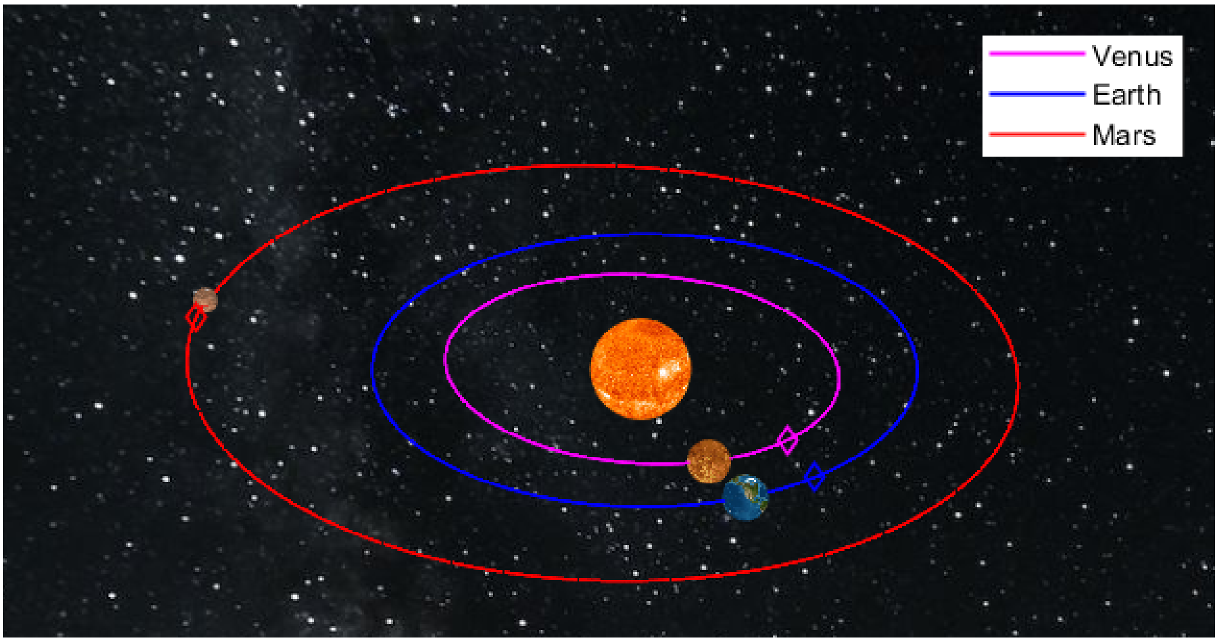
AER E 351 Astrodynamics I Project Report

Nathan Denny

November 2022

Department of Aerospace Engineering

Iowa State University



Contents

1 Introduction

There are many ways to transfer from one planet to another. These different ways offer different benefits, such as time or fuel savings. In this report I analyze three potential missions from Earth to Mars. Each mission will depart from Earth on 5 Jun 2004 at 01:52:21 UT and arrive to Mars on 14 May 2005 at 13:23:33 UT. The spacecraft will leave from a parking orbit of four times the radius of Earth and arrive in a parking orbit four times the radius of Mars.

- **Part 1**
  The mission is to leave Earth and arrive at Mars using two impulses.
- **Part 2**
  The mission is to leave Earth and arrive at Venus on 20 Nov 2004 at 15:10:59 UT, and then transfer to Mars. This requires three impulses.
- **Part 3**
  The mission is to leave Earth and arrive at Venus on 20 Nov 2004 at 15:10:59 UT. A flyby of Venus will then put the spacecraft on a transfer orbit to Mars. This requires two impulses.

The goal of the analysis is to determine which mission will require the least Δv. In the process, the transfers can be compared and analyzed for a greater understanding of interplanetary transfers.

