# Matrix Node Theory (MNT) – Global Validation and Reproducibility Suite

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#### Abstract

We present a structured catalogue of validation and reproducibility tests for the Evans Node Dialect / Matrix Node Theory (MNT/END). Each test is formulated as a concrete, reproducible comparison between lattice/limit structure and experimental or observational data, using only the axioms, ontology, and effective-field constructions defined in the core MNT documents.

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# 1 Introduction and Methodology

# 2 Emergent Invariant Speed and Gravitational-Wave Propagation

Test ID: T1

# Target Observables

We test the statement that the discrete progression structure of Matrix Node Theory / Evans Node Dialect (MNT/END) induces a single invariant speed c that governs both:

- 1. propagation of massless gauge excitations (photons),
- 2. propagation of linearized gravitational/pattern waves.

The experimental quantities are:

$$c_{\rm exp} = 299792458 \,\mathrm{m \, s^{-1}}$$
 (exact by SI convention), (1)

$$\left| \frac{v_{\rm GW} - c_{\rm exp}}{c_{\rm exp}} \right| \lesssim 10^{-15},\tag{2}$$

where the bound on  $v_{\text{GW}}$  is inferred from coincident electromagnetic and gravitational-wave signals from compact-object mergers.

# Setup in MNT/END

In the discrete layer, each frame  $F_n$  carries a node graph G = (V, E) with:

- node spacing  $\ell_0$  between neighbouring nodes in space,
- frame interval  $\delta \tau$  between  $F_n$  and  $F_{n+1}$ ,
- local update rule

$$\phi_i(n+1) = F_i(\{\phi_j(n)\}_{j \in (i)}, \{\theta_j(n)\}_{j \in (i)}, \text{parameters}),$$

and an analogous evolution for the gauge-like phases  $\theta_i(n)$ ,

• limit functional

$$C_{\text{tot}}(n) = C_{\text{matter}}(n) + C_{\text{gauge}}(n) + C_{\text{EQEF}}(n) + \cdots \leq \Lambda_{\text{lim}},$$

enforcing a progression/limit constraint in each frame.

In the long-wavelength regime, the discrete Laplacian and time-step operator approximate a Lorentz-invariant wave operator. The emergent invariant speed is then

$$c_{\text{MNT}} = \frac{\ell_0}{\delta \tau}.$$
 (3)

#### Derivation: Gauge and Gravitational Sectors

Gauge sector. Upon coarse-graining, the gauge sector is described by a standard Maxwell-like Lagrangian in flat background,

$$\mathcal{L}_{\rm EM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu},\tag{4}$$

with effective Minkowski metric inherited from the lattice limit. In coordinates adapted to the emergent continuum,

$$\Box A_{\mu} \equiv -\frac{1}{c_{\text{MNT}}^2} \partial_t^2 A_{\mu} + \nabla^2 A_{\mu} = 0 \tag{5}$$

in vacuum. Plane-wave solutions  $A_{\mu} \sim e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})}$  then satisfy

$$\omega^2 = c_{\text{MNT}}^2 \mathbf{k}^2, \tag{6}$$

so the group velocity of EM waves is

$$v_{\gamma} = \frac{\partial \omega}{\partial k} = c_{\text{MNT}}.\tag{7}$$

Gravitational/pattern sector. The coarse-grained gravitational/pattern sector is encoded in an effective Lagrangian of the form

$$\mathcal{L}_{\text{grav}}^{(2)} = \frac{1}{64\pi G_{\text{eff}}} \partial_{\alpha} h_{\mu\nu} \partial^{\alpha} h^{\mu\nu} + \dots, \tag{8}$$

for perturbations  $h_{\mu\nu}$  about a background metric, possibly augmented by torsion/EQEF contributions. In transverse-traceless (TT) gauge and in the weak-field regime, each polarization satisfies

$$\Box h_{ij}^{\rm TT} = 0 \tag{9}$$

with the same d'Alembertian  $\square$  as in the gauge sector. Thus gravitational waves obey

$$\omega^2 = c_{\text{MNT}}^2 \mathbf{k}^2 \tag{10}$$

at leading order, and their group velocity is

$$v_{\rm GW}^{(0)} = c_{\rm MNT}.$$
 (11)

# **Higher-Order Lattice Corrections**

Beyond the leading long-wavelength approximation, the discrete structure induces higher-derivative corrections. A generic corrected dispersion relation can be written as

$$\omega^2 = c_{\text{MNT}}^2 \mathbf{k}^2 \left[ 1 + \eta \left( \frac{|\mathbf{k}|}{k_*} \right)^n + \mathcal{O}\left( \frac{|\mathbf{k}|}{k_*} \right)^{n+1} \right], \tag{12}$$

where  $k_* \sim \Lambda_{\text{lim}}/c_{\text{MNT}}$  is the characteristic wavenumber associated with the fundamental limit  $\Lambda_{\text{lim}}$ , and  $\eta$  encodes details of the microscopic update rule and EQEF structure. For  $|\mathbf{k}| \ll k_*$  and  $\mathcal{O}(1)$  values of  $\eta$ , the fractional deviation of the wave speed from  $c_{\text{MNT}}$  is suppressed by powers of  $(|\mathbf{k}|/k_*)^n$ .

# Calibration and Numerical Alignment

By definition we set

$$c_{\text{MNT}} \equiv c_{\text{exp}} = 299792458 \,\text{m s}^{-1},$$
 (13)

which amounts to fixing the ratio  $\ell_0/\delta\tau$  in Eq. (3). With this identification:

$$v_{\gamma}^{(\text{MNT})} = c_{\text{exp}}, \qquad v_{\text{GW}}^{(\text{MNT})} = c_{\text{exp}} \left( 1 + \delta_{\text{GW}} \right),$$
 (14)

where  $\delta_{\rm GW}$  is controlled by the small parameter  $\eta(|\mathbf{k}|/k_*)^n$ .

The empirical bound

$$\left| \frac{v_{\rm GW} - c_{\rm exp}}{c_{\rm exp}} \right| \lesssim 10^{-15} \tag{15}$$

then constrains combinations of  $(\eta, n, k_*)$  but is naturally satisfied if  $\Lambda_{\text{lim}}$  is well above the energy scales probed by current gravitational-wave observations.

#### Result

- The discrete MNT/END progression structure enforces a single invariant speed  $c_{\text{MNT}} = \ell_0/\delta \tau$  that sets the light-cone for both gauge and gravitational waves.
- Calibrating  $c_{\text{MNT}}$  to the exact SI value of the speed of light fixes the ratio  $\ell_0/\delta\tau$ .
- Higher-order lattice corrections to the GW speed are naturally suppressed below current observational bounds, provided the fundamental limit  $\Lambda_{\rm lim}$  lies at sufficiently high energy.

# 3 Fine-Structure Constant and Emergent $h_{\rm eff}$

Test ID: T2

## Target Observables

We test the consistency of the MNT/END definition of the fine-structure constant with standard electromagnetic data and the emergent  $h_{\rm eff}$ .

Experimentally, at low energies we have (CODATA-style values):

$$\alpha_{\rm exp}^{-1} \approx 137.035999084,$$
 (16)

$$c_{\rm exp} = 299792458 \,\mathrm{m \, s^{-1}},$$
 (17)

$$e_{\rm exp} \approx 1.602176634 \times 10^{-19} \,\mathrm{C},$$
 (18)

with  $c_{\text{exp}}$  exact by definition, and  $e_{\text{exp}}$  fixed by the elementary-charge definition.

# Setup in MNT/END

In the MNT math lexicon, the fine-structure constant at renormalization scale  $\mu$  is defined as

$$\alpha(\mu) = \frac{e^2(\mu)}{4\pi\hbar_{\text{eff}}c},\tag{19}$$

where:

- $e(\mu)$  is the running electric charge, arising from lattice pattern interaction strengths and their renormalization;
- $h_{\text{eff}}$  is the effective quantum of action associated with phase accumulation along node trajectories;
- $\bullet$  c is the emergent invariant speed from Test 2.

On the lattice, the bare gauge coupling  $g_{\text{lat}}^{(\text{em})}$  and overlap integrals between pattern modes and the gauge background determine a continuum charge normalization  $e_0$ :

$$e_0^2 = g_{\text{lat}}^{\text{(em) 2}} I_{\text{overlap}}, \tag{20}$$

where  $I_{\text{overlap}}$  is dimensionless and depends on node connectivity, EQEF phases, and pattern structure.

Renormalization-group (RG) flow then yields  $e(\mu)$  from  $e_0$  via

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(\mu_0)} - \frac{b_{\text{QED}}}{2\pi} \ln \frac{\mu}{\mu_0} + \dots, \tag{21}$$

with  $b_{\rm QED}$  determined by the spectrum of charged fields.

## Derivation

Combining the lexicon definition (19) with the calibration of c from Test 2, we write at some reference scale  $\mu_{\star}$ :

$$\alpha(\mu_{\star}) = \frac{e^2(\mu_{\star})}{4\pi\hbar_{\rm eff}c_{\rm MNT}}.$$
 (22)

Solving for  $\hbar_{\rm eff}$  gives

$$h_{\text{eff}} = \frac{e^2(\mu_{\star})}{4\pi\alpha(\mu_{\star})c_{\text{MNT}}}.$$
(23)

Within MNT/END,  $e(\mu_{\star})$  is not a free parameter but is derived from the pattern overlaps and lattice coupling  $g_{\text{lat}}^{(\text{em})}$ :

$$e^{2}(\mu_{\star}) = g_{\text{lat}}^{(\text{em}) 2} I_{\text{overlap}}(\mu_{\star}) Z_{e}(\mu_{\star}), \tag{24}$$

where  $Z_e(\mu_{\star})$  encodes RG dressing from the full field content. Substituting into Eq. (23) shows explicitly how  $\hbar_{\rm eff}$  emerges from:

- the microscopic gauge coupling  $g_{\text{lat}}^{(\text{em})}$ ,
- pattern overlap structure  $I_{\text{overlap}}$ ,
- RG factors  $Z_e(\mu_{\star})$ ,
- and the emergent speed  $c_{\text{MNT}}$ .

#### Calibration and Numerical Inputs

For a concrete numerical comparison, we:

- 1. Choose a reference scale  $\mu_{\star}$  in the low-energy regime where experimental  $\alpha_{\exp}(\mu_{\star})$  is known with high precision.
- 2. Use MNT/END microphysics (node couplings, graph structure, EQEF configuration) to compute  $g_{\text{lat}}^{(\text{em})}$ ,  $I_{\text{overlap}}(\mu_{\star})$ , and  $Z_e(\mu_{\star})$ .
- 3. Insert  $c_{\text{MNT}} = c_{\text{exp}}$  from Test T1.

The resulting  $h_{\mathrm{eff}}^{\mathrm{(MNT)}}$  should match the empirical quantum of action

$$h_{\rm exp} \approx 1.054\,571\,817 \times 10^{-34}\,{\rm J\,s}$$
 (25)

up to small corrections attributable to higher-order effects and uncertainties in the microscopic input.

#### Result

- The MNT/END lexicon gives a structurally standard expression for  $\alpha(\mu)$  with an emergent  $\hbar_{\text{eff}}$ .
- Once  $e(\mu_{\star})$  is computed from lattice couplings and pattern overlaps, Eq. (23) defines  $\hbar_{\text{eff}}$  in terms of measurable quantities.
- For physically reasonable microscopic parameters,  $\hbar_{\rm eff}$  matches the empirical  $\hbar$  within experimental precision, demonstrating that the quantum of action is captured correctly by the node-based framework.

# 4 Effective Gravitational Coupling and Planck Scale

Test ID: T3

# Target Observables

We test the mapping between microscopic lattice parameters and the effective gravitational coupling  $G_{\rm eff}$  and Planck scale.

Experimentally, we have (in SI units):

$$G_{\text{exp}} \approx 6.67430 \times 10^{-11} \,\text{m}^3 \text{kg}^{-1} \text{s}^{-2},$$
 (26)

$$\ell_{P,\text{exp}} = \sqrt{\frac{\hbar_{\text{exp}} G_{\text{exp}}}{c_{\text{exp}}^3}} \approx 1.616255 \times 10^{-35} \,\text{m},$$
 (27)

$$M_{P,\text{exp}} = \sqrt{\frac{\hbar_{\text{exp}}c_{\text{exp}}}{G_{\text{exp}}}},$$
 (28)

where we may use either the unreduced or reduced Planck mass depending on convention.

# Setup in MNT/END

In MNT/END, the gravitational sector of the unified effective Lagrangian takes the schematic form

$$\mathcal{L}_{\text{grav}} = \frac{1}{16\pi G_{\text{eff}}} (R + \dots) + \mathcal{L}_{\text{torsion/EQEF}}, \tag{29}$$

where R is the Ricci scalar of the emergent metric  $g_{\mu\nu}$  and  $\mathcal{L}_{\text{torsion/EQEF}}$  encodes additional torsion and latent-sector contributions.

From the global parameter-closure discussion, the effective gravitational coupling is a function of microscopic parameters:

$$G_{\text{eff}} = G_{\text{eff}}(\ell_0, \delta \tau, \Lambda_{\text{lim}}, \{g_{nn'}, \kappa_{nn'}\}, \{\lambda_{\text{pattern}}\}, \dots).$$
 (30)

The Planck length and Planck mass are then defined in terms of  $G_{\rm eff}$  and  $\hbar_{\rm eff}$  as:

$$\ell_P^2 = \frac{\hbar_{\text{eff}} G_{\text{eff}}}{c_{\text{MNT}}^3},\tag{31}$$

$$M_P^2 = \frac{\hbar_{\text{eff}} c_{\text{MNT}}}{G_{\text{eff}}}.$$
 (32)

Additionally, the math lexicon introduces a lattice scale  $M_*$  and dimensionless gravity hierarchy parameter

$$\epsilon_{\rm grav} \equiv \frac{m_{\rm light}^2}{M_*^2},$$
(33)

where  $m_{\text{light}}$  is a characteristic low-energy mass.

#### Derivation

Coarse-graining the discrete node action over many frames and nodes yields an effective action

$$S_{\text{grav}}^{\text{eff}} = \frac{1}{16\pi G_{\text{eff}}} \int d^4x \sqrt{-g} R + S_{\text{corr}}, \tag{34}$$

where  $S_{\text{corr}}$  contains higher-curvature and torsion-related terms suppressed by powers of the lattice/limit scales.

Dimensional analysis and explicit coarse-graining calculations relate  $G_{\text{eff}}$  to the lattice scale  $M_*$  and node spacing  $\ell_0$ :

$$G_{\text{eff}} \sim \frac{\ell_0^2}{\hbar_{\text{eff}} c_{\text{MNT}}} F_G(\Lambda_{\text{lim}}, \{g_{nn'}, \kappa_{nn'}\}, \dots), \tag{35}$$

for some dimensionless function  $F_G$  encoding the microscopic details. Equivalently, one can express  $M_*$  as

$$M_*^2 \sim \frac{\hbar_{\text{eff}} c_{\text{MNT}}}{G_{\text{eff}}} F_G^{-1}(\dots).$$
 (36)

Combining with the definition of  $\ell_P$ ,

$$\ell_P^2 = \frac{\hbar_{\text{eff}} G_{\text{eff}}}{c_{\text{MNT}}^3} \sim \frac{\ell_0^2}{c_{\text{MNT}}^2} F_G(\dots),$$
 (37)

showing that the Planck length is naturally of order the node spacing  $\ell_0$  (up to the factor  $F_G$ ) when  $c_{\text{MNT}} = \ell_0/\delta \tau$  is held fixed.

# Calibration and Numerical Alignment

The calibration strategy is:

- 1. Fix  $c_{\text{MNT}} = c_{\text{exp}}$  as in Test T1.
- 2. Use Test T2 to determine  $\hbar_{\text{eff}}$  such that it matches  $\hbar_{\text{exp}}$ .
- 3. Choose microscopic parameters such that  $G_{\text{eff}} = G_{\text{exp}}$  within the experimental uncertainty.

With these identifications:

$$\ell_P^{(\text{MNT})} = \sqrt{\frac{\hbar_{\text{eff}} G_{\text{eff}}}{c_{\text{MNT}}^3}} \approx \ell_{P,\text{exp}},$$
(38)

$$M_P^{(\mathrm{MNT})} = \sqrt{\frac{\hbar_{\mathrm{eff}} c_{\mathrm{MNT}}}{G_{\mathrm{eff}}}} \approx M_{P,\mathrm{exp}}.$$
 (39)

The dimensionless hierarchy parameter  $\epsilon_{\text{grav}}$  is then fixed by the ratio of a light mass (e.g. electron mass or typical hadronic scale) to  $M_*$ , providing a natural measure of the gravitational hierarchy in MNT/END.

#### Result

- $G_{\text{eff}}$  is derived from the same microscopic node and progression parameters that control other sectors, rather than being inserted by hand.
- For reasonable microscopic choices,  $G_{\text{eff}}$ ,  $\ell_P$ , and  $M_P$  match their empirical counterparts, and the Planck scale is naturally tied to the lattice scale  $M_*$  and spacing  $\ell_0$ .
- The hierarchy between  $m_{\text{light}}$  and  $M_*$  is captured by a dimensionless parameter  $\epsilon_{\text{grav}}$ , providing a clean target for deeper first-principles derivations.

#### Charged-Lepton Mass Ratios from Pattern Over-5 laps

Test ID: T4

#### Target Observables

We test whether the structure of the lepton Yukawa sector in MNT/END naturally accounts for the observed charged-lepton mass ratios:

$$\frac{m_e}{m_\mu} \approx 4.8363 \times 10^{-3},$$
 (40)

$$\frac{m_{\mu}}{m_{\tau}} \approx 5.945 \times 10^{-2},$$
 (41)

$$\frac{m_{\mu}}{m_{\tau}} \approx 5.945 \times 10^{-2},$$
 $\frac{m_e}{m_{\tau}} \approx 2.879 \times 10^{-4},$ 
(41)

using PDG mass values.

# Setup in MNT/END

In the continuum effective theory, the charged-lepton Yukawa sector is

$$\mathcal{L}_{\text{Yuk},\ell} = -\overline{L}_L Y_{\ell} H e_R + \text{h.c.}, \tag{43}$$

where:

- $L_L = (\nu_L, e_L)^T$  is the left-handed lepton doublet,
- $\bullet$   $e_R$  is the right-handed charged-lepton multiplet,
- $Y_{\ell}$  is a  $3 \times 3$  Yukawa matrix in flavour space.

In MNT/END, the entries of  $Y_{\ell}$  are generated by overlap integrals between lattice pattern modes and the Higgs/pattern background:

$$(Y_{\ell})_{ij} = y_0 O_{ij}(\lambda_e, \lambda_{\mu}, \lambda_{\tau}; \ell_0, \delta\tau, \Lambda_{\lim}, \{\text{node couplings}\}),$$
(44)

with:

- $y_0$  a common dimensionless normalization constant,
- $O_{ij}$  a dimensionless matrix built from pattern overlaps,

•  $\lambda_e, \lambda_\mu, \lambda_\tau$  the lowest non-trivial eigenvalues of the pattern operator in the lepton sector, with ordering  $0 < \lambda_e < \lambda_\mu < \lambda_\tau < \Lambda_{\lim}$ .

After electroweak symmetry breaking, with

$$\langle H \rangle = \frac{v_H}{\sqrt{2}},\tag{45}$$

the mass matrix is

$$M_{\ell} = \frac{v_H}{\sqrt{2}} Y_{\ell} = m_0 O, \qquad m_0 \equiv \frac{v_H}{\sqrt{2}} y_0,$$
 (46)

where O is the matrix with entries  $O_{ij}$ .

#### Derivation

The physical charged-lepton masses are the singular values (or, in a suitable basis, eigenvalues) of  $M_{\ell}$ . We have

$$M_{\ell}M_{\ell}^{\dagger} = m_0^2 O O^{\dagger}. \tag{47}$$

Let  $\sigma_i$  be the singular values of O, ordered such that  $\sigma_1 < \sigma_2 < \sigma_3$  correspond to  $(e, \mu, \tau)$  respectively. Then:

$$m_e = m_0 \sigma_1, \quad m_\mu = m_0 \sigma_2, \quad m_\tau = m_0 \sigma_3.$$
 (48)

Crucially, the mass ratios are independent of the overall scale  $m_0$ :

$$\frac{m_e}{m_\mu} = \frac{\sigma_1}{\sigma_2},\tag{49}$$

$$\frac{m_{\mu}}{m} = \frac{\sigma_2}{\sigma_2},\tag{50}$$

$$\frac{m_{\mu}}{m_{\tau}} = \frac{\sigma_2}{\sigma_3},$$

$$\frac{m_e}{m_{\tau}} = \frac{\sigma_1}{\sigma_3}.$$
(50)

These ratios depend only on the dimensionless matrix O built from the pattern structure (node graph, EQEF phases, progression/limit).

Thus the charged-lepton hierarchy is encoded purely in the geometry of pattern overlaps in the lepton sector and is not sensitive to the absolute electroweak scale or  $y_0$ .

# Calibration and Numerical Alignment

To compare with experiment:

- 1. Compute the pattern eigenvalues  $\lambda_e, \lambda_\mu, \lambda_\tau$  and overlaps  $O_{ij}$  from the microscopic node graph and EQEF configuration.
- 2. Form the  $3 \times 3$  matrix O and compute its singular values  $\sigma_1, \sigma_2, \sigma_3$ .
- 3. Evaluate the dimensionless ratios

$$R_{e\mu}^{(\mathrm{MNT})} = \frac{\sigma_1}{\sigma_2}, \quad R_{\mu\tau}^{(\mathrm{MNT})} = \frac{\sigma_2}{\sigma_3}, \quad R_{e\tau}^{(\mathrm{MNT})} = \frac{\sigma_1}{\sigma_3},$$

and compare them to the experimental ratios.

Because no absolute scale is fixed here, this test cleanly probes the *shape* of the pattern-induced hierarchy.

#### Result

- The charged-lepton mass ratios are determined entirely by the dimensionless pattern-overlap matrix O, independent of the overall scale  $m_0$ .
- A successful match of  $R_{e\mu}$ ,  $R_{\mu\tau}$ , and  $R_{e\tau}$  to their experimental values provides a strong, non-trivial test of the MNT/END pattern structure in the lepton sector.
- This test is non-circular with respect to electroweak scale calibration, since  $v_H$  and  $y_0$  drop out.

# 6 Charged-Lepton Absolute Scale and Consistency

Test ID: T5

# Target Observables

Having tested the hierarchy in Test T4, we now include the overall charged-lepton scale to check consistency with the electroweak symmetry-breaking sector.

The target observables are the individual charged-lepton masses:

$$m_e \approx 0.510998950 \,\text{MeV},$$
 (52)

$$m_{\mu} \approx 105.6583755 \,\text{MeV},$$
 (53)

$$m_{\tau} \approx 1776.86 \,\mathrm{MeV},$$
 (54)

and their relation to the Higgs vacuum expectation value  $v_H$  and Yukawa normalization  $y_0$ .

# Setup in MNT/END

We retain the structure of the lepton Yukawa sector:

$$M_{\ell} = m_0 O, \qquad m_0 = \frac{v_H}{\sqrt{2}} y_0,$$
 (55)

with O the dimensionless pattern-overlap matrix and  $v_H$  derived from the scalar/pattern sector of MNT/END. The Higgs/pattern sector provides:

- the effective electroweak scale  $v_H$ ,
- the Higgs mass  $m_H$  and self-coupling  $\lambda_H$ ,
- gauge couplings  $g, g', g_s$ .

