Data Visualization Using the Fourier Transform

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Basic Ideas

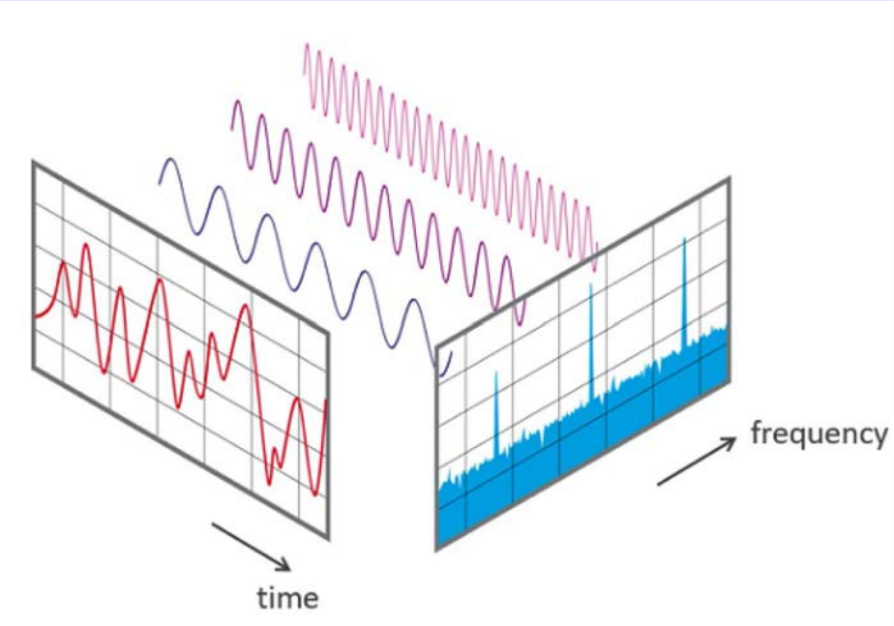
The Concept

Unknown signal (red)

Fourier fits data exactly (purple waves)

The transformed data (blue)

The transformed result shows
three periodic tones (blue spikes) and
random Gaussian noise (blue horizontal)



The Fourier Transform Definition (1)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi kn}{N}}$$

The Fourier Transform (2)

$$X(k) = \sum_{n=0}^{N-1} x(n) \times \left(cos(\frac{2\pi nk}{N}) - i \times sin(\frac{2\pi nk}{N}) \right)$$

This equation is a recipe for making something like a spreadsheet.
The recipe multiplies each “x” data value by sines and cosines, in a systematic way.
The “x” data is a ROW of numbers. The recipe creates ROWS and COLUMNS, like a spreadsheet.

Example Notation for an N-Point Transform

a = 2*pi/N

K = [0:N-1] going to right (frequency bin)

J = [0:N-1] going down the rows

i = sqrt(-1)

x = x-axis values to be transformed

C = cos

S = sin

Transform Arranged in Matrix Form for N =10

Column 1	Column 2	Column 3	.	Column 9	Column 10
x0*[C()-i*S()]	x0*[C()-i*S()]	x0*[C()-i*S()]	.	x0*[C()-i*S()]	x0*[C()-i*S()]
x1*[C()-i*S()]	x1*[C()-i*S()]	x1*[C()-i*S()]	.	x1*[C()-i*S()]	x1*[C()-i*S()]
x2*[C()-i*S()]	x2*[C()-i*S()]	x2*[C()-i*S()]	.	x2*[C()-i*S()]	x2*[C()-i*S()]
x3*[C()-i*S()]	x3*[C()-i*S()]	x3*[C()-i*S()]	.	x3*[C()-i*S()]	x3*[C()-i*S()]
x4*[C()-i*S()]	x4*[C()-i*S()]	x4*[C()-i*S()]	.	x4*[C()-i*S()]	x4*[C()-i*S()]
x5*[C()-i*S()]	x5*[C()-i*S()]	x5*[C()-i*S()]	.	x5*[C()-i*S()]	x5*[C()-i*S()]
x6*[C()-i*S()]	x6*[C()-i*S()]	x6*[C()-i*S()]	.	x6*[C()-i*S()]	x6*[C()-i*S()]
x7*[C()-i*S()]	x7*[C()-i*S()]	x7*[C()-i*S()]	.	x7*[C()-i*S()]	x7*[C()-i*S()]
x8*[C()-i*S()]	x8*[C()-i*S()]	x8*[C()-i*S()]	.	x8*[C()-i*S()]	x8*[C()-i*S()]
x9*[C()-i*S()]	x9*[C()-i*S()]	x9*[C()-i*S()]	.	x9*[C()-i*S()]	x9*[C()-i*S()]

The matrix has N rows, and N columns. Each column is identical so far.
“C” means cosine and “S” means sine.
Each “C()” and “S()” expression will have a unique angle inserted inside the parenthesis.

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The Angles To Be Inserted Into the Matrix for N = 10

a*0*0	a*0*1	a*0*2	.	a*0*8	a*0*9
a*1*0	a*1*1	a*1*2	.	a*1*8	a*1*9
a*2*0	a*2*1	a*2*2	.	a*2*8	a*2*9
a*3*0	a*3*1	a*3*2	.	a*3*8	a*3*9
a*4*0	a*4*1	a*4*2	.	a*4*8	a*4*9
a*5*0	a*5*1	a*5*2	.	a*5*8	a*5*9
a*6*0	a*6*1	a*6*2	.	a*6*8	a*6*9
a*7*0	a*7*1	a*7*2	.	a*7*8	a*7*9
a*8*0	a*8*1	a*8*2	.	a*8*8	a*8*9
a*9*0	a*9*1	a*9*2	.	a*9*8	a*9*9

Note the symmetry and asymmetry.

The Transformation

With the angles inserted in the Matrix expressions,

each element in each column is now distinct.

Each matrix column is summed, giving N transformed values.

Each value describes the energy of the “x” values in terms of frequency.

