphs at small s need not enter the smooth asymptotic window.

> **FAILURE RULE.** A curvature term in A_2 does not derive Lorentzian gravity or its coefficient.

Atlas code E09. *Return to Part 4: Graph evolution and continuum control.*

E10 — Renormalized parameter card

DEFINED

Canonical statement

$$\theta_R(\mu) = R_\mu[\theta(a), a)$$

> **FIRST LOOK.** Bare graph parameters become physical only through declared renormalization conditions.

Dependency. E04 and independent observables.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Wilson (1974); effective-field-theory matching.

E10 working page — Renormalized parameter card

Construction or derivation

Select independent observables, solve for bare parameters at each resolution, and record scheme, scale, branch, and uncertainty.

Dimensions and normalization

Regulator powers, renormalization conditions, and graph-family comparisons must be expressed through fixed dimensionless physical ratios.

Decisive calculation

Predict additional observables across resolutions without retuning and verify stable limits.

Adversarial edge case

Using the target observable as a renormalization condition turns it into calibration.

> **FAILURE RULE.** A parameter is predicted only when its target or algebraic equivalent did not choose the card.

Atlas code E10. Return to Part 4: Graph evolution and continuum control.

Part 5. Gauge structure and matter targets

This part contains 10 canonical dossiers. Equations have one home, dependencies are explicit, and later uses add consequences rather than repeat definitions.

G01 — Abelian link covariance

DERIVED

Canonical statement

$q_i \mapsto e^{i\alpha_i} q_i, U_{ij} \mapsto e^{i\alpha_i} U_{ij} e^{-i\alpha_j}$

> **FIRST LOOK.** A link variable transports phase so a neighbor difference transforms at one node.

Dependency. oriented graph links.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Wilson (1974).

G01 working page — Abelian link covariance

Construction or derivation

Apply the transformation to U_ij q_j-q_i and show that its norm is exactly invariant at finite spacing.

Dimensions and normalization

Link phases are dimensionless; continuum fields, couplings, masses, and operators receive units only through the declared spacing and normalization.

Decisive calculation

Randomize node phases and verify action invariance to numerical precision.

Adversarial edge case

Omitting the inverse orientation rule U_ji=U_ij^{-1} breaks path consistency.

> **FAILURE RULE.** This identity establishes lattice covariance only; it does not select the observed gauge group.

Atlas code G01. Return to Part 5: Gauge structure and matter targets.

G02 — Non-Abelian plaquette holonomy

DERIVED

Canonical statement

$U_p = U_{ij} U_{jk} U_{k\ell} U_{\ell i}, ReTr\left(I - U_p\right)$

FIRST LOOK. A closed link product records curvature and transforms by conjugation.

Dependency. G01 generalized to a compact group.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Wilson (1974).

G02 working page — Non-Abelian plaquette holonomy

Construction or derivation

Track group action at every endpoint and show cancellation of all interior transformations around the loop.

Dimensions and normalization

Link phases are dimensionless; continuum fields, couplings, masses, and operators receive units only through the declared spacing and normalization.

Decisive calculation

Generate random group elements and verify gauge invariance of the real trace.

Adversarial edge case

Loop orientation reverses holonomy and can expose inconsistent conventions.

> **FAILURE RULE.** Plaquette covariance does not determine group, representation, coupling, or matter content.

Atlas code G02. Return to Part 5: Gauge structure and matter targets.

G03 — Finite-spacing gauge action

PROPOSED

Canonical statement

$$S_g = \frac{\beta}{N} \sum_p ReTr\left(I - U_p\right)$$

> **FIRST LOOK.** A gauge-invariant action becomes dynamics only after its normalization and graph support are fixed.

Dependency. G02 and a chosen loop set.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Wilson (1974); lattice gauge theory texts.

G03 working page — Finite-spacing gauge action

Construction or derivation

Choose plaquettes or generalized cycles, weights, orientation multiplicity, boundary treatment, and the relation of beta to continuum coupling.

Dimensions and normalization

Link phases are dimensionless; continuum fields, couplings, masses, and operators receive units only through the declared spacing and normalization.

Decisive calculation

Recover the weak-field continuum operator and test scaling of Wilson loops.

Adversarial edge case

Irregular graphs may have no canonical elementary plaquette.

> **FAILURE RULE.** Do not claim Yang-Mills recovery until locality, continuum scaling, and coefficient matching are demonstrated.

Atlas code G03. Return to Part 5: Gauge structure and matter targets.

G04 — Gauge fixing and ghost gate

CLOSURE

Canonical statement

$$1 = \Delta_{FP}[A) \int D g \, \delta\left(F\left[A^{g}\right)\right)$$

> **FIRST LOOK.** Gauge redundancy must be handled without counting equivalent configurations as distinct physics.

Dependency. G03 continuum or path-integral quantization.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. Faddeev-Popov and BRST literature.

G04 working page — Gauge fixing and ghost gate

Construction or derivation

Specify whether the formulation is Hamiltonian, gauge-fixed path integral, or gauge-invariant observable based; derive the corresponding measure.

Dimensions and normalization

Link phases are dimensionless; continuum fields, couplings, masses, and operators receive units only through the declared spacing and normalization.

Decisive calculation

Reproduce a benchmark propagator and a gauge-independent observable in two gauges.

Adversarial edge case

Gribov copies can invalidate naive uniqueness of gauge fixing.

> **FAILURE RULE.** Quantum gauge claims remain open until measure, fixing, ghosts, and unitarity are coherently treated.

Atlas code G04. Return to Part 5: Gauge structure and matter targets.

G05 — Lattice Dirac operator

PROPOSED

Canonical statement

$$S_f = a^d \sum_{x,y} \acute{\psi}_x D_{xy}[U)\psi_y$$

> **FIRST LOOK.** Fermions require an explicit local operator, inner product, boundary conditions, and gauge representation.

Dependency. G03 and spinor state space.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Nielsen and Ninomiya (1981); lattice-fermion texts.

G05 working page — Lattice Dirac operator

Construction or derivation

Define D, gamma conventions, locality decay, hermiticity property, free spectrum, and gauge covariance.

Dimensions and normalization

Link phases are dimensionless; continuum fields, couplings, masses, and operators receive units only through the declared spacing and normalization.

Decisive calculation

Diagonalize the free operator and verify the target continuum branch and normalization.

Adversarial edge case

Naive discretization creates extra low-energy zeros.

> **FAILURE RULE.** No statement about quarks, leptons, or generations is accepted without the full operator spectrum.

Atlas code G05. Return to Part 5: Gauge structure and matter targets.

G06 — Fermion-doubling obstruction

DERIVED

Canonical statement

$$D(k) = i\sum_{\mu} \gamma_{\mu} a^{-1} \sin(k_{\mu} a)$$

FIRST LOOK. A periodic local chiral discretization produces additional zeros at Brillouin-zone corners.

Dependency. G05 naive translationally invariant lattice.

The conclusion follows only within the displayed assumptions and named benchmark.

Primary literature route. Nielsen and Ninomiya (1981 I, II).

G06 working page — Fermion-doubling obstruction

Construction or derivation

List all momenta for which every sine vanishes and count their chirality contributions.

Dimensions and normalization

Link phases are dimensionless; continuum fields, couplings, masses, and operators receive units only through the declared spacing and normalization.

Decisive calculation

Plot the full free spectrum rather than only the neighborhood of k=0.

Adversarial edge case

Boundary, locality, or symmetry assumptions can change how the theorem applies.

> **FAILURE RULE.** A scalar sign change does not bypass the obstruction without a complete fermion construction.

Atlas code G06. Return to Part 5: Gauge structure and matter targets.

G07 — Ginsparg-Wilson closure route

CONDITIONAL

Canonical statement

$$D\gamma_5 + \gamma_5 D = a D \gamma_5 D$$

FIRST LOOK. Modified finite-spacing chirality can preserve an exact lattice symmetry and the correct index.

Dependency. G05 and a local overlap-type operator.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Ginsparg and Wilson (1982); Neuberger (1998).

G07 working page — Ginsparg-Wilson closure route

Construction or derivation

Construct D, establish locality in the allowed gauge background, verify the relation, and compute the index.

Dimensions and normalization

Link phases are dimensionless; continuum fields, couplings, masses, and operators receive units only through the declared spacing and normalization.

Decisive calculation

Test residuals, spectral circle, zero-mode chirality, and continuum scaling on controlled backgrounds.

Adversarial edge case

Locality can fail on rough gauge configurations.

> **FAILURE RULE.** Chirality advances only after unwanted light mirrors decouple and anomalies cancel.

Atlas code G07. Return to Part 5: Gauge structure and matter targets.

G08 — Domain-wall closure route

CONDITIONAL

Canonical statement

$$D_{d+1}\Psi = 0, m(s) \to \pm m_0$$

> **FIRST LOOK.** A sign-changing mass may localize boundary modes, but finite separation leaves residual mixing.

Dependency. G05 plus an auxiliary direction.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Kaplan (1992).

G08 working page — Domain-wall closure route

Construction or derivation

State the higher-dimensional operator, wall profile, boundary conditions, separation, gauge coupling, and projection to physical modes.

Dimensions and normalization

Link phases are dimensionless; continuum fields, couplings, masses, and operators receive units only through the declared spacing and normalization.

Decisive calculation

Compute the complete spectrum and measure residual mass as wall separation grows.