2 Part 1

2.1 Earth Departure

The initial state of the spacecraft will be a circular parking orbit around Earth of radius four times the radius of the Earth. The spacecraft must leave this parking orbit on 5 Jun 2004 at 01:52:21.000 Universal Time UT, and arrive to the position of Mars on 14 May 2005 at 13:23:33 UT. To solve for the velocity of the spacecraft leaving Earth and arriving at Mars, a Lambert's Problem Solver is used. The inputs are simply the positions of the departure and arrival planets and the time between them.

Using NASA Horizon's Ephemeris, I obtained that the position and velocity of Earth on 5 Jun 2004 at 01:52:21 Universal Time UT in an ecliptic (x,y,z) Sun-centered plane is:

Earth Position: [-32430684.31, -103625896.9, 454387.1234] km

Earth Velocity: [33.18507753, -10.60723692, -2.06050762] km/s

Using the Lambert's Problem Solver, I was able to obtain the velocity of departure from the Earth and the velocity of arrival at Mars. These values allowed me to compute v_∞, the hyperbolic excess velocity of the spacecraft.

Considering the departure from Earth, $\vec{v}_{\infty,dep}$ is obtained

$$\vec{v}_{\infty,dep} = \vec{V}_{sc} - \vec{V}_{planet} \tag{1}$$

where $\vec{V}_{sc}$ is velocity of the spacecraft and $\vec{V}_{planet}$ is the velocity of the planet.

To find the Δv of the maneuver from the cirular parking orbit into the hyperbolic trajectory, the velocity on the hyperbolic trajectory at the position of the burn must be computed

$$v_{p,hyp,dep} = \sqrt{v_{\infty,dep}^2 + \frac{2\mu_{earth}}{r_p}} \tag{2}$$

where v_{p,hyp_dep} is the velocity at the perigee of the escape hyperbola, μ_{earth} is the gravitational parameter of the Earth, and r_p is the radius of the perigee of the hyperbola.

The velocity on the parking orbit is

$$v_{c,earth} = \sqrt{\frac{\mu_{earth}}{r_{p,c}}} \tag{3}$$

where $v_{c,earth}$ is the velocity on the circular parking orbit and $r_{p,c}$ is the radius of the perigee on the circular parking orbit. In this case, $r_{p,c}$ would be four times Earth's radius.

Finally, using Equations 2 and 3, the Δv of the burn into the interplanetary transfer orbit is

$$\Delta v_{dep} = v_{p,hyp,dep} - v_{c,earth} \tag{4}$$

In executing these steps, I determined that Δv_{dep} is equal to 30.2511 km/s.

2.2 Mars Arrival

The spacecraft will need to insert into a circular capture orbit around Mars with a radius equal to four times Mars's radius. To calculate the ΔV required for the maneuver, the hyperbolic excess speed of arrival must be found. This can be computed using the speed of the spacecraft on arrival to Mars that was solved for using the Lambert's Problem solver.

The hyperbolic excess speed on arrival to Mars is

$$\vec{v}_{\infty,arr} = \vec{V}_{sc,arr} - \vec{V}_{planet} \tag{5}$$

where $\vec{V}_{sc,arr}$ is velocity of the spacecraft on arrival and $\vec{V}_{planet}$ is the velocity of the planet (in this case, Mars).

I then found the velocity of the spacecraft at the periapsis of the hyperbola around Mars

$$v_{p,hyp,arr} = \sqrt{v_{\infty,arr}^2 + \frac{2\mu_{mars}}{r_p}} \tag{6}$$

where $v_{p,hyp,arr}$ is the velocity at the periapsis of the hyperbola, μ_{mars} is the gravitational parameter of Mars, and r_p is the radius of the periapsis of the hyperbola.

Next, the velocity on the circular parking orbit is found

$$v_{c,mars} = \sqrt{\frac{\mu_{mars}}{r_{p,c}}} \tag{7}$$

where $v_{c,mars}$ is the velocity on the circular parking orbit and $r_{p,c}$ is the radius of the perigee on the circular parking orbit. In this case, $r_{p,c}$ would be four times Mars's radius.