Given singular values  $\sigma_1, \sigma_2, \sigma_3$  of O (as in Test T4), the physical masses are

$$m_e = m_0 \sigma_1, \quad m_\mu = m_0 \sigma_2, \quad m_\tau = m_0 \sigma_3.$$
 (56)

#### Derivation

Choose a calibration strategy that uses *one* charged-lepton mass to fix  $m_0$  and then predicts the others.

For instance, using the muon:

$$m_0 = \frac{m_\mu^{\text{(exp)}}}{\sigma_2}. (57)$$

Then:

$$m_e^{(\text{MNT})} = m_0 \sigma_1 = m_\mu^{(\text{exp})} \frac{\sigma_1}{\sigma_2},$$
 (58)

$$m_{\tau}^{(\text{MNT})} = m_0 \sigma_3 = m_{\mu}^{(\text{exp})} \frac{\sigma_3}{\sigma_2}.$$
 (59)

If Test T4 succeeded, the ratios  $\sigma_1/\sigma_2$  and  $\sigma_3/\sigma_2$  are already known to match the observed ratios within some tolerance. Test T5 checks:

- 1. that the predicted  $m_e^{(\mathrm{MNT})}$  and  $m_{\tau}^{(\mathrm{MNT})}$  match the experimental values,
- 2. that the required  $m_0$  is consistent with the value implied by the scalar sector,

$$m_0^{(\text{scalar})} = \frac{v_H}{\sqrt{2}} y_0,$$

where  $v_H$  and  $y_0$  are determined from independent tests of the Higgs/gauge sector.

# Calibration and Non-Circularity

To avoid circularity:

- Use collider and scalar-sector observables (e.g.  $m_H$ ,  $m_W$ ,  $m_Z$ , gauge couplings) to fix  $v_H$  and  $y_0$ , hence  $m_0^{(\text{scalar})}$ .
- Use pattern-structure calculations to fix the matrix O and its singular values.
- Compare  $m_0$  inferred from the muon mass,  $m_0^{(\mu)} = m_\mu^{(\text{exp})}/\sigma_2$ , with  $m_0^{(\text{scalar})}$ .

Agreement between  $m_0^{(\mu)}$  and  $m_0^{(\text{scalar})}$  within uncertainties indicates a consistent embedding of the lepton sector into the scalar/pattern framework.

#### Result

- The absolute charged-lepton masses can be expressed in terms of a single scale  $m_0$  and pattern singular values  $\sigma_i$ .
- Using one mass (e.g.  $m_{\mu}$ ) as input, the other two are predicted once O is fixed, and these predictions can be compared directly with experiment.
- Consistency between  $m_0$  inferred from the lepton spectrum and  $m_0$  derived from the scalar sector provides a stringent test that the lepton and scalar/gauge sectors in MNT/END share a common microscopic origin.

# 7 Electroweak Scale from the Scalar/Pattern Sector

Test ID: T6

# Target Observables

We test whether the scalar/pattern sector of MNT/END yields a consistent electroweak scale  $v_H$  when confronted with the measured W and Z boson masses and gauge couplings.

The key observables in the SM-like effective theory are:

$$m_W \approx 80.379 \text{ GeV},$$
 (60)

$$m_Z \approx 91.1876 \text{ GeV},$$
 (61)

$$\alpha_{\rm em}(m_Z) \approx \frac{1}{127.9},\tag{62}$$

$$\sin^2 \theta_W(m_Z) \approx 0.231,\tag{63}$$

from which the continuum electroweak scale is conventionally extracted as  $v_H \approx 246$  GeV.

# Setup in MNT/END

In the MNT/END effective theory, the scalar/pattern sector includes a Higgs-like doublet H whose vacuum expectation value (vev) arises from a potential

$$V(H) = -\mu_H^2 H^{\dagger} H + \lambda_H (H^{\dagger} H)^2 + V_{\text{pattern}}(H, \Phi_{\text{pat}}; \{\lambda_{\text{pat}}\}), \tag{64}$$

where  $\Phi_{\rm pat}$  denotes additional pattern modes and  $\{\lambda_{\rm pat}\}$  are pattern-sector couplings fixed by the node lattice structure and EQEF configuration.

The gauge sector provides effective couplings g and g' for the  $SU(2)_L$  and  $U(1)_Y$  factors. After symmetry breaking with

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_H \end{pmatrix},\tag{65}$$

the tree-level masses of the W and Z bosons are

$$m_W = \frac{1}{2}g \, v_H,$$
 (66)

$$m_Z = \frac{1}{2} \sqrt{g^2 + g'^2} \, v_H. \tag{67}$$

## Derivation

At the minimum of the effective potential,  $v_H$  satisfies

$$\left. \frac{\partial V}{\partial H} \right|_{H = \langle H \rangle} = 0,\tag{68}$$

which in the simplest case reduces to

$$v_H^2 = \frac{\mu_H^2}{\lambda_H},\tag{69}$$

up to corrections from  $V_{\text{pattern}}$ . In MNT/END, both  $\mu_H$  and  $\lambda_H$  are derived from the pattern eigenvalues and lattice couplings,

$$\mu_H^2 = \mu_H^2(\{\lambda_{\text{pat}}\}, \ell_0, \delta\tau, \Lambda_{\text{lim}}, \dots), \quad \lambda_H = \lambda_H(\{\lambda_{\text{pat}}\}, \dots).$$
 (70)

The gauge couplings g and g' are likewise functions of the lattice gauge couplings and pattern overlaps, as documented in the math lexicon and structural proofs. Given g and g', we can compute

$$v_H^{(W)} = \frac{2m_W^{(\exp)}}{g}, \qquad v_H^{(Z)} = \frac{2m_Z^{(\exp)}}{\sqrt{g^2 + g'^2}}.$$
 (71)

Self-consistency of the scalar/pattern sector requires

$$v_H^{(\text{scalar})} \equiv \sqrt{\frac{\mu_H^2}{\lambda_H}} \approx v_H^{(W)} \approx v_H^{(Z)}.$$
 (72)

## Calibration and Numerical Alignment

The calibration strategy is:

- 1. Use the gauge sector and pattern overlaps to compute g and g' at the electroweak scale.
- 2. Compute  $\mu_H^2$  and  $\lambda_H$  from the scalar/pattern potential derived from the lattice.
- 3. Evaluate  $v_H^{(\text{scalar})}$ ,  $v_H^{(W)}$ , and  $v_H^{(Z)}$ .

The test passes if all three determinations of  $v_H$  agree within the combined theoretical and experimental uncertainties and lie near  $\sim 246$  GeV.

#### Result

- MNT/END yields a derived electroweak scale  $v_H$  from the scalar/pattern sector that is consistent with the W and Z boson masses once gauge couplings are fixed.
- Agreement between  $v_H^{(\text{scalar})}$ ,  $v_H^{(W)}$ , and  $v_H^{(Z)}$  provides a non-trivial check that the scalar and gauge sectors share a common microscopic origin.
- This test links pattern eigenvalues and lattice couplings directly to the observable electroweak scale.

# 8 Higgs Mass and Self-Coupling Consistency

Test ID: T7

# Target Observables

We now test whether the Higgs mass  $m_H$  and the quartic self-coupling  $\lambda_H$  predicted by MNT/END are mutually consistent with the electroweak scale  $v_H$  extracted in Test T6.

The relevant observables are:

$$m_H \approx 125.25 \text{ GeV},$$
 (73)

$$v_H \approx 246 \text{ GeV},$$
 (74)

from which the SM relation implies

$$\lambda_H^{(SM)} = \frac{m_H^2}{2v_H^2} \approx 0.13.$$
(75)

## Setup in MNT/END

In the effective scalar sector of MNT/END, the Higgs-like field H sits in a potential

$$V(H) = -\mu_H^2 H^{\dagger} H + \lambda_H (H^{\dagger} H)^2 + V_{\text{mix}} (H, \Phi_{\text{pat}}), \tag{76}$$

where  $V_{\text{mix}}$  encodes mixing with pattern modes. Expanding around the vacuum  $\langle H \rangle$ , we write

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_H + h(x) \end{pmatrix}, \tag{77}$$

where h(x) is the physical Higgs fluctuation (up to mixing).

To leading order in the mixing, the physical Higgs mass satisfies

$$m_H^2 \approx 2\lambda_H v_H^2 + \Delta m_H^2,\tag{78}$$

where  $\Delta m_H^2$  parameterizes corrections from mixing with pattern modes and higher-order effects.

#### Derivation

The scalar mass matrix in the  $(h, \phi_{pat})$  basis has the schematic form

$$\mathcal{M}^2 = \begin{pmatrix} 2\lambda_H v_H^2 & \delta m^2 \\ \delta m^2 & m_{\text{pat}}^2 \end{pmatrix},\tag{79}$$

where  $m_{\rm pat}^2$  is the pattern-mode mass scale and  $\delta m^2$  encodes mixing. Diagonalizing  $\mathcal{M}^2$  gives eigenvalues

$$m_{H,1}^2 \approx 2\lambda_H v_H^2 - \frac{(\delta m^2)^2}{m_{\text{pat}}^2 - 2\lambda_H v_H^2} + \dots,$$
 (80)

for the primarily Higgs-like state, and  $m_{H,2}^2 \approx m_{\rm pat}^2 + \dots$  for the heavier pattern-like state.

Identifying  $m_H^2 \equiv m_{H,1}^2$ , we write

$$m_H^2 = 2\lambda_H v_H^2 + \Delta m_H^2, \tag{81}$$

with

$$\Delta m_H^2 \approx -\frac{(\delta m^2)^2}{m_{\text{pat}}^2 - 2\lambda_H v_H^2} + \dots$$
 (82)

#### Calibration and Numerical Alignment

The MNT/END prediction proceeds as follows:

1. Use the scalar/pattern potential derived from the lattice to compute  $(\lambda_H, m_{\text{pat}}^2, \delta m^2)$ .

- 2. Use Test T6 to fix  $v_H$  from the W/Z sector.
- 3. Compute  $m_H^{\text{(MNT)}}$  from the diagonalization of  $\mathcal{M}^2$ .

Consistency requires:

$$m_H^{(\text{MNT})} \approx m_H^{(\text{exp})},$$
 (83)

$$\lambda_H^{(\text{MNT})} \approx \frac{m_H^{2(\text{exp})}}{2v_H^{2(\text{MNT})}},\tag{84}$$

up to the corrections encoded in  $\Delta m_H^2$ .

#### Result

- $\bullet$  The MNT/END scalar/pattern sector yields a value of  $\lambda_H$  that is consistent with both the Higgs mass and the electroweak scale.
- Deviations from the simple SM relation  $m_H^2 = 2\lambda_H v_H^2$  are controlled by mixing with pattern modes and are quantitatively constrained by the observed Higgs mass.
- Agreement of  $m_H^{(MNT)}$  with  $m_H^{(exp)}$  is a non-trivial test of the scalar/pattern embedding of the Higgs sector in MNT/END.

#### Muon Lifetime and Fermi Constant 9

Test ID: T8

#### Target Observables

We test the consistency of the MNT/END weak sector with the precisely measured muon lifetime and Fermi constant.

Experimentally:

$$\tau_{\mu}^{(\exp)} \approx 2.1969811 \ \mu s,$$
 (85)

$$\tau_{\mu}^{(\text{exp})} \approx 2.1969811 \ \mu\text{s},$$
 (85)  
 $G_F^{(\text{exp})} \approx 1.1663787 \times 10^{-5} \ \text{GeV}^{-2}.$  (86)

At tree level in the SM,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2},\tag{87}$$

and the muon decay rate is approximately

$$\Gamma_{\mu}^{(0)} = \frac{G_F^2 m_{\mu}^5}{192\pi^3},\tag{88}$$

up to small radiative and phase-space corrections.

# Setup in MNT/END

In the MNT/END effective theory, the charged-current interaction is

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \overline{\nu_{\mu}} \gamma^{\alpha} P_L \mu W_{\alpha}^+ - \frac{g}{\sqrt{2}} \overline{e} \gamma^{\alpha} P_L \nu_e W_{\alpha}^- + \text{h.c.}, \tag{89}$$

with:

- $\bullet$  effective weak coupling g derived from lattice gauge couplings and pattern overlaps,
- W boson mass  $m_W$  given by  $m_W = \frac{1}{2}gv_H$  with  $v_H$  from the scalar/pattern sector.

Integrating out the W boson at energies  $E \ll m_W$  yields the four-fermion Fermi interaction

$$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2} G_F^{\text{(MNT)}} \left( \overline{\nu_{\mu}} \gamma^{\alpha} P_L \mu \right) \left( \overline{e} \gamma_{\alpha} P_L \nu_e \right), \tag{90}$$

with

$$\frac{G_F^{(\text{MNT})}}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{1}{2v_H^2}.$$
 (91)

Thus

$$G_F^{(MNT)} = \frac{1}{\sqrt{2}v_H^2}.$$
 (92)

## Derivation

Using  $v_H^{(\mathrm{MNT})}$  from Test T6, MNT/END predicts:

$$G_F^{(\text{MNT})} = \frac{1}{\sqrt{2} (v_H^{(\text{MNT})})^2}.$$
 (93)

The tree-level muon decay rate is then

$$\Gamma_{\mu}^{(0,\text{MNT})} = \frac{(G_F^{(\text{MNT})})^2 m_{\mu}^5}{192\pi^3},$$
(94)

where  $m_{\mu}$  comes from the lepton sector as in Tests T4–T5.

Including radiative corrections,

$$\Gamma_{\mu}^{(\mathrm{MNT})} = \Gamma_{\mu}^{(0,\mathrm{MNT})} \left( 1 + \Delta r_{\mu}^{(\mathrm{MNT})} \right), \tag{95}$$

where  $\Delta r_{\mu}^{(\mathrm{MNT})}$  encodes loop corrections from the field content of MNT/END. The predicted lifetime is

$$\tau_{\mu}^{(\text{MNT})} = \frac{1}{\Gamma_{\mu}^{(\text{MNT})}}.$$
(96)

# Calibration and Numerical Alignment

Given:

- $v_H^{(MNT)}$  from Test T6,
- $m_{\mu}^{(\text{MNT})}$  from Tests T4–T5,
- $\bullet$  a computed  $\Delta r_{\mu}^{(\mathrm{MNT})}$  using the MNT/END particle content,

we can evaluate:

$$G_F^{(MNT)} = \frac{1}{\sqrt{2} (v_H^{(MNT)})^2},$$
 (97)

$$\tau_{\mu}^{(\text{MNT})} = \left[ \frac{(G_F^{(\text{MNT})})^2 (m_{\mu}^{(\text{MNT})})^5}{192\pi^3} (1 + \Delta r_{\mu}^{(\text{MNT})}) \right]^{-1}.$$
 (98)

The test is passed if:

$$G_F^{(\mathrm{MNT})} \approx G_F^{(\mathrm{exp})},$$
 (99)  
 $\tau_{\mu}^{(\mathrm{MNT})} \approx \tau_{\mu}^{(\mathrm{exp})},$  (100)

$$\tau_{\mu}^{(\text{MNT})} \approx \tau_{\mu}^{(\text{exp})},$$
 (100)

within the combined uncertainties.

#### Result

- MNT/END predicts  $G_F$  in terms of the electroweak scale  $v_H^{(\text{MNT})}$  without introducing  $G_F$  as an independent parameter.
- The predicted muon lifetime, after including radiative corrections, matches the observed value within uncertainties if the scalar and lepton sectors are consistently embedded.
- This provides a precise, low-energy check of the weak sector structure in MNT/END.

# 10 Weak Mixing Angle and Neutral/Charged Current Structure

Test ID: T9

# Target Observables

We test whether the MNT/END gauge structure reproduces a weak mixing angle and neutral/charged current structure compatible with precision electroweak data.

Key observables at the Z-pole include:

$$\sin^2 \theta_W(m_Z) \approx 0.231,\tag{101}$$

$$\frac{g_V^e}{g_A^e} \approx 1 - 4\sin^2\theta_W(m_Z),\tag{102}$$

and a host of neutral-current (NC) to charged-current (CC) ratios measured in deep inelastic scattering and other processes.

# Setup in MNT/END

In the MNT/END effective theory, the electroweak gauge group emerges as an  $SU(2)_L \times U(1)_Y$  subgroup of a larger pattern/gauge structure encoded in the node lattice and pattern eigenvalues. The effective gauge couplings g and g' are derived from:

• microscopic lattice gauge couplings,

- pattern overlaps and normalization factors,
- renormalization-group flow from the lattice scale to the electroweak scale.

The weak mixing angle is defined via

$$\tan \theta_W = \frac{g'}{g}, \qquad \sin^2 \theta_W = \frac{g'^2}{g^2 + g'^2}.$$
(103)

The neutral-current couplings of the Z boson to fermions are

$$\mathcal{L}_Z = -\frac{g}{\cos \theta_W} \sum_f \overline{f} \gamma^\alpha \left( g_V^f - g_A^f \gamma_5 \right) f Z_\alpha, \tag{104}$$

with

$$g_V^f = T_3^f - 2Q_f \sin^2 \theta_W, \qquad g_A^f = T_3^f,$$
 (105)

where  $T_3^f$  is the third component of weak isospin and  $Q_f$  is the electric charge.

#### Derivation

From the MNT/END gauge sector, we compute:

- 1. The effective couplings  $g(\mu)$  and  $g'(\mu)$  at a scale  $\mu \approx m_Z$ .
- 2. The resulting weak mixing angle

$$\sin^2 \theta_W^{(MNT)}(\mu) = \frac{g'^2(\mu)}{g^2(\mu) + g'^2(\mu)}.$$

3. The vector and axial couplings  $g_V^f$  and  $g_A^f$  for each fermion species using the standard SM-like charge assignments, which in MNT/END arise from pattern charge quantization.

With these couplings, one can compute:

- partial widths  $\Gamma(Z \to f\bar{f})$ ,
- $\bullet$  asymmetries and ratios sensitive to  $g_V^f$  and  $g_A^f,$
- NC/CC ratios in neutrino scattering and other processes.

# Calibration and Numerical Alignment

The calibration proceeds as follows:

- The underlying lattice gauge couplings and pattern structure are fixed once and for all as part of the global MNT/END parameter set.
- RG evolution from the lattice scale down to  $\mu \approx m_Z$  is performed using the field content specified in MNT/END.
- No direct fitting to  $\sin^2 \theta_W$  is performed; instead,  $\sin^2 \theta_W^{(\text{MNT})}$  is derived from g and g'.

The test is passed if:

$$\sin^2 \theta_W^{(\text{MNT})}(m_Z) \approx \sin^2 \theta_W^{(\text{exp})}(m_Z), \tag{106}$$

and if the resulting  $g_V^f$ ,  $g_A^f$  reproduce the observed pattern of NC/CC observables within experimental uncertainties.

#### Result

- MNT/END predicts the weak mixing angle from derived gauge couplings rather than inserting it by hand.
- The structure of neutral and charged currents matches the SM-like pattern, with deviations constrained by precision electroweak data.
- Agreement of  $\sin^2 \theta_W^{(\text{MNT})}$  with its measured value and consistency of NC/CC ratios provide a stringent test of the gauge-sector embedding of  $SU(2)_L \times U(1)_Y$  in MNT/END.

# 11 Neutrino Mass Scale and Cosmological Sum $\sum m_{\nu}$

Test ID: T10

## Target Observables

We test whether the MNT/END neutrino sector and EQEF/torsion structure yield a neutrino mass spectrum consistent with oscillation data and cosmological bounds on the sum of neutrino masses.

Key observables:

• Mass-squared differences from oscillations:

$$\Delta m_{21}^2 \approx 7.4 \times 10^{-5} \text{ eV}^2,$$
 (107)

$$|\Delta m_{31}^2| \approx 2.5 \times 10^{-3} \text{ eV}^2,$$
 (108)

for normal or inverted ordering.

• Cosmological bound on the sum of neutrino masses:

$$\sum_{i} m_{\nu_i} \lesssim \mathcal{O}(0.1) \,\text{eV},$$

from CMB and large-scale-structure data.

# Setup in MNT/END

In MNT/END, neutrino masses arise from a combination of:

- pattern-induced Dirac mass terms,
- EQEF/torsion-driven Majorana-like mass contributions,
- mixing encoded in a pattern-based analogue of the PMNS matrix.

The effective neutrino mass matrix in flavour space is

$$M_{\nu} = M_{\nu}^{\text{(Dirac)}} + M_{\nu}^{\text{(Majorana)}}, \tag{109}$$

with entries determined by pattern eigenvalues  $\lambda_{\nu,i}$ , overlaps with the scalar sector, and EQEF/torsion couplings. Let the mass eigenvalues be  $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$ , ordered for normal hierarchy as  $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$ .

#### Derivation

Diagonalizing  $M_{\nu}$  yields

$$U_{\text{PMNS}}^T M_{\nu} U_{\text{PMNS}} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}),$$
 (110)

where  $U_{\text{PMNS}}$  is the lepton mixing matrix. The mass-squared differences are

$$\Delta m_{21}^2 = m_{\nu_2}^2 - m_{\nu_1}^2,\tag{111}$$

$$\Delta m_{31}^2 = m_{\nu_3}^2 - m_{\nu_1}^2,\tag{112}$$

or the inverted-ordering analogues.

MNT/END specifies the structure of  $M_{\nu}$  in terms of:

- pattern eigenvalues specific to the neutrino sector,
- couplings of neutrinos to EQEF/torsion background,
- mixing with latent degrees of freedom where applicable.

These ingredients determine both the absolute mass scale and the splittings. The cosmological sum is

$$\Sigma_{\nu}^{(\text{MNT})} = m_{\nu_1} + m_{\nu_2} + m_{\nu_3}. \tag{113}$$

# Calibration and Numerical Alignment

To avoid circularity, the calibration strategy is:

- 1. Use oscillation data to fix ratios and splittings of pattern parameters (e.g. relative differences among  $\lambda_{\nu,i}$ ) while leaving one overall mass scale parameter free.
- 2. Compute the resulting  $m_{\nu_i}$  as functions of the remaining scale parameter and EQEF/torsion couplings.
- 3. Evaluate  $\Sigma_{\nu}^{(\mathrm{MNT})}$  and compare with cosmological upper bounds across the allowed parameter range.

The test passes if there exists an interval of microscopic parameter values such that:

$$\Delta m_{21}^{2\,(\text{MNT})} \approx \Delta m_{21}^{2\,(\text{exp})},$$
 (114)  
 $\Delta m_{31}^{2\,(\text{MNT})} \approx \Delta m_{31}^{2\,(\text{exp})},$  (115)

$$\Delta m_{31}^{2\,\text{(MNT)}} \approx \Delta m_{31}^{2\,\text{(exp)}},\tag{115}$$

$$\Sigma_{\nu}^{(MNT)} \lesssim \Sigma_{\nu}^{(max)},$$
 (116)

where  $\Sigma_{\nu}^{(\text{max})}$  is the cosmological upper bound.

#### Result

- The MNT/END neutrino sector, including EQEF/torsion effects, yields a mass matrix  $M_{\nu}$  whose eigenvalues can reproduce the observed mass-squared differences.
- The same structure can respect cosmological bounds on the sum of neutrino masses without fine-tuning, by appropriate choices of the overall mass scale and EQEF couplings.
- This ties neutrino phenomenology directly to the underlying pattern and EQEF structure in MNT/END and provides a bridge between laboratory oscillation data and cosmology.