Adversarial edge case

Additional walls or finite extent can restore unwanted mirror modes.

> **FAILURE RULE.** A wall profile is not a Standard Model derivation until spectrum and anomaly inflow are complete.

Atlas code G08. Return to Part 5: Gauge structure and matter targets.

G09 — Anomaly cancellation ledger

CLOSURE

Canonical statement

$$\sum_f tr\left(T_f^a\{T_f^b,T_f^c\}\right)=0$$

FIRST LOOK. Quantum gauge consistency constrains the complete chiral matter content.

Dependency. G05-G08 and representations.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. Anomaly and chiral-gauge-theory literature.

G09 working page — Anomaly cancellation ledger

Construction or derivation

List every left-handed representation, multiplicity, charge, global anomaly condition, and regulator contribution.

Dimensions and normalization

Link phases are dimensionless; continuum fields, couplings, masses, and operators receive units only through the declared spacing and normalization.

Decisive calculation

Compute cubic, mixed, gravitational, and global anomaly tests using exact rational charges.

Adversarial edge case

A heavy mirror state may not decouple harmlessly if it carries anomaly flow.

> **FAILURE RULE.** Any uncancelled anomaly falsifies the proposed quantum gauge sector.

Atlas code G09. Return to Part 5: Gauge structure and matter targets.

G10 — Mass and mixing overlap map

PROPOSED

Canonical statement

$$\left(M_f\right)_{ab} = g_f \int d^d x \sqrt{|g|}\,\acute{\psi}_a C(x)\,\psi_b$$

> **FIRST LOOK.** An overlap becomes predictive only when profiles, couplings, and labels are fixed without target masses.

Dependency. normalized modes and fixed capacity profile.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Standard mixing-matrix and flavor-physics methodology.

G10 working page — Mass and mixing overlap map

Construction or derivation

Derive modes and normalization, compute the matrix, diagonalize it, and count independent inputs against mass ratios and phases.

Dimensions and normalization

Link phases are dimensionless; continuum fields, couplings, masses, and operators receive units only through the declared spacing and normalization.

Decisive calculation

Reserve selected masses and mixing elements as held-out observables.

Adversarial edge case

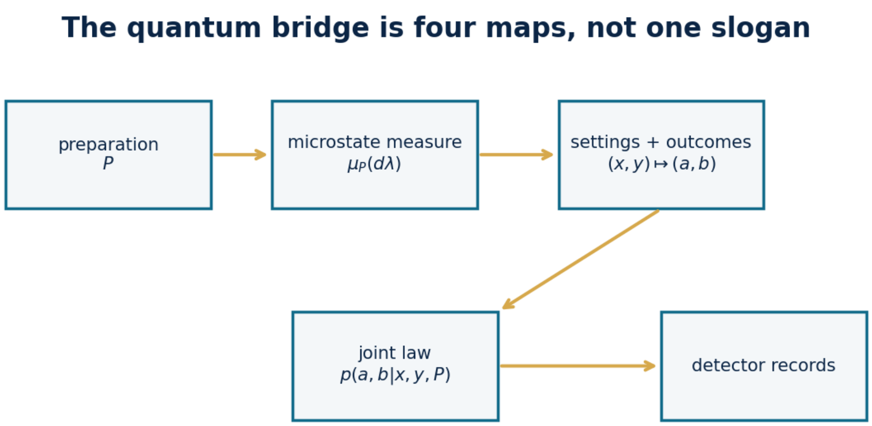
Relabeling modes after seeing data silently uses the target.

> **FAILURE RULE.** Three geometric axes do not establish three generations; the full spectrum and anomaly ledger must decide.

Atlas code G10. Return to Part 5: Gauge structure and matter targets.

Part 6. Quantum map and entanglement

This part contains 10 canonical dossiers. Equations have one home, dependencies are explicit, and later uses add consequences rather than repeat definitions.



Technical orientation figure for Part 6

Q01 — Preparation measure

PROPOSED

Canonical statement

$$P \mapsto \mu_P(d\lambda), \int_{\Lambda} \mu_P(d\lambda) = 1$$

> **FIRST LOOK.** A deterministic dynamics still needs a reproducible distribution over prepared microstates.

Dependency. F09 and a complete microstate space.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Probability foundations and statistical mechanics.

Q01 working page — Preparation measure

Construction or derivation

Define the laboratory preparation, induced ensemble, hidden variables, equilibrium or typicality argument, and independence assumptions.

Dimensions and normalization

Probabilities normalize to one, settings and outcomes are labels, and detector records carry the units of the instrument response.

Decisive calculation

Generate repeated preparations and compare empirical microstate summaries with the proposed measure.

Adversarial edge case

An unknown preparation bias can imitate a non-Born outcome distribution.

> **FAILURE RULE.** No probability claim begins until the same preparation rule applies across measurement contexts.

Atlas code Q01. Return to Part 6: Quantum map and entanglement.

Q02 — Outcome basin map

PROPOSED

Canonical statement

$$(\lambda, x) \mapsto a = A(\lambda, x)$$

> **FIRST LOOK.** Outcomes require explicit basins or detector records, not verbal collapse.

Dependency. Q01 and measurement setting x.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Dynamical-systems basin theory and measurement models.

Q02 working page — Outcome basin map

Construction or derivation

Specify apparatus degrees of freedom, coupling, stopping criterion, basin boundaries, and coarse record.

Dimensions and normalization

Probabilities normalize to one, settings and outcomes are labels, and detector records carry the units of the instrument response.

Decisive calculation

Simulate perturbations near boundaries and measure outcome stability and resolution dependence.

Adversarial edge case

Fractal basin boundaries can make frequencies precision dependent.

> **FAILURE RULE.** A context-specific fitted basin is not a universal measurement rule.

Atlas code Q02. Return to Part 6: Quantum map and entanglement.

Q03 — Born-frequency gate

CLOSURE

Canonical statement

$$p(k|P,M)=Tr(\rho_P E_k)$$

> **FIRST LOOK.** The Born rule is closed only if one preparation and outcome construction yields quantum probabilities across a declared class.

Dependency. Q01-Q02 plus a Hilbert-space bridge.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. Gleason (1957).

Q03 working page — Born-frequency gate

Construction or derivation

Derive Hilbert space, states, effects, the measure over microstates, and the basin-to-projector map without fitting each context.

Dimensions and normalization

Probabilities normalize to one, settings and outcomes are labels, and detector records carry the units of the instrument response.

Decisive calculation

Test multiple noncommuting measurements and composite systems with the same frozen construction.

Adversarial edge case

Gleason's theorem assumes quantum event structure and does not create it from a generic capacity boundary.

> **FAILURE RULE.** Context-by-context numerical agreement does not close the universal Born bridge.

Atlas code Q03. Return to Part 6: Quantum map and entanglement.

Q04 — Bell-CHSH statistic

TEST

Canonical statement

$S = E_{00} + E_{01} + E_{10} - E_{11}$

> **FIRST LOOK.** A complete model must state which Bell assumption it changes and calculate the observable joint law.

Dependency. Q01-Q03 and two separated settings.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Bell (1964); Clauser et al. (1969).

Q04 working page — Bell-CHSH statistic

Construction or derivation

Define source, settings, outcomes, spacetime or update dependencies, sample selection, and correlation estimator.

Dimensions and normalization

Probabilities normalize to one, settings and outcomes are labels, and detector records carry the units of the instrument response.

Decisive calculation

Generate all four setting pairs and verify estimator coverage before comparing with the bounds 2 and 2 sqrt 2.

Adversarial edge case

Postselection or setting-dependent loss can create an apparent violation.

> **FAILURE RULE.** A local measurement-independent hidden-variable model with |S| greater than 2 is internally inconsistent.

Atlas code Q04. Return to Part 6: Quantum map and entanglement.

Q05 — Measurement-independence audit

DEFINED

Canonical statement

$\mu(\lambda|x,y,P)=\mu(\lambda|P)$

> **FIRST LOOK.** Bell analysis distinguishes locality from independence of the prepared state and later settings.

Dependency. Q04.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Bell-test methodology and causal inference.

Q05 working page — Measurement-independence audit

Construction or derivation

Draw the causal or conditional-dependency graph and identify every common cause, seed, schedule, and selection variable.

Dimensions and normalization

Probabilities normalize to one, settings and outcomes are labels, and detector records carry the units of the instrument response.

Decisive calculation

Test setting predictors and source-setting correlations in simulated and laboratory records.

Adversarial edge case

A deterministic pseudorandom setting generator may share hidden state with the source in the model.

> **FAILURE RULE.** If independence is denied, the replacement causal mechanism and its testable restrictions must be published.

Atlas code Q05. Return to Part 6: Quantum map and entanglement.

Q06 — Operational no-signalling

TEST

Canonical statement

$$\sum_b p(a,b|x,y,P) = p(a|x,P) \forall y$$

> **FIRST LOOK.** No-signalling is a marginal equality, not merely a statement about finite propagation speed.

Dependency. Q04 joint probabilities.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Bell and no-signalling literature.

Q06 working page — Operational no-signalling

Construction or derivation

Derive the marginal analytically where possible and specify finite-sample estimators with multiplicity correction.

Dimensions and normalization

Probabilities normalize to one, settings and outcomes are labels, and detector records carry the units of the instrument response.

Decisive calculation

Compare remote-setting marginals under blinded labels and injected signalling alternatives.

Adversarial edge case

A global implicit constraint solve can transmit setting dependence in one update.