Elementary Example

A time series

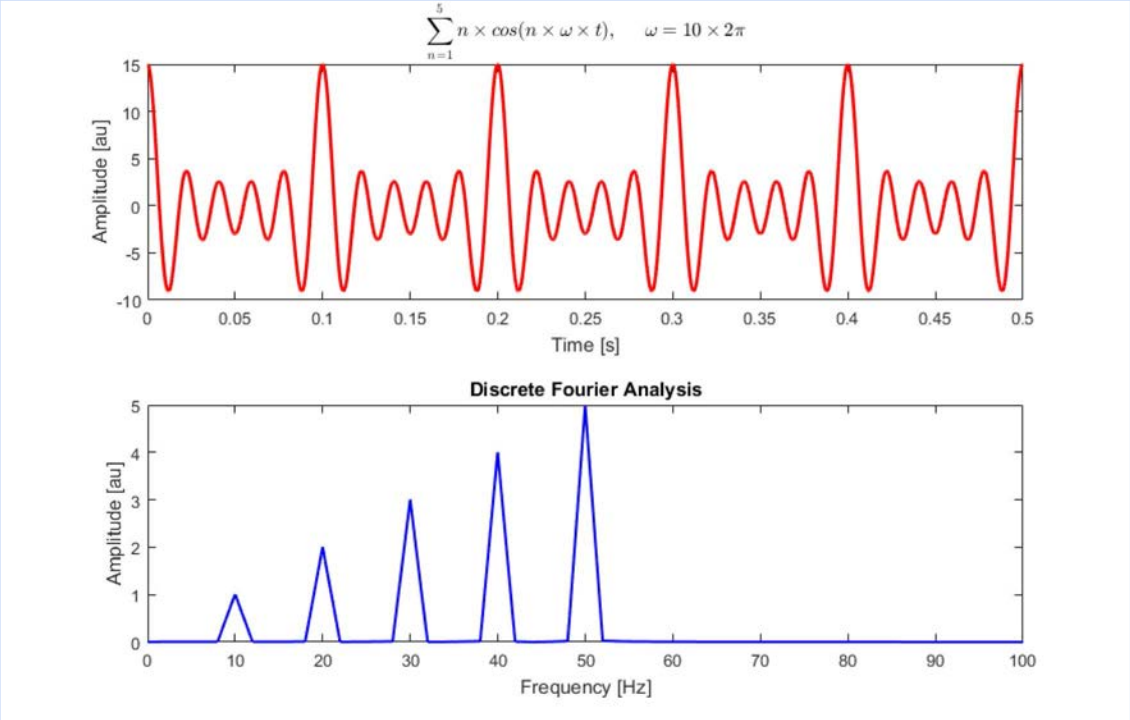
with multiple waves:

Fourier Transform

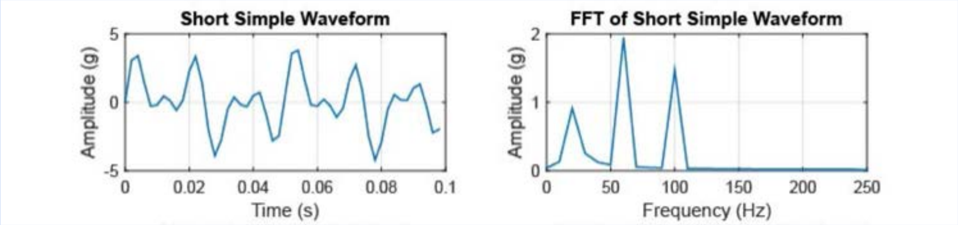
shows the frequency

and amplitude

of each wave.



Data Length Example #1



On the left, the data sample is too short.
On the right, the Fourier transform “spikes” are too wide near the base.

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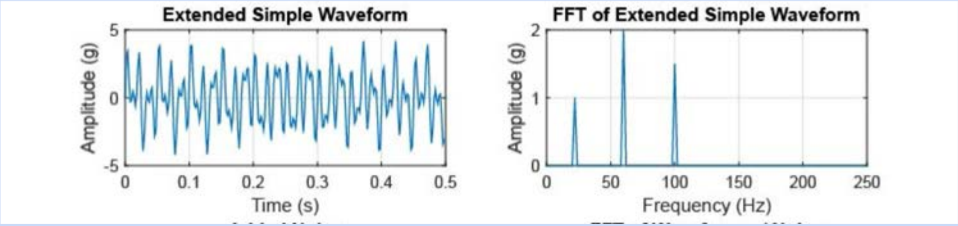
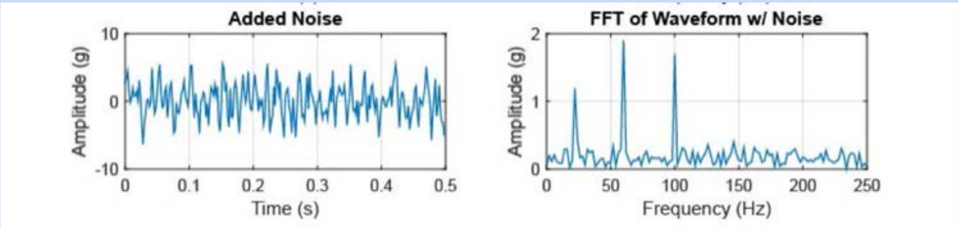
Data Length Example #2



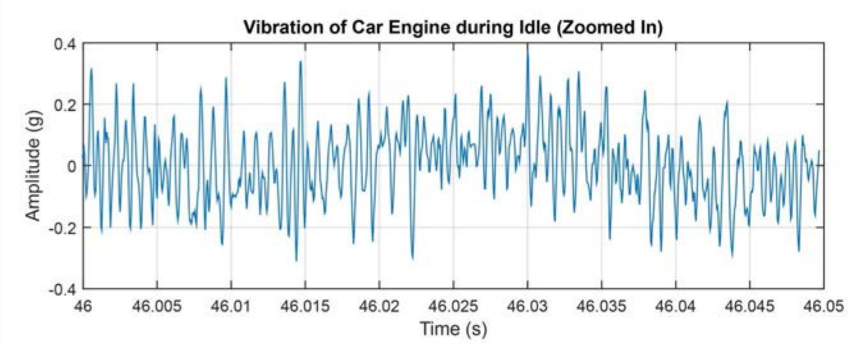
On the left, the data sample is more representative of the dynamics.
On the right, we see concentrated energy at distinct frequencies.

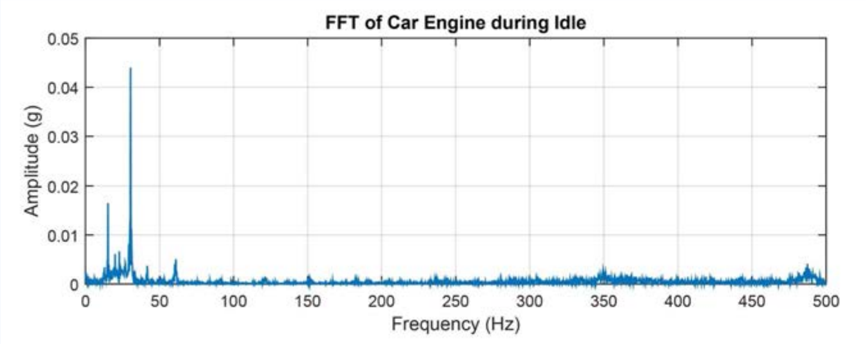
White Noise Example



On the left, Gaussian (random) white noise is in the time series.
The Fourier spectrum shows the signal and the noise separately.