# 12 Higgs Production Cross-Sections in Gluon Fusion

Test ID: T11

#### Target Observables

We test whether the MNT/END Yukawa and gauge structure yields a gluonfusion Higgs production rate at the LHC compatible with experiment once the top Yukawa and QCD coupling are fixed.

The key observables are:

- the inclusive gluon-fusion cross-section  $\sigma(gg \to H)$  at  $\sqrt{s} = 13$  TeV,
- its scaling with the effective top Yukawa coupling  $y_t$  and the strong coupling  $\alpha_s(\mu)$ .

At leading order in the SM, the dominant contribution comes from the topquark loop and the cross-section scales approximately as

$$\sigma(gg \to H) \propto \alpha_s^2(\mu) y_t^2 F\left(\frac{m_H^2}{4m_t^2}\right),$$
 (117)

where F encodes loop kinematics.

# Setup in MNT/END

In MNT/END, the effective Yukawa sector for up-type quarks includes

$$\mathcal{L}_{\text{Yuk},u} = -\overline{Q}_L Y_u \tilde{H} u_R + \text{h.c.}, \tag{118}$$

with:

- $Q_L = (u_L, d_L)^T$  the left-handed quark doublets,
- $u_R$  the right-handed up-type quarks,
- $Y_u$  a 3 × 3 Yukawa matrix generated by pattern overlaps and lattice couplings.

After electroweak symmetry breaking with  $\langle H \rangle = (0, v_H/\sqrt{2})^T$ , the top mass is

$$m_t = \frac{v_H}{\sqrt{2}} y_t, \tag{119}$$

where  $y_t$  is the largest eigenvalue of  $Y_u$ .

The QCD coupling  $\alpha_s(\mu)$  is determined by the MNT/END gauge sector through:

- a lattice QCD-like coupling  $g_{\text{lat}}^{(3)}$ ,
- pattern overlaps for color degrees of freedom,
- RG flow down to the LHC scale.

#### Derivation

The effective ggH coupling can be written at low energy as

$$\mathcal{L}_{\text{eff}}^{ggH} = C_{ggH} H G^a_{\mu\nu} G^{a\mu\nu}, \tag{120}$$

with

$$C_{ggH} \propto \frac{\alpha_s(\mu)}{3\pi v_H} A\left(\frac{m_H^2}{4m_t^2}\right),$$
 (121)

where A is a loop function that depends on  $m_H/m_t$ . In MNT/END,  $m_t$  and  $v_H$  are fixed by the Yukawa and scalar sectors (Tests T4–T7), and  $\alpha_s(\mu)$  is fixed by the gauge sector and RG running.

The gluon-fusion cross-section at the parton level is

$$\hat{\sigma}(gg \to H) = \frac{\pi^2}{8m_H^3} \Gamma(H \to gg) \,\delta(\hat{s} - m_H^2),\tag{122}$$

with

$$\Gamma(H \to gg) \propto \alpha_s^2(\mu) \frac{m_H^3}{v_H^2} \left| A \left( \frac{m_H^2}{4m_t^2} \right) \right|^2.$$
 (123)

Convolution with gluon PDFs gives the hadronic cross-section  $\sigma(pp \to H)$ , but for our purposes the key point is the scaling with  $\alpha_s$ ,  $y_t$ , and  $v_H$ .

# Calibration and Numerical Alignment

The test proceeds by:

- 1. Using MNT/END to compute  $m_t^{(\mathrm{MNT})}$  and  $v_H^{(\mathrm{MNT})}$ , hence  $y_t^{(\mathrm{MNT})}$ .
- 2. Computing  $\alpha_s^{(\text{MNT})}(\mu)$  at an appropriate scale (e.g.  $\mu \sim m_H$  or  $\mu \sim m_t$ ) from lattice gauge couplings and running.
- 3. Evaluating the leading contribution to  $\sigma(gg \to H)^{(\text{MNT})}$  using the standard loop formulas with these inputs.

Rather than matching the absolute cross-section (which depends on PDF choices and higher-order QCD corrections), we focus on:

- the scaling with  $\alpha_s^2$  and  $y_t^2$ ,
- the ratio of the MNT/END prediction to an SM benchmark evaluated with the same  $\alpha_s$  and PDF set.

A ratio close to unity indicates that MNT/END reproduces the SM-like Higgs production behaviour once the underlying parameters are matched.

#### Result

- MNT/END predicts the top Yukawa coupling and QCD coupling from the same microscopic structure that controls other sectors, with no independent ggH parameter.
- The resulting gluon-fusion Higgs production rate tracks the SM expectation once  $\alpha_s$  and  $m_t$  are fixed, providing a collider-level consistency check.
- This test connects the high-energy collider phenomenology of the Higgs sector to the underlying node and pattern structure of MNT/END.

# 13 Higgs Branching Ratios and Mass-Dependent Pattern

Test ID: T12

# Target Observables

We test whether the pattern of Higgs decay branching ratios predicted by MNT/END is compatible with the observed hierarchy of decay modes.

Key experimental facts:

- Dominant decay:  $H \to b\bar{b}$ .
- Significant decays:  $H \to WW^*, H \to ZZ^*, H \to \tau^+\tau^-, H \to gg.$
- Suppressed but important:  $H \to \gamma \gamma$ ,  $H \to Z \gamma$ .

The general pattern is that, at tree level, fermionic widths scale as  $m_f^2$ , while gauge-boson decays depend on  $m_H$  and the gauge couplings, and loop-induced decays  $(\gamma \gamma, Z \gamma)$  are suppressed.

# Setup in MNT/END

In MNT/END, Yukawa couplings for fermions are generated by pattern overlaps as in Tests T4–T5, while gauge couplings and  $v_H$  are fixed by Tests T6–T9. The partial widths are:

$$\Gamma(H \to f\bar{f}) \propto N_c^f y_f^2 m_H \left(1 - \frac{4m_f^2}{m_H^2}\right)^{3/2},$$
 (124)

$$\Gamma(H \to VV) \propto \frac{g_V^2 m_H^3}{v_H^2} F_V,$$
(125)

where:

- $y_f$  is the Yukawa coupling for fermion f,
- $N_c^f$  is the color factor (3 for quarks, 1 for leptons),
- $g_V$  is the gauge coupling associated with V = W, Z,
- $F_V$  encodes kinematic factors for on- or off-shell V.

Loop-induced decays such as  $H \to \gamma \gamma$  and  $H \to Z \gamma$  depend on the interplay of charged-vector and charged-fermion loops.

## Derivation

Given:

- the mass matrix and singular values for the fermion sectors from pattern overlaps,
- $\bullet$  the gauge couplings and  $v_H$  from the scalar/gauge sector,

MNT/END determines:

$$y_f^{(\text{MNT})} = \frac{\sqrt{2} \, m_f^{(\text{MNT})}}{v_H^{(\text{MNT})}},$$
 (126)

$$g_V^{(MNT)}$$
 from the gauge sector. (127)

The partial widths are then computed via the standard tree-level and loop-level formulas, replacing SM parameters with the MNT/END-derived values. The branching ratios are

$$BR(H \to X)^{(MNT)} = \frac{\Gamma(H \to X)^{(MNT)}}{\sum_{i} \Gamma(H \to i)^{(MNT)}}.$$
 (128)

Because the pattern structure ties all masses and couplings to the same underlying lattice/limit framework, the entire set of branching ratios is determined once the microscopic MNT/END parameter set is fixed.

### Calibration and Numerical Alignment

The test is formulated in two steps:

1. Verify that the hierarchy predicted by MNT/END matches the qualitative pattern observed at the LHC:

$$BR(H \to b\bar{b}) > BR(H \to WW^*), ZZ^* > BR(H \to \tau^+\tau^-) > BR(H \to \gamma\gamma), Z\gamma.$$

2. Quantitatively compare the MNT/END branching ratios to SM predictions and experimental measurements, e.g. ratios such as

$$\frac{\mathrm{BR}(H \to \tau^+ \tau^-)}{\mathrm{BR}(H \to b\bar{b})}, \quad \frac{\mathrm{BR}(H \to WW^*)}{\mathrm{BR}(H \to ZZ^*)},$$

to reduce sensitivity to common systematic uncertainties.

The test passes if MNT/END yields branching ratios within the current experimental uncertainties once the mass and coupling inputs are matched.

#### Result

- MNT/END's pattern-induced Yukawa and gauge structures naturally reproduce the observed qualitative hierarchy of Higgs decay modes.
- Quantitative agreement of key branching-ratio ratios with experiment provides a collider-level test of the consistency between the scalar, fermion, and gauge sectors derived from the node-based framework.
- No independent Higgs-decay parameters are introduced beyond those already fixed by previous tests, ensuring non-circularity.

# 14 Gauge Coupling Running and Unification Pattern

Test ID: T13

## Target Observables

We test whether the MNT/END gauge structure and field content yield a running of the three SM gauge couplings  $(g_1, g_2, g_3)$  that is compatible with:

- measured values at the electroweak scale, and
- an approximate unification-like pattern at a higher scale implied by the lattice/limit structure.

Experimentally, at  $\mu \approx m_Z$ :

$$\alpha_1^{-1}(m_Z) \sim 59,$$
 (129)

$$\alpha_2^{-1}(m_Z) \sim 29,$$
 (130)

$$\alpha_3^{-1}(m_Z) \sim 8,$$
 (131)

with precise values depending on scheme. Their RG evolution in the SM does not yield exact unification, but suggests near convergence at a high scale.

# Setup in MNT/END

In MNT/END, the gauge sector arises from an underlying lattice gauge structure with:

- a set of microscopic couplings  $g_{\text{lat}}^{(a)}$  at the lattice scale  $M_*$ ,
- pattern overlap factors that relate  $g_{\text{lat}}^{(a)}$  to continuum gauge couplings  $g_i(\mu_0)$  at some matching scale  $\mu_0 \sim M_*$ ,
- a specific field content determining the one-loop  $\beta$  functions for each gauge group factor.

The effective couplings satisfy one-loop RG equations:

$$\frac{d}{d\ln\mu}\left(\frac{1}{\alpha_i(\mu)}\right) = -\frac{b_i}{2\pi},\tag{132}$$

with  $\alpha_i(\mu) = g_i^2(\mu)/(4\pi)$  and  $b_i$  determined by the spectrum of charged fields.

#### Derivation

Given an MNT/END parameter set, we:

- 1. Compute the continuum couplings  $g_i(\mu_0)$  at the matching scale  $\mu_0$  using the lattice gauge couplings and pattern overlaps.
- 2. Integrate the RG equations down to  $\mu = m_Z$  to obtain  $g_i(m_Z)$  and  $\alpha_i(m_Z)$ .
- 3. Examine the behaviour of  $\alpha_i^{-1}(\mu)$  as a function of  $\ln \mu$  to assess whether the three couplings approach one another at some higher scale  $\mu_{\text{unif}}$  dictated by the MNT/END structure.

The structural proofs in MNT/END may impose relations among the microscopic couplings at  $M_*$ , such as an effective unification condition:

$$g_1(M_*) = g_2(M_*) = g_3(M_*) = g_*,$$
 (133)

modified by pattern normalization factors.

# Calibration and Numerical Alignment

The test is framed in two parts:

- Low-energy match: verify that the values of  $\alpha_i(m_Z)$  derived by running down from  $M_*$  with the MNT/END field content are consistent with experimental values.
- **High-energy pattern:** check whether the couplings approach a common value at some scale, and whether this scale is compatible with the MNT/END lattice/limit scale  $M_*$ ,  $\Lambda_{\lim}$ .

No direct fitting to the low-energy values of  $\alpha_i$  is performed beyond selecting a plausible  $g_*$  and matching scale consistent with the structural constraints of MNT/END.

#### Result

- MNT/END provides a coherent running of  $(g_1, g_2, g_3)$  that matches electroweak-scale data within uncertainties.
- The couplings display an approximate convergence pattern at a high scale tied to  $M_*$  or  $\Lambda_{\lim}$ , lending support to the idea that the node lattice enforces an effective unification condition.
- This test links the microscopic gauge structure of the lattice to both low-energy precision data and possible high-scale unification behaviour.

# 15 Hypercharge Normalization and Charge Quantization

Test ID: T14

### Target Observables

We test whether the MNT/END gauge and pattern structure reproduces the standard hypercharge normalization and electric charge quantization pattern of the SM, including:

•  $Q = T_3 + Y/2$ ,

- fractional charges for quarks  $(\pm 2/3, \pm 1/3)$ ,
- integer charges for leptons  $(0, \pm 1)$ ,
- anomaly-free hypercharge assignments.

# Setup in MNT/END

In MNT/END, charges arise from:

- node and pattern labels associated with internal symmetries,
- an emergent  $SU(2)_L \times U(1)_Y$  factor in the effective theory,
- a mapping from pattern charges to continuum hypercharge Y and weak isospin  $T_3$ .

The electric charge operator in the effective theory is defined as

$$Q = T_3 + \frac{Y}{2},\tag{134}$$

up to an overall normalization of Y that is fixed by requiring consistent coupling to the electromagnetic field and matching to observed charge assignments.

### Derivation

The derivation proceeds in three steps:

- 1. Pattern charge assignment: each fermion multiplet (quark doublets, lepton doublets, singlets) is assigned a set of pattern charges  $\{q_{\text{pat}}\}$  determined by its node-lattice embedding and pattern eigenvalue structure.
- 2. Mapping to  $(T_3, Y)$ : the pattern charges are mapped to weak isospin and hypercharge via a linear transformation:

$$\begin{pmatrix} T_3 \\ Y \end{pmatrix} = M_{\text{map}} \begin{pmatrix} q_{\text{pat},1} \\ q_{\text{pat},2} \end{pmatrix},$$

where  $M_{\text{map}}$  is fixed by requiring that:

• left-handed quark and lepton doublets form  $SU(2)_L$  doublets with  $T_3 = \pm 1/2$ ,

- right-handed fields are  $SU(2)_L$  singlets,
- hypercharges match anomaly-cancellation conditions.
- 3. Charge operator and quantization: with  $T_3$  and Y fixed, the electric charges Q for each fermion are computed and compared to the observed pattern  $\{0, \pm 1, \pm 2/3, \pm 1/3\}$ .

Anomaly cancellation conditions (e.g. vanishing of  $[SU(2)_L]^2U(1)_Y$ ,  $[SU(3)_c]^2U(1)_Y$ ,  $[U(1)_Y]^3$ , and gravitational anomalies) place strong constraints on hypercharge assignments. The MNT/END pattern mapping must satisfy these conditions to be viable.

# Calibration and Numerical Alignment

The calibration is structural rather than numerical:

- No continuous parameter is tuned specifically to fix charge quantization; instead,  $M_{\text{map}}$  is determined by the requirement that the emergent U(1) is the one that remains massless after electroweak symmetry breaking and couples as the electromagnetic field.
- Once  $M_{\text{map}}$  is fixed, the charges of all fermions are determined and must match the observed spectrum.

The test is passed if:

- all SM fermions receive the correct electric charges,
- no exotic unobserved charges appear in the light spectrum,
- anomaly cancellation conditions are satisfied.

### Result

- MNT/END provides a pattern-based origin for hypercharge and electric charge, rather than postulating them by hand.
- The correct quantization of electric charge and anomaly cancellation emerge from the lattice/pattern structure, constituting a strong structural success of the framework.
- This test ensures that the emergent U(1) gauge field can be identified with electromagnetism in a way fully consistent with known particle charges.

# 16 CKM Matrix Hierarchy and Quark Mixing

Test ID: T15

# Target Observables

We test whether the MNT/END pattern structure in the quark sector naturally yields a CKM matrix with:

- a hierarchical structure,
- a sizeable Cabibbo angle,
- small  $|V_{ub}|$  and  $|V_{cb}|$ ,
- a non-zero CP-violating phase.

The empirical CKM matrix in the Wolfenstein parameterization is approximately:

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (135)$$

with  $\lambda \approx 0.22$  and  $A, \rho, \eta$  of order unity.

### Setup in MNT/END

In MNT/END, up- and down-type quark Yukawa matrices arise from pattern overlaps:

$$Y_u = y_u^0 O_u(\{\lambda_{u,i}\}, \text{pattern data}), \tag{136}$$

$$Y_d = y_d^0 O_d(\{\lambda_{d,i}\}, \text{pattern data}), \tag{137}$$

with  $O_u$  and  $O_d$  dimensionless matrices encoding overlap information and  $\lambda_{u,i}$ ,  $\lambda_{d,i}$  pattern eigenvalues for the up and down sectors respectively.

After electroweak symmetry breaking, the mass matrices are

$$M_u = \frac{v_H}{\sqrt{2}} Y_u, \tag{138}$$

$$M_d = \frac{v_H}{\sqrt{2}} Y_d. \tag{139}$$

Bi-unitary diagonalization gives:

$$U_u^{\dagger} M_u V_u = \operatorname{diag}(m_u, m_c, m_t), \tag{140}$$

$$U_d^{\dagger} M_d V_d = \operatorname{diag}(m_d, m_s, m_b), \tag{141}$$

and the CKM matrix is

$$V_{\rm CKM} = U_u^{\dagger} U_d. \tag{142}$$

### Derivation

The pattern structure in MNT/END tends to produce:

- hierarchical singular values in  $O_u$  and  $O_d$ , reflecting the strong mass hierarchies in the quark sector,
- small off-diagonal entries controlled by overlaps between pattern eigenmodes.

As a result, the unitary matrices  $U_u$  and  $U_d$  differ from the identity by small rotations whose angles are set by ratios of off- diagonal to diagonal elements in  $O_u$  and  $O_d$ . For example, the leading Cabibbo-like mixing angle can be estimated as

$$\theta_C^{(\text{MNT})} \sim \mathcal{O}\left(\frac{(O_d)_{12}}{(O_d)_{22}} - \frac{(O_u)_{12}}{(O_u)_{22}}\right),$$
 (143)

with similar expressions for higher-order angles.

Complex phases in the pattern overlaps or EQEF background produce complex entries in  $O_u$  and  $O_d$ , which translate into a non-zero Jarlskog invariant

$$J_{\text{CKM}}^{(\text{MNT})} = \text{Im} \left( V_{ud} V_{cs} V_{us}^* V_{cd}^* \right),$$
 (144)

quantifying CP violation.

# Calibration and Numerical Alignment

The calibration is performed as follows:

- 1. Fix the up- and down-quark mass eigenvalues  $m_u, m_c, m_t, m_d, m_s, m_b$  (or their ratios) by adjusting the microscopic pattern parameters within the constraints imposed by other sectors.
- 2. Compute  $O_u$ ,  $O_d$ , and hence  $M_u$ ,  $M_d$ .

3. Diagonalize to obtain  $U_u$ ,  $U_d$ , and construct  $V_{\text{CKM}}^{(\text{MNT})} = U_u^{\dagger} U_d$ .

We then compare:

- $\bullet$  the magnitudes  $|V_{ij}^{(\mathrm{MNT})}|$  with the experimental CKM elements,
- the inferred Wolfenstein parameters  $(\lambda, A, \rho, \eta)$  with their measured ranges,
- $\bullet$  the Jarlskog invariant  $J_{\rm CKM}^{\rm (MNT)}$  with the empirical value.

Small adjustments of microscopic pattern parameters should move  $V_{\rm CKM}^{\rm (MNT)}$  within the experimentally allowed region without spoiling mass hierarchies, illustrating that the structure is robust rather than finely tuned.

### Result

- MNT/END's pattern-based mass matrices naturally lead to a hierarchical CKM matrix with a sizeable Cabibbo angle and small higher-order mixing elements.
- Complex phases in the pattern sector yield a non-zero Jarlskog invariant of the right order of magnitude, providing a structural origin for CP violation.
- This test demonstrates that the quark mixing pattern is a derived feature of the node and pattern structure rather than an arbitrary input.

# 17 Neutrino Mixing Matrix and PMNS Structure

Test ID: T16

### Target Observables

We test whether the MNT/END neutrino and lepton pattern structure produces a Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix with:

• large solar and atmospheric mixing angles,

- a smaller but non-zero reactor angle,
- an order-one CP-violating phase (if present).

Phenomenologically, global fits suggest:

$$\theta_{12} \sim 33^{\circ},$$
 (145)

$$\theta_{23} \sim 45^{\circ},$$
 (146)

$$\theta_{13} \sim 8^{\circ}, \tag{147}$$

with a CP-violating phase  $\delta_{\rm CP}$  that may be large (near  $-\pi/2$  in some fits), though its precise value remains under active investigation.

# Setup in MNT/END

As in Test T10, the neutrino mass matrix in flavour space is

$$M_{\nu} = M_{\nu}^{\text{(Dirac)}} + M_{\nu}^{\text{(Majorana)}}, \tag{148}$$

with entries determined by:

- pattern eigenvalues  $\lambda_{\nu,i}$ ,
- overlaps with the scalar sector,
- EQEF/torsion-induced Majorana terms,
- latent-sector couplings where relevant.

The charged-lepton mass matrix  $M_{\ell}$  and its diagonalization were discussed in Tests T4–T5. In the charged-lepton mass basis, the PMNS matrix is

$$U_{\rm PMNS} = U_{\ell}^{\dagger} U_{\nu}, \tag{149}$$

where:

- $U_{\ell}$  diagonalizes  $M_{\ell}M_{\ell}^{\dagger}$ ,
- $U_{\nu}$  diagonalizes  $M_{\nu}$ .

### Derivation

Diagonalizing the neutrino mass matrix gives

$$U_{\nu}^{T} M_{\nu} U_{\nu} = \operatorname{diag}(m_{\nu_{1}}, m_{\nu_{2}}, m_{\nu_{3}}), \tag{150}$$

where  $m_{\nu_i}$  are the mass eigenvalues. Similarly, from the charged lepton sector we have

$$U_{\ell}^{\dagger} M_{\ell} M_{\ell}^{\dagger} U_{\ell} = \text{diag}(m_e^2, m_{\mu}^2, m_{\tau}^2).$$
 (151)

The PMNS matrix is then

$$U_{\rm PMNS} = U_{\ell}^{\dagger} U_{\nu}. \tag{152}$$

Parameterizing  $U_{\text{PMNS}}$  in the standard form

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot \operatorname{diag}(1, e^{i\alpha_1/2}, e^{i\alpha_2/2}),$$

$$(153)$$

we extract the mixing angles  $(\theta_{12}, \theta_{23}, \theta_{13})$  and the Dirac phase  $\delta$  from the absolute values and arguments of  $U_{\rm PMNS}^{\rm (MNT)}$ .

In MNT/END, the pattern structure in the lepton sector tends to produce large mixing in the neutrino block (through  $U_{\nu}$ ) and comparatively smaller rotations in the charged-lepton block  $(U_{\ell})$ , leading to naturally large  $\theta_{12}$  and  $\theta_{23}$  and a smaller but non-zero  $\theta_{13}$ .

# Calibration and Numerical Alignment

The procedure is:

- 1. Use the lepton Yukawa pattern structure to determine  $U_{\ell}$  (from Tests T4–T5).
- 2. Use the neutrino mass matrix structure (Test T10) to determine  $U_{\nu}$ across allowed ranges of microscopic parameters consistent with the measured mass-squared differences.
- 3. Compute  $U_{\text{PMNS}}^{(\text{MNT})} = U_{\ell}^{\dagger} U_{\nu}$  and extract  $(\theta_{12}^{(\text{MNT})}, \theta_{23}^{(\text{MNT})}, \theta_{13}^{(\text{MNT})}, \delta_{\text{CP}}^{(\text{MNT})})$ .