> **FAILURE RULE.** Any reproducible remote-setting dependence beyond the registered tolerance falsifies operational no-signalling.

Atlas code Q06. Return to Part 6: Quantum map and entanglement.

Q07 — Contextuality gate

CLOSURE

Canonical statement

$v(A)$ independent of commuting context

> **FIRST LOOK.** Quantum event structure constrains whether outcomes can be assigned independently of measurement context.

Dependency. Q02-Q03 for compatible observables.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. Kochen and Specker (1967).

Q07 working page — Contextuality gate

Construction or derivation

Define compatible measurement implementations, shared observables, operational equivalences, and the ontological response map.

Dimensions and normalization

Probabilities normalize to one, settings and outcomes are labels, and detector records carry the units of the instrument response.

Decisive calculation

Evaluate a contextuality inequality with the same apparatus model across all contexts.

Adversarial edge case

Small implementation differences can break the operational equivalence required by the theorem.

> **FAILURE RULE.** The theory must either reproduce contextual statistics or expose and test the assumption it rejects.

Atlas code Q07. Return to Part 6: Quantum map and entanglement.

Q08 — Decoherence functional

CONDITIONAL

Canonical statement

$$\rho_S(t) = Tr_E\left[U(t)\,\rho_{SE}(0)\,U^{\dagger}(t)\right)$$

> **FIRST LOOK.** Environmental suppression of interference explains effective classical records only after the quantum structure exists.

Dependency. a quantum-state bridge and environment split.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Open-quantum-systems and decoherence literature.

Q08 working page — Decoherence functional

Construction or derivation

Specify the system-environment factorization, interaction, initial correlations, pointer observable, and recurrence scale.

Dimensions and normalization

Probabilities normalize to one, settings and outcomes are labels, and detector records carry the units of the instrument response.

Decisive calculation

Compute off-diagonal decay and compare with exact finite-environment evolution.

Adversarial edge case

Decoherence alone selects no unique outcome in a single run.

> **FAILURE RULE.** Do not use decoherence as a substitute for the missing preparation and outcome maps.

Atlas code Q08. Return to Part 6: Quantum map and entanglement.

Q09 — Detector-record map

DEFINED

Canonical statement

$(a,v) \mapsto d \sim p(d \mid a,v)$

> **FIRST LOOK.** Latent outcomes reach evidence only through resolution, inefficiency, noise, and selection.

Dependency. Q02 and calibrated apparatus nuisance variables.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Detector calibration and inverse-problem methodology.

Q09 working page — Detector-record map

Construction or derivation

Specify transfer functions, thresholds, dead time, calibration covariance, and missing-data policy.

Dimensions and normalization

Probabilities normalize to one, settings and outcomes are labels, and detector records carry the units of the instrument response.

Decisive calculation

Inject known latent outcomes and verify recovery, confusion matrix, and uncertainty coverage.

Adversarial edge case

Outcome-dependent selection can alter observed frequencies.

> **FAILURE RULE.** A model-to-data comparison that omits detector response is not a test of the latent theory.

Atlas code Q09. Return to Part 6: Quantum map and entanglement.

Q10 — Statistical distinguishability

TEST

Canonical statement

$$D_{KL}(p_{END} \parallel p_0) = \sum_d p_{END}(d) \ln \frac{p_{END}(d)}{p_0(d)}$$

FIRST LOOK. A proposed difference matters only if detector-level distributions can separate it.

Dependency. Q09 and a baseline model.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Information theory and hypothesis testing.

Q10 working page — Statistical distinguishability

Construction or derivation

Compute divergence or expected log-likelihood ratio over the calibrated response, including nuisance profiling.

Dimensions and normalization

Probabilities normalize to one, settings and outcomes are labels, and detector records carry the units of the instrument response.

Decisive calculation

Use simulated experiments to estimate power, error rates, and sample size before opening held-out records.

Adversarial edge case

A difference concentrated in an unobservable tail may have no practical test power.

> **FAILURE RULE.** Do not call two theories empirically distinct without a feasible statistic and declared decision error.

Atlas code Q10. Return to Part 6: Quantum map and entanglement.

Part 7. Operational geometry and gravitation

This part contains 10 canonical dossiers. Equations have one home, dependencies are explicit, and later uses add consequences rather than repeat definitions.

R01 — Radar-time metric data

PROPOSED

Canonical statement

$\tau_{\pm}(x, y) = \tau_{receive} \pm \tau_{send}$

> **FIRST LOOK.** Geometry begins with repeatable exchanges between persistent clocks and reflectors.

Dependency. F05 clocks and F06 influence paths.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Operational relativity and radar-coordinate literature.

R01 working page — Radar-time metric data

Construction or derivation

Define emission, reflection, reception, clock synchronization, path selection, and uncertainty before assigning coordinates.

Dimensions and normalization

Operational intervals carry clock and rod units; curvature, action, and source terms must be checked in one declared signature and unit system.

Decisive calculation

Repeat reciprocal and nonreciprocal exchanges and test reconstruction under clock replacement.

Adversarial edge case

Multiple paths or dispersive propagation can make one radar interval multivalued.

> **FAILURE RULE.** Do not infer a metric where exchange records fail a smooth local quadratic approximation.

Atlas code R01. Return to Part 7: Operational geometry and gravitation.

R02 — World-function reconstruction

CONDITIONAL

Canonical statement

$$g_{\mu\nu}(x) = -\left.\frac{\partial^2 \sigma(x,y)}{\partial x^\mu \partial y^\nu}\right)_{y=x}$$

FIRST LOOK. A two-point interval function determines a local metric only in a smooth regime.

Dependency. R01 and a smooth coincidence limit.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Synge world-function and differential-geometry literature.

R02 working page — World-function reconstruction

Construction or derivation

Fit sigma from exchange data, enforce symmetry or document its failure, differentiate with uncertainty, and state coordinate transformations.

Dimensions and normalization

Operational intervals carry clock and rod units; curvature, action, and source terms must be checked in one declared signature and unit system.

Decisive calculation

Validate the procedure on synthetic curved spacetimes with the same sampling geometry.

Adversarial edge case

Noise amplification in second derivatives can create spurious signature changes.

> **FAILURE RULE.** Metric language remains conditional until signature, rank, smoothness, and transformation behavior are stable.

Atlas code R02. Return to Part 7: Operational geometry and gravitation.

R03 — Tetrad and transport map

PROPOSED

Canonical statement

$$g_{\mu\nu} = e^{a}{}_{\mu} e^{b}{}_{\nu} \eta_{ab}$$

FIRST LOOK. Matter and spin require local frames and a rule for transporting them.

Dependency. R02 and local frame observables.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Tetrad and Cartan-geometry literature.

R03 working page — Tetrad and transport map

Construction or derivation

Construct orthonormal frames from operational intervals, define link transport, and separate frame gauge from physical holonomy.

Dimensions and normalization

Operational intervals carry clock and rod units; curvature, action, and source terms must be checked in one declared signature and unit system.

Decisive calculation

Transport frames around shrinking loops and compare holonomy with fitted curvature.

Adversarial edge case

Frame sign and orientation ambiguities can create discontinuities.

> **FAILURE RULE.** A metric alone does not supply the spin connection or its dynamics.

Atlas code R03. Return to Part 7: Operational geometry and gravitation.

R04 — Torsion as loop nonclosure

PROPOSED

Canonical statement

$T^a = d\,e^a + \omega^a{}_b \wedge e^b$

> **FIRST LOOK.** Torsion measures translational nonclosure after a connection and frame are defined.

Dependency. R03.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Hehl et al. (1976).

R04 working page — Torsion as loop nonclosure

Construction or derivation

Specify a discrete loop, parallel transport, closure vector, area normalization, and continuum extrapolation.

Dimensions and normalization

Operational intervals carry clock and rod units; curvature, action, and source terms must be checked in one declared signature and unit system.

Decisive calculation

Use reversed and deformed loops to separate torsion signal from discretization error.

Adversarial edge case

Finite polygon nonclosure can arise from curvature or coarse sampling rather than torsion.

> **FAILURE RULE.** Do not infer torsion from a picture; require a stable tensorial limit and matter coupling.

Atlas code R04. Return to Part 7: Operational geometry and gravitation.

R05 — Regge curvature comparison

CONDITIONAL

Canonical statement

$$S_{Regge} = \frac{1}{8\pi G} \sum_h A_h \delta_h$$

> **FIRST LOOK.** Deficit angles provide a benchmark discretization of curvature, not an automatic END derivation.

Dependency. a simplicial or dual-cell geometric map.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Regge (1961).

R05 working page — Regge curvature comparison

Construction or derivation

Map graph data to edge lengths satisfying simplex inequalities, calculate hinges and deficits, and test refinement.

Dimensions and normalization

Operational intervals carry clock and rod units; curvature, action, and source terms must be checked in one declared signature and unit system.

Decisive calculation

Recover flat space and a known curved benchmark before using the construction dynamically.

Adversarial edge case

Arbitrary weighted graphs need not define consistent simplices.

> **FAILURE RULE.** If the graph-to-length map is nonunique, gravitational predictions inherit that unresolved freedom.

Atlas code R05. Return to Part 7: Operational geometry and gravitation.

R06 — Induced curvature term

CONDITIONAL

Canonical statement

$\Gamma_{eff} \supset c_R \int d^4 x \sqrt{|g|} R$

> **FIRST LOOK.** A curvature invariant can appear in an effective action, but its sign and coefficient depend on the complete construction.