Diagnostic Interpretation Example

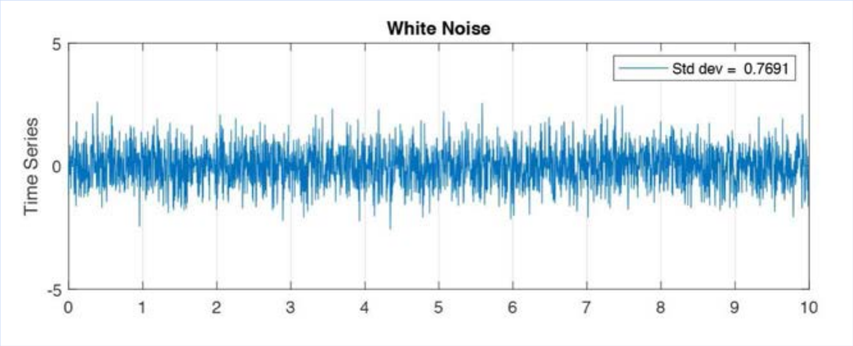


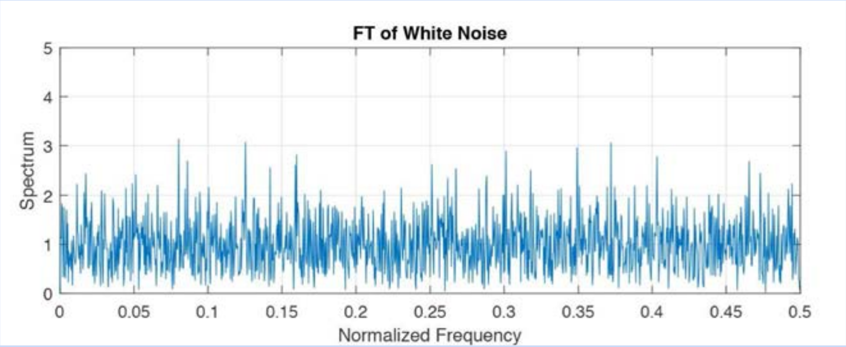


Left image: A tiny portion of a time series (idling car engine, with white noise).

Right image: Two main "spikes" and some noise.
Spike #1: At 15 Hz (900 RPM). Engine idle, crankshaft rotation speed.
Spike #2: At 30 Hz (1800 RPM). Dynamics.
Two pairs of pistons are moving out of phase with each other, and
Two revolutions of the crankshaft are needed for all 4 cylinders to fire.

White Noise





White noise, aka Gaussian noise, aka random noise.

We usually cannot predict random components.

Common in real physical systems.

On the left, in a time series. On the right, in a spectrum.

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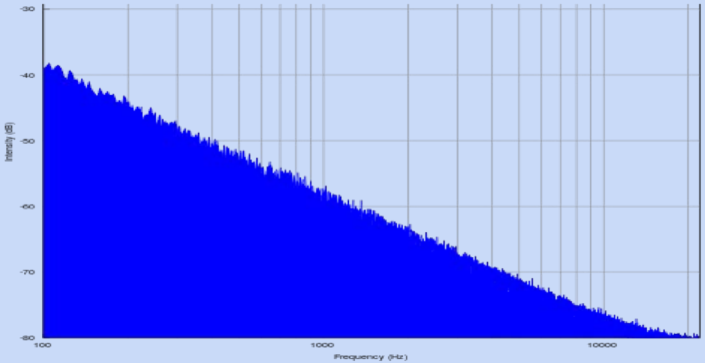
Brownian Noise



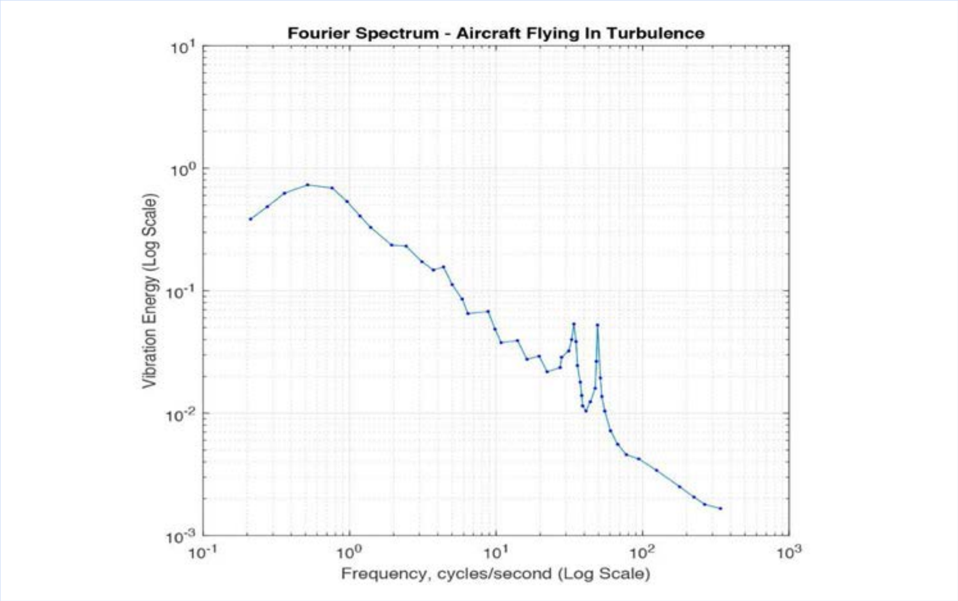
Brownian noise, aka random walk noise, 1/f noise, a Wiener process.

This energy *might* have useful predictive information buried in it.

Common in real physical systems.

Above, a spectrum for Brownian noise. Slope of minus 1 is typical.

Brownian Energy Example #1



Brownian energy, and two structural vibration spikes.

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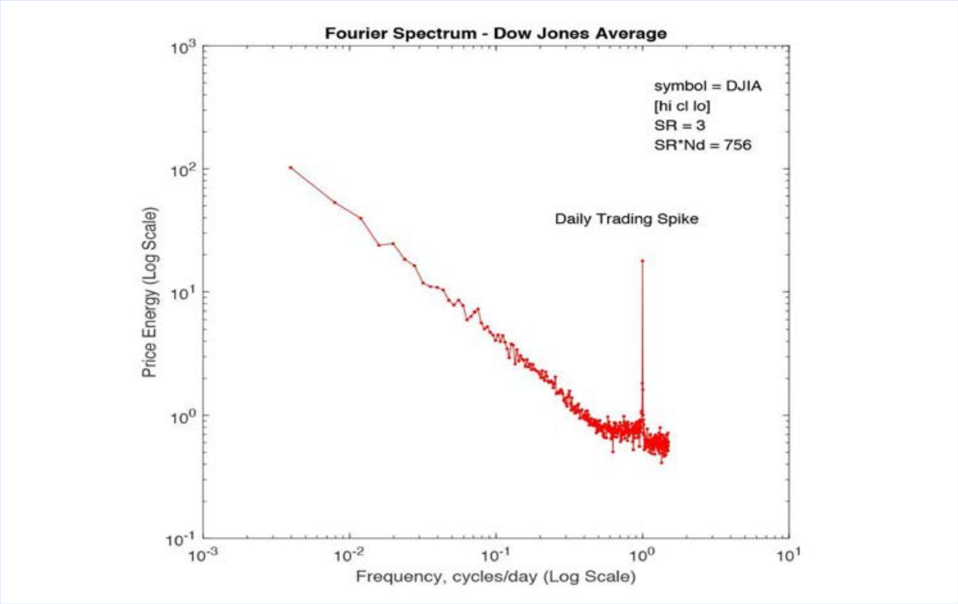
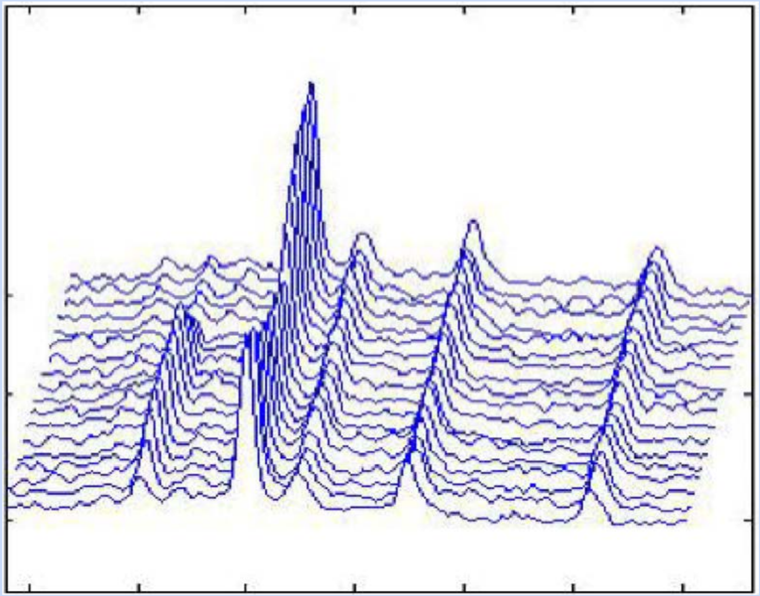
Brownian Energy Example #2