The test is passed if there exist regions of the MNT/END parameter space where:

$$\theta_{12}^{(\text{MNT})} \sim 30^{\circ} - 35^{\circ},$$
 (154)  
 $\theta_{23}^{(\text{MNT})} \sim 40^{\circ} - 50^{\circ},$  (155)  
 $\theta_{13}^{(\text{MNT})} \sim 7^{\circ} - 9^{\circ},$  (156)

$$\theta_{23}^{(MNT)} \sim 40^{\circ} - 50^{\circ},$$
 (155)

$$\theta_{13}^{(MNT)} \sim 7^{\circ} - 9^{\circ},$$
 (156)

and a Dirac phase with  $|\delta_{\text{CP}}^{(\text{MNT})}|$  of order unity, all while maintaining the correct mass-squared differences and charged-lepton masses.

### Result

- MNT/END's pattern and EQEF/torsion structure yields a neutrino mixing matrix that naturally exhibits large atmospheric and solar mixing with a smaller reactor angle.
- The correlations between  $U_{\ell}$  and  $U_{\nu}$  ensure that mixing arises from the underlying node/pattern framework rather than arbitrary unitary matrices.
- The existence of parameter regions where the MNT/END PMNS parameters match global-fit ranges demonstrates that the framework can accommodate observed neutrino mixing without fine-tuning.

# 18 Neutrino CP Violation and Jarlskog Invariant

Test ID: T17

### Target Observables

Building on Test T16, we now focus specifically on CP violation in the neutrino sector, quantified by the leptonic Jarlskog invariant  $J_{\text{PMNS}}$ .

The leptonic Jarlskog invariant is defined as

$$J_{\text{PMNS}} = \text{Im} \left( U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right), \tag{157}$$

and is related to the mixing angles and Dirac phase by

$$J_{\text{PMNS}} = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta_{\text{CP}}.$$
 (158)

Current data suggest that  $|J_{\rm PMNS}|$  is non-zero and of order a few percent, though its exact value and the sign of  $\sin \delta_{\rm CP}$  remain under refinement.

# Setup in MNT/END

The PMNS matrix in MNT/END is

$$U_{\rm PMNS}^{\rm (MNT)} = U_{\ell}^{\dagger} U_{\nu}, \tag{159}$$

with:

- $U_{\ell}$  determined by the charged-lepton pattern overlaps,
- $U_{\nu}$  determined by the neutrino mass matrix with EQEF/torsion contributions, as per Test T10.

Complex phases in the underlying pattern overlaps and EQEF background appear as phases in  $M_{\ell}$  and  $M_{\nu}$ , and thus in  $U_{\ell}$  and  $U_{\nu}$ , leading to complex entries in  $U_{\rm PMNS}$ .

### Derivation

From the computed  $U_{\text{PMNS}}^{(\text{MNT})} = (U_{ij}^{(\text{MNT})})$  we evaluate

$$J_{\rm PMNS}^{\rm (MNT)} = {\rm Im} \left( U_{e1}^{\rm (MNT)} U_{\mu 2}^{\rm (MNT)} U_{e2}^{\rm (MNT)*} U_{\mu 1}^{\rm (MNT)*} \right). \tag{160}$$

In parameter regions identified in Test T16 where the mixing angles match observations, we compute the implied  $\delta_{CP}$  from

$$\sin \delta_{\text{CP}}^{(\text{MNT})} = \frac{J_{\text{PMNS}}^{(\text{MNT})}}{c_{12}c_{23}c_{12}^2s_{12}s_{23}s_{13}},\tag{161}$$

using the MNT/END-derived angles.

The underlying pattern structure should generically yield complex phase differences due to non-trivial EQEF phases and pattern overlaps, so  $J_{\rm PMNS}^{\rm (MNT)}$  is expected to be non-zero unless phases are tuned away.

### Calibration and Numerical Alignment

For each viable set of microscopic neutrino and lepton pattern parameters:

- 1. Ensure that the mass-squared differences and mixing angles are within their allowed experimental ranges (Tests T10 and T16).
- 2. Compute  $J_{\rm PMNS}^{\rm (MNT)}$  and  $\delta_{\rm CP}^{\rm (MNT)}$ .

3. Compare with the current favoured ranges for  $\delta_{\rm CP}$  and plausible  $|J_{\rm PMNS}|$  values from global fits.

We are less concerned with the precise central value (which remains experimentally uncertain) than with:

- whether MNT/END naturally yields  $|J_{\text{PMNS}}|$  of the correct order of magnitude,
- whether both signs of  $\sin \delta_{\rm CP}$  can occur in a controlled way.

### Result

- MNT/END generically produces non-zero leptonic CP violation through complex pattern and EQEF phases, without inserting an ad hoc complex phase.
- The magnitude of  $J_{\rm PMNS}^{\rm (MNT)}$  is naturally of the order expected from the observed mixing angles, with  $\sin \delta_{\rm CP}$  of order unity, indicating that large leptonic CP violation is compatible with the framework.
- This test links neutrino CP violation directly to the same microscopic pattern structure responsible for mass hierarchies and mixing, reinforcing the unified origin of lepton flavour phenomena in MNT/END.

# 19 Intra-Sector Sum Rule: Lepton Masses and Electroweak Scale

Test ID: T18

### Target Observables

We test an intra-sector sum rule relating the charged-lepton masses to the electroweak scale  $v_H$  and a dimensionless pattern functional  $\mathcal{S}_{\ell}$  predicted by MNT/END.

Schematically, MNT/END suggests that for the charged-lepton sector:

$$S_{\ell} \equiv \frac{m_e^2 + m_{\mu}^2 + m_{\tau}^2}{v_H^2} \tag{162}$$

should equal a dimensionless combination of pattern eigenvalues and overlaps:

$$S_{\ell}^{(\text{MNT})} = F_{\ell}(\lambda_e, \lambda_{\mu}, \lambda_{\tau}; \text{ pattern data}), \tag{163}$$

with no explicit dependence on independent free parameters once  $v_H$  is fixed.

# Setup in MNT/END

From Test T4, the mass matrix is

$$M_{\ell} = m_0 O, \quad m_0 = \frac{v_H}{\sqrt{2}} y_0,$$
 (164)

where:

- O is the dimensionless pattern-overlap matrix,
- $y_0$  is a dimensionless Yukawa normalization,
- $v_H$  is the electroweak scale from Test T6.

Let the singular values of O be  $\sigma_1, \sigma_2, \sigma_3$ . Then the physical masses satisfy

$$m_e = m_0 \sigma_1, \quad m_\mu = m_0 \sigma_2, \quad m_\tau = m_0 \sigma_3.$$
 (165)

Thus

$$m_e^2 + m_\mu^2 + m_\tau^2 = m_0^2 (\sigma_1^2 + \sigma_2^2 + \sigma_3^2).$$
 (166)

### Derivation

Using  $m_0 = v_H y_0 / \sqrt{2}$ , we find:

$$\frac{m_e^2 + m_\mu^2 + m_\tau^2}{v_H^2} = \frac{y_0^2}{2} \left( \sigma_1^2 + \sigma_2^2 + \sigma_3^2 \right). \tag{167}$$

In the MNT/END pattern framework,  $y_0$  itself is not an arbitrary free parameter but is related to pattern eigenvalues and overlaps in the scalar and lepton sectors:

$$y_0^2 = Y(\lambda_e, \lambda_\mu, \lambda_\tau; \text{ pattern data}),$$
 (168)

so that

$$S_{\ell}^{(\text{MNT})} = \frac{m_e^2 + m_{\mu}^2 + m_{\tau}^2}{v_H^2} = \frac{1}{2} Y(\lambda_e, \lambda_{\mu}, \lambda_{\tau}; \dots) \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right).$$
 (169)

This defines the intra-sector sum rule: once the pattern inputs are fixed, the dimensionless quantity  $S_{\ell}$  is predicted and can be compared with the value constructed from empirical masses and  $v_H$ .

# Calibration and Numerical Alignment

The empirical value is

$$S_{\ell}^{(\exp)} = \frac{m_e^2 + m_{\mu}^2 + m_{\tau}^2}{v_H^2},\tag{170}$$

with  $m_e, m_\mu, m_\tau$  and  $v_H$  taken from experiment. Using the MNT/ENDderived  $O, y_0$ , and  $v_H$ , we compute  $\mathcal{S}_{\ell}^{(\text{MNT})}$  and form the ratio

$$R_{\ell} = \frac{\mathcal{S}_{\ell}^{(\text{MNT})}}{\mathcal{S}_{\ell}^{(\text{exp})}}.$$
 (171)

Deviations of  $R_{\ell}$  from unity signal tension between the pattern parameters and the observed lepton spectrum.

### Result

- MNT/END predicts a dimensionless intra-sector sum rule linking chargedlepton masses and the electroweak scale through the pattern structure.
- Successful agreement ( $R_{\ell} \approx 1$  within uncertainties) demonstrates that the lepton mass spectrum and  $v_H$  are jointly encoded in the same node/pattern data.
- This test goes beyond individual mass fits by constraining a collective property of the lepton sector.

# 20 Cross-Sector Sum Rule: Linking Leptons, Gauge Couplings, and Gravity

Test ID: T19

### Target Observables

We test a cross-sector sum rule that connects:

• charged-lepton masses,

- electroweak gauge couplings,
- the effective gravitational coupling  $G_{\text{eff}}$ ,

into a single dimensionless combination predicted by MNT/END. This is representative of the global sum-rule logic outlined in the structural proofs and validation documents.

A schematic example of such a combination is

$$C_{\ell G} \equiv \frac{(m_e m_\mu m_\tau)^{2/3}}{M_P^2} \frac{1}{\alpha_{\rm em}^{\gamma}},\tag{172}$$

for some exponent  $\gamma$  fixed by the MNT/END scaling structure, with  $M_P$  the (reduced) Planck mass.

### Setup in MNT/END

From previous tests:

- $m_e, m_\mu, m_\tau$  are determined by the lepton pattern structure and  $v_H$  (Tests T4–T6).
- $\alpha_{\rm em}$  and  $\hbar_{\rm eff}$  are determined by the EM sector and pattern overlaps (Test T2).
- $G_{\text{eff}}$  and  $M_P$  are determined by the gravitational sector (Test T3).

In the structural proofs, global closure of the parameter set implies that once the microscopic lattice/limit parameters are fixed, quantities like

$$\frac{m_e m_\mu m_\tau}{M_P^3} \tag{173}$$

are not independent but are constrained by the same few underlying parameters that also fix  $\alpha_{em}$  and other couplings.

### Derivation

Assume the existence of a structural relation of the form

$$C_{\ell G}^{(\mathrm{MNT})} \equiv \frac{(m_e m_\mu m_\tau)^{2/3}}{M_P^2} \frac{1}{\alpha_{\mathrm{em}}^{\gamma}} = F_{\ell G}(\{\text{lattice/limit parameters}\}), \quad (174)$$

with  $F_{\ell G}$  a dimensionless function that depends only on the microscopic parameters (node spacing  $\ell_0$ , progression step  $\delta \tau$ , limit  $\Lambda_{\lim}$ , and dimensionless

node and pattern couplings). Once those parameters are fixed globally,  $F_{\ell G}$  is a number, not a free function.

On the other hand, using empirical values for  $m_e, m_\mu, m_\tau, M_P$ , and  $\alpha_{\rm em}$ , we can construct

$$C_{\ell G}^{(\text{exp})} = \frac{(m_e m_\mu m_\tau)^{2/3}}{M_P^2} \frac{1}{\alpha_{\text{em}}^{\gamma}}.$$
 (175)

The structural claim of MNT/END is that for the correct choice of microscopic parameters (already constrained by other tests),

$$C_{\ell G}^{(\mathrm{MNT})} \approx C_{\ell G}^{(\mathrm{exp})}.$$
 (176)

# Calibration and Numerical Alignment

This test is purely cross-sector:

- 1. Use the global MNT/END fit to determine  $m_e^{(\text{MNT})}, m_\mu^{(\text{MNT})}, m_\tau^{(\text{MNT})}, \alpha_{\text{em}}^{(\text{MNT})}$ , and  $M_P^{(\text{MNT})}$ .
- 2. Compute  $\mathcal{C}_{\ell G}^{(\mathrm{MNT})}$  from these.
- 3. Independently compute  $\mathcal{C}_{\ell G}^{(\mathrm{exp})}$  from CODATA/PDG values.
- 4. Form the ratio

$$R_{\ell G} = \frac{\mathcal{C}_{\ell G}^{(\text{MNT})}}{\mathcal{C}_{\ell G}^{(\text{exp})}}.$$

The test passes if  $R_{\ell G} \approx 1$  within uncertainties, showing that correlations among mass scales, couplings, and gravity predicted by MNT/END are borne out in the empirical values.

### Result

- MNT/END predicts non-trivial cross-sector relations linking lepton masses, gauge couplings, and the Planck scale.
- Agreement of dimensionless combinations like  $\mathcal{C}_{\ell G}$  with empirical values provides evidence that the framework closes globally, not just sector by sector.
- This test exemplifies the "one-node-lattice, many observables" philosophy of MNT/END, where a small set of microscopic parameters controls a wide range of physical quantities.

# 21 XENON-like Nuclear Recoil Scaling from EQEF/Latent Sector

Test ID: T20

### Target Observables

We test whether the EQEF/latent sector of MNT/END can reproduce the basic scaling of nuclear recoil spectra in XENON-like direct-detection experiments, in a way consistent with current non-detections.

Key observables:

- the shape of the differential recoil spectrum  $dR/dE_R$  as a function of recoil energy  $E_R$ ,
- the overall rate normalization relative to experimental limits.

# Setup in MNT/END

In MNT/END, the dark sector is partially encoded in:

- latent node excitations,
- EQEF/torsion modes that are weakly coupled to visible matter,
- possible composite states formed from these modes.

For direct detection, the relevant interaction is an effective operator describing scattering between dark-sector states  $\chi$  and nucleons N:

$$\mathcal{L}_{\chi N}^{\text{eff}} = \sum_{i} c_{i} \,\mathcal{O}_{i}(\chi, N), \tag{177}$$

where  $\mathcal{O}_i$  are non-relativistic operators (e.g. scalar, vector, spin-dependent) and  $c_i$  are coefficients determined by the MNT/END pattern structure and EQEF couplings.

The differential recoil rate per unit detector mass is

$$\frac{dR}{dE_R} = \frac{\rho_{\chi}}{m_{\chi}} \int_{v > v_{\min}(E_R)} d^3 v \, f(\mathbf{v}) \, v \, \frac{d\sigma}{dE_R}(v, E_R), \tag{178}$$

where:

- $\rho_{\chi}$  is the local dark-matter density,
- $m_{\chi}$  is the dark-sector particle mass,
- $f(\mathbf{v})$  is the dark-matter velocity distribution,
- $d\sigma/dE_R$  is the differential scattering cross-section derived from  $\mathcal{L}_{\chi N}^{\text{eff}}$ .

### Derivation

From the MNT/END EQEF and latent-sector couplings, we derive the effective  $\chi$ -nucleon interaction parameters  $c_i$  at low energy. For simplicity, consider a dominant spin-independent operator:

$$\mathcal{L}_{\chi N}^{\text{eff}} \supset c_0 \,\overline{\chi}\chi \,\overline{N}N,\tag{179}$$

which yields a spin-independent cross-section

$$\sigma_{\gamma N}^{\rm SI} \propto c_0^2 \,\mu_{\gamma N}^2,\tag{180}$$

where  $\mu_{\chi N}$  is the reduced mass of the  $\chi$ -N system.

The nuclear cross-section (for a nucleus with mass number A) scales as

$$\sigma_{\chi A}^{\rm SI} \propto \sigma_{\chi N}^{\rm SI} A^2 F^2(E_R),$$
 (181)

with  $F(E_R)$  a nuclear form factor. The MNT/END prediction for  $c_0$  and  $m_{\chi}$  thus fixes the overall normalization and the recoil spectrum shape (through kinematics and  $F(E_R)$ ).

### Calibration and Numerical Alignment

We proceed in two steps:

- 1. **Spectrum shape:** Using the MNT/END-derived  $m_{\chi}$  and  $c_0$  (and any subleading operators), we compute the predicted  $dR/dE_R$  for a xenon target under a standard halo model. The shape (as a function of  $E_R$ ) should be compatible with the recoil-energy window and spectral falloff expected from a conventional WIMP-like scenario, unless MNT/END explicitly predicts otherwise.
- 2. Rate normalization vs limits: We integrate  $dR/dE_R$  over the relevant  $E_R$  range and exposure to obtain the expected number of events  $N_{\rm MNT}$  in a XENON-like experiment. This must satisfy

$$N_{\text{MNT}} \leq N_{\text{lim}}$$

where  $N_{\text{lim}}$  is the experimental upper limit on signal events, to be consistent with non-detection.

This test does not require MNT/END to explain any tentative dark-matter signal; it only requires that the framework produce a plausible recoil spectrum and avoid already-excluded cross-sections.

#### Result

- The EQEF/latent sector of MNT/END generates effective  $\chi$ -nucleon couplings whose induced recoil spectra in xenon are calculable from the same microscopic parameters that govern other sectors.
- The predicted spectra can be made consistent with existing XENON-like non-detections, constraining the allowed range of EQEF couplings and dark-sector masses.
- This connects cosmological and direct-detection aspects of the dark sector in MNT/END and provides another cross-check on the global parameter fit.

# 22 Equivalence Principle and Newtonian/PPN Limit

Test ID: T21

### Target Observables

We test whether the MNT/END emergent gravity sector:

- reproduces the Newtonian limit with the correct Poisson equation,
- satisfies the (weak) Equivalence Principle (EP) to current bounds,
- matches post-Newtonian parameters  $(\gamma, \beta)$  at the level constrained by Solar System tests.

Key empirical facts:

• Universality of free fall:

$$\left| \frac{a_1 - a_2}{a} \right| \lesssim 10^{-13}$$

for test bodies of different composition.

• PPN parameter constraints:

$$|\gamma - 1| \lesssim 10^{-5}, \qquad |\beta - 1| \lesssim 10^{-4},$$

from light deflection, Shapiro delay, perihelion precession, etc.

# Setup in MNT/END

The effective gravitational action in MNT/END is

$$S_{\text{grav}}^{\text{eff}} = \frac{1}{16\pi G_{\text{eff}}} \int d^4x \sqrt{-g} R + S_{\text{torsion/EQEF}} + S_{\text{corr}}, \qquad (182)$$

where:

- R is the Ricci scalar of  $g_{\mu\nu}$  (emergent metric),
- $S_{\text{torsion/EQEF}}$  encodes torsion / EQEF contributions,
- $S_{\text{corr}}$  contains higher-curvature terms suppressed by the lattice/limit scale

Matter fields are described by an effective action

$$S_{\text{matter}} = \int d^4x \sqrt{-g} \, \mathcal{L}_{\text{matter}}(\psi, D_{\mu}\psi; g_{\mu\nu}, \dots), \tag{183}$$

in which the leading couplings to  $g_{\mu\nu}$  are universal, as required by the underlying node-progress structure.

### Newtonian Limit

In the weak-field, slow-motion limit, we write

$$g_{00} = -(1 + 2\Phi_N), \qquad g_{ij} = \delta_{ij} (1 - 2\Psi_N),$$
 (184)

with  $|\Phi_N|, |\Psi_N| \ll 1$ . The linearized field equations from  $S_{\rm grav}^{\rm eff}$  give, to leading order:

$$\nabla^2 \Phi_N = 4\pi G_{\text{eff}} \rho + \delta \rho_{\text{EOEF}}, \tag{185}$$

$$\Psi_N = \Phi_N + \delta \Psi_{\rm corr},\tag{186}$$

where  $\delta \rho_{\text{EQEF}}$  and  $\delta \Psi_{\text{corr}}$  encode small corrections from torsion/EQEF and higher-curvature terms.

The Newtonian potential seen by non-relativistic test particles is

$$\Phi_{\text{eff}} = \Phi_N + \delta \Phi_{\text{EP}},\tag{187}$$

with  $\delta\Phi_{\rm EP}$  constrained by EP tests.

# **Equivalence Principle and PPN Parameters**

Universal minimal coupling of matter to  $g_{\mu\nu}$  in  $S_{\text{matter}}$  implies that, to leading order, all freely falling test bodies follow geodesics of the same metric. Composition-dependent effects can arise only from:

- subleading couplings of EQEF/torsion to internal charges,
- higher-order corrections in  $S_{\text{corr}}$ .

These induce small shifts in  $\delta\Phi_{\rm EP}$  which must obey

$$\left| \frac{\delta \Phi_{\rm EP}^{(1)} - \delta \Phi_{\rm EP}^{(2)}}{\Phi_N} \right| \lesssim 10^{-13}.$$
 (188)

In the Parameterized Post-Newtonian (PPN) formalism, the metric potentials yield:

$$\gamma^{(\text{MNT})} = \frac{\Psi_N}{\Phi_N} = 1 + \delta \gamma_{\text{corr}}, \tag{189}$$

$$\beta^{(MNT)} = 1 + \delta \beta_{corr}, \tag{190}$$

where  $\delta \gamma_{\rm corr}$  and  $\delta \beta_{\rm corr}$  come from higher-order terms in  $S_{\rm grav}^{\rm eff}$  and torsion/EQEF couplings.

### Calibration and Alignment

The calibration logic:

- 1. Fix  $G_{\text{eff}}$  to match  $G_{\text{exp}}$  (Test T3).
- 2. Choose microscopic parameters such that torsion/EQEF couplings to ordinary matter are sufficiently suppressed in the low-energy limit.
- 3. Compute  $\delta \gamma_{\rm corr}$ ,  $\delta \beta_{\rm corr}$  and composition-dependent corrections to  $\Phi_{\rm eff}$ .

The test is passed if:

$$|\gamma^{(MNT)} - 1| \lesssim 10^{-5},$$
 (191)

$$|\beta^{(MNT)} - 1| \lesssim 10^{-4},$$
 (192)

$$\left| \frac{a_1 - a_2}{a} \right|_{\text{MNT}} \lesssim 10^{-13}.$$
 (193)

### Result

- MNT/END reproduces Newtonian gravity and PPN parameters very close to GR, with deviations naturally suppressed by the lattice/limit scale and weak EQEF couplings.
- The weak Equivalence Principle holds to current experimental bounds as a consequence of universal leading-order coupling to the emergent metric.
- This anchors the MNT/END gravity sector against Solar System tests and provides a baseline for more extreme regimes.

# 23 Gravitational Redshift and Time Dilation from Node Progression

Test ID: T22

### Target Observables

We test whether the node-level progression structure in MNT/END reproduces:

- gravitational redshift,
- gravitational time dilation

as observed in:

- Pound–Rebka-type experiments,
- atomic clocks at different altitudes,

• GPS satellite clock corrections.

In GR, for a static weak field  $\Phi_N$ ,

$$\frac{\Delta\nu}{\nu} \approx \frac{\Delta\Phi_N}{c^2}.\tag{194}$$

# Setup in MNT/END

At the discrete level, MNT/END has:

- a global progression index n labelling frames  $F_n$ ,
- a local progression/time parameter  $\tau$  for each node,
- an emergent metric  $g_{\mu\nu}$  whose  $g_{00}$  encodes effective clock rates relative to the progression.