Dependency. E09 plus a quantum measure and field content.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Vassilevich (2003); induced-gravity literature.

R06 working page — Induced curvature term

Construction or derivation

Specify operator, measure, regulator, matter spectrum, boundary terms, analytic continuation, and renormalization prescription.

Dimensions and normalization

Operational intervals carry clock and rod units; curvature, action, and source terms must be checked in one declared signature and unit system.

Decisive calculation

Compute c_R in two regulators and show that the renormalized prediction is scheme coherent.

Adversarial edge case

Power-divergent terms can be dominated by regulator choices.

> **FAILURE RULE.** Matching c_R to 1/(16 pi G) is calibration unless microscopic inputs were fixed independently.

Atlas code R06. Return to Part 7: Operational geometry and gravitation.

R07 — Einstein-dynamics gate

CLOSURE

Canonical statement

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

> **FIRST LOOK.** Recovering an R term is not enough; the complete field equations and source response must follow.

Dependency. R02-R06 and a Lorentzian action.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. Einstein-Hilbert and effective-gravity literature.

R07 working page — Einstein-dynamics gate

Construction or derivation

Vary the renormalized action with boundary conditions, identify stress energy, constraints, extra modes, and conservation identities.

Dimensions and normalization

Operational intervals carry clock and rod units; curvature, action, and source terms must be checked in one declared signature and unit system.

Decisive calculation

Solve weak-field and homogeneous benchmarks and compare with independent continuum calculations.

Adversarial edge case

Higher-curvature or nonlocal terms can introduce additional propagating modes.

> **FAILURE RULE.** Einstein language is rejected if extra modes or coefficients remain hidden in an unspecified ellipsis.

Atlas code R07. Return to Part 7: Operational geometry and gravitation.

R08 — Newtonian limit

CONDITIONAL

Canonical statement

$$\nabla^2 \Phi = 4\pi G \rho$$

> **FIRST LOOK.** The inverse-square regime follows only after identifying potential, source density, and coupling.

Dependency. R07 and a weak stationary field.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Standard general-relativity weak-field derivations.

R08 working page — Newtonian limit

Construction or derivation

Linearize the metric, choose gauge, isolate the slowly moving limit, and derive the Poisson equation including extra fields.

Dimensions and normalization

Operational intervals carry clock and rod units; curvature, action, and source terms must be checked in one declared signature and unit system.

Decisive calculation

Calculate point, extended, and screened sources and compare boundary-sensitive solutions.

Adversarial edge case

A fitted Schwarzschild radius simply reuses measured G and mass.

> **FAILURE RULE.** The gravity bridge must predict deviations or derive G without circular use of the target.

Atlas code R08. Return to Part 7: Operational geometry and gravitation.

R09 — Post-Newtonian dossier

TEST

Canonical statement

$$g_{00} = -1 + 2U - 2\beta U^2 + \cdots, g_{ij} = (1 + 2\gamma U)\delta_{ij} + \cdots$$

> **FIRST LOOK.** Solar-system viability is summarized by coefficients that must be derived from the same field equations.

Dependency. R07-R08.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Will (2014).

R09 working page — Post-Newtonian dossier

Construction or derivation

Perform the velocity and potential expansion, solve constraints, and map every extra field to PPN parameters.

Dimensions and normalization

Operational intervals carry clock and rod units; curvature, action, and source terms must be checked in one declared signature and unit system.

Decisive calculation

Compute light deflection, time delay, and perihelion observables with covariance.

Adversarial edge case

Screening can make parameters environment dependent.

> **FAILURE RULE.** A theory fails this gate if its derived PPN region cannot meet the registered observational bounds.

Atlas code R09. Return to Part 7: Operational geometry and gravitation.

R10 — Strong-field and wave sector

CLOSURE

Canonical statement

$$h_A'' + \Gamma_A h_A' + \omega_A^2(k,\eta) h_A = 0$$

> **FIRST LOOK.** A viable gravity theory must specify polarizations, propagation, damping, generation, and compact-object solutions.

Dependency. R07 on a dynamical background.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. GWOSC and tests-of-general-relativity literature.

R10 working page — Strong-field and wave sector

Construction or derivation

Linearize about relevant backgrounds, classify modes, derive source coupling, and solve or simulate the nonlinear strong-field regime.

Dimensions and normalization

Operational intervals carry clock and rod units; curvature, action, and source terms must be checked in one declared signature and unit system.

Decisive calculation

Recover a general-relativistic injection pipeline before searching for END-specific residuals.

Adversarial edge case

Agreement of propagation speed alone does not test waveform generation or extra polarizations.

> **FAILURE RULE.** No gravitational-wave claim is complete without a source-to-strain map and consistency across detectors and event classes.

Atlas code R10. Return to Part 7: Operational geometry and gravitation.

Part 8. EQEF cosmology and the dark sector

This part contains 10 canonical dossiers. Equations have one home, dependencies are explicit, and later uses add consequences rather than repeat definitions.

K01 — Coarse residual-capacity field

PROPOSED

Canonical statement

$$\chi(x) = C_R\left[\{r_i\}_{i \in B_R(x)}\right)$$

> **FIRST LOOK.** Residual capacity becomes cosmological only after a scale-controlled field construction.

Dependency. C06 and a continuum coarse map.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Coarse-graining and effective-field-theory literature.

K01 working page — Coarse residual-capacity field

Construction or derivation

Define blocks, weights, interpolation, normalization, and stochastic residual; test transformation and refinement.

Dimensions and normalization

Cosmological background and perturbation variables must use a single convention for scale factor, conformal or proper time, and gravitational units.

Decisive calculation

Compare coarse fields across block sizes and graph families at fixed physical smoothing scale.

Adversarial edge case

Averaging a bounded variable can produce a smooth field with no autonomous dynamics.

> **FAILURE RULE.** Do not assign dark-sector meaning until chi has a derived action and detector-independent observables.

Atlas code K01. Return to Part 8: EQEF cosmology and the dark sector.

K02 — Leading EQEF action

PROPOSED

Canonical statement

$$S_\chi = \int d^4 x \sqrt{-g}\left[-1/2\, Z_\chi (\partial \chi)^2 - U(\chi) - \xi \chi^2 R + \cdots\right)$$

> **FIRST LOOK.** The most general low-order scalar action displays kinetic, potential, curvature, and higher-operator obligations.

Dependency. K01 and R07.

This is a candidate law or bridge whose empirical status remains open.

Primary literature route. Scalar-tensor and effective-field-theory literature.

K02 working page — Leading EQEF action

Construction or derivation

Derive or match every coefficient, state field range and symmetries, and bound omitted operators.

Dimensions and normalization

Cosmological background and perturbation variables must use a single convention for scale factor, conformal or proper time, and gravitational units.

Decisive calculation

Check ghost freedom, boundedness, and radiative stability over the claimed regime.

Adversarial edge case

A sign error in Z or a steep negative potential destabilizes the background.

> **FAILURE RULE.** Slow-roll algebra does not derive this action from capacity dynamics.

Atlas code K02. Return to Part 8: EQEF cosmology and the dark sector.

K03 — Homogeneous background

CONDITIONAL

Canonical statement

$\rho_\chi = 1/2\, Z_\chi \dot{\chi}^2 + U,\ p_\chi = 1/2\, Z_\chi \dot{\chi}^2 - U$

FIRST LOOK. Negative pressure follows from potential domination, not from the name residual capacity.

Dependency. K02 in FLRW symmetry.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Standard scalar-field cosmology.

K03 working page — Homogeneous background

Construction or derivation

Vary the action, derive Friedmann and scalar equations, specify initial conditions, and identify attractors.

Dimensions and normalization

Cosmological background and perturbation variables must use a single convention for scale factor, conformal or proper time, and gravitational units.

Decisive calculation

Integrate across initial conditions and require a broad stable basin compatible with expansion history.

Adversarial edge case

Fine-tuned initial conditions can mimic an attractor over a short redshift interval.

> **FAILURE RULE.** Do not call w near -1 derived unless the microscopic model selects the potential and initial regime.

Atlas code K03. Return to Part 8: EQEF cosmology and the dark sector.

K04 — Scalar perturbation system

CLOSURE

Canonical statement

$$Q_s \dot{R}^2 - Q_s c_s^2 a^{-2} (\nabla R)^2$$

FIRST LOOK. Background agreement is insufficient; perturbations determine stability, clustering, and structure growth.

Dependency. K02-K03.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. Cosmological-perturbation and EFT-of-dark-energy literature.

K04 working page — Scalar perturbation system

Construction or derivation

Expand the action to quadratic order, solve nondynamical constraints, and identify gauge-invariant propagating variables.

Dimensions and normalization

Cosmological background and perturbation variables must use a single convention for scale factor, conformal or proper time, and gravitational units.

Decisive calculation

Check kinetic eigenvalues, gradient speeds, and growth against a Boltzmann-code benchmark.

Adversarial edge case

A stable homogeneous solution can contain a ghost or gradient instability.

> **FAILURE RULE.** The cosmology gate fails anywhere in the fitted domain where Q_s or c_s squared is nonpositive.

Atlas code K04. Return to Part 8: EQEF cosmology and the dark sector.