To compute Δv_{arr}, Equations 6 and 7 are used

$$\Delta v_{arr} = v_{p,hyp,arr} - v_{c,mars} \tag{8}$$

I found that Δv_{arr} is equal to 24.148 km/s. This number is so high because the spacecraft is arriving to Mars with a velocity that is not nearly in the same direction to Mars's velocity. Since the two velocities are not nearly similar in direction, the spacecraft's velocity has to be drastically corrected to capture around Mars. Figure 1 clearly shows the oblique angle that the spacecraft is arriving to Mars with. This is the same case as to why the departure from Earth is such an expensive maneuver.

2.3 Transfer

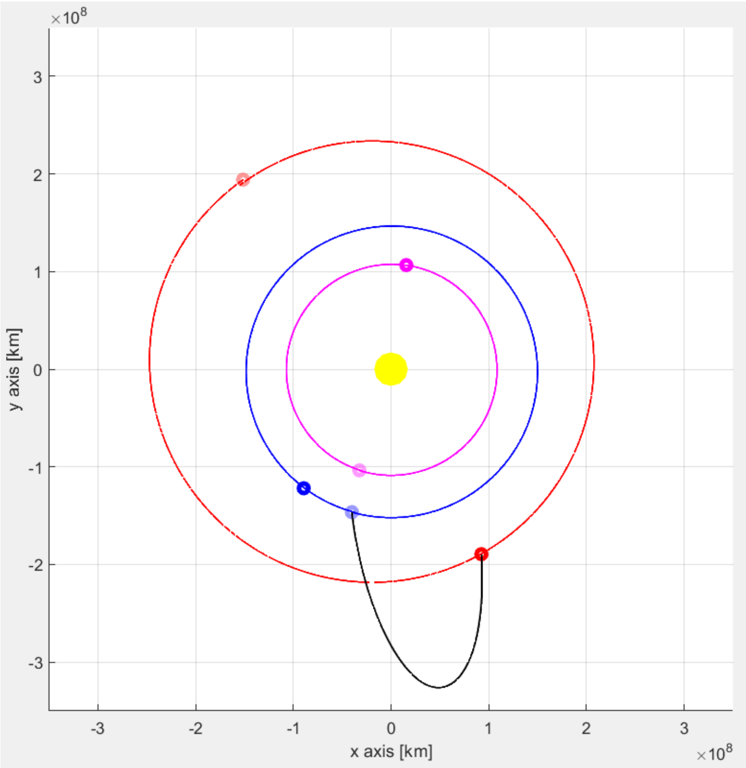


Figure 1: Part 1 Transfer 2D Plot

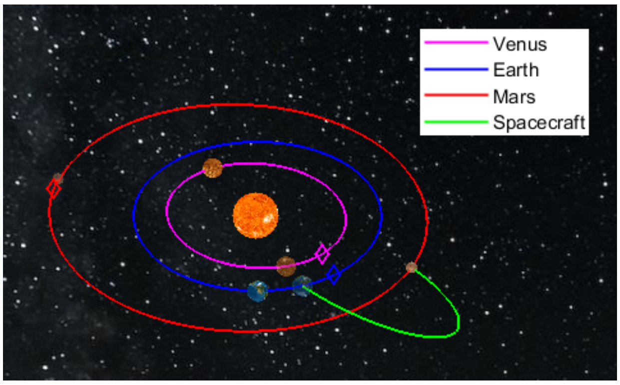


Figure 2: Part 1 Transfer 3D Plot

2.4 Δv Considerations

The total Δv required for this interplanetary mission is about 54.3991 km/s, making the two-burn maneuver a very expensive mission.

3 Part 2

3.1 Earth Departure

The spacecraft must depart Earth on 5 Jun 2004 at 01:52:21 UT and arrive to Venus on 20 Nov 2004 at 15:10:59 UT. This would involve a maneuver to escape Earth, and a maneuver to equalize the spacecraft's velocity with Venus's orbital velocity.