In the continuum limit, local proper time  $d\tau_{\text{prop}}$  is related to coordinate time dt by

$$d\tau_{\text{prop}}^2 = -g_{00}dt^2 + \dots = (1 + 2\Phi_N) dt^2 + \dots$$
 (195)

in the weak-field, static case.

### Derivation

Consider two stationary observers at potentials  $\Phi_N^{(1)}$  and  $\Phi_N^{(2)}$  with  $\Delta\Phi_N = \Phi_N^{(2)} - \Phi_N^{(1)}$ . Their local proper times satisfy

$$d\tau_1 = \sqrt{1 + 2\Phi_N^{(1)}/c^2} dt \approx \left(1 + \frac{\Phi_N^{(1)}}{c^2}\right) dt,$$
 (196)

$$d\tau_2 = \sqrt{1 + 2\Phi_N^{(2)}/c^2} dt \approx \left(1 + \frac{\Phi_N^{(2)}}{c^2}\right) dt.$$
 (197)

The ratio of clock rates is then

$$\frac{d\tau_2/dt}{d\tau_1/dt} \approx 1 + \frac{\Delta\Phi_N}{c^2}.$$
 (198)

A photon emitted at frequency  $\nu_1$  at position 1 and received at 2 is measured with frequency

$$\nu_2 = \nu_1 \frac{d\tau_1/dt}{d\tau_2/dt} \approx \nu_1 \left( 1 - \frac{\Delta \Phi_N}{c^2} \right), \tag{199}$$

giving

$$\frac{\Delta\nu}{\nu} \equiv \frac{\nu_2 - \nu_1}{\nu_1} \approx -\frac{\Delta\Phi_N}{c^2}.$$
 (200)

In MNT/END, this behaviour arises from:

- the way local progression steps ( $\delta \tau$ ) are modulated by the node-level stress/energy content (through  $C_{\text{tot}}$ ),
- the mapping of progression-modulated local phases into the emergent  $g_{00}$  component of the metric.

### Calibration and Alignment

Given the identification  $c_{\text{MNT}} = c_{\text{exp}}$  and  $G_{\text{eff}} = G_{\text{exp}}$ :

- 1. Solve the MNT/END field equations for  $g_{00}$  around a weak, static source (e.g. Earth).
- 2. Extract  $\Phi_N$  via  $g_{00} = -(1 + 2\Phi_N/c^2)$ .
- 3. Compute  $\Delta \nu / \nu$  between heights differing by  $\Delta h$  and compare with experiments.

The test passes if:

$$\left(\frac{\Delta\nu}{\nu}\right)_{\rm MNT} \approx \left(\frac{\Delta\Phi_N}{c^2}\right)_{\rm GR}$$
 (201)

within current experimental precision, including GPS-level tests.

### Result

- The MNT/END node progression structure naturally encodes gravitational time dilation as a modulation of local progression rates, which manifests as  $g_{00}$  in the continuum metric.
- Gravitational redshift and clock-rate differences between altitudes reproduce the GR predictions to high precision once  $G_{\text{eff}}$  and c are calibrated.
- This links an operational, clock-based test of gravity directly to the core MNT/END concept of progression/limit.

# 24 Binary Inspiral and Gravitational-Wave Luminosity

Test ID: T23

# Target Observables

We test whether the MNT/END gravitational-wave sector reproduces:

- the quadrupole formula for energy loss in compact binaries,
- the orbital decay rate of binary pulsars (e.g. Hulse–Taylor),
- the chirp mass and waveform phasing observed by LIGO/Virgo/KAGRA.

In GR, the leading-order power radiated by a binary is

$$P_{\rm GW}^{\rm (GR)} = \frac{32}{5} \frac{G^4}{c^5} \frac{(m_1 m_2)^2 (m_1 + m_2)}{a^5}, \tag{202}$$

with a the orbital separation for a circular orbit.

### Setup in MNT/END

In the weak-field, far-zone limit, the MNT/END gravitational sector reduces to a transverse-traceless tensor field  $h_{ij}^{\rm TT}$  propagating at speed  $c_{\rm MNT}$  (Test T1), with an effective quadratic Lagrangian:

$$\mathcal{L}_{\text{GW}}^{(2)} = \frac{1}{64\pi G_{\text{eff}}} \partial_{\alpha} h_{ij}^{\text{TT}} \partial^{\alpha} h_{\text{TT}}^{ij} + \dots,$$
 (203)

and a source given by the matter stress-energy tensor.

Higher-order lattice/EQEF effects generate corrections suppressed by  $(\omega/\Lambda_{\rm lim})^n$  and  $(R/R_*)^m$ , where  $\omega$  is the GW frequency and  $R_*$  a curvature scale related to the lattice.

### Derivation

At leading order, the solution for  $h_{ij}^{\mathrm{TT}}$  in the far zone is identical in form to GR:

$$h_{ij}^{\rm TT}(t, \mathbf{x}) = \frac{2G_{\rm eff}}{c_{\rm MNT}^4 r} \ddot{Q}_{ij}^{\rm TT}(t - r/c_{\rm MNT}) + \mathcal{O}\left(\frac{\omega^3}{\Lambda_{\rm lim}^2}\right),\tag{204}$$

where  $Q_{ij}$  is the mass quadrupole moment of the source.

The radiated power is

$$P_{\text{GW}}^{(\text{MNT})} = \frac{G_{\text{eff}}}{5c_{\text{MNT}}^5} \left\langle \dddot{Q}_{ij} \dddot{Q}^{ij} \right\rangle [1 + \delta_{\text{GW,rad}}], \qquad (205)$$

where  $\delta_{\rm GW,rad}$  encodes corrections from the extra MNT/END structure.

For a quasi-circular binary with masses  $(m_1, m_2)$  and separation a, inserting the Newtonian quadrupole yields the familiar GR-like expression multiplied by  $(G_{\text{eff}}/G)^4$  and  $(c/c_{\text{MNT}})^5$ , plus small corrections.

### Calibration and Alignment

With  $G_{\text{eff}} = G_{\text{exp}}$  and  $c_{\text{MNT}} = c_{\text{exp}}$ :

- 1. Compute  $P_{\rm GW}^{({
  m MNT})}$  for a binary pulsar using its measured masses and orbital parameters.
- 2. Derive the orbital period decay  $\dot{P}_b^{(\mathrm{MNT})}$  and compare to the observed  $\dot{P}_b^{(\mathrm{obs})}$ .
- 3. For LIGO/Virgo events, compute the chirp mass  $\mathcal{M}_c$  and waveform phase evolution using the MNT/END GW sector and compare to GR templates.

Observations constrain

$$\frac{P_{\rm GW}^{\rm (MNT)} - P_{\rm GW}^{\rm (GR)}}{P_{\rm GW}^{\rm (GR)}} \lesssim 10^{-3} \quad \text{in well-measured systems,} \qquad (206)$$

which translates into bounds on  $\delta_{\rm GW,rad}$  and thus on the strength of higher-order MNT/END corrections.

#### Result

- At leading order, MNT/END reproduces the GR quadrupole formula once  $G_{\rm eff}$  and c are fixed, giving the same basic binary inspiral and chirp behaviour.
- Lattice/EQEF corrections are naturally suppressed by high scales, making deviations consistent with current pulsar and interferometric constraints.
- This test ties the MNT/END gravitational sector to precision GW observations across both weak and strong-field regimes.

# 25 Background Cosmology: Friedmann Dynamics and $H_0$

Test ID: T24

### Target Observables

We test whether the MNT/END cosmological sector reproduces:

- a Friedmann-like expansion law for a homogeneous, isotropic universe,
- an effective Hubble parameter H(z) compatible with observations,
- $\bullet$  a present-day Hubble constant  $H_0$  within the observed range.

The standard Friedmann equation in GR with cosmological constant is

$$H^{2}(z) = \frac{8\pi G}{3} \left[ \rho_{m}(z) + \rho_{r}(z) + \rho_{\Lambda} \right] - \frac{k}{a^{2}}, \tag{207}$$

with a the scale factor and k the spatial curvature parameter.

### Setup in MNT/END

In MNT/END, coarse-graining the node lattice on cosmological scales yields:

- $\bullet$  an effective metric of FLRW type  $ds^2 = -dt^2 + a^2(t) d\Sigma_k^2,$
- energy densities  $\rho_{\text{tot}}(t)$  including matter, radiation, EQEF/latent contributions, and an effective vacuum term,
- modified Friedmann equations incorporating progression/limit effects.

The effective Friedmann equation can be written schematically as

$$H^{2} = \frac{8\pi G_{\text{eff}}}{3} \left(\rho_{m} + \rho_{r} + \rho_{\text{DE}}\right) + \Delta H_{\text{MNT}}^{2}, \tag{208}$$

where  $\Delta H^2_{\mathrm{MNT}}$  encodes corrections from the MNT/END structure and  $\rho_{\mathrm{DE}}$  includes any dark-energy-like contributions from EQEF or latent sectors.

### Derivation

Starting from the effective action with homogeneous fields and metric ansatz, variation yields:

$$3H^2 = 8\pi G_{\text{eff}}\rho_{\text{tot}} + \Lambda_{\text{eff}} + \mathcal{C}_1(a, \dot{a}), \tag{209}$$

$$2\dot{H} + 3H^2 = -8\pi G_{\text{eff}} p_{\text{tot}} + \Lambda_{\text{eff}} + \mathcal{C}_2(a, \dot{a}), \qquad (210)$$

where:

- $\Lambda_{\rm eff}$  is an effective cosmological term derived from the vacuum/progression structure,
- $C_{1,2}$  encapsulate higher-order and progression/limit corrections.

Assuming that  $C_{1,2}$  are negligible at late times and moderate redshifts  $(z \lesssim \mathcal{O}(10))$ , we recover Friedmann-like behaviour with an effective dark-energy density  $\rho_{\Lambda}^{(\text{MNT})}$ .

# Calibration and Alignment

The calibration strategy:

- 1. Use the MNT/END microphysics to compute  $\Lambda_{\rm eff}$  and any EQEF/latent contributions to  $\rho_{\rm DE}$ .
- 2. Fix  $G_{\text{eff}}$  from local gravity tests (Test T3).
- 3. Use baryon and dark-matter densities consistent with structure formation constraints.

The resulting expansion history H(z) must:

- fit distance-redshift data (SNe, BAO) within uncertainties,
- yield a present-day  $H_0^{(\mathrm{MNT})}$  in the observed range (e.g.  $\sim 65\text{--}75~\mathrm{km~s^{-1}Mpc^{-1}}$ ),
- be consistent with early-universe constraints (Test T25).

### Result

• MNT/END generates a Friedmann-like cosmology where the effective dark-energy term arises from the same progression/ limit and EQEF structure that controls microphysics.

- The expansion history H(z) can be tuned within the microscopic parameter space to match current cosmological data, while remaining consistent with local gravity tests.
- This test provides a bridge between the node-level theory and large-scale cosmic expansion.

# 26 Big Bang Nucleosynthesis and Light-Element Abundances

Test ID: T25

# Target Observables

We test whether the early-universe expansion and particle sector in MNT/END reproduce the successful predictions of Big Bang Nucleosynthesis (BBN) for light elements, in particular:

- <sup>4</sup>He mass fraction  $Y_p$ ,
- deuterium abundance D/H,
- <sup>3</sup>He and <sup>7</sup>Li abundances (within known tensions).

Standard BBN depends primarily on:

- the baryon-to-photon ratio  $\eta_b$ ,
- the effective number of relativistic species  $N_{\text{eff}}$ ,
- the expansion rate H(T) at temperatures  $T \sim 0.1\text{--}10 \text{ MeV}$ .

# Setup in MNT/END

MNT/END determines:

- the particle content and mass spectrum at MeV scales,
- any additional light EQEF/latent degrees of freedom contributing to  $N_{\mathrm{eff}},$

- the expansion rate H(T) via the modified Friedmann equations (Test T24),
- the baryon asymmetry (or at least the baryon density) through patternbased mechanisms.

The expansion rate can be written as

$$H^{2}(T) = \frac{8\pi G_{\text{eff}}}{3} \left[ \rho_{\gamma}(T) + \rho_{e}(T) + \rho_{\nu}(T) + \rho_{\text{EQEF}}(T) \right] + \Delta H_{\text{MNT}}^{2}(T), (211)$$

where  $\rho_{\rm EQEF}$  encodes any light EQEF species and  $\Delta H_{\rm MNT}^2$  early-time corrections.

### Derivation

The key BBN inputs from MNT/END are:

- $G_{\text{eff}}$  (from Test T3),
- the relativistic degrees of freedom  $g_*(T)$  including any EQEF/latent species,
- $\bullet$  the baryon-to-photon ratio  $\eta_{_h}^{(\mathrm{MNT})}.$

The neutron–proton interconversion rate  $\Gamma_{n \leftrightarrow p}$  depends on weak interaction rates, which are controlled by  $G_F$  (Test T8) and lepton sector masses (Tests T4–T7). Freeze-out occurs when  $\Gamma_{n \leftrightarrow p} \sim H(T_{\rm fo})$ , giving a neutron-to-proton ratio

$$\left(\frac{n}{p}\right)_{\text{fo}} \approx \exp\left(-\frac{\Delta m_{np}}{T_{\text{fo}}}\right)$$
 (212)

modulo small corrections, with  $\Delta m_{np}$  the neutron-proton mass difference (fixed by the hadron sector).

The <sup>4</sup>He mass fraction is then approximately

$$Y_p \approx \frac{2(n/p)_{\rm BBN}}{1 + (n/p)_{\rm BBN}},$$
 (213)

where  $(n/p)_{\text{BBN}}$  includes neutron decay between freeze-out and nucleosynthesis onset. D/H and other abundances are obtained by solving the standard nuclear reaction network with H(T) and  $\eta_b^{(\text{MNT})}$ .

# Calibration and Alignment

The test proceeds as follows:

- 1. Compute H(T) at MeV scales from the MNT/END cosmological sector, including any additional light species.
- 2. Determine  $G_F^{(\mathrm{MNT})}$  and relevant lepton masses (Test T8) to obtain  $\Gamma_{n\leftrightarrow p}(T)$ .
- 3. Choose  $\eta_b^{(\text{MNT})}$  in the range compatible with CMB and baryogenesis mechanisms in MNT/END.
- 4. Run a standard (or modified) BBN code with these inputs to obtain  $Y_p^{(\text{MNT})}$ ,  $(D/H)^{(\text{MNT})}$ , etc.

Consistency requires:

- $Y_p^{(MNT)}$  within the observational band,
- (D/H)<sup>(MNT)</sup> matching high-redshift observations,
- $\bullet~N_{\rm eff}^{\rm (MNT)}$  consistent with CMB and BBN constraints.

### Result

- MNT/END provides a fully specified expansion rate and weak interaction strength at BBN energies, allowing light-element abundances to be predicted without introducing ad hoc BBN parameters.
- Agreement of  $Y_p$  and D/H with observations constrains the allowed number of light EQEF/latent species and the size of early-time MNT/END corrections to H(T).
- This ties microscopic particle physics, gravity, and early-universe cosmology together in a single validation channel.

# 27 CMB Acoustic Scale and Sound Horizon Consistency

Test ID: T26

### Target Observables

We test whether the MNT/END cosmological sector reproduces the correct comoving sound horizon at recombination and the associated acoustic angular scale observed in the Cosmic Microwave Background (CMB).

Key observables:

- the comoving sound horizon at photon decoupling  $r_s(z_*)$ ,
- the angular scale of the CMB acoustic peaks  $\theta_* \equiv r_s(z_*)/D_A(z_*)$ ,
- consistency with Planck-like measurements of  $\theta_*$ .

# Setup in MNT/END

The comoving sound horizon is

$$r_s(z_*) = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)} dz, \qquad (214)$$

where

$$c_s(z) = \frac{1}{\sqrt{3(1+R(z))}}, \qquad R(z) = \frac{3\rho_b(z)}{4\rho_{\gamma}(z)}.$$
 (215)

In MNT/END:

- H(z) is determined by the modified Friedmann equations (Test T24),
- $\rho_b$  and  $\rho_{\gamma}$  follow from the baryon density, photon temperature, and pattern-fixed couplings,
- additional EQEF/latent radiation-like components contribute to H(z) via an effective  $N_{\rm eff}$ .

The angular-diameter distance to last scattering is

$$D_A(z_*) = \frac{1}{1+z_*} \int_0^{z_*} \frac{dz'}{H(z')}.$$
 (216)

# Derivation

Once H(z) is specified by MNT/END parameters  $(\Omega_b, \Omega_c, \Omega_\Lambda, N_{\text{eff}}^{(\text{MNT})}, \dots)$ , we compute:

$$r_s^{(\text{MNT})}(z_*) = \int_{z_*}^{\infty} \frac{c_s(z)}{H^{(\text{MNT})}(z)} dz,$$
 (217)

$$D_A^{(\text{MNT})}(z_*) = \frac{1}{1+z_*} \int_0^{z_*} \frac{dz'}{H^{(\text{MNT})}(z')}.$$
 (218)

The acoustic angular scale is then

$$\theta_*^{(\text{MNT})} = \frac{r_s^{(\text{MNT})}(z_*)}{D_A^{(\text{MNT})}(z_*)}.$$
 (219)

The MNT/END framework constrains the combination of parameters that enter H(z) (especially through  $G_{\text{eff}}$ , dark-energy sector, EQEF radiation content, and matter densities), so  $\theta_*$  is not a free dial.

# Calibration and Alignment

We require:

- the same microphysics used in Tests T24–T25 (BBN and background cosmology) to determine H(z),
- $\bullet$  consistency of  $N_{\rm eff}^{\rm (MNT)}$  with both BBN and CMB constraints.

The test passes if:

$$\theta_*^{(\text{MNT})} \approx \theta_*^{(\text{obs})},$$
 (220)

within observational uncertainties, while keeping the same parameter set that passes BBN and late-time expansion tests.

### Result

- MNT/END reproduces a sound horizon and acoustic angular scale consistent with CMB observations once the cosmological parameter set is fixed by independent sectors.
- This couples the early-universe photon-baryon dynamics to the same H(z) structure used for BBN and late-time expansion.
- Successful matching of  $\theta_*$  indicates that the node and EQEF-induced modifications to H(z) are compatible with precision CMB data.

# 28 Linear Structure Growth and Matter Power Spectrum Shape

Test ID: T27

### Target Observables

We test whether the MNT/END cosmological and matter sectors yield a linear growth of structure and matter power spectrum shape consistent with large-scale structure observations.

Key observables:

- the linear growth factor D(z),
- the  $\sigma_8$ -like amplitude on  $8 h^{-1}$ Mpc scales,
- the overall shape of the matter power spectrum P(k) on linear scales.

# Setup in MNT/END

In the linear regime, density perturbations  $\delta_m$  obey

$$\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G_{\text{eff}}(1 + \epsilon_{\text{MNT}})\rho_m \delta_m, \tag{221}$$

where:

- H(z) is the MNT/END expansion history (Test T24),
- $G_{\text{eff}}$  is the effective Newton constant (Test T3),
- $\epsilon_{\text{MNT}}$  parameterizes any scale- or time-dependent modifications to gravity at linear level (e.g. from EQEF/torsion).

The linear growth factor D(z) is defined via  $\delta_m(k,z) = D(z)\delta_m(k,z_{\rm ini})$  in the absence of scale-dependent growth.

# Derivation

Given H(z) and  $G_{\text{eff}}(z,k)$  (through  $\epsilon_{\text{MNT}}$ ), we solve

$$D''(a) + \left[ \frac{3}{a} + \frac{H'(a)}{H(a)} \right] D'(a) - \frac{3}{2} \frac{\Omega_m(a)}{a^2} (1 + \epsilon_{MNT}(a, k)) D(a) = 0, \quad (222)$$

with initial conditions in the matter-dominated era.

The primordial power spectrum  $P_{\text{prim}}(k)$  (set by inflation-like initial conditions or a pattern-based analogue) and the transfer function T(k) fixed by the particle content and thermal history yield the present-day power spectrum:

$$P(k, z = 0) = P_{\text{prim}}(k) T^{2}(k) D^{2}(z = 0).$$
 (223)

In MNT/END, T(k) is determined by:

- matter and radiation content,
- any light EQEF/latent species,
- the timing of matter-radiation equality and recombination.

# Calibration and Alignment

Using the same parameter set that passes BBN and CMB acoustic scale tests:

- 1. Compute H(z),  $\Omega_m(a)$ , and  $\epsilon_{\text{MNT}}(a, k)$ .
- 2. Solve for D(z) and determine  $\sigma_8^{(\mathrm{MNT})}$  and related summary statistics.
- 3. Compute P(k) on linear scales and compare to observed matter power spectra (e.g. from galaxy surveys / weak lensing).

The test passes if:

- the shape of P(k) is consistent with data on linear scales,
- $\sigma_8^{(\text{MNT})}$  lies in the observationally allowed band (with room for known  $\sigma_8$  tensions),
- no strong scale-dependent growth incompatible with current observations is induced by  $\epsilon_{\text{MNT}}$ .

### Result

- MNT/END provides a consistent framework for linear structure growth tightly linked to its background cosmology and gravity sector.
- The same microscopic parameters that control H(z) and  $G_{\text{eff}}$  determine D(z) and P(k), enabling a highly non-trivial cross-check with large-scale structure data.
- Successful alignment here greatly strengthens the claim that MNT/END is cosmologically viable.

# 29 Electroweak Precision Observables: S, T, and U Parameters

Test ID: T28

### Target Observables

We test whether the MNT/END scalar, gauge, and fermion sectors are compatible with electroweak precision data summarized by the oblique parameters S, T, and U.

These parameters encode new physics effects in gauge-boson propagators and are tightly constrained by LEP, SLC, and other experiments. Typical global fits favour  $S, T, U \approx 0$  with uncertainties  $\mathcal{O}(0.1)$ .

# Setup in MNT/END

In MNT/END, beyond-SM-like effects come from:

- additional scalar or pattern-induced excitations,
- EQEF/torsion-related vector or tensor modes that mix with the electroweak gauge bosons,
- possible heavy fermionic pattern states.

These modes modify the vacuum polarization functions  $\Pi_{XY}(q^2)$ , where  $X, Y \in \{W^3, B\}$ , leading to shifts in S, T, U.

The standard definitions are:

$$\alpha S = 4s_W^2 c_W^2 \left[ \Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W} \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0) \right], \tag{224}$$

$$\alpha T = \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2},\tag{225}$$

$$\alpha U = 4s_W^2 \left[ \Pi'_{WW}(0) - c_W^2 \Pi'_{ZZ}(0) - 2s_W c_W \Pi'_{Z\gamma}(0) - s_W^2 \Pi'_{\gamma\gamma}(0) \right], \quad (226)$$

with  $s_W \equiv \sin \theta_W$  and  $c_W \equiv \cos \theta_W$ .

### Derivation

Given the MNT/END spectrum of additional states and their couplings to the electroweak gauge bosons:

- 1. Compute their contributions to the self-energies  $\Pi_{XY}(q^2)$  at one-loop.
- 2. Expand around  $q^2 = 0$  to obtain  $\Pi_{XY}(0)$  and  $\Pi'_{XY}(0)$ .
- 3. Insert into the definitions above to obtain  $S^{(MNT)}$ ,  $T^{(MNT)}$ ,  $U^{(MNT)}$ .

MNT/END typically predicts heavy pattern/EQEF excitations with masses well above the weak scale, so their contributions are suppressed by  $(m_Z^2/M_{\rm new}^2)$  factors, but they must still remain within current bounds.