Brownian energy, white noise, and day trading energy spike.

Waterfall Plots -- Visualization of Changes Over Time



Stack the Fourier transforms to observe spectrum energy changes over all frequencies.

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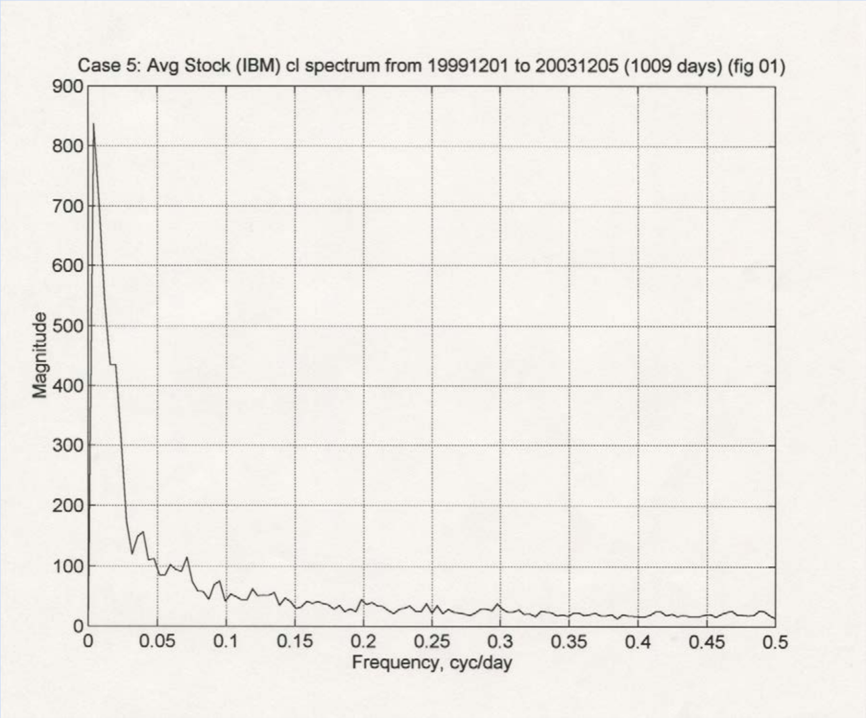
Example 1 of 6



Average waterfall spectrum of IBM close, from Dec 01 1999 to Dec 05 2003 (1009 days - no stacking).

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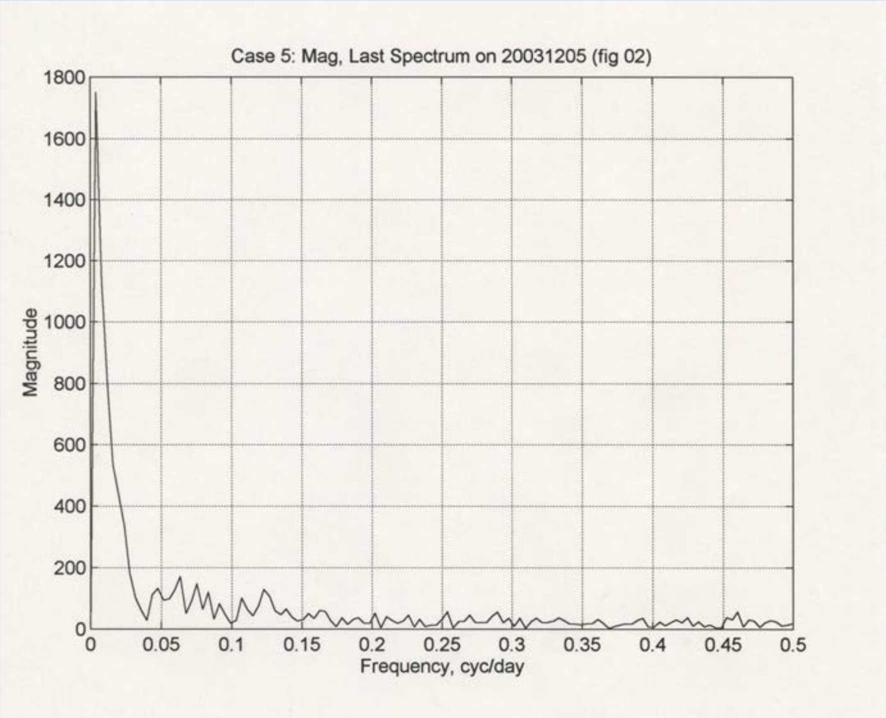

Last waterfall spectrum of IBM on Dec 05 2003 (1 day stacked).