K05 — Dark-sector sound speed

CONDITIONAL

Canonical statement

$$c_s^2 = \frac{\delta p_\chi}{\delta \rho_\chi}\Big|_{rest}$$

> **FIRST LOOK.** Pressure support controls whether the field clusters like matter or remains smooth like dark energy.

Dependency. K04.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Dark-energy perturbation literature.

K05 working page — Dark-sector sound speed

Construction or derivation

Derive the rest-frame ratio from the quadratic action, including noncanonical and curvature couplings.

Dimensions and normalization

Cosmological background and perturbation variables must use a single convention for scale factor, conformal or proper time, and gravitational units.

Decisive calculation

Propagate the resulting transfer functions and compare lensing and clustering observables jointly.

Adversarial edge case

Adiabatic and physical sound speeds need not coincide.

> **FAILURE RULE.** A single EQEF field cannot be called both cold dark matter and smooth dark energy without the required scale-dependent behavior.

Atlas code K05. Return to Part 8: EQEF cosmology and the dark sector.

K06 — Effective dark-matter map

CLOSURE

Canonical statement

$$k^2 \Phi = -4\pi G a^2 \mu(k,a) \rho_m \Delta_m$$

> **FIRST LOOK.** Replacing particle dark matter requires the same model to reproduce lensing, dynamics, and growth.

Dependency. K04-K05.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. Modified-gravity and large-scale-structure methodology.

K06 working page — Effective dark-matter map

Construction or derivation

Derive modified Poisson and slip functions from the field equations, then predict halos and linear structure with one parameter card.

Dimensions and normalization

Cosmological background and perturbation variables must use a single convention for scale factor, conformal or proper time, and gravitational units.

Decisive calculation

Test rotation curves only after CMB, BAO, lensing, and cluster constraints share the same cosmology.

Adversarial edge case

A baryon-tuned galactic fit may fail early-universe acoustic peaks.

> **FAILURE RULE.** The dark-matter replacement fails if one frozen model cannot fit gravitational and clustering channels together.

Atlas code K06. Return to Part 8: EQEF cosmology and the dark sector.

K07 — Early-universe gate

CLOSURE

Canonical statement

$$P_R(k) = A_s(k/k_*)^{n_s - 1 + \cdots}$$

FIRST LOOK. An origin model must address initial perturbations, horizon structure, relics, and thermal history.

Dependency. K02-K04 at high energy.

This is an explicit missing bridge; no downstream claim may silently assume it.

Primary literature route. Early-universe and primordial-spectrum literature.

K07 working page — Early-universe gate

Construction or derivation

Specify the phase, background solution, perturbation vacuum or ensemble, reheating map, and transition to standard radiation domination.

Dimensions and normalization

Cosmological background and perturbation variables must use a single convention for scale factor, conformal or proper time, and gravitational units.

Decisive calculation

Calculate scalar and tensor spectra plus non-Gaussianity without inserting measured amplitudes as outputs.

Adversarial edge case

A low-energy effective action may be invalid at the proposed origin scale.

> **FAILURE RULE.** Do not claim an inflation replacement until the complete perturbation and thermal history is executable.

Atlas code K07. Return to Part 8: EQEF cosmology and the dark sector.

K08 — BAO distance map

TEST

Canonical statement

$D_M(z)/r_d, D_H(z)/r_d = c/[H(z)r_d]$

> **FIRST LOOK.** Expansion predictions reach BAO data through transverse and radial distance ratios.

Dependency. K03 plus a sound-horizon convention.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Official DESI DR1/DR2 cosmology products.

K08 working page — BAO distance map

Construction or derivation

Derive H(z), specify whether r_d is predicted or externally calibrated, and evaluate the released covariance.

Dimensions and normalization

Cosmological background and perturbation variables must use a single convention for scale factor, conformal or proper time, and gravitational units.

Decisive calculation

Validate the baseline likelihood before comparing the frozen EQEF model with DESI products.

Adversarial edge case

Changing the sound-horizon prior can move apparent late-time conclusions.

> **FAILURE RULE.** Calibration with BAO distances is not a held-out prediction of the same distances.

Atlas code K08. Return to Part 8: EQEF cosmology and the dark sector.

K09 — CMB transfer map

TEST

Canonical statement

$$C_\ell^{XY} = 4\pi \int d\ln k \, P(k)\, \Delta_\ell^X(k)\, \Delta_\ell^Y(k)$$

> **FIRST LOOK.** CMB claims require perturbation transfer functions, foreground treatment, and likelihood conventions.

Dependency. K04 and a complete thermal history.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Planck Legacy Archive and Planck 2018 likelihood literature.

K09 working page — CMB transfer map

Construction or derivation

Implement the model in a validated Boltzmann solver and reproduce a standard baseline before activating END terms.

Dimensions and normalization

Cosmological background and perturbation variables must use a single convention for scale factor, conformal or proper time, and gravitational units.

Decisive calculation

Compare TT, TE, EE, lensing, and cross-covariances under one nuisance treatment.

Adversarial edge case

Fitting only a compressed parameter table can hide spectral residuals.

> **FAILURE RULE.** A model that cannot reproduce the released spectra and likelihood is excluded regardless of background elegance.

Atlas code K09. Return to Part 8: EQEF cosmology and the dark sector.

K10 — Weak-lensing and shear map

TEST

Canonical statement

$$C_\ell^{ij} = \int d\chi \frac{W_i(\chi) W_j(\chi)}{\chi^2} P_\delta((\ell + 1/2)/\chi, z)$$

> **FIRST LOOK.** Lensing tests geometry and growth simultaneously and is sensitive to model-dependent nonlinear structure.

Dependency. K04-K06 and survey response.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Official Euclid Q1/Q2 archive and lensing methodology.

K10 working page — Weak-lensing and shear map

Construction or derivation

Propagate the frozen background and perturbations through redshift distributions, masks, shear calibration, and covariance.

Dimensions and normalization

Cosmological background and perturbation variables must use a single convention for scale factor, conformal or proper time, and gravitational units.

Decisive calculation

Use synthetic catalogs to validate selection, tomography, and blinding before opening cosmology summaries.

Adversarial edge case

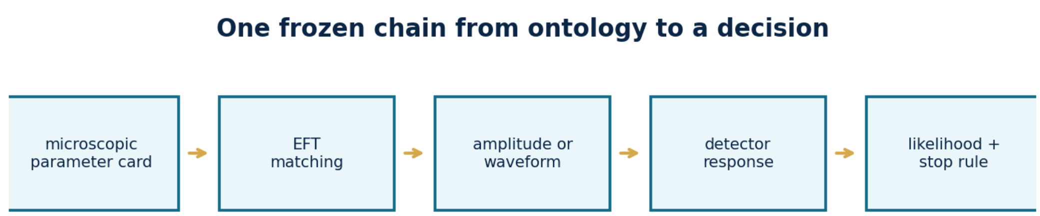
Baryonic feedback can imitate scale-dependent modified gravity.

> **FAILURE RULE.** No Euclid confirmation claim is permitted without the released data model, known-issues treatment, and nuisance audit.

Atlas code K10. Return to Part 8: EQEF cosmology and the dark sector.

Part 9. Phenomenology and public-data dossiers

This part contains 10 canonical dossiers. Equations have one home, dependencies are explicit, and later uses add consequences rather than repeat definitions.



Technical orientation figure for Part 9

D01 — Universal forward-model contract

DEFINED

Canonical statement

$$\theta_{micro} \xrightarrow{C} \theta_{EFT} \xrightarrow{A} s \xrightarrow{D} \mu \xrightarrow{L} \text{decision}$$

> **FIRST LOOK.** Every empirical claim uses one versioned chain from microscopic state to a statistical decision.

Dependency. F09 and a frozen parameter card.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Cowan et al. (2011); experiment-specific analysis documentation.

D01 working page — Universal forward-model contract

Construction or derivation

Record the theory bridge, amplitude or waveform, detector response, likelihood, software versions, units, and uncertainty at every arrow.

Dimensions and normalization

The theory output, detector prediction, observed data, and nuisance parameters must use the same binning, units, and covariance convention.

Decisive calculation

Run an end-to-end synthetic example whose truth is known and require unbiased recovery with nominal coverage.

Adversarial edge case

A correct microscopic calculation can still yield a wrong detector prediction through selection or calibration.

> **FAILURE RULE.** If any arrow is missing, the result constrains a template rather than tests the full theory.

Atlas code D01. Return to Part 9: Phenomenology and public-data dossiers.

D02 — GWOSC dispersion test

TEST

Canonical statement

$$E^2 = p^2 c^2 + A_\alpha p^\alpha c^\alpha$$

> **FIRST LOOK.** Propagation dispersion produces a frequency-dependent phase that can be tested across gravitational-wave events.

Dependency. D01 and a fixed micro-to-waveform coefficient.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. GWOSC API v2 and official event releases; LIGO-Virgo-KAGRA tests of general relativity.

D02 working page — GWOSC dispersion test

Construction or derivation

Derive the waveform phase convention, source-frame and redshift factors, priors, calibration model, waveform alternatives, and event partition.

Dimensions and normalization

The theory output, detector prediction, observed data, and nuisance parameters must use the same binning, units, and covariance convention.

Decisive calculation

Reproduce a general-relativistic baseline, inject known coefficients into off-source data, and verify hierarchical coverage.

Adversarial edge case

Waveform-systematic error can imitate a propagation phase.

> **FAILURE RULE.** Reject the frozen coefficient if detectors, waveform families, or event classes are mutually inconsistent beyond tolerance.