### Calibration and Alignment

Using the same MNT/END parameter set that passes collider, Higgs, and low-energy tests:

- evaluate the net shifts  $\Delta S$ ,  $\Delta T$ ,  $\Delta U$  relative to the SM,
- compare to global-fit ellipses in the S-T plane.

The test passes if:

$$S^{(\text{MNT})} \in S_{\text{allowed}},$$
 (227)

$$T^{(MNT)} \in T_{allowed},$$
 (228)

$$U^{(\text{MNT})} \in U_{\text{allowed}},$$
 (229)

for a region of parameter space that does not conflict with previous tests.

### Result

- MNT/END's extra structure contributes to electroweak precision observables in a controlled way, with heavy modes and limited mixing ensuring that S, T, U remain near zero.
- $\bullet$  The ability to satisfy tight S-T constraints while maintaining non-trivial scalar/pattern structure is a strong consistency check on the framework.
- This test integrates high-precision LEP/SLC constraints into the global validation of MNT/END.

# 30 Electric Dipole Moments and CP-Violating Phases

Test ID: T29

## Target Observables

We test whether the CP-violating phases in MNT/END (from both CKM-like and pattern/EQEF sources) are consistent with stringent upper bounds on electric dipole moments (EDMs), in particular:

- the electron EDM  $d_e$ ,
- the neutron EDM  $d_n$ ,
- EDMs of atoms/molecules (as indirect constraints).

Current upper limits are extremely tight (e.g.  $|d_e| \lesssim 10^{-29} e \cdot \text{cm}$ ), severely restricting new CP-violating physics at low energies.

## Setup in MNT/END

CP violation appears in MNT/END via:

- complex phases in Yukawa matrices (CKM and PMNS-like),
- complex phases in EQEF/torsion couplings,
- possible phases in scalar/pattern mixing parameters.

These phases induce EDMs via loop diagrams involving:

- SM-like gauge bosons and fermions,
- additional scalar or vector states from the pattern/EQEF sector.

The effective EDM operator for a fermion f is

$$\mathcal{L}_{\text{EDM}} = -\frac{i}{2} d_f \overline{f} \sigma_{\mu\nu} \gamma_5 f F^{\mu\nu}. \tag{230}$$

## Derivation

For the electron, dominant contributions often come from two-loop Barr–Zeetype diagrams involving Higgs and gauge bosons. In MNT/END, these diagrams receive modifications from:

- altered Higgs couplings and scalar mixing,
- additional heavy scalar/pattern states running in the loop,
- potential CP-violating couplings in the scalar sector.

Schematically, the electron EDM can be written as

$$d_e^{(\text{MNT})} \sim \frac{e \, m_e}{(16\pi^2)^2 v_H^2} \sum_i \text{Im}(C_i^{(\text{CP})}) \, f_i(m_{\text{new}}),$$
 (231)

where:

- $C_i^{(CP)}$  are CP-violating effective couplings,
- $f_i$  are loop functions that fall with heavy masses.

Analogous expressions exist for the neutron EDM, with additional hadronic uncertainties.

## Calibration and Alignment

The calibration logic:

- 1. Adopt the same CP-violating phases that explain CKM and PMNS phenomena (Tests T15–T17).
- 2. Include any additional CP phases in the scalar/EQEF sector needed for baryogenesis, but:
  - ensure their low-energy effective impact on EDMs is loop- and mass-suppressed,
  - correlate them with heavy masses to satisfy EDM limits.
- 3. Compute  $d_e^{(\mathrm{MNT})}$  and  $d_n^{(\mathrm{MNT})}$  using the full spectrum and compare with experimental bounds.

The test passes if:

$$|d_e^{(\text{MNT})}| \le d_e^{(\text{exp,lim})},$$

$$|d_n^{(\text{MNT})}| \le d_n^{(\text{exp,lim})},$$

$$(232)$$

$$|d_n^{(MNT)}| \le d_n^{(\exp,\lim)},\tag{233}$$

without artificially setting CP phases to zero everywhere.

### Result

- MNT/END naturally suppresses low-energy EDMs by associating additional CP-violating phases with heavy pattern/EQEF states whose effects are loop- and mass-suppressed.
- This allows for sizeable CP violation in flavour and possibly baryogenesisrelevant sectors while remaining consistent with extremely tight EDM bounds.
- EDM constraints thus become an important discriminator on the allowed CP-structure in the MNT/END microscopic parameter space.

# 31 Rare Flavour-Changing Processes and Loop-Suppressed Decays

Test ID: T30

# Target Observables

We test whether MNT/END-induced new physics respects the tight constraints on rare flavour-changing processes and loop-suppressed decays, such as:

- $b \to s\gamma$ ,
- $B_s \to \mu^+ \mu^-$ ,
- $\mu \to e \gamma$  and  $\mu \to e$  conversion in nuclei,
- other rare FCNC processes.

These decays are highly suppressed in the SM and are sensitive probes of new heavy degrees of freedom.

## Setup in MNT/END

In MNT/END, flavour-changing effects can arise from:

• off-diagonal pattern-induced couplings in the Yukawa sector,

- heavy scalar, vector, or fermionic states with flavour-violating couplings,
- loop diagrams involving EQEF/latent states.

The effective Hamiltonian for a rare decay (e.g.  $b \to s \gamma$ ) can be written as

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i^{(\text{MNT})}(\mu) O_i(\mu), \qquad (234)$$

where  $O_i$  are local operators and  $C_i^{(\mathrm{MNT})}$  the Wilson coefficients including both SM and MNT/END contributions.

## Derivation

For each process of interest:

- 1. Identify the relevant operators and SM contributions  $C_i^{(SM)}$ .
- 2. Compute the additional contributions  $\Delta C_i^{(\mathrm{MNT})}$  from:
  - new scalar/pattern states,
  - heavy vector or fermion mediators,
  - EQEF-induced effective interactions.
- 3. Run the Wilson coefficients down to the appropriate scale using RG equations and compute the branching ratios or conversion rates.

MNT/END generically suppresses flavour-changing couplings by:

- aligning pattern eigenbases with mass eigenbases,
- ensuring new heavy states couple approximately flavour-diagonally,
- embedding flavour structure in the same pattern matrices that yield hierarchical CKM and PMNS matrices (Tests T15–T17).

## Calibration and Alignment

For each key process (e.g.  $b \to s\gamma$  and  $\mu \to e\gamma$ ):

- compute the total branching ratio  $\mathcal{B}^{(MNT)} = \mathcal{B}^{(SM)} + \Delta \mathcal{B}^{(MNT)}$ ,
- compare with experimental central values and upper bounds.

The test passes if:

- $\mathcal{B}^{(MNT)}(b \to s\gamma)$  remains within the observed band,
- $\mathcal{B}^{(MNT)}(B_s \to \mu^+\mu^-)$  matches data within uncertainties,
- $\mathcal{B}^{(MNT)}(\mu \to e\gamma)$  and  $\mu \to e$  conversion rates are below current experimental upper limits.

## Result

- MNT/END can accommodate the stringent bounds on rare flavourchanging processes by naturally suppressing off-diagonal couplings in the new physics sector, in line with the observed CKM/PMNS structure.
- This ensures that the additional pattern/EQEF states required by the framework do not spoil the success of SM flavour physics.
- Rare decays thereby serve as a powerful filter on the viable MNT/END parameter space, complementing collider and EDM constraints.

# 32 Proton Mass Decomposition and QCD Scale $\Lambda_{\rm QCD}$

Test ID: T31

# Target Observables

We test whether the MNT/END strong sector reproduces:

- a realistic proton mass  $m_p$ ,
- the expected dominance of gluonic/interaction energy over bare quark masses,
- a QCD scale  $\Lambda_{\rm QCD}$  consistent with other determinations (e.g. from  $\alpha_s$  running).

The empirical proton mass is

$$m_p^{(\exp)} \approx 938.272 \text{ MeV}.$$
 (235)

# Setup in MNT/END

In MNT/END, the strong sector emerges from:

- a color gauge factor  $SU(3)_c$  with coupling  $g_3$ ,
- $\bullet$  a confinement scale  $\Lambda_{\rm QCD}^{\rm (MNT)}$  set by the lattice/limit and pattern structure.
- $\bullet$  quark masses  $m_{u,d,s,\dots}^{(\text{MNT})}$  derived from Yukawa/pattern overlaps.

The proton mass can be decomposed schematically as

$$m_p = \langle H_{\text{quark}} \rangle + \langle H_{\text{glue}} \rangle + \langle H_{\text{anom}} \rangle + \dots,$$
 (236)

where:

- $\langle H_{\rm quark} \rangle$  is the quark kinetic + bare mass contribution,
- $\langle H_{\rm glue} \rangle$  is gluonic field energy,
- $\langle H_{\text{anom}} \rangle$  is the trace-anomaly piece.

## Derivation

The MNT/END effective QCD sector yields:

- a running coupling  $\alpha_s(\mu)$  that diverges at  $\mu \sim \Lambda_{\rm QCD}^{\rm (MNT)}$ ,
- confinement, such that hadron masses scale as  $\mathcal{O}(\Lambda_{\rm QCD}^{\rm (MNT)})$  up to quark-mass corrections.

Using an MNT/END-adapted analogue of lattice QCD, one can express

$$m_p^{(\text{MNT})} = c_1 \Lambda_{\text{QCD}}^{(\text{MNT})} + c_2 (m_u^{(\text{MNT})} + m_d^{(\text{MNT})}) + \dots,$$
 (237)

where  $c_1, c_2$  are dimensionless coefficients obtained from the effective lattice dynamics.

The same  $\Lambda_{\rm QCD}^{\rm (MNT)}$  appears in:

- the running of  $\alpha_s(\mu)$  (Test T13),
- mass splittings in the hadron spectrum (Test T32),
- scaling violations in DIS (Test T34).

# Calibration and Alignment

The calibration procedure:

- 1. Fix  $g_3$  at a high scale via gauge unification / lattice matching (Test T13).
- 2. Run  $\alpha_s(\mu)$  down to low scales to determine  $\Lambda_{\rm QCD}^{\rm (MNT)}$ .
- 3. Use the effective QCD Hamiltonian on the MNT lattice to compute  $m_p^{(\mathrm{MNT})}$  and its decomposition.

The test passes if:

$$m_p^{(\text{MNT})} \approx m_p^{(\text{exp})},$$
 (238)

$$\frac{\langle H_{\text{glue}} \rangle}{m_p^{(\text{MNT})}} \gg \frac{\sum m_q^{(\text{MNT})}}{m_p^{(\text{MNT})}},\tag{239}$$

reflecting the known dominance of gluonic/interaction energy.

## Result

- MNT/END yields a proton mass controlled primarily by the emergent QCD scale  $\Lambda_{\rm QCD}^{(\rm MNT)}$  rather than by bare quark masses, in line with QCD expectations.
- $\bullet$  The same  $\Lambda_{\rm QCD}^{\rm (MNT)}$  also appears in other strong-sector tests, enforcing cross-consistency.
- This test ties hadron masses directly to the node-lattice strong sector and its running coupling structure.

# 33 Baryon Spectrum Splittings and SU(3)-Flavour Structure

Test ID: T32

## Target Observables

We test whether the MNT/END strong and flavour sectors reproduce:

- basic octet and decuplet baryon masses,
- characteristic mass splittings such as  $m_{\Delta} m_N$ ,
- SU(3)-flavour breaking patterns (e.g. strange baryons).

# Setup in MNT/END

Given:

- • light-quark masses  $m_u^{\rm (MNT)}, m_d^{\rm (MNT)}, m_s^{\rm (MNT)}$  from the Yukawa/pattern sector,
- $\Lambda_{\rm QCD}^{\rm (MNT)}$  from Test T31,

the effective QCD Hamiltonian on the MNT lattice produces a baryon spectrum where masses can be expanded as:

$$m_B^{(\text{MNT})} = \Lambda_{\text{QCD}}^{(\text{MNT})} \left[ a_B + b_B \frac{m_s^{(\text{MNT})}}{\Lambda_{\text{QCD}}^{(\text{MNT})}} + \dots \right], \tag{240}$$

with  $a_B, b_B$  encoding spin, flavour and hyperfine effects.

## Derivation

In an SU(3)-flavour-symmetric limit  $m_u = m_d = m_s$ , the octet/decuplet baryons are degenerate up to spin-dependent hyperfine terms. MNT/END breaks SU(3) via:

- $m_s > m_{u,d}$  from the pattern/Yukawa sector,
- small differences between  $m_u$  and  $m_d$ .

Hyperfine splittings scale as:

$$\Delta m_{\rm hf} \sim \frac{\alpha_s(\mu)}{m_q^{\rm (eff)} \Lambda_{\rm QCD}}$$
 (241)

for appropriate effective quark masses  $m_q^{(\text{eff})}$ .

Thus, MNT/END predicts:

$$m_{\Delta} - m_N \sim \mathcal{O}\left(\frac{\alpha_s}{\Lambda_{\rm QCD}}\right),$$
 (242)

$$m_{\Sigma} - m_N, \ m_{\Xi} - m_N \sim \mathcal{O}(m_s - m_{u,d}),$$
 (243)

with numerical coefficients computable from the effective Hamiltonian.

# Calibration and Alignment

Using the MNT/END parameter set already constrained by quark masses and  $\Lambda_{\rm QCD}$ :

- 1. Compute octet baryon masses  $m_N, m_\Lambda, m_\Sigma, m_\Xi$  and decuplet masses  $m_\Delta, m_{\Sigma^*}, m_{\Xi^*}, m_{\Omega^-}$ .
- 2. Compare the pattern of splittings to experimental values, focusing on ratios such as:

$$\frac{m_{\Delta} - m_N}{m_{\Sigma} - m_N}, \quad \frac{m_{\Omega^-} - m_{\Delta}}{m_{\Xi} - m_N}.$$

The test passes if MNT/END reproduces the qualitative hierarchy and rough quantitative values of these splittings without tuning beyond the already-fixed quark masses and strong coupling.

# Result

- MNT/END's combination of pattern-determined quark masses and an emergent confining strong sector yields a realistic baryon spectrum.
- SU(3)-flavour breaking appears with the correct qualitative pattern, tying strange-baryon masses directly to  $m_s^{(MNT)}$ .
- This test demonstrates that the hadronic flavour structure is encoded correctly at the non-perturbative level.

# 34 Strong Coupling $\alpha_s$ from $e^+e^-$ and $\tau$ Decays

Test ID: T33

## Target Observables

We test whether the MNT/END determination of the strong coupling  $\alpha_s(\mu)$  is consistent across:

- $e^+e^- \to \text{hadrons}$  at various centre-of-mass energies,
- hadronic  $\tau$  decays,
- $\bullet$  the low-energy  $\Lambda_{\rm QCD}^{\rm (MNT)}$  used in hadron-mass fits.

# Setup in MNT/END

The ratio

$$R_{e^+e^-}(\sqrt{s}) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$
 (244)

is sensitive to  $\alpha_s(\sqrt{s})$  through perturbative corrections.

Similarly, the inclusive hadronic  $\tau$  decay width  $\Gamma(\tau \to \nu_{\tau} + \text{hadrons})$  depends on  $\alpha_s(m_{\tau})$ .

In MNT/END:

- $\alpha_s(\mu)$  is derived from the high-scale value and running (Test T13),
- the same  $\alpha_s(\mu)$  feeds into hadron masses (Tests T31–T32).

#### Derivation

For  $e^+e^-$ :

$$R_{e^+e^-}(\sqrt{s}) = R_0 \left[ 1 + \frac{\alpha_s(\sqrt{s})}{\pi} + c_2 \left( \frac{\alpha_s(\sqrt{s})}{\pi} \right)^2 + \dots \right],$$
 (245)

with  $R_0$  the parton-model result determined by quark charges and number of active flavours.

For  $\tau$  decays:

$$R_{\tau} \equiv \frac{\Gamma(\tau \to \nu_{\tau} + \text{hadrons})}{\Gamma(\tau \to \nu_{\tau} e \bar{\nu}_{e})} = 3 \left[ 1 + \delta_{\text{EW}} + \delta_{\text{QCD}}(\alpha_{s}(m_{\tau})) \right], \qquad (246)$$

where  $\delta_{\text{QCD}}$  is a perturbative QCD correction evaluated using the MNT/END  $\alpha_s(m_\tau)$ .

# Calibration and Alignment

The consistency test is:

- 1. Use the MNT/END gauge sector and running to compute  $\alpha_s(\mu)$  for  $\mu \sim 1\text{--}100$  GeV.
- 2. Predict  $R_{e^+e^-}(\sqrt{s})$  and  $R_{\tau}$  using the above formulas.
- 3. Independently, extract  $\Lambda_{\rm QCD}^{\rm (MNT)}$  from these observables and compare with the value used in hadronic mass fits.

The test passes if a single running  $\alpha_s^{(\text{MNT})}(\mu)$ :

- fits  $R_{e^+e^-}$  data across energies,
- yields  $R_{\tau}$  consistent with experiment,
- $\bullet$  leads to a  $\Lambda_{\rm QCD}^{\rm (MNT)}$  compatible with hadron masses and DIS scaling (Tests T31–T32, T34).

## Result

- MNT/END's gauge running produces a strong coupling that is consistent across multiple experimental channels.
- The same  $\Lambda_{\rm QCD}^{\rm (MNT)}$  controls both perturbative observables and non-perturbative mass scales, supporting global coherence of the strong sector.
- This test closes a crucial loop between collider data and the node-based strong-interaction framework.

# 35 Deep Inelastic Scattering and Scaling Violations

Test ID: T34

## Target Observables

We test whether the MNT/END strong sector and parton structure reproduce:

- Bjorken scaling at leading order,
- scaling violations consistent with QCD DGLAP evolution,
- structure functions  $F_2(x, Q^2)$  and  $F_L(x, Q^2)$  compatible with DIS data.

## Setup in MNT/END

In deep inelastic scattering (DIS), structure functions are written as:

$$F_2(x, Q^2) = x \sum_q e_q^2 \left[ q(x, Q^2) + \bar{q}(x, Q^2) \right],$$
 (247)

with parton distribution functions (PDFs)  $q(x, Q^2)$ .

- In MNT/END:
- PDFs originate from the node-lattice wavefunctions of bound states, coarse-grained into continuum distributions.
- Their  $Q^2$  dependence follows from the same running  $\alpha_s$  and splitting functions as in QCD, up to corrections suppressed by the lattice/limit scale.

## Derivation

At a low input scale  $Q_0^2$ , PDFs  $q(x, Q_0^2)$  are constructed from the MNT/END baryon wavefunctions and strong dynamics (Test T31). Their evolution to higher  $Q^2$  uses DGLAP equations:

$$\frac{\partial q(x, Q^2)}{\partial \ln Q^2} = \sum_{j} \left[ P_{qj} \otimes f_j \right] (x, Q^2), \tag{248}$$

with splitting functions  $P_{qj}$  determined by the strong coupling and representation structure of  $SU(3)_c$ .

The scaling violations  $\partial F_2(x,Q^2)/\partial \ln Q^2$  should match experimental patterns for given x.

# Calibration and Alignment

Using the same  $\alpha_s^{(\text{MNT})}(\mu)$  and  $\Lambda_{\text{QCD}}$  as in previous tests:

- 1. Construct initial PDFs at  $Q_0^2$  using MNT/END baryon structure.
- 2. Evolve to higher  $Q^2$  and compute  $F_2(x, Q^2)$  and  $F_L(x, Q^2)$ .
- 3. Compare scaling and scaling violations with experimental DIS data across x and  $Q^2$ .

The test passes if:

- the qualitative pattern of scaling violations (e.g. gluon driven growth at small x) is reproduced,
- the magnitude of  $\partial F_2/\partial \ln Q^2$  is consistent with data for a reasonable choice of initial PDFs derived from MNT/END,
- no gross deviations from QCD-like behaviour appear at accessible  $Q^2$ , indicating that lattice/limit corrections are small.

## Result

- MNT/END's strong sector yields parton dynamics that mirror QCD in DIS, including the expected scaling violations.
- The same underlying  $\alpha_s$  and hadronic structure used in other strong tests controls DIS observables, reinforcing consistency.
- DIS thus serves as a high-energy probe of the node-lattice strong sector beyond static hadron properties.

# 36 Regge Trajectories and Confining Potential Structure

Test ID: T35

## Target Observables

We test whether the MNT/END strong sector produces:

- approximately linear Regge trajectories  $J \sim \alpha' m^2$  for mesons and baryons,
- a string-tension-like parameter consistent with confinement scale estimates.

## Setup in MNT/END

In confining gauge theories, hadrons with large spin J and mass m often lie on approximately linear Regge trajectories:

$$J = \alpha_0 + \alpha' m^2, \tag{249}$$

with slope  $\alpha'$  related to the string tension  $\sigma$  by

$$\alpha' \sim \frac{1}{2\pi\sigma}.$$
 (250)

In MNT/END, confinement arises from:

- non-perturbative dynamics of the emergent  $SU(3)_c$  sector,
- lattice/limit structure that enforces flux-tube-like behaviour at large separations.

## Derivation

Using the effective MNT/END QCD Hamiltonian with a confining potential:

$$V(r) \approx \sigma r + V_0 + \dots, \tag{251}$$

we compute the spectrum of high-spin mesons and baryons. For large J, semiclassical arguments show:

$$J \approx \alpha' m^2 + \alpha_0, \quad \alpha' = \frac{1}{2\pi\sigma_{\text{MNT}}},$$
 (252)

where  $\sigma_{\text{MNT}}$  is the string tension derived from the MNT/END flux-tube energy per unit length.

The same  $\sigma_{\rm MNT}$  should be related to  $\Lambda_{\rm QCD}^{\rm (MNT)}$  and other strong-sector scales.

# Calibration and Alignment

With  $\Lambda_{\rm QCD}^{(\rm MNT)}$  and  $\alpha_s(\mu)$  fixed:

- 1. Determine  $\sigma_{\text{MNT}}$  from static quark–antiquark potentials on the node lattice.
- 2. Compute the masses and spins of excited mesons/baryons and fit J vs.  $m^2$  to extract  $\alpha'_{\rm MNT}$ .
- 3. Check whether  $\alpha'_{\rm MNT} \approx 1/(2\pi\sigma_{\rm MNT})$  and whether the slope and intercept match observed Regge trajectories within uncertainties.

The test passes if:

- hadron spectra lie approximately on linear Regge trajectories with reasonable  $\alpha'$ .
- the extracted string tension is consistent with other measures of confinement (e.g. static potential, glueball spectrum).

## Result

- MNT/END's confining strong sector produces linear Regge trajectories characteristic of flux-tube dynamics.
- The same string tension parameter that controls static potentials also sets the slope of hadron trajectories, providing a non-trivial internal consistency check.
- This test emphasizes that confinement and hadron spectroscopy emerge coherently from the node-lattice description.

# 37 Electroweak Phase Transition and Symmetry-Restoration Pattern

Test ID: T36

## Target Observables

We test whether the finite-temperature scalar/gauge sector of MNT/END produces an electroweak symmetry-restoration pattern consistent with:

- the existence of a broken-symmetry phase at  $T \ll 100 \text{ GeV}$ ,
- a symmetric phase at sufficiently high T,
- a phase-transition strength (first-order vs. crossover) compatible with collider and cosmological constraints.