NASA's Horizons Ephemeris is used to find the position and velocity of Venus on 20 Nov 2004 at 15:10:59 in an ecliptic (x,y,z) Sun-centered plane.

Venus Position = [-102350900.4, 32501461.54, 6352131.88] km

Venus Velocity = [-10.76766311, -33.53992208, 0.16266872] km/s

A Lambert's Problem solver is then used to compute the departure velocity of the spacecraft from Earth and the arrival velocity of the spacecraft at Venus. The velocity of the spacecraft departing Earth allows for the computation of the Δv required to depart Earth. The steps taken to compute this are the exact same presented in §2.1. Using Equations 1, 2, 3, and 4, I found that the Δv required to go from Earth to Venus as described is equal to 3.28045 km/s.

3.2 Venus Maneuvers

At Venus, two burns take place. The first is a burn from our first transfer orbit to equalize Venus's orbital velocity (insert the spacecraft into Venus's orbit). The second is to move the spacecraft from Venus to Mars.

The first of these maneuvers is simple. Using the velocity of the spacecraft approaching Venus that is supplied by the Lambert's Problem Solver, I can simply subtract Venus's orbital velocity to find the $\Delta\vec{v}$ necessary

$$\Delta\vec{v}_{venus,arr} = \vec{V}_{venus} - \vec{V}_{sc,arr} \tag{9}$$

where $\vec{V}_{venus}$ is the orbital velocity of Venus and $\vec{V}_{sc,arr}$ is the velocity of the spacecraft arriving at Venus.

The second maneuver is solved using the Lambert's Problem solver. The Lambert's Problem solver provides the velocity of the spacecraft leaving Venus on its way to Mars. Therefore,

$$\Delta\vec{v}_{venus,dep} = \vec{V}_{venus,dep} - \vec{V}_{venus} \tag{10}$$

where $V_{venus,dep}$ is the velocity of the spacecraft leaving Venus on its way to Mars.

The total Δv for the maneuvers at Venus can be computed by adding Equations 9 and 10

$$\Delta\vec{v}_{venus,total} = \vec{V}_{venus} - \vec{V}_{sc,arr} + \vec{V}_{venus,dep} - \vec{V}_{venus} \tag{11}$$

$$\Delta\vec{v}_{venus,total} = \vec{V}_{venus,dep} - \vec{V}_{sc,arr} \tag{12}$$

where $\Delta\vec{v}_{venus,total}$ is the total Δv required at Venus.

The value of the total Δv required at Venus is equal to 3.85562 km/s.

3.3 Mars Arrival

To arrive at Mars, we must use the same techniques demonstrated in §2.2. The Lambert's Problem Solver provides the velocity of the spacecraft on its arrival at Mars, and then using Equations 5, 6, 7, and 8 one can find the Δv required to insert the spacecraft into the circular parking orbit around Mars. After following these steps, I computed the Δv required for Mars arrival to be 4.86478 km/s.