Atlas code D02. Return to Part 9: Phenomenology and public-data dossiers.

D03 — CERN neutral-scalar template

CONDITIONAL

Canonical statement

$$L \supset -1/2\, m_S^2 S^2 - \sum_f y_{Sf} S \acute{f} f - \frac{c_g}{4\Lambda} S G_{\mu\nu}^a G^{a\mu\nu}$$

> **FIRST LOOK.** A resonance search becomes an END test only when mass, width, and couplings are fixed or related before the signal region is opened.

Dependency. D01 and a micro-to-EFT matching relation.

The result is valid only if the named construction gate is satisfied.

Primary literature route. CERN Open Data research CMS routes; Cowan et al. (2011); Gross and Vitells (2011).

D03 working page — CERN neutral-scalar template

Construction or derivation

Generate signal and backgrounds with declared orders, detector simulation, triggers, object definitions, luminosity, and nuisance correlations.

Dimensions and normalization

The theory output, detector prediction, observed data, and nuisance parameters must use the same binning, units, and covariance convention.

Decisive calculation

Validate cut flow and standard candles, then compute observed and expected intervals with the look-elsewhere range fixed.

Adversarial edge case

A broad resonance can be absorbed by background-function flexibility.

> **FAILURE RULE.** An upper limit on freely scanned couplings is not confirmation of END.

Atlas code D03. Return to Part 9: Phenomenology and public-data dossiers.

D04 — CERN contact-interaction test

TEST

Canonical statement

$$L_{EFT} \supset \frac{C}{\Lambda^2}\left(\acute{q}\,\gamma_\mu q\right)\left(\acute{\ell}\,\gamma^\mu \ell\right)$$

> **FIRST LOOK.** A high-mass spectrum can test a capacity-induced contact term only inside the EFT validity region.

Dependency. D01 and fixed C over Lambda squared.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. CERN Open Data and standard SMEFT methodology.

D04 working page — CERN contact-interaction test

Construction or derivation

Define parton-level operator, interference, running, event generation, detector selection, binning, and correlated systematic model.

Dimensions and normalization

The theory output, detector prediction, observed data, and nuisance parameters must use the same binning, units, and covariance convention.

Decisive calculation

Publish the fraction outside sqrt(s-hat) less than Lambda-valid and repeat under stricter truncations.

Adversarial edge case

Using events above the cutoff can create artificially strong limits.

> **FAILURE RULE.** The EFT interpretation is invalid if the accepted sample is dominated by energies outside the registered validity gate.

Atlas code D04. Return to Part 9: Phenomenology and public-data dossiers.

D05 — XENONnT recoil dossier

TEST

Canonical statement

$$\frac{dR}{dE_R} = \frac{\rho_\chi}{m_\chi m_T} \int_{v > v_{min}} d^3v \, v \, f(v) \frac{d\sigma_T}{dE_R}$$

> **FIRST LOOK.** A dark-sector excitation reaches XENONnT through a recoil spectrum, yield model, detector response, and likelihood.

Dependency. D01 and a stable END excitation.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. XENON public-data and recast products; XENON Collaboration (2025).

D05 working page — XENONnT recoil dossier

Construction or derivation

Declare halo assumptions, nuclear response, charge and light yields, efficiencies, backgrounds, and the official recast interface.

Dimensions and normalization

The theory output, detector prediction, observed data, and nuisance parameters must use the same binning, units, and covariance convention.

Decisive calculation

Reproduce a published benchmark and inject custom spectra before scanning the frozen END region.

Adversarial edge case

Yield uncertainty near threshold can dominate a light-particle interpretation.

> **FAILURE RULE.** END fails this channel if a parameter region required by another frozen prediction is excluded at the registered level.

Atlas code D05. Return to Part 9: Phenomenology and public-data dossiers.

D06 — DUNE oscillation programme

CONDITIONAL

Canonical statement

$$i\frac{d\nu}{dx}=\left[\frac{U\,diag\left(0,\Delta m_{21}^2,\Delta m_{31}^2\right)U^{\dagger}}{2E}+diag\left(a,0,0\right)+H_{END}\right)\nu$$

FIRST LOOK. DUNE remains a forecast or simulation route until the relevant public experimental data exist.

Dependency. D01 and a frozen neutrino Hamiltonian.

The result is valid only if the named construction gate is satisfied.

Primary literature route. Official DUNE publication and public-plot database.

D06 working page — DUNE oscillation programme

Construction or derivation

Use official flux, response, systematic, and analysis products; include near-detector constraints, migration, cross sections, and matter density.

Dimensions and normalization

The theory output, detector prediction, observed data, and nuisance parameters must use the same binning, units, and covariance convention.

Decisive calculation

Reproduce an official sensitivity under identical assumptions before inserting H_END.

Adversarial edge case

New physics can be degenerate with CP phase, mass ordering, or cross-section systematics.

> **FAILURE RULE.** Never label pseudo-data or sensitivity curves as validation with DUNE data.

Atlas code D06. Return to Part 9: Phenomenology and public-data dossiers.

D07 — DESI expansion and growth

TEST

Canonical statement

$$d = \left(D_M / r_d , D_H / r_d , f \sigma_8 \right)_z$$

FIRST LOOK. A frozen cosmology predicts a correlated distance-and-growth vector across redshift.

Dependency. K03-K08 and D01.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Official DESI DR1 and DR2 data and cosmology documentation.

D07 working page — DESI expansion and growth

Construction or derivation

Use official release products, covariance, fiducial corrections, window functions, and declared external datasets.

Dimensions and normalization

The theory output, detector prediction, observed data, and nuisance parameters must use the same binning, units, and covariance convention.

Decisive calculation

Reproduce the collaboration baseline before replacing the expansion and growth functions.

Adversarial edge case

Combining releases or tracers without cross-covariance double-counts information.

> **FAILURE RULE.** Report tension or exclusion without retuning the EQEF potential after viewing held-out redshift bins.

Atlas code D07. Return to Part 9: Phenomenology and public-data dossiers.

D08 — Planck CMB likelihood dossier

TEST

Canonical statement

$$-2\ln L_{CMB} = \left(C - \widehat{C}\right)^{T} \Sigma^{-1} \left(C - \widehat{C}\right) + \cdots$$

> **FIRST LOOK.** CMB evidence uses spectra, masks, foreground nuisance parameters, calibration, and released likelihood choices.

Dependency. K09 and D01.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Planck Legacy Archive, including official PR4 frequency maps.

D08 working page — Planck CMB likelihood dossier

Construction or derivation

Record map or likelihood release, multipole ranges, foreground model, beam and calibration priors, and low-ell treatment.

Dimensions and normalization

The theory output, detector prediction, observed data, and nuisance parameters must use the same binning, units, and covariance convention.

Decisive calculation

Recover an official baseline and parameter covariance before introducing END transfer functions.

Adversarial edge case

Compressed Gaussian summaries can fail for low multipoles or extended models.

> **FAILURE RULE.** Do not use Planck-derived parameters as both calibration inputs and predicted outputs.

Atlas code D08. Return to Part 9: Phenomenology and public-data dossiers.

D09 — Euclid imaging dossier

CONDITIONAL

Canonical statement

$$d = \left(\xi_{+}^{ij}(\theta), \xi_{-}^{ij}(\theta), n(z), \mathrm{calibration}\right)$$

> **FIRST LOOK.** Euclid imaging becomes a theory test only after the released data model and known issues enter the forward model.

Dependency. K10 and D01.

The result is valid only if the named construction gate is satisfied.

Primary literature route. ESA Euclid Q1 documentation, Q1 papers, Q2 release route, and archive access.

D09 working page — Euclid imaging dossier

Construction or derivation

Version catalogs, masks, photometric redshifts, point-spread-function corrections, shear calibration, selection, and covariance.

Dimensions and normalization

The theory output, detector prediction, observed data, and nuisance parameters must use the same binning, units, and covariance convention.

Decisive calculation

Validate on survey simulations and null tests before fitting cosmology or END parameters.

Adversarial edge case

Residual PSF or redshift bias can mimic scale-dependent growth.

> **FAILURE RULE.** Q1 or Q2 products must be named by actual release; forecasts cannot be relabeled as observations.

Atlas code D09. Return to Part 9: Phenomenology and public-data dossiers.

D10 — Transient timing test

TEST

Canonical statement

$$t_{obs} = t_{emit} + (1+z)\,\Delta t_{source} + \Delta t_{prop} + \Delta t_{inst}$$

> **FIRST LOOK.** Multi-messenger timing separates source, propagation, cosmological, and instrumental delays only probabilistically.

Dependency. D01 and a frozen propagation law.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Time-domain astronomy and multi-messenger methodology.

D10 working page — Transient timing test

Construction or derivation

Specify source-lag population, redshift, clock corrections, event association, selection, trials, and propagation coefficient.

Dimensions and normalization

The theory output, detector prediction, observed data, and nuisance parameters must use the same binning, units, and covariance convention.

Decisive calculation

Fit a calibration population and predict held-out transients across source classes and distances.

Adversarial edge case

Unknown intrinsic lags can dominate one celebrated event.

> **FAILURE RULE.** A propagation claim fails if its coefficient is not stable across source classes or requires event-by-event sign choices.

Atlas code D10. Return to Part 9: Phenomenology and public-data dossiers.