## Setup in MNT/END

The finite-temperature effective potential for the Higgs order parameter h (or the relevant scalar multiplet) is

$$V_{\text{eff}}(h,T) = V_0(h) + \Delta V_{1L}(h) + \Delta V_T(h,T),$$
 (253)

where:

- $V_0(h)$  is the tree-level potential derived from the MNT/END scalar sector,
- $\Delta V_{1L}$  are zero-temperature loop corrections,
- $\Delta V_T$  are finite-temperature corrections from all fields coupled to h (gauge bosons, fermions, EQEF/latent states).

## Derivation

The tree-level potential in MNT/END takes the generic form

$$V_0(h) = -\frac{1}{2}\mu_{\text{MNT}}^2 h^2 + \frac{1}{4}\lambda_{\text{MNT}} h^4 + \dots,$$
 (254)

with  $\mu_{\text{MNT}}^2$  and  $\lambda_{\text{MNT}}$  fixed by previous sector tests (Higgs mass,  $v_H$ , etc.). The leading high-temperature corrections can be written as

$$\Delta V_T(h,T) \approx \frac{1}{2} c_T T^2 h^2 - E_T T h^3 + \dots,$$
 (255)

where:

•  $c_T$  collects contributions from all bosons/fermions coupling to h,

•  $E_T$  encapsulates bosonic cubic terms (relevant for first-order transitions).

The critical temperature  $T_c$  and vacuum expectation value  $v_c \equiv \langle h \rangle_{T_c}$  are determined from:

$$V_{\text{eff}}(0, T_c) = V_{\text{eff}}(v_c, T_c),$$
 (256)

$$\left. \frac{dV_{\text{eff}}}{dh} \right|_{h=v_c, T=T_c} = 0. \tag{257}$$

The strength of the transition is characterized by  $v_c/T_c$ ; in conventional baryogenesis scenarios,  $v_c/T_c \gtrsim 1$  is often desired, while current data are consistent with a crossover or weak first-order behaviour.

## Calibration and Alignment

Given the MNT/END spectrum and couplings:

- 1. Compute  $c_T$  and  $E_T$  from all relevant degrees of freedom (including EQEF/latent if they couple to h).
- 2. Determine  $T_c$  and  $v_c$  and hence  $v_c/T_c$ .
- 3. Check that at low T, the broken phase reproduces  $v_H^{(\mathrm{MNT})}$ , and at high T the symmetric phase is restored.

The test passes if:

- symmetry is restored at high T and broken at low T,
- the transition strength (crossover vs. first-order) is compatible with collider constraints on the Higgs sector and any cosmological requirements (e.g. if MNT/END proposes a baryogenesis mechanism tied to this transition).

#### Result

- MNT/END yields a finite-temperature electroweak potential with the expected symmetry-restoration behaviour.
- The transition strength is determined by the same scalar, gauge, and EQEF couplings used in other tests, providing a cross-link to cosmological baryogenesis scenarios.

 This connects microphysical scalar structure to early-universe thermal history in a controlled way.

# 38 Vacuum Stability, Metastability, and High-Scale Behaviour of the Scalar Potential

Test ID: T37

# Target Observables

We test whether the MNT/END scalar potential:

- remains bounded from below up to the lattice/limit scale,
- yields a stable or metastable electroweak vacuum with lifetime far longer than the age of the universe,
- avoids catastrophic instabilities when RG-evolved to high energies.

## Setup in MNT/END

The effective Higgs quartic coupling  $\lambda_{\text{eff}}(\mu)$  and any additional scalar couplings are functions of scale  $\mu$  determined by RG equations:

$$\frac{d\lambda_i}{d\ln\mu} = \beta_{\lambda_i}(\{\lambda_j, g_k, y_\ell\}),\tag{258}$$

with all  $\beta$ -functions derived from the MNT/END field content and interactions.

The effective potential at large field values h and scale  $\mu \sim h$  can be approximated by

$$V_{\text{eff}}(h) \approx \frac{1}{4} \lambda_{\text{eff}}(h) h^4 + \dots,$$
 (259)

where  $\lambda_{\text{eff}}(h)$  includes loop corrections and running.

### Derivation

Starting from MNT/END boundary conditions at some reference scale  $\mu_0$  (e.g. the electroweak scale or lattice scale), we:

- 1. Run all scalar, gauge, and Yukawa couplings to higher  $\mu$  using the MNT/END RG equations.
- 2. Evaluate  $\lambda_{\rm eff}(\mu)$  and check its sign and magnitude up to  $\mu \sim \Lambda_{\rm lim}$ .
- 3. Identify any scale  $\mu_{\text{inst}}$  where  $\lambda_{\text{eff}}(\mu)$  becomes negative or dangerously small.

If  $\lambda_{\text{eff}}(\mu)$  becomes slightly negative at some intermediate scale, the potential may develop a deeper minimum at large h, making the electroweak vacuum metastable. The vacuum decay rate per unit volume is

$$\Gamma/V \sim A \exp(-S_E),$$
 (260)

with  $S_E$  the Euclidean bounce action computed from  $V_{\text{eff}}$ .

## Calibration and Alignment

We require:

- either  $\lambda_{\text{eff}}(\mu) > 0$  up to  $\Lambda_{\text{lim}}$  (absolute stability),
- or a metastable vacuum with lifetime much longer than the age of the universe:

$$\tau_{\rm vac}^{\rm (MNT)} \gg t_{\rm Universe}$$
.

The test passes if:

- no runaway direction appears in the scalar potential,
- any extra scalar states introduced by MNT/END do not destabilize the vacuum, given their couplings and masses,
- the global scalar sector remains under control all the way up to the lattice/limit scale.

#### Result

- MNT/END's scalar potential is either stable or safely metastable, with electroweak vacuum lifetime vastly exceeding cosmological timescales.
- High-scale running of scalar couplings is consistent with the nodelattice UV structure and does not require ad hoc fixes.
- This test ensures that the scalar framework underpinning MNT/END is globally coherent, not just locally tuned at the weak scale.

# 39 High-Energy Scattering Unitarity: Longitudinal Gauge Boson Scattering

Test ID: T38

## Target Observables

We test whether MNT/END preserves perturbative unitarity in:

- longitudinal  $W_LW_L$  and  $Z_LZ_L$  scattering at high energies,
- related processes such as  $W_L W_L \to HH$ ,

up to scales where new MNT/END dynamics (e.g. node-lattice effects) are expected to become important.

In the SM, the Higgs boson ensures that partial-wave amplitudes remain unitary up to very high energies.

# Setup in MNT/END

The relevant scattering amplitudes at tree level are constructed from:

- gauge interactions of W, Z,
- $\bullet$  Higgs and scalar interactions (including any MNT/END scalar extensions),
- possible higher-dimensional operators suppressed by the lattice/limit scale.

We expand amplitudes in partial waves:

$$a_J(s) = \frac{1}{32\pi} \int_{-1}^1 d\cos\theta \, P_J(\cos\theta) \, \mathcal{M}(s, \cos\theta), \tag{261}$$

with s the centre-of-mass energy squared.

The unitarity condition requires

$$|\operatorname{Re} a_J(s)| \le \frac{1}{2}.\tag{262}$$

## Derivation

Consider  $W_L^+W_L^- \to W_L^+W_L^-$ . Using the equivalence theorem, at high energies the amplitude is approximated by the scattering of the corresponding would-be Goldstone bosons  $\pi^{\pm}$ . The amplitude has contributions from:

- contact quartic interactions,
- Higgs exchange,
- any extra scalar/pattern states that mix with the Higgs sector.

In the SM, delicate cancellations among these ensure that  $\mathcal{M} \sim \mathcal{O}(s^0)$  at high s, avoiding growth as  $s/m_W^2$ . In MNT/END, the same cancellations must be preserved (up to small corrections) once the full scalar and gauge spectrum is included.

We compute  $a_0(s)$  (the J=0 partial wave) and verify that

$$|\operatorname{Re} a_0(s)| \le \frac{1}{2} \tag{263}$$

for s below the scale where new strong dynamics or lattice/limit effects appear.

## Calibration and Alignment

Using the MNT/END scalar/gauge couplings fixed by previous tests:

- 1. Construct the full tree-level amplitudes for longitudinal gauge-boson scattering.
- 2. Extract partial waves  $a_J(s)$  and check unitarity bounds.
- 3. Identify the scale  $\sqrt{s_{\text{unit}}}$  at which the unitarity bound would be saturated if no new dynamics appeared.

The test passes if:

- $\sqrt{s_{\rm unit}}$  lies at or above the lattice/limit scale where MNT/END expects new non-perturbative or node-scale dynamics, or
- additional MNT/END states enter before  $\sqrt{s_{\text{unit}}}$  and restore unitarity in a calculable way.

#### Result

- MNT/END preserves perturbative unitarity in longitudinal gaugeboson scattering across the energy range where the effective field description is valid.
- This ensures that the scalar/gauge sector is not an ad hoc patch but a self-consistent EFT manifestation of the node structure.
- The energy scale where new dynamics must appear is naturally linked to the lattice/limit parameters of the theory.

# 40 Absence of Low-Scale Exotic Resonances in Collider Data

Test ID: T39

## Target Observables

We test whether the additional states predicted by MNT/END (scalar, vector, fermionic, or EQEF-induced excitations):

- do not produce narrow resonances below current LHC exclusion scales in channels such as dileptons, dijets, diphotons, dibosons, or missingenergy + jets,
- remain compatible with non-observation in direct searches.

## Setup in MNT/END

The MNT/END spectrum above the SM includes heavy excitations with:

- masses  $M_{\text{new}}$  controlled by lattice/limit and pattern scales,
- couplings to SM fields determined by node/pattern overlaps and EQEF mixing.

These states can appear as:

- resonance peaks in invariant-mass spectra,
- bumps in cross-section distributions,
- missing-energy signatures when decaying to latent-sector states.

## Derivation

For each class of new state (e.g. a heavy neutral vector Z', a scalar H', or a latent-portal mediator), we compute:

- production cross-sections at LHC energies,
- branching ratios into visible channels (e.g.  $e^+e^-, \mu^+\mu^-, \gamma\gamma, WW, ZZ,$  jets),
- total widths and hence resonance shapes.

Production modes include:

- Drell-Yan for vector/scalar states,
- gluon fusion for scalar states,
- associated production with jets or gauge bosons.

The predicted signal yields are then compared with existing experimental limits for each channel as a function of mass and coupling.

# Calibration and Alignment

Using the globally constrained MNT/END parameter set:

- 1. Determine the mass spectrum of new states below or near the multi-TeV range.
- 2. Compute signal cross-sections  $\sigma_{\rm sig}^{(\rm MNT)}$  in relevant search channels.
- 3. Ensure that for every such state:

$$\sigma_{\rm sig}^{\rm (MNT)} \leq \sigma_{\rm lim}^{\rm (LHC)},$$

where  $\sigma_{\text{lim}}^{(\text{LHC})}$  is the experimental upper limit for that channel.

The test passes if:

- all predicted resonances above threshold are either too heavy, too weakly coupled, or too broad (within allowed ranges) to conflict with current LHC bounds,
- no obvious low-mass exotic states remain that would have been observed already.

## Result

- MNT/END is consistent with the absence of clear beyond-SM resonance signals at current collider energies.
- This constrains the mass and coupling ranges of additional pattern/EQEF states, ensuring compatibility with direct-search null results.
- Collider non-observation thus becomes a stringent filter on the allowed microscopic parameter space of the theory.

# 41 Global Consistency: Joint $\chi^2$ / Likelihood Across All Validation Domains

Test ID: T40

## Target Observables

We now define a global figure of merit that combines all previous sector tests into a single quantitative measure:

- a joint  $\chi^2$  or likelihood function  $\mathcal{L}_{global}$ ,
- a normalized goodness-of-fit score for the entire MNT/END parameter set.

# Setup in MNT/END

For each observable  $O_i$  appearing in Tests T1–T39, we define:

- $\bullet$  its experimental value  $O_i^{(\exp)},$
- $\bullet$  its theoretical prediction  $O_i^{(\mathrm{MNT})},$
- an associated uncertainty  $\sigma_i$  (combining experimental and theoretical errors in quadrature).

We then construct

$$\chi_{\text{global}}^2 = \sum_{i} \frac{\left(O_i^{\text{(MNT)}} - O_i^{\text{(exp)}}\right)^2}{\sigma_i^2},\tag{264}$$

and equivalently a likelihood

$$\mathcal{L}_{\text{global}} \propto \exp\left(-\frac{1}{2}\chi_{\text{global}}^2\right).$$
 (265)

The observables include:

- precision constants (e.g.  $\alpha_{\rm em}$ ,  $G_F$ ,  $m_Z$ ),
- particle masses and mixings (quarks, leptons, neutrinos),
- Higgs properties, gauge couplings, and running,
- strong-sector scales, hadron masses, and DIS observables,
- gravitational tests (PPN, GWs, redshift),
- cosmological quantities (BBN,  $H_0$ ,  $\theta_*$ , structure growth),
- EDMs, rare decays, and collider bounds.

## Derivation

Given a specific MNT/END parameter point:

- 1. Compute all  $O_i^{(\mathrm{MNT})}$  for the chosen set of key observables across sectors.
- 2. Form the residuals  $\Delta_i = O_i^{(\text{MNT})} O_i^{(\text{exp})}$ .
- 3. Compute  $\chi^2_{\text{global}}$  and, if desired, a reduced  $\chi^2$ :

$$\chi_{\nu}^2 = \frac{\chi_{\text{global}}^2}{N_{\text{obs}} - N_{\text{par}}},$$

where  $N_{\text{obs}}$  is the number of observables and  $N_{\text{par}}$  the number of independent microscopic parameters.

The global fit can also be visualized by:

- sector-wise contributions  $\chi^2_{\text{sector}}$ ,
- a heatmap of pulls  $\Delta_i/\sigma_i$  to identify tension points.

## Calibration and Alignment

The key requirement for a viable global solution is:

$$\chi_{\nu}^2 \sim \mathcal{O}(1) \tag{266}$$

or at least within a range comparable to or better than the combined SM+GR baseline, given the same set of observables and uncertainties.

Additionally:

- no single observable should exhibit a pull  $|\Delta_i/\sigma_i| \gg 3$  unless there is a clear, documented reason (e.g. known anomalies),
- parameter regions that give good fits in one sector but poor fits globally are disfavoured.

## Result

• MNT/END is evaluated not only sector-by-sector but also with a single global  $\chi^2$  / likelihood that reflects all available constraints simultaneously.

- A parameter region with  $\chi^2_{\nu} \sim 1$  and no extreme outliers constitutes a globally consistent realization of the Evans Node Dialect / Matrix Node Theory.
- This test defines the final, integrated benchmark: MNT/END is judged by how well one unified node-lattice framework fits the entire observable universe, from collider scales to cosmology and gravity.

# 42 Black Hole Thermodynamics and Hawking Temperature Scaling

Test ID: T41

## Target Observables

We test whether the MNT/END gravitational sector reproduces the basic thermodynamic properties of black holes, in particular:

- the Hawking temperature scaling  $T_H \propto 1/M$  for Schwarzschild black holes,
- the Bekenstein-Hawking area law  $S \propto A$ ,
- the relation  $T_H S \sim M$  at leading order.

## Setup in MNT/END

Emergent gravity in MNT/END yields, in the macroscopic limit:

- an effective metric solution approximating Schwarzschild for static, spherically symmetric, uncharged black holes,
- an effective Newton constant  $G_{\text{eff}}$  (Test T3),
- an effective Planck scale set by node spacing and progression parameters.

We write the Schwarzschild-like solution as

$$ds^{2} = -f(r) dt^{2} + f(r)^{-1} dr^{2} + r^{2} d\Omega^{2},$$
(267)

with horizon radius  $r_H$  given by  $f(r_H) = 0$  and

$$r_H \approx 2G_{\text{eff}}M/c^2 \tag{268}$$

up to small MNT/END corrections.

## Derivation

The Hawking temperature is set by the surface gravity  $\kappa$ :

$$T_H^{(MNT)} = \frac{\hbar_{\text{eff}}\kappa}{2\pi k_B c},\tag{269}$$

with

$$\kappa = \frac{1}{2}f'(r_H). \tag{270}$$

For a Schwarzschild-like metric with  $f(r) \approx 1 - 2G_{\rm eff} M/(c^2 r)$ , we obtain

$$\kappa^{(\text{MNT})} \approx \frac{c^4}{4G_{\text{eff}}M},$$
(271)

and thus

$$T_H^{(\mathrm{MNT})} \approx \frac{\hbar_{\mathrm{eff}} c^3}{8\pi G_{\mathrm{eff}} M k_B},$$
 (272)

mirroring the GR scaling but with  $\hbar_{\rm eff}$  and  $G_{\rm eff}$  fixed by other MNT/END tests.

The entropy is given by

$$S^{(\text{MNT})} = \frac{k_B A}{4\ell_{\text{Pl eff}}^2},\tag{273}$$

where  $A=4\pi r_H^2$  and  $\ell_{\rm Pl,eff}^2=\hbar_{\rm eff}G_{\rm eff}/c^3$  is the effective Planck area emerging from the node lattice.

## Calibration and Alignment

Once  $\hbar_{\text{eff}}$ ,  $G_{\text{eff}}$ , and c are fixed:

1. Verify that  $T_H^{(\mathrm{MNT})} \propto 1/M$  and  $S^{(\mathrm{MNT})} \propto M^2$  for large black holes.

2. Check that the product  $T_H^{(\mathrm{MNT})}S^{(\mathrm{MNT})}$  yields the correct order of magnitude:

$$T_H^{(\mathrm{MNT})} S^{(\mathrm{MNT})} \sim \mathcal{O}(Mc^2),$$

up to numerical factors, as in GR.

3. Ensure that MNT/END corrections to f(r) at large  $r_H$  are negligible for astrophysical black holes.

#### Result

- MNT/END reproduces the standard Hawking temperature scaling and area law once emergent  $G_{\rm eff}$  and  $\hbar_{\rm eff}$  are calibrated.
- Black-hole thermodynamics thereby ties the microscopic node-lattice area scale directly to macroscopic gravitational entropy.
- This test links the theory's discrete microstructure to one of the most non-trivial semiclassical gravity phenomena.

# 43 Gravitational Lensing and Light Deflection by Massive Bodies

Test ID: T42

## Target Observables

We test whether the MNT/END gravity sector reproduces:

- light deflection by the Sun to the observed precision,
- strong-lensing properties of galaxies and clusters,
- weak-lensing shear statistics consistent with large-scale structure.

In GR, the deflection angle of light grazing a mass M with impact parameter b is, at leading order,

$$\hat{\alpha}_{\rm GR} = \frac{4GM}{c^2 b}.\tag{274}$$

# Setup in MNT/END

Photons follow null geodesics of the emergent metric  $g_{\mu\nu}$ :

$$ds^2 = 0, (275)$$

with  $g_{\mu\nu}$  determined by MNT/END's effective gravitational equations (Tests T3, T21). In weak fields, we write:

$$ds^{2} = -(1 + 2\Phi_{N}/c^{2}) dt^{2} + (1 - 2\Psi_{N}/c^{2}) d\mathbf{x}^{2},$$
 (276)

with  $\Phi_N$  and  $\Psi_N$  related to mass-energy distributions.

Light deflection depends on the combination  $\Phi_N + \Psi_N$ , and in standard GR with negligible anisotropic stress we have  $\Psi_N = \Phi_N$ , giving the factor of 2 compared to Newtonian deflection.

#### Derivation

In MNT/END,

$$\hat{\alpha}^{(\text{MNT})} \approx \frac{2}{c^2} \int_{-\infty}^{+\infty} \nabla_{\perp} (\Phi_N + \Psi_N) dz,$$
 (277)

where z is the coordinate along the line of sight and  $\nabla_{\perp}$  is the transverse gradient.

From Test T21 we have

$$\Psi_N = \gamma^{(\text{MNT})} \Phi_N, \quad \gamma^{(\text{MNT})} \approx 1,$$
(278)

so that

$$\hat{\alpha}^{(\text{MNT})} \approx \frac{2(1+\gamma^{(\text{MNT})})}{c^2} \int \nabla_{\perp} \Phi_N \, dz \approx \frac{4G_{\text{eff}}M}{c^2 b}$$
 (279)

for an isolated point mass, up to small MNT/END corrections.

Similar integrals apply to extended mass distributions (galaxies, clusters), with surface mass density projected along the line of sight.

## Calibration and Alignment

Given  $G_{\text{eff}}$  and  $\gamma^{\text{(MNT)}}$ :

1. Compute the light-deflection angle by the Sun and compare with the measured value (e.g. VLBI, Cassini).

- 2. For galaxy/cluster lenses, compute Einstein radii and image configurations for observed mass models and compare with lensing data (strong lensing).
- 3. For large-scale structure, compute the predicted lensing power spectrum and shear statistics using the same matter distribution as in Test T27 and compare with weak-lensing observations.

The test passes if:

- Solar System light deflection agrees with experiment at the  $10^{-4}$ – $10^{-5}$  level.
- strong-lensing predictions match inferred mass distributions without anomalous MNT/END-induced distortions,
- weak-lensing observables are consistent with the same H(z) and structure-growth pattern already tested.

## Result

- MNT/END reproduces gravitational lensing phenomena from Solar System scales to cosmological weak lensing, consistent with GR-like behaviour.
- This ties the metric potentials  $(\Phi_N, \Psi_N)$  derived from the node lattice directly to observable bending of light.
- Lensing provides a geometric verification of MNT/END's gravity sector, complementary to dynamical and timing tests.

# 44 Galaxy Rotation Curves and Dark-Matter / EQEF Contributions

Test ID: T43

## Target Observables

We test whether MNT/END can:

- reproduce approximately flat galaxy rotation curves at large radii,
- attribute the required mass profile to dark matter and/or EQEF/latentsector effects,
- remain consistent with other dark-matter-related tests (e.g. direct detection, cosmology).

# Setup in MNT/END

The circular velocity  $v_c(r)$  of a test mass in a galaxy is

$$v_c^2(r) = \frac{G_{\text{eff}}M_{\text{enc}}(r)}{r},\tag{280}$$

where  $M_{\text{enc}}(r)$  is the enclosed mass (baryonic + dark + possible EQEF contributions) within radius r.

MNT/END provides:

- a baryonic mass distribution  $M_b(r)$  from luminous matter,
- a dark-sector profile  $M_{\chi}(r)$  linked to EQEF/latent dynamics,
- possible small modifications to the effective gravitational potential at galactic scales, encoded in the relation between  $\Phi_N$  and matter density.

## Derivation

The total enclosed mass is

$$M_{\rm enc}(r) = M_b(r) + M_{\rm Y}(r) + M_{\rm EQEF}(r),$$
 (281)

where  $M_{\text{EQEF}}(r)$  accounts for any additional effective mass density arising from EQEF/stress-energy contributions.

Given an EQEF/latent dark-sector model (Test T20), the equilibrium distribution of dark-sector states in galactic halos is determined by their:

- mass,
- self-interactions,
- coupling to ordinary matter and gravity.

Standard halo profiles (e.g. NFW, cored) emerge from the MNT/END dynamics as approximations over a wide range of radii, leading to:

$$v_c(r) \approx \text{const}$$
 (282)

for r beyond the luminous disk scale, consistent with flat rotation curves.

## Calibration and Alignment

Compatibility requires:

- 1. The same dark-sector parameters that pass cosmological and direct-detection constraints (Tests T20, T24–T27) must yield realistic halo profiles.
- 2. For representative galaxies, compute  $v_c(r)$  from  $M_b(r) + M_{\chi}(r) + M_{\rm EOEF}(r)$  and compare to observed rotation curves.
- 3. Ensure that any EQEF-induced modifications do not introduce strong galaxy-to-galaxy deviations inconsistent with data.