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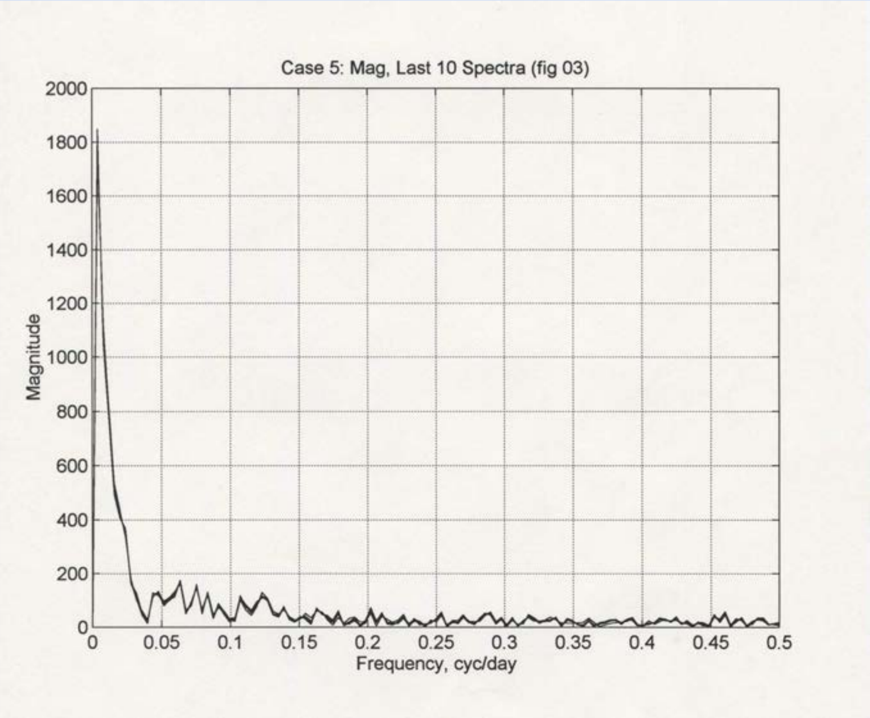
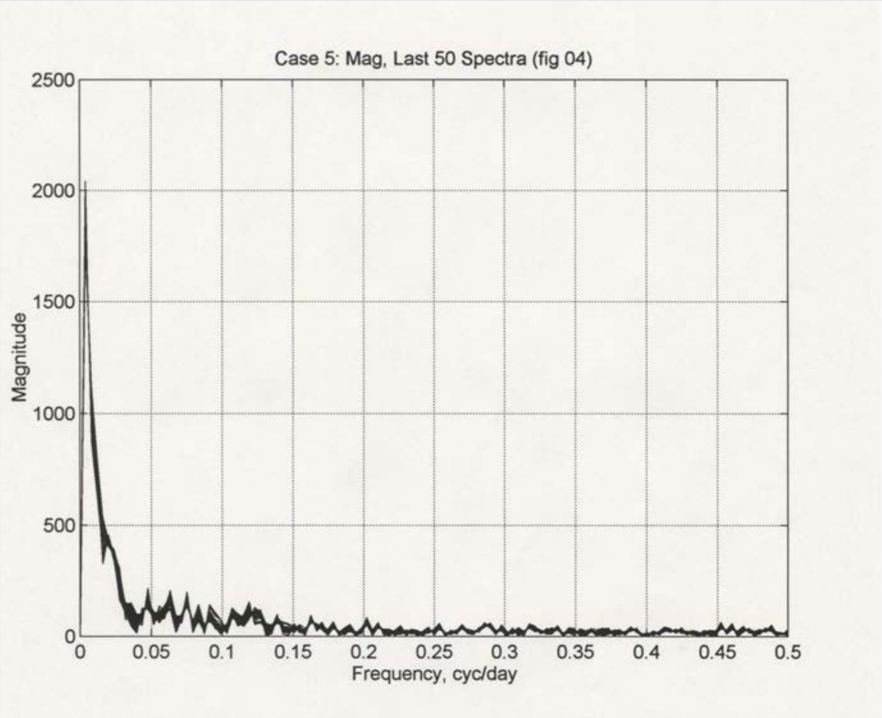
Example 3 of 6



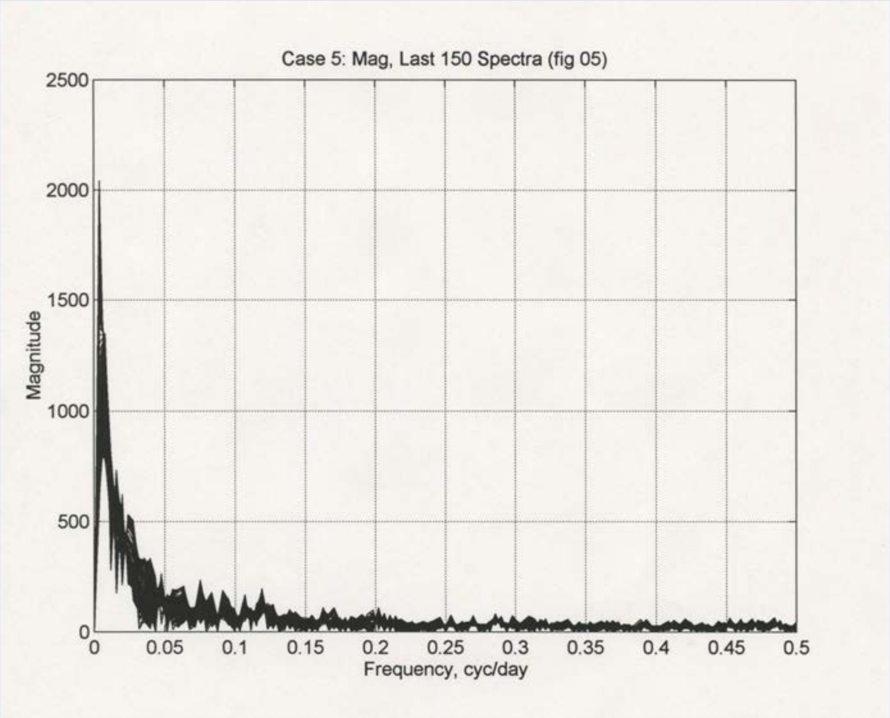
Last 10 waterfall spectra of IBM (10 days stacked).

Example 4 of 6



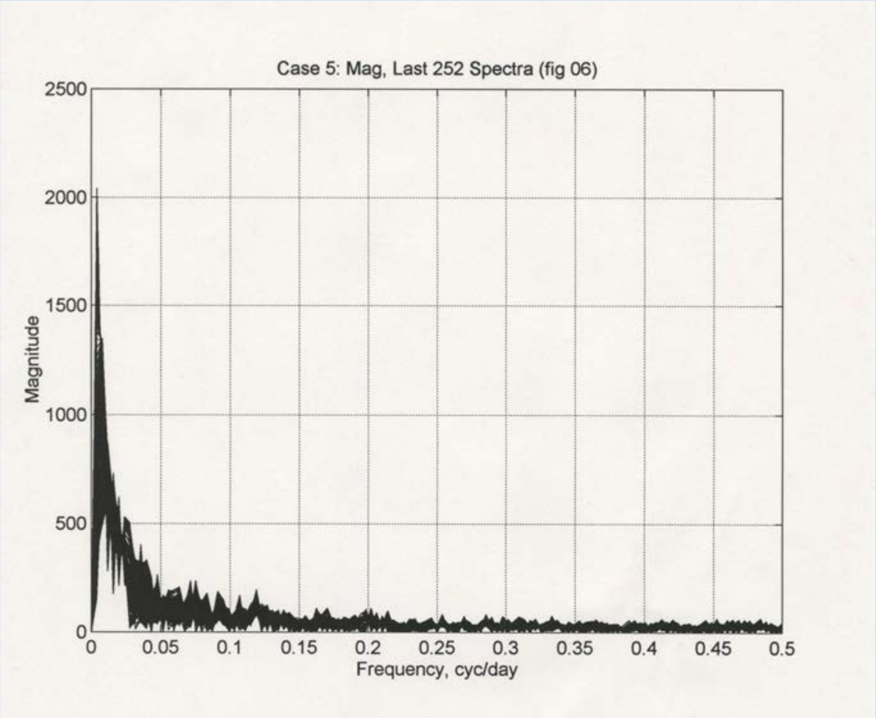
Last 50 waterfall spectra of IBM (50 days stacked).