Part 10. Closure, inference, and reproducibility

This part contains 10 canonical dossiers. Equations have one home, dependencies are explicit, and later uses add consequences rather than repeat definitions.

A01 — Immutable parameter card

DEFINED

Canonical statement

$\Theta = (X, U, \Gamma, D, E_*, B, \theta, \mathrm{precision}, \mathrm{version})$

FIRST LOOK. One machine-readable card fixes the model before calibration and held-out comparison.

Dependency. all foundational definitions.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Reproducible-research and configuration-management standards.

A01 working page — Immutable parameter card

Construction or derivation

Record domains, algorithms, tolerances, boundary conditions, seeds, hashes, parameter sources, units, and allowed ranges.

Dimensions and normalization

Audit quantities are dimensionless where possible; hashes, versions, units, priors, and transformations remain explicit metadata.

Decisive calculation

Recreate the card in a clean environment and reproduce canonical checksums and benchmark outputs.

Adversarial edge case

A silent default or library-version change can alter a chaotic trajectory.

> **FAILURE RULE.** Any post-unblinding card change creates a new model version and cannot retain the old prediction label.

Atlas code A01. Return to Part 10: Closure, inference, and reproducibility.

A02 — Dependency DAG

DEFINED

Canonical statement

$$G_{dep}=\left(V_{claim},E_{uses}\right)$$

FIRST LOOK. Definitions, inputs, calibrations, derivations, and tests form a directed dependency graph.

Dependency. A01 and canonical claim identifiers.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Data provenance and build-system dependency methods.

A02 working page — Dependency DAG

Construction or derivation

Give every node a status and version; add an edge for each logical or numerical input; reject undeclared cycles.

Dimensions and normalization

Audit quantities are dimensionless where possible; hashes, versions, units, priors, and transformations remain explicit metadata.

Decisive calculation

Delete or perturb one upstream node and verify automated propagation of affected claims.

Adversarial edge case

A circular constant derivation may hide across several innocent-looking identities.

> **FAILURE RULE.** A claim cannot be DERIVED if its target or algebraic equivalent is an ancestor used to select the construction.

Atlas code A02. Return to Part 10: Closure, inference, and reproducibility.

A03 — Dimensional and circularity audit

TEST

Canonical statement

$A\theta = b, null(A)$ records free scales

> **FIRST LOOK.** Dimensional algebra detects inconsistency and shows which scales cannot be predicted from the available inputs.

Dependency. A02 and a unit registry.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Buckingham-Pi theorem and metrological standards.

A03 working page — Dimensional and circularity audit

Construction or derivation

Encode base-dimension exponents, solve the linear system, and trace each numerical value to unit definition, measurement, fit, or theory.

Dimensions and normalization

Audit quantities are dimensionless where possible; hashes, versions, units, priors, and transformations remain explicit metadata.

Decisive calculation

Run the audit automatically for every displayed equation and constant card.

Adversarial edge case

Natural units can conceal that c, hbar, or G entered as measured conversions.

> **FAILURE RULE.** A unit conversion or algebraic identity is never reported as a new prediction.

Atlas code A03. Return to Part 10: Closure, inference, and reproducibility.

A04 — Calibration and holdout partition

DEFINED

Canonical statement

$D_{cal} \cap D_{test} = \varnothing$

> **FIRST LOOK.** Inputs used to choose the model are separated from records used to test the frozen result.

Dependency. A01-A03 and a dataset registry.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Preregistration and predictive-model-assessment literature.

A04 working page — Calibration and holdout partition

Construction or derivation

Partition by preregistered event identifiers, release dates, sites, or regimes; record all exploratory access.

Dimensions and normalization

Audit quantities are dimensionless where possible; hashes, versions, units, priors, and transformations remain explicit metadata.

Decisive calculation

Simulate distribution shift and verify that the test remains interpretable under the registered split.

Adversarial edge case

Repeatedly checking the holdout leaks information even without explicit fitting.

> **FAILURE RULE.** Once the holdout influences a choice, it becomes calibration data and a fresh test set is required.

Atlas code A04. Return to Part 10: Closure, inference, and reproducibility.

A05 — Likelihood construction

DEFINED

Canonical statement

$$L(\theta,\eta)=\prod_b Pois(n_b|\mu_b(\theta,\eta))\prod_k \pi_k(\eta_k)$$

> **FIRST LOOK.** A likelihood encodes the sampling model, nuisance constraints, and correlations used for decisions.

Dependency. D01 and calibrated detector response.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Cowan et al. (2011); Chernoff (1954); Feldman and Cousins (1998).

A05 working page — Likelihood construction

Construction or derivation

Specify binning, event selection, parameter boundaries, constraint terms, correlations, and low-count treatment.

Dimensions and normalization

Audit quantities are dimensionless where possible; hashes, versions, units, priors, and transformations remain explicit metadata.

Decisive calculation

Generate pseudo-experiments and verify estimator bias, interval coverage, and test-statistic distribution.

Adversarial edge case

Asymptotic formulae can fail at boundaries, low counts, or weak identifiability.

> **FAILURE RULE.** Use simulation-calibrated inference whenever registered regularity checks fail.

Atlas code A05. Return to Part 10: Closure, inference, and reproducibility.

A06 — Injection and coverage campaign

TEST

Canonical statement

$$\Pr_{\theta_0}\left[\theta_0 \in I\left(D\right)\right] = 1 - \alpha$$

> **FIRST LOOK.** Synthetic truth tests the entire pipeline before real held-out data are interpreted.

Dependency. A05.

The statement becomes informative only under a frozen analysis and failure rule.

Primary literature route. Frequentist coverage and simulation-based calibration literature.

A06 working page — Injection and coverage campaign

Construction or derivation

Inject across parameter, nuisance, noise, graph, and detector regimes; preserve the same reconstruction and selection.

Dimensions and normalization

Audit quantities are dimensionless where possible; hashes, versions, units, priors, and transformations remain explicit metadata.

Decisive calculation

Report conditional and marginal coverage plus failure regions rather than one average number.

Adversarial edge case

Convenient injections drawn from the inference prior can miss adversarial corners.

> **FAILURE RULE.** No interval or upper limit is accepted until its coverage is demonstrated in the claimed regime.

Atlas code A06. Return to Part 10: Closure, inference, and reproducibility.

A07 — Nuisance provenance table

DEFINED

Canonical statement

$$\eta_k \leftarrow (\mathrm{source}, \mathrm{version}, \pi_k, \mathrm{correlation}, \mathrm{scope})$$

FIRST LOOK. Every nuisance parameter has a source, constraint, correlation, and domain of applicability.

Dependency. A05-A06.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Experimental-systematics and hierarchical-modeling practice.

A07 working page — Nuisance provenance table

Construction or derivation

Record whether each nuisance is measured, simulated, theoretical, or ad hoc; preserve covariance and transformation conventions.

Dimensions and normalization

Audit quantities are dimensionless where possible; hashes, versions, units, priors, and transformations remain explicit metadata.

Decisive calculation

Remove, widen, and shift nuisances in controlled stress tests and compare with influence diagnostics.

Adversarial edge case

Duplicated nuisance constraints can double-count auxiliary information.

> **FAILURE RULE.** A result is not reproducible if its nuisance model cannot be reconstructed from the archive.

Atlas code A07. Return to Part 10: Closure, inference, and reproducibility.

A08 — Preregistered stop rules

DEFINED

Canonical statement

$$\delta(D) \in \{\mathrm{retain}, \mathrm{constrain}, \mathrm{falsify}, \mathrm{inconclusive}\}$$

> **FIRST LOOK.** A test is decisive only when outcomes and allowed conclusions are fixed before unblinding.

Dependency. A04-A07.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Neyman-Pearson decision theory and preregistration practice.

A08 working page — Preregistered stop rules

Construction or derivation

Specify primary statistic, threshold, multiplicity, consistency checks, null-reporting rule, and handling of unforeseen data defects.

Dimensions and normalization

Audit quantities are dimensionless where possible; hashes, versions, units, priors, and transformations remain explicit metadata.

Decisive calculation

Apply the decision function to simulated nulls and alternatives and publish its operating characteristics.

Adversarial edge case

A post-hoc exception can rescue almost any model.

> **FAILURE RULE.** Changes after unblinding create a new exploratory analysis and cannot be called the original prediction.

Atlas code A08. Return to Part 10: Closure, inference, and reproducibility.

A09 — Theory-wide falsification matrix

DEFINED

Canonical statement

$F_{ij} = 1$ if test j excludes claim i

> **FIRST LOOK.** A matrix records which observation constrains which frozen claim and how failure propagates.

Dependency. all closure and data dossiers.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. Model criticism and falsification methodology.

A09 working page — Theory-wide falsification matrix

Construction or derivation

Map claims to coefficients, datasets, thresholds, alternatives, and downstream dependents; distinguish local retirement from programme-wide failure.

Dimensions and normalization

Audit quantities are dimensionless where possible; hashes, versions, units, priors, and transformations remain explicit metadata.

Decisive calculation

Inject hypothetical failures and verify the dependency DAG retires exactly the affected conclusions.

Adversarial edge case

Vague programme language can insulate every core statement from any result.

> **FAILURE RULE.** A scientific theory must contain at least one reachable observation that can retire a substantive frozen claim.

Atlas code A09. Return to Part 10: Closure, inference, and reproducibility.