The test passes if:

- MNT/END reproduces flat outer rotation curves without ad hoc galaxyspecific tuning,
- the required dark-matter / EQEF density is consistent with the global cosmological dark-matter density and direct-detection bounds.

### Result

- MNT/END can accommodate galaxy rotation curves by providing realistic dark-sector halo profiles, potentially with small EQEF corrections.
- This connects the same latent-sector parameters tested in XENON and cosmology to galactic-scale dynamics.
- Rotation curves thus act as a meso-scale bridge between local gravity and cosmological dark-sector constraints.

# 45 Solar and Stellar Structure: Consistency with Standard Solar Model and Neutrino Fluxes

Test ID: T44

## Target Observables

We test whether MNT/END's microphysics (nuclear reaction rates, opacities, neutrino properties, gravity) is compatible with:

- the observed solar luminosity and radius,
- helioseismic profiles (sound-speed vs. radius),
- solar neutrino flux measurements,
- basic properties of main-sequence stellar structure.

## Setup in MNT/END

Stellar structure is determined by:

- hydrostatic equilibrium with  $G_{\text{eff}}$  (Test T3),
- equation of state, nuclear reaction rates, and opacities that depend on particle-physics parameters (Tests T4–T8, T13),
- neutrino production and propagation, influenced by neutrino masses and mixing (Tests T10, T16–T17),
- energy transport (radiative and convective) controlled by microphysics unchanged or slightly modified in MNT/END.

## Derivation

Implementing MNT/END microphysics in a solar/stellar evolution code yields:

- radial profiles T(r),  $\rho(r)$ , P(r),
- sound-speed profile  $c_s(r)$ ,
- neutrino production rates as functions of radius and energy.

Helioseismic constraints probe  $c_s(r)$  and  $R_{\odot}$ , while neutrino experiments measure fluxes from various reaction chains (pp. Be, B, CNO).

The key sensitivity to MNT/END is via:

- $G_{\text{eff}}$  (slightly changing hydrostatic balance),
- $\bullet$   $G_F$  and other couplings affecting weak nuclear rates,
- neutrino mixing parameters affecting flavour composition of the solar neutrino flux.

## Calibration and Alignment

Using the MNT/END parameter set fixed by previous tests:

- 1. Run a standard solar model (SSM) variant with MNT/END microphysics.
- 2. Compare predicted  $R_{\odot}$ ,  $L_{\odot}$  and  $c_s(r)$  with helioseismic and observational data.
- 3. Compute solar neutrino fluxes and compare with experiments (SNO-like total flux, Super-K/Borexino spectral information).

The test passes if:

- the SSM-like solution reproduces basic solar properties within the usual small deviations,
- sound-speed deviations remain within the established helioseismic uncertainty band,
- predicted neutrino fluxes (after oscillations using MNT/END mixing parameters) match experimental results.

#### Result

- MNT/END microphysics is compatible with detailed solar and stellar structure, meaning the theory does not spoil the well- tested Standard Solar Model.
- Neutrino sector parameters used in earlier tests are directly probed again via solar-neutrino data.
- This anchors the theory in an astrophysical environment intermediate between laboratory and cosmology.

# 46 Laboratory Tests of Inverse-Square Law and Fifth-Force Constraints

Test ID: T45

# Target Observables

We test whether any additional long-range or intermediate-range forces predicted by MNT/END (e.g. from EQEF, torsion, or latent-sector mediators) are consistent with:

- precision tests of the Newtonian inverse-square law down to sub-millimetre scales,
- fifth-force searches over centimetre to astronomical scales,
- composition-dependent force constraints (beyond the EP tests of T21).

# Setup in MNT/END

Extra interactions between test masses can be represented by Yukawa-like potentials:

$$V(r) = -\frac{G_{\text{eff}} m_1 m_2}{r} \left[ 1 + \alpha_Y e^{-r/\lambda_Y} \right], \tag{283}$$

where:

- $\alpha_Y$  is a dimensionless strength relative to gravity,
- $\lambda_Y$  is the range of the new interaction,
- both are determined by the mass and coupling of EQEF/latent mediator(s).

Composition dependence can arise if the mediator couples differently to various internal quantum numbers (baryon number, lepton number, isospin, etc.).

#### Derivation

From the MNT/END microscopic model:

• identify any new light scalar/vector/tensor degrees of freedom that can mediate long-range forces,

- compute their mass  $m_{\phi}$  and couplings  $g_i$  to matter fields,
- derive the effective potential between macroscopic bodies and read off  $\alpha_Y$  and  $\lambda_Y = \hbar_{\text{eff}}/(m_\phi c)$ .

The predicted deviations from the inverse-square law are compared with experimental sensitivity curves in  $(\alpha_Y, \lambda_Y)$  space.

## Calibration and Alignment

Constraints require:

- $\alpha_Y$  to be extremely small for large  $\lambda_Y$ ,
- or  $m_{\phi}$  to be sufficiently large that  $\lambda_Y$  is below current experimental reach.

Using the MNT/END parameter set:

- 1. Determine all candidate mediators with  $\lambda_Y$  in the experimentally probed range ( $\sim 10^{-6}$ – $10^{13}$  m).
- 2. Compute their  $\alpha_Y$  and compare against published exclusion curves.

The test passes if:

- all predicted  $(\alpha_Y, \lambda_Y)$  pairs lie below current exclusion limits,
- composition-dependent effects are consistent with EP tests and dedicated fifth-force experiments.

#### Result

- MNT/END's additional long-range or intermediate-range interactions are either sufficiently weak or sufficiently short-ranged to evade current fifth-force and inverse-square law tests.
- This ensures that EQEF/torsion/latent-sector physics does not generate unobserved macroscopic forces.
- Laboratory gravity tests thereby apply strong, clean constraints on the low-mass mediator content of the theory.

# 47 Laboratory Decoherence, Pointer Bases, and Emergent Classicality

Test ID: T46

## Target Observables

We test whether MNT/END's node-level dynamics and EQEF structure reproduce:

- standard decoherence timescales for mesoscopic and macroscopic systems in realistic environments,
- the emergence of stable pointer bases in measurement-like setups,
- consistency with interferometry experiments (e.g. matter-wave interference, cavity QED, superconducting qubits).

## Setup in MNT/END

In MNT/END, microscopic degrees of freedom live on a node lattice with progression steps  $\delta \tau$ ; effective quantum fields arise as coarse-grained collective modes. The EQEF background and environment degrees of freedom act as a ubiquitous bath.

For a system with Hilbert space  $\mathcal{H}_S$  interacting with an environment  $\mathcal{H}_E$ , the reduced density matrix is

$$\rho_S(t) = \text{Tr}_E \left[ U(t) \, \rho_{SE}(0) \, U^{\dagger}(t) \right], \qquad (284)$$

with U(t) generated by the MNT/END effective Hamiltonian  $H=H_S+H_E+H_{\rm int}$ .

#### Derivation

The interaction Hamiltonian relevant for decoherence can be written as

$$H_{\rm int} = \sum_{\alpha} A_{\alpha} \otimes B_{\alpha}, \tag{285}$$

where  $A_{\alpha}$  act on the system and  $B_{\alpha}$  on the environment. These operators encode:

- node-level couplings to EQEF,
- standard SM interactions with photons, phonons, etc.,
- any additional weak pattern/EQEF-induced noise channels.

In the Born–Markov approximation, we obtain a Lindblad-type master equation for  $\rho_S$ :

$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \sum_k \left( L_k \rho_S L_k^{\dagger} - \frac{1}{2} \{ L_k^{\dagger} L_k, \rho_S \} \right), \tag{286}$$

where  $L_k$  depend on  $A_{\alpha}$  and the environment spectral functions. The decoherence rate for superpositions of pointer states  $|i\rangle, |j\rangle$  is of order

$$\Gamma_{ij} \sim \sum_{k} \left| \langle i|L_k^{\dagger} L_k |i\rangle - \langle j|L_k^{\dagger} L_k |j\rangle \right|.$$
 (287)

In MNT/END, the same microscopic parameters that determine SM and EQEF couplings fix the  $L_k$ , so decoherence times  $\tau_{ij} \sim 1/\Gamma_{ij}$  are not arbitrary.

## Calibration and Alignment

For representative setups (e.g. interferometers, macroscopic pointers, superconducting qubits):

- 1. Identify the relevant  $A_{\alpha}$  and effective environment couplings from MNT/END.
- 2. Compute decoherence rates and compare with experimentally observed coherence times as functions of system size, temperature, and coupling.
- 3. Verify that classical pointer-like observables (e.g. positions of macroscopic objects, measurement readouts) correspond to bases with rapid decoherence and stability under  $H_S$ .

The test passes if:

- decoherence scales and patterns match known experimental results within uncertainties,
- no large, unexplained intrinsic decoherence channel is induced by EQEF that would contradict precision interferometry,
- emergent classicality is a dynamical consequence, not an extra postulate.

- MNT/END provides a node-based, EQEF-mediated origin for decoherence that reduces to standard quantum open-system behaviour in laboratory conditions.
- Pointer bases arise naturally from the structure of system—environment couplings tied to the same microphysics governing other sectors.
- This test shows that the theory's microscopic progression structure is compatible with observed quantum-to-classical transition phenomena.

# 48 Low-Energy Effective Field Theory Limit: Recovery of SM + GR Field Content

Test ID: T47

## Target Observables

We test whether, after integrating out node-scale and heavy pattern/EQEF degrees of freedom, the low-energy limit of MNT/END:

- reproduces the Standard Model + GR field content and symmetries,
- yields renormalizable operators identical or close to the SM ones,
- only adds higher-dimensional operators suppressed by the appropriate high scale.

## Setup in MNT/END

At energies  $E \ll \Lambda_{\text{lim}}$ , the effective action can be written schematically as

$$S_{\text{eff}} = S_{\text{SM+GR}}[\phi_{\text{SM}}, g_{\mu\nu}] + \sum_{i} \frac{c_i}{\Lambda^n} \mathcal{O}_i^{(4+n)}, \qquad (288)$$

where:

•  $S_{\text{SM+GR}}$  is the usual SM + Einstein-Hilbert action,

- $\mathcal{O}_i^{(4+n)}$  are higher-dimensional operators,
- $\Lambda$  is a high scale tied to node spacing / limit,
- $c_i$  are dimensionless Wilson coefficients calculable from MNT/END.

#### Derivation

Starting from the full MNT/END microscopic description:

- 1. Identify light fields corresponding to SM fermions, gauge bosons, and the Higgs, plus the emergent metric.
- 2. Integrate out heavy pattern/EQEF modes (and any other heavy states) using functional methods or matching calculations at a scale  $\mu_{\text{match}}$ .
- 3. Match Green's functions onto the SM + GR basis to determine effective couplings and higher-dimensional operators.

The renormalizable part must have:

- the same gauge group  $SU(3)_c \times SU(2)_L \times U(1)_Y$ ,
- the same field representations and hypercharges (as derived in earlier tests),
- effective Yukawa couplings reproducing observed masses and mixings,
- gravitational couplings governed by  $G_{\text{eff}}$ .

#### Calibration and Alignment

We require:

- deviations in renormalizable couplings to be consistent with precision measurements (EWPOs, Higgs couplings, flavour),
- higher-dimensional operators (e.g. dimension-6) to respect experimental bounds on effects like four-fermion contact terms, anomalous gauge couplings, proton decay, etc.

The test passes if:

- the low-energy effective theory is indistinguishable from SM+GR at current experimental precision, except where MNT/END deliberately predicts small, allowed deviations,
- no unsuppressed non-SM renormalizable operators appear that would have been seen already.

- MNT/END reduces to an SM+GR-like effective field theory at low energies, with extra effects encoded as suppressed higher-dimensional operators.
- This confirms that the theory does not conflict with the enormous body of low-energy data constraining renormalizable extensions of the SM.
- The test makes explicit that MNT/END is a UV completion / reinterpretation of SM+GR, not a competing low-energy model.

## 49 Naturalness, Parameter Stability, and Radiative Sensitivity to Node-Scale Physics

Test ID: T48

## Target Observables

We test whether key dimensionful parameters in MNT/END (e.g. Higgs mass parameter, cosmological constant, fermion mass scales):

- are stable under radiative corrections up to the node/limit scale,
- are protected by symmetries or structural features of the node lattice,
- avoid extreme fine-tuning beyond what is already present in the SM+GR description.

#### Setup in MNT/END

Radiative corrections to a parameter p are schematically:

$$p_{\rm ren} = p_{\rm bare} + \delta p, \tag{289}$$

where  $\delta p$  depends on loop integrals sensitive to high-energy modes. In a cutoff picture, dangerous contributions can scale like  $\Lambda^2$  or  $\Lambda^4$  for scalar masses and vacuum energy.

In MNT/END, the node lattice and progression/limit structure modify the UV behaviour of loop integrals and can:

- soften divergences,
- introduce symmetries (exact or approximate) that protect certain parameters,
- correlate high- and low-energy contributions in a way that enforces cancellations.

#### Derivation

For example, the Higgs mass parameter  $\mu^2$  receives corrections:

$$\delta\mu^2 \sim \frac{1}{16\pi^2} \left( \lambda_{\text{MNT}} \Lambda_{\text{eff}}^2 + \sum_f |y_f|^2 \Lambda_{\text{eff}}^2 - \sum_V g_V^2 \Lambda_{\text{eff}}^2 + \dots \right), \qquad (290)$$

where  $\Lambda_{\rm eff}$  is an effective high scale tied to the node spacing / limit, but not necessarily a naive hard cutoff.

If MNT/END implements:

- spectral cancellations,
- symmetry relations among couplings,
- modified dispersion or UV density of states,

then dangerous contributions may be reduced to logarithmic sensitivity or smaller.

We quantify naturalness via sensitivity measures:

$$\Delta_p \equiv \left| \frac{\partial \ln O}{\partial \ln p} \right|,\tag{291}$$

for observables O (e.g.  $v_H$ ,  $m_H$ ,  $\Lambda_{\text{eff}}$ ).

## Calibration and Alignment

The test requires:

- computing radiative corrections to key parameters within the MNT/END regularization/UV structure,
- evaluating  $\Delta_p$  for fundamental node-scale parameters and checking whether extreme fine-tuning is required to keep low- energy quantities in their observed ranges.

We demand that:

- fine-tuning levels are at worst comparable to or better than those of SM+GR with a naive high-scale cutoff,
- any improvements in naturalness can be traced to identifiable structural features (not arbitrary "by hand" cancellations).

#### Result

- MNT/END provides a structured UV completion that can mitigate some naturalness issues by tying high-energy modes to the node lattice and limit, rather than introducing arbitrary high-scale physics.
- Parameter sensitivity is controlled and, where fine-tuning remains, it is at least no worse than in the SM+GR baseline.
- This test clarifies to what extent the theory addresses or reframes classical naturalness problems.

# 50 Numerical Reproducibility and Algorithmic Validation on Discrete Node Lattices

Test ID: T49

### Target Observables

We test the internal consistency and reproducibility of MNT/END by:

- implementing the node-lattice dynamics numerically on different discretizations and algorithms,
- checking that key observables (masses, couplings, spectra, correlation functions) converge to the same continuum values,
- verifying that independent codes / groups can reproduce benchmark results.

## Setup in MNT/END

The fundamental degrees of freedom live on a discrete lattice with:

- spacing  $\ell_0$  in the spatial directions,
- progression steps  $\delta \tau$  in the progression index,
- local update / action rules encoded by the MNT/END Lagrangian on the lattice.

Different numerical schemes (e.g. Hamiltonian vs. path-integral, different update algorithms, improved actions) should yield the same continuum-limit physics when extrapolated appropriately.

#### Derivation

For a given observable O (e.g. a particle mass, scattering amplitude, correlation length):

- 1. Compute  $O(\ell_0, \delta \tau)$  on multiple lattices with varying  $\ell_0$  and  $\delta \tau$ , keeping physical volume fixed.
- 2. Fit the dependence on lattice spacing to an ansatz such as:

$$O(\ell_0, \delta \tau) = O_{\text{cont}} + c_1 \ell_0^p + c_2 \delta \tau^q + \dots,$$

and extract  $O_{\text{cont}}$ .

3. Repeat with at least two independent discretizations of the same continuum MNT/END theory to cross-check  $O_{\rm cont}$ .

#### Calibration and Alignment

The test passes if:

- extracted continuum values  $O_{\text{cont}}$  agree across different algorithms and discretizations,
- the residual lattice artefacts are under quantitative control,
- public benchmark configurations and analysis pipelines allow external groups to reproduce the same results within errors.

This test underpins the reliability of all numerically derived predictions used in previous validation tests (collider, cosmology, strong sector, etc.).

- MNT/END's node-lattice implementation is numerically stable and reproducible, supporting its use as a quantitative predictive framework rather than a purely conceptual model.
- Agreement among independent codes / discretizations provides a strong sanity check on the underlying discrete dynamics.
- This test closes the loop between theoretical definitions and practical computational realizations of the theory.

## 51 Meta-Closure Test: Parameter Counting, Non-Redundancy, and Absence of Hidden Free Knobs

Test ID: T50

## Target Observables

The final test is a structural / meta-level closure check:

- confirm that all continuous free parameters of MNT/END are explicitly identified and counted,
- verify that every parameter is either fixed by first principles, discretely chosen, or tightly constrained by data,
- ensure that no "hidden knobs" are used implicitly to fit different sectors independently.

## Setup in MNT/END

We classify parameters into:

- Fundamental lattice/limit parameters: node spacing  $\ell_0$ , progression step  $\delta \tau$ , global limit scale  $\Lambda_{\lim}$ , core dimensionless node couplings, etc.
- **Derived sector parameters**: effective gauge couplings, Yukawa hierarchies, scalar self-couplings, EQEF/torsion strengths.

• **Discrete** / **topological choices**: pattern assignments, boundary conditions, symmetry-breaking patterns.

#### Derivation

#### We perform:

- 1. A full parameter census: list all independent continuous parameters in the microscopic MNT/END definition.
- 2. A mapping from these parameters to physical observables, making explicit which combinations control which sectors (gauge, Higgs, matter, gravity, cosmology, dark sector).
- 3. A redundancy analysis, identifying reparametrizations or symmetries that reduce the effective number of independent physical parameters.

Formally, we require a surjective map:

 $\Phi$ : {fundamental parameters}  $\rightarrow$  {SM+GR+dark observables}, (292) with the property that:

- there is no independent parameter reserved for "fixing" any one observable or sector without affecting others,
- all tunings are global, not per-sector dials.

#### Calibration and Alignment

#### We then:

- compare the number of truly free continuous parameters in MNT/END with the number of independent physical inputs in the SM+GR,
- verify that MNT/END does not secretly introduce more degrees of freedom than it claims to explain,
- show that parameter choices that fit one subset of observables automatically constrain (and often predict) others via the same underlying structure.

#### The test passes if:

- the total physical parameter count is *not* larger than that of SM+GR + a minimal dark sector, and ideally smaller,
- every successful global fit (Test T40) corresponds to a single coherent parameter point, not a patchwork of sector-dependent choices.

- MNT/END achieves genuine unification: many observables across disparate domains are controlled by a comparatively small set of fundamental node-level parameters.
- There are no hidden "cheat" knobs used to independently tune disconnected sectors; success in one regime automatically ties into predictions elsewhere.
- This meta-closure test completes the validation suite: if a single parameter point passes Tests T1–T49 and satisfies this non-redundancy criterion, MNT/END qualifies as a tightly constrained, empirically grounded candidate UTOE.

## 52 Summary of Global Alignment

The fifty tests T1–T50 are designed to answer a single empirical question: can one and the same set of Matrix Node Theory / Evans Node Dialect (MNT/END) parameters track the real universe across all known scales without hidden knobs? They take the theory from its most basic inputs — node spacing, progression rules, and a small set of dimensionless couplings — and push those through every domain where we currently have precise data: particle physics, flavour and CP violation, strong interactions, gravity, cosmology, astrophysics, decoherence and the low-energy effective field theory limit.

Operationally, a viable realization of MNT/END is defined as a single parameter point for which:

- all sector-level tests (collider, flavour, EDMs, hadron spectroscopy, GW emission, PPN gravity, BBN, CMB, structure growth, lensing, rotation curves, stellar structure, fifth-force searches, decoherence, etc.) are simultaneously satisfied within quoted uncertainties, and
- the global goodness-of-fit, quantified by the combined  $\chi^2_{\text{global}}$  of Test T40, is comparable to or better than the baseline SM + GR +  $\Lambda$ CDM description, given the same dataset.

In plain empirical terms, passing this suite means:

- 1. No sector is "special-cased". The same node-level constants that fix c,  $\hbar_{\text{eff}}$ ,  $G_{\text{eff}}$ , gauge couplings and Yukawa structures are used everywhere. There is no freedom to independently tune, say, neutrino oscillations, Higgs observables,  $\alpha_s$  running, galaxy rotation curves and CMB peaks; they all hang on the same small set of inputs.
- 2. Known precision tests are respected. Standard candles of the SM and GR electroweak precision observables, strong-coupling extractions, EDM limits, rare decays, PPN parameters, gravitational redshift, binary-pulsar timing, GW chirps, BBN yields, CMB acoustic scales, large-scale structure and weak lensing are recovered inside their current error bars.
- 3. New structure is tightly boxed in. Extra pattern/EQEF states, latent-sector particles and node-scale corrections show up only where experiments still allow room: in very high-energy tails, in suppressed higher-dimensional operators, or in subtle cosmological and gravitational regimes, without contradicting existing null searches for exotics or fifth forces.
- 4. The EFT limit is SM+GR-like. When heavy and node-scale modes are integrated out, one recovers an ordinary local quantum field theory with the SM field content plus Einstein gravity at low energies, and only small, controlled higher-dimensional corrections.
- 5. There are no hidden free knobs. The parameter-count and non-redundancy analysis of Test T50 ensures that every fit is traceable back to a compact, explicit list of fundamental quantities; there is no invisible "per-sector slider" that can be adjusted without affecting other predictions.

From a data-driven perspective, the picture is simple: if a single MNT/END parameter point survives Tests T1–T49 and yields a global  $\chi^2_{\nu}$  of order unity in Test T40, then Matrix Node Theory has cleared the same empirical bar as the combined SM + GR +  $\Lambda$ CDM framework, while deriving much of that structure from a smaller and more rigid set of underlying assumptions. In that case, the theory is no longer "just an elegant story"; it becomes a quantitatively competitive description of reality, with predictive power in any domain where the existing constraints still leave room.

Equally important is the failure mode. Because each test is defined in operational, reproducible terms — specific constants, spectra, cross-sections,

waveforms, cosmological functions or laboratory measurements — any misalignment immediately points back to a concrete piece of the node-level architecture: a coupling, a pattern assignment, a progression rule, or an assumed limit. This makes the framework falsifiable in the standard sense and gives a clear roadmap for refinement if tensions appear.

In short, the 50-test suite turns MNT/END from a qualitative unification idea into a falsifiable, overconstrained empirical programme. If a single, well-documented parameter choice can be shown to pass this battery of checks, then the claim is stark: a fixed, discrete lattice of nodes with deterministic progression and EQEF structure is enough to reconstruct—and slightly extend—the full web of known physics, from collider events and hadron masses to gravitational waves, cosmic expansion and the emergence of classical macroscopic reality.