Example 5 of 6



Last 150 waterfall spectra of IBM (150 days stacked).

Example 6 of 6



Last 252 waterfall spectra of IBM (252 days stacked).

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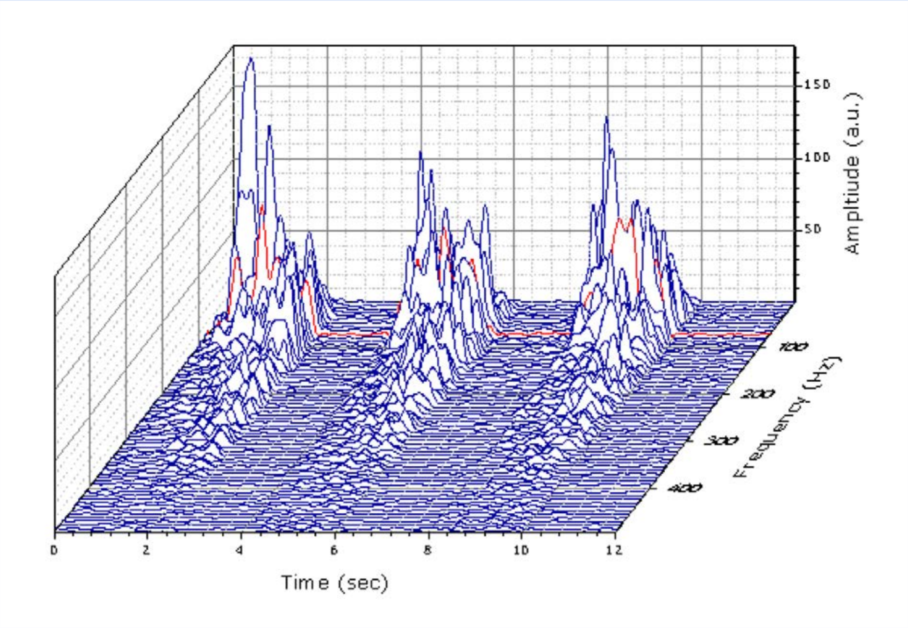
Waterfall Plots -- Visualization of Changes Over Time



Stack the Fourier transforms to observe energy changes at ONE particular frequency.

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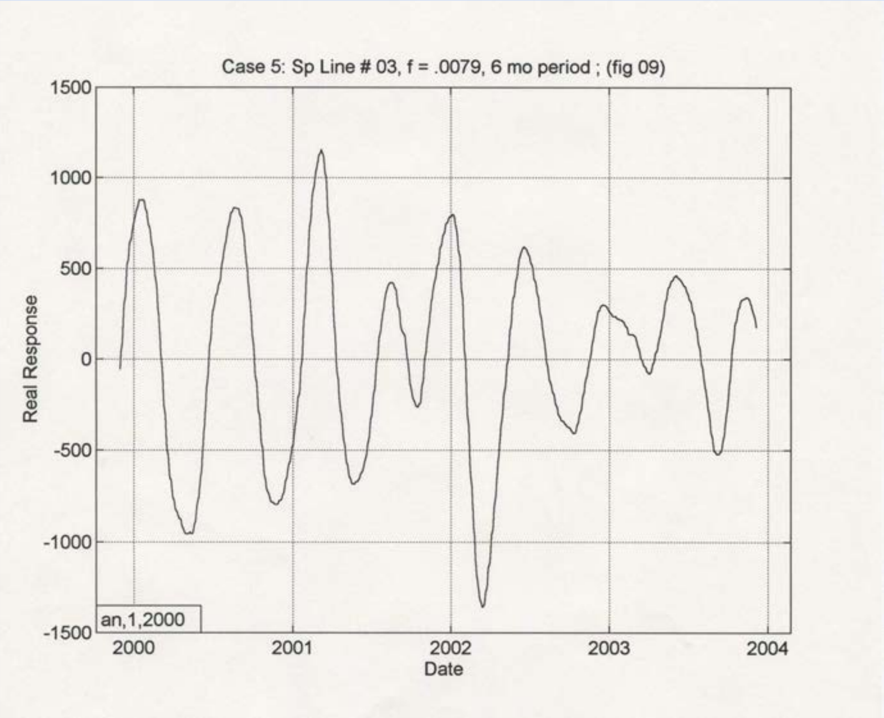
Example 1 of 4



IBM spectral line #3: Quasi-periodic, NOT random (1009 days stacked).