A10 — Reproducibility archive

DEFINED

Canonical statement

$$R = (\text{data IDs}, \text{code hash}, \text{environment}, \Theta, \text{workflow}, \text{outputs})$$

> **FIRST LOOK.** Independent reproduction requires durable inputs, exact code, environment, workflow, and expected outputs.

Dependency. A01-A09.

This page fixes notation and scope; it does not claim that nature realizes the construction.

Primary literature route. FAIR principles and reproducible-computing standards.

A10 working page — Reproducibility archive

Construction or derivation

Archive legal redistributable data or stable identifiers, lock dependencies, store parameter cards, and provide one-command benchmark checks.

Dimensions and normalization

Audit quantities are dimensionless where possible; hashes, versions, units, priors, and transformations remain explicit metadata.

Decisive calculation

Rebuild in a clean environment and compare hashes, numerical tolerances, figures, tables, and decision output.

Adversarial edge case

A notebook that depends on mutable remote state is not a durable computational record.

> **FAILURE RULE.** A TESTED label is withheld until an independent group can execute the archived chain.

Atlas code A10. Return to Part 10: Closure, inference, and reproducibility.

Bibliographic and data spine

Entries are cited by stable identifier. A DOI is supplied when verified and applicable; official data services are cited by their stable route.

1. E. Noether, “Invariant Variation Problems,” Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen (1918). English translation DOI: 10.1080/00411457108231446.
2. J. E. Marsden and M. West, “Discrete mechanics and variational integrators,” Acta Numerica 10, 357-514 (2001). DOI: 10.1017/S096249290100006X.
3. F. R. K. Chung, Spectral Graph Theory, CBMS Regional Conference Series 92 (1997). ISBN: 978-0-8218-0315-8.
4. K. G. Wilson, “Confinement of quarks,” Physical Review D 10, 2445-2459 (1974). DOI: 10.1103/PhysRevD.10.2445.
5. P. H. Ginsparg and K. G. Wilson, “A remnant of chiral symmetry on the lattice,” Physical Review D 25, 2649-2657 (1982). DOI: 10.1103/PhysRevD.25.2649.
6. H. B. Nielsen and M. Ninomiya, “Absence of neutrinos on a lattice: I. Proof by homotopy theory,” Nuclear Physics B 185, 20-40 (1981). DOI: 10.1016/0550-3213(81)90361-8.
7. H. B. Nielsen and M. Ninomiya, “Absence of neutrinos on a lattice: II. Intuitive topological proof,” Nuclear Physics B 193, 173-194 (1981). DOI: 10.1016/0550-3213(81)90524-1.
8. H. Neuberger, “Exactly massless quarks on the lattice,” Physics Letters B 417, 141-144 (1998). DOI: 10.1016/S0370-2693(97)01368-3.
9. D. B. Kaplan, “A method for simulating chiral fermions on the lattice,” Physics Letters B 288, 342-347 (1992). DOI: 10.1016/0370-2693(92)91112-M.
10. T. Regge, “General relativity without coordinates,” Il Nuovo Cimento 19, 558-571 (1961). DOI: 10.1007/BF02733251.
11. D. V. Vassilevich, “Heat kernel expansion: user’s manual,” Physics Reports 388, 279-360 (2003). DOI: 10.1016/j.physrep.2003.09.002.
12. F. W. Hehl, P. von der Heyde, G. D. Kerlick, and J. M. Nester, “General relativity with spin and torsion,” Reviews of Modern Physics 48, 393-416 (1976). DOI: 10.1103/RevModPhys.48.393.
13. J. S. Bell, “On the Einstein Podolsky Rosen paradox,” Physics Physique Fizika 1, 195-200 (1964). DOI: 10.1103/PhysicsPhysiqueFizika.1.195.
14. J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, “Proposed experiment to test local hidden-variable theories,” Physical Review Letters 23, 880-884 (1969). DOI: 10.1103/PhysRevLett.23.880.
15. A. M. Gleason, “Measures on the closed subspaces of a Hilbert space,” Journal of Mathematics and Mechanics 6, 885-893 (1957). DOI: 10.1512/iumj.1957.6.56050.
16. S. Kochen and E. P. Specker, “The problem of hidden variables in quantum mechanics,” Journal of Mathematics and Mechanics 17, 59-87 (1967). DOI: 10.1512/iumj.1968.17.17004.
17. J. Ambjorn, J. Jurkiewicz, and R. Loll, “Spectral dimension of the universe,” Physical Review Letters 95, 171301 (2005). DOI: 10.1103/PhysRevLett.95.171301.

18. C. M. Will, "The confrontation between general relativity and experiment," Living Reviews in Relativity 17, 4 (2014). DOI: 10.12942/lrr-2014-4.
19. G. Cowan, K. Cranmer, E. Gross, and O. Vitells, "Asymptotic formulae for likelihood-based tests of new physics," European Physical Journal C 71, 1554 (2011). DOI: 10.1140/epjc/s10052-011-1554-0.
20. O. Vitells and E. Gross, "Estimating the significance of a signal in a multi-dimensional search," Astroparticle Physics 35, 230-234 (2011). DOI: 10.1016/j.astropartphys.2011.08.005.
21. G. J. Feldman and R. D. Cousins, "Unified approach to the classical statistical analysis of small signals," Physical Review D 57, 3873-3889 (1998). DOI: 10.1103/PhysRevD.57.3873.
22. H. Chernoff, "On the distribution of the likelihood ratio," Annals of Mathematical Statistics 25, 573-578 (1954). DOI: 10.1214/aoms/1177728725.
23. J. Neyman and E. S. Pearson, "On the problem of the most efficient tests of statistical hypotheses," Philosophical Transactions A 231, 289-337 (1933). DOI: 10.1098/rsta.1933.0009.
24. Gravitational Wave Open Science Center, read-only API v2 and official event releases: https://gwosc.org/api/.
25. CMS Collaboration, research-use 2016 proton-proton collision and simulation releases, CERN Open Data Portal: https://opendata.cern.ch/research/CMS.
26. ATLAS Collaboration, 13 TeV samples with at least two leptons, CERN Open Data Portal (2020). DOI: 10.7483/OPENDATA.ATLAS.GQ1W.I9VI.
27. XENON Collaboration, "First Search for Light Dark Matter in the Neutrino Fog with XENONnT," Physical Review Letters 134, 111802 (2025). DOI: 10.1103/PhysRevLett.134.111802.
28. XENON Collaboration public data and official likelihood/recast routes: https://xenonexperiment.org/public-data/.
29. DUNE Collaboration publication and public-plot database: https://dune-data.fnal.gov/dunepublicplot/.
30. DESI Collaboration official DR1 and DR2 data and cosmology documentation: https://data.desi.lbl.gov/doc/.
31. European Space Agency, Planck Legacy Archive, official mission products including PR4 frequency maps: https://pla.esac.esa.int/.
32. European Space Agency, Euclid Quick Data Release documentation, papers, known issues, and data access: https://www.cosmos.esa.int/web/euclid/data.
33. Planck Collaboration, "Planck 2018 results. VI. Cosmological parameters," Astronomy & Astrophysics 641, A6 (2020); correction 2021. DOI: 10.1051/0004-6361/201833910.
34. N. Metropolis et al., "Equation of state calculations by fast computing machines," Journal of Chemical Physics 21, 1087-1092 (1953). DOI: 10.1063/1.1699114.
35. S. Duane et al., "Hybrid Monte Carlo," Physics Letters B 195, 216-222 (1987). DOI: 10.1016/0370-2693(87)91197-X.
36. H. Akaike, "A new look at the statistical model identification," IEEE Transactions on Automatic Control 19, 716-723 (1974). DOI: 10.1109/TAC.1974.1100705.

37. G. Schwarz, “Estimating the dimension of a model,” Annals of Statistics 6, 461-464 (1978). DOI: 10.1214/aos/1176344136.
38. R. T. Rockafellar, Convex Analysis, Princeton University Press (1970). ISBN: 978-0-691-01586-6.
39. E. Hairer, C. Lubich, and G. Wanner, Geometric Numerical Integration, 2nd ed., Springer (2006). DOI: 10.1007/3-540-30666-8.
40. FAIR Principles: M. D. Wilkinson et al., Scientific Data 3, 160018 (2016). DOI: 10.1038/sdata.2016.18.
41. Jordan Ryan Evans, END/MNT public repository record, Zenodo. DOI: 10.5281/zenodo.15617243.
42. Jordan Ryan Evans, END/MNT public repository record, Zenodo. DOI: 10.5281/zenodo.17730781.

Release acceptance record

☐ One immutable microscopic parameter card exists.

☐ Every state variable has a domain, unit, and executable update.

☐ Graph evolution is permutation-invariant and locally computable.

☐ Capacity handling has a proved balance or declared dissipation.

☐ Benchmark proofs have independent numerical tests.

☐ Continuum claims use multi-resolution extrapolation and more than one graph family.

☐ Gauge and fermion sectors pass locality, index, spectrum, and anomaly checks.

☐ Quantum claims include preparation, outcome, joint-probability, and detector maps.

☐ Gravity claims include an operational metric, dynamics, coefficients, and PPN tests.

☐ EQEF claims include action, background, perturbations, and stability.

☐ Every constant has a dependency and circularity audit.

☐ Every experiment has a frozen forward model, nuisance table, likelihood, and stop rule.

☐ Null and falsified regions are published without post-unblinding relabeling.

Open boxes are not editorial defects; they are the precise boundary between the present research programme and a closed physical theory.