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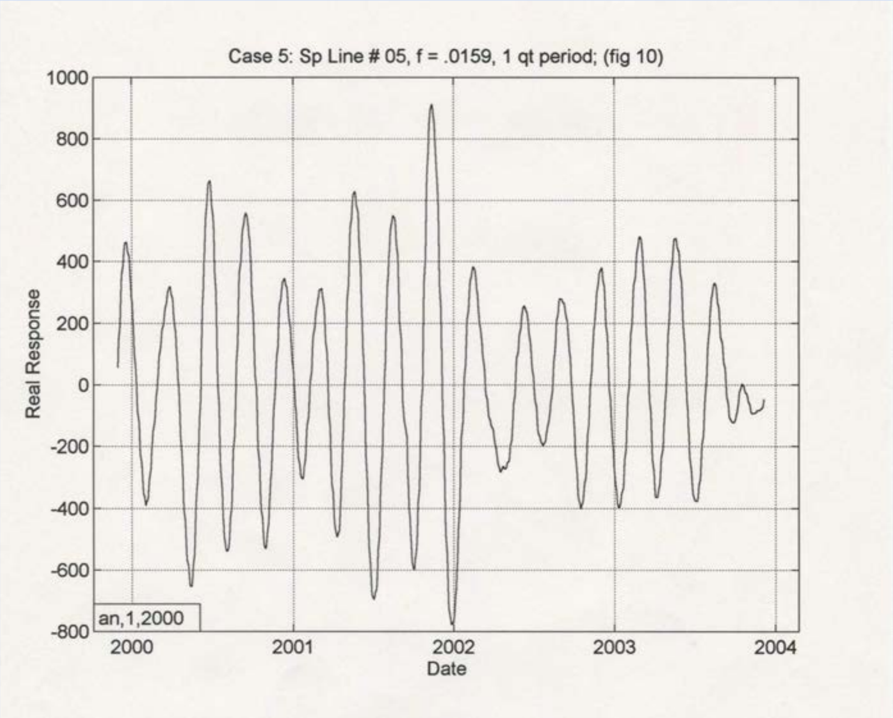
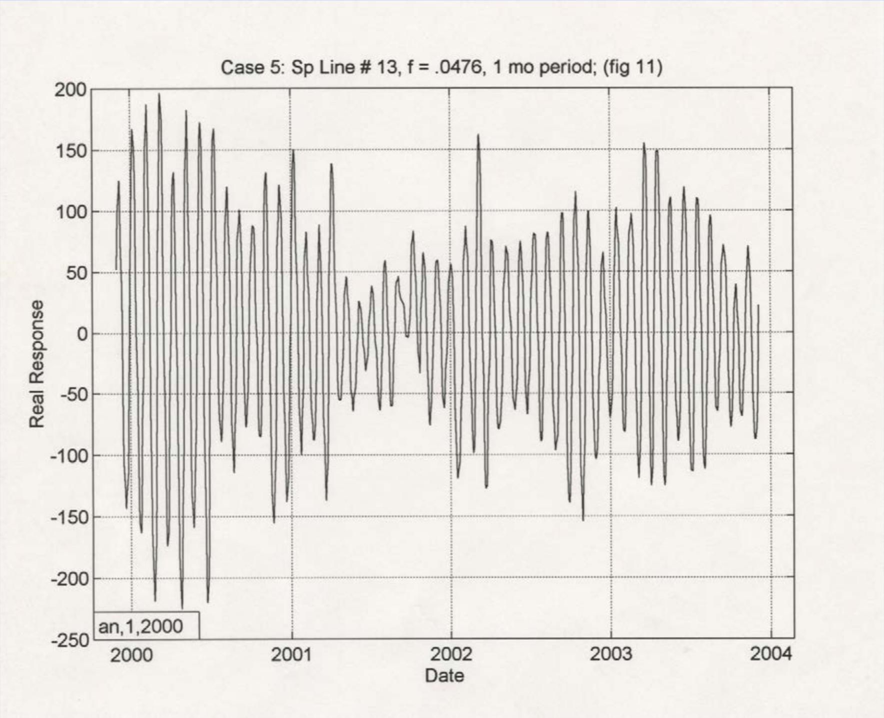
Example 2 of 4



IBM spectral line #5: Quasi-periodic, NOT random (1009 days stacked).

Example 3 of 4



IBM spectral line #13: Quasi-periodic, NOT random (1009 days stacked).

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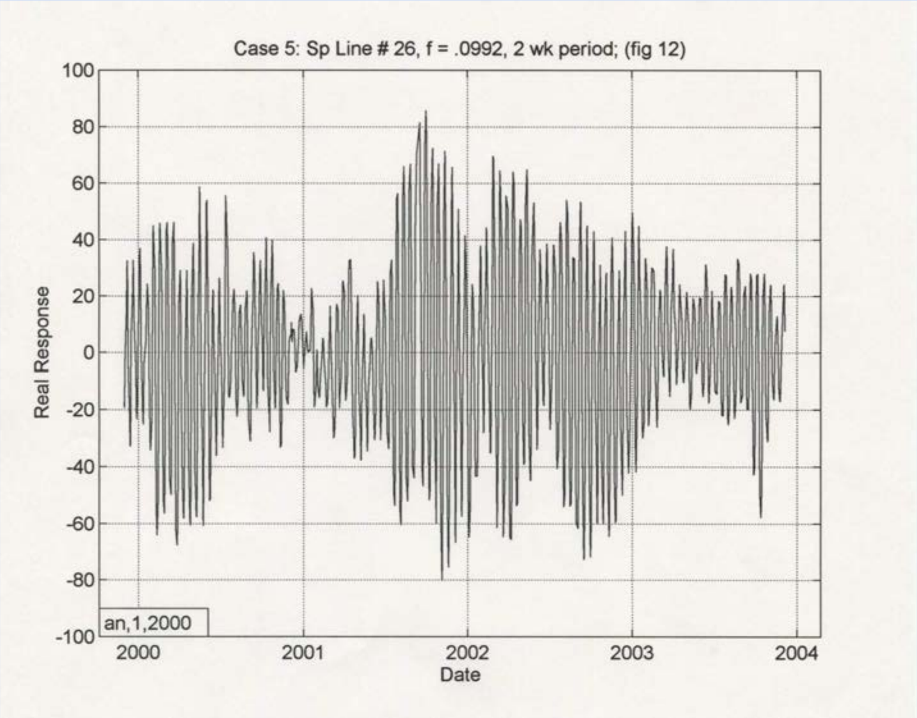
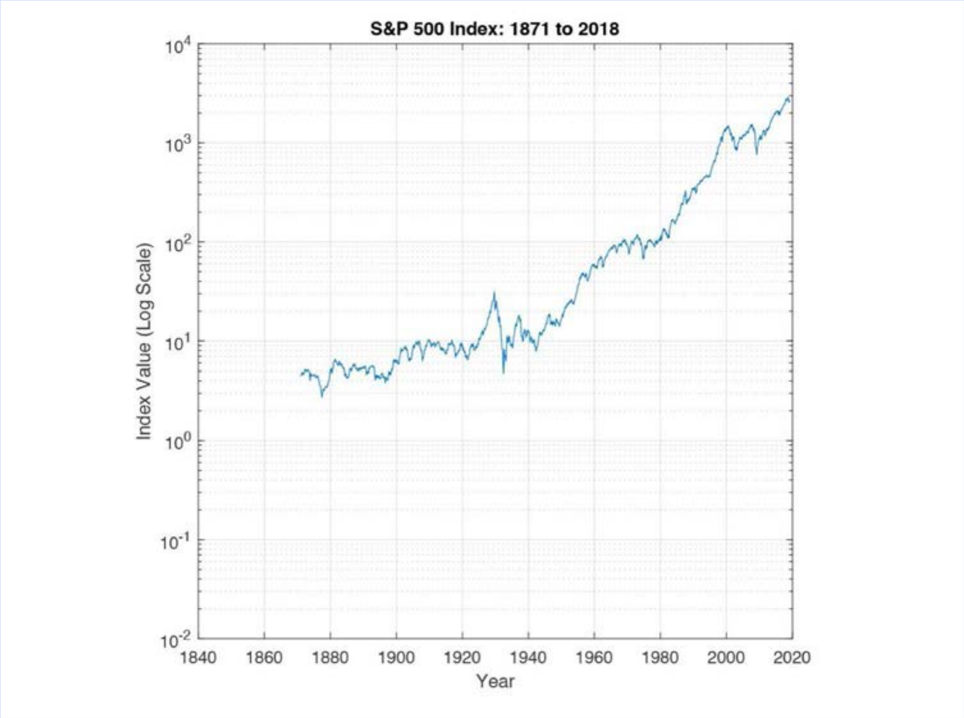
Example 4 of 4

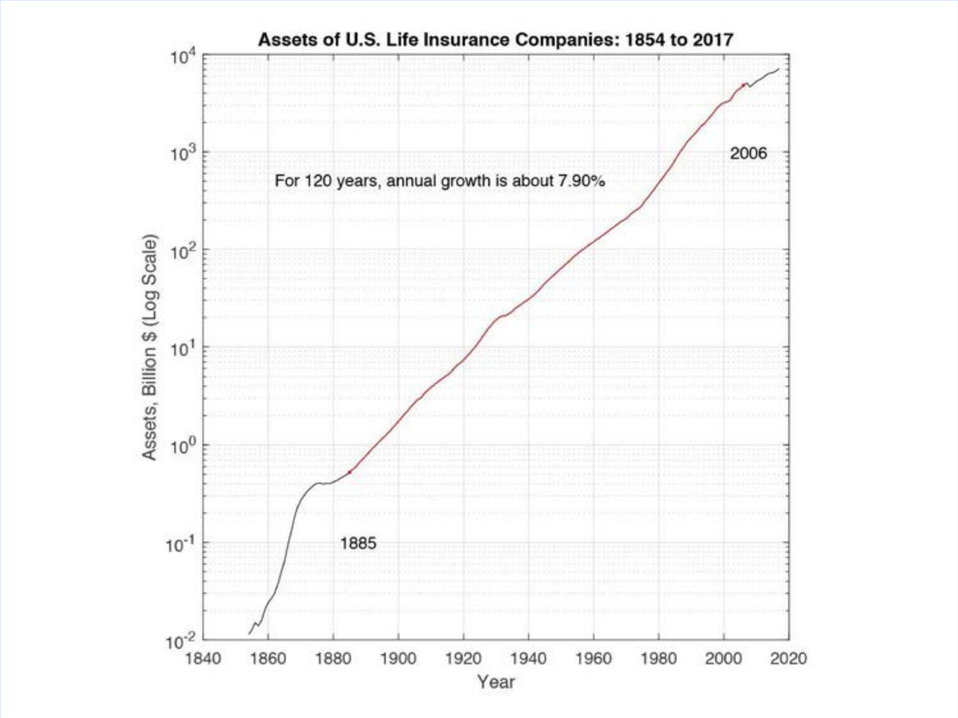


IBM spectral line #26: Quasi-periodic, NOT random (1009 days stacked).

Volatility Implies Little Prediction Skill



Smooth Growth Implies Prediction Skill



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Digital Signal Processing (DSP)

Related to Fourier Transforms and Frequency Response Ideas.

Aims to extract what we want to see.

Removes unwanted content, such as noise.

Scientific Design.

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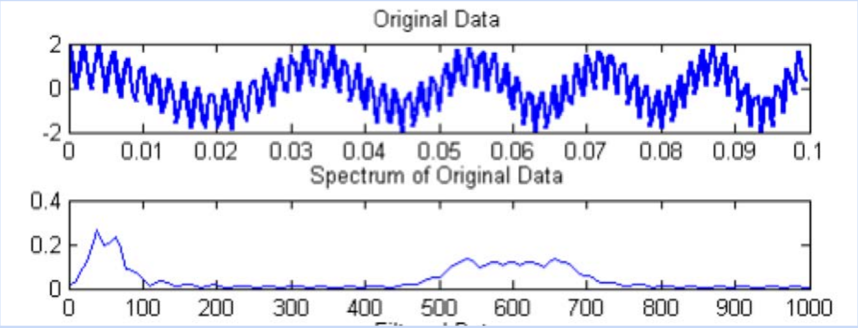
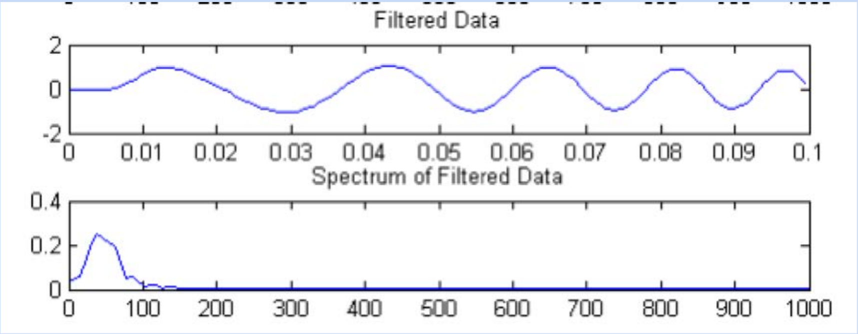
Example DSP To Remove Unwanted Content





On the left, the original signal.

On the right, a clean signal with useful information.

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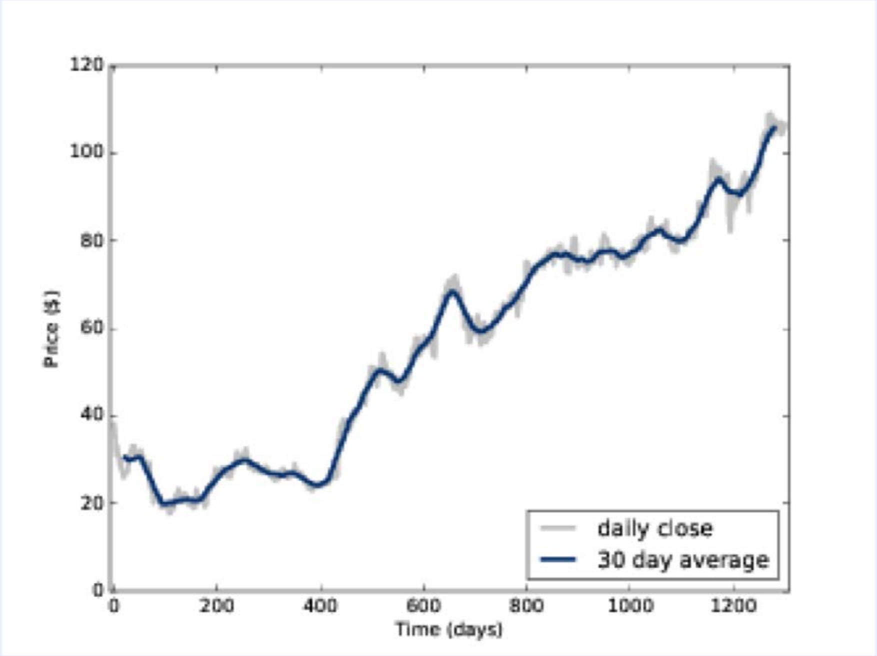
Example DSP To Expose and Remove Noise Simultaneously (For Trading)



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Summary

- The Fourier Transform uses sines and cosines to fit the data perfectly.
- The spectrum shows the energy of the data as a function of frequency.
- For best results, take a data sample long enough to be representative.
- Gaussian (random) white noise usually cannot be predicted directly.
- Brownian noise has a slope of minus 1. Information might be buried in it.
- A waterfall plot shows the dynamic changes of the system over time.
- DSP helps extract the information from the data that we want to see.
- Statistical methods cannot do that.

References

https://en.wikipedia.org/wiki/Fourier_transform

https://github.com/trekhleb/javascript-algorithms/tree/master/src/algorithms/math/fourier-transform

https://www.originlab.com/index.aspx?go=Products/Origin/DataAnalysis/SignalProcessing