3.4 Transfers

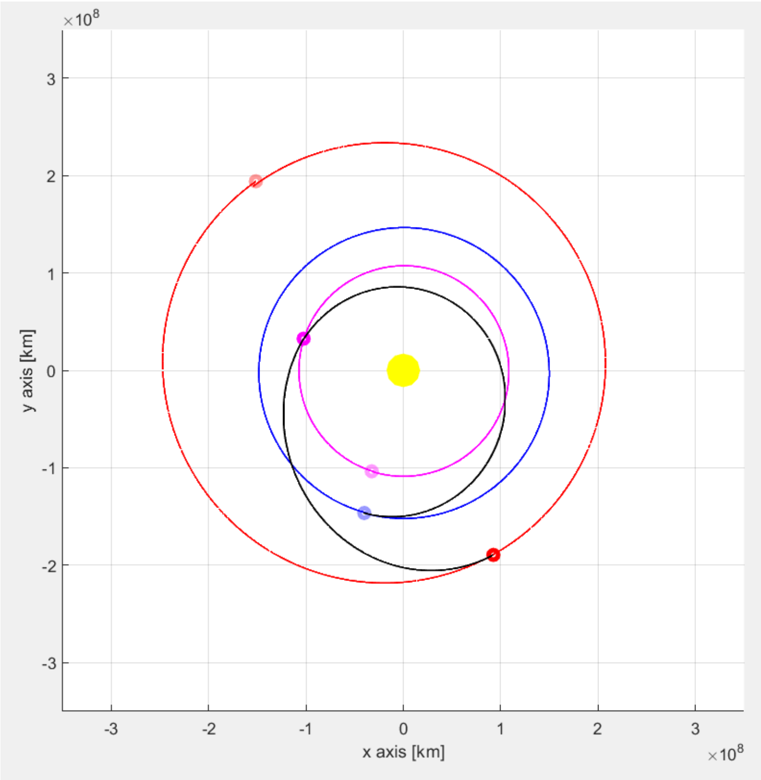


Figure 3: Part 2 Transfer 2D Plot

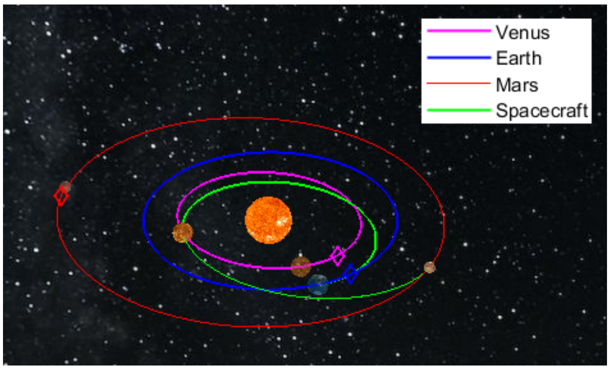


Figure 4: Part 2 Transfer 3D Plot

3.5 Δv Considerations

The total Δv required for this mission is about 12 km/s. This is considerably cheaper than the 54.3991 km/s required for the first mission.

4 Part 3

4.1 Earth Depature

The spacecraft must depart from Earth on 5 Jun 2004 at 01:52:21 UT and arrive to Venus on 20 Nov 2004 at 15:10:59 UT in order to complete the flyby. This maneuver is the exact same maneuver as completed in part §3.1. Equations 1, 2, 3, 4 are used to solve for the final Δv required for this maneuver. The Δv required is computed as 3.28045 km/s.

4.2 Venus Flyby

To perform the flyby of Venus, certain orbital parameters of the transfer orbit (from Earth to Venus) must be computed. First, the true anomaly of Venus on the transfer orbit must be found.

This is completed by using the fact that the Earth is the apoapsis of the transfer orbit, so 180°subtracted by the obtuse angle between the Earth and Venus will be the true anomaly.

$$\theta_{venus} = 180 - \arccos(\frac{\vec{r}_{earth} \cdot \vec{r}_{venus}}{r_{earth} r_{venus}}) \tag{13}$$

where θ_{venus} is the true anomaly of Venus on the transfer orbit, $\vec{r}_{earth}$ is the position of the Earth on 5 Jun 2004 at 01:52:21 UT, and $\vec{r}_{venus}$ is the position of Venus on 20 Nov 2004 at 15:10:59 UT.

Using the true anomaly of Venus, the eccentricity of the transfer orbit can be found

$$e_{trans} = \frac{r_{earth} - r_{venus}}{r_{earth} + r_{venus}\cos(\theta_{venus})} \tag{14}$$

The specific angular momentum of the transfer orbit can be then be found

$$h_{trans} = \sqrt{\mu_{sun} r_{earth}(1 - e_{trans})} \tag{15}$$

The velocity of the spacecraft in terms of perpendicular and radial components can then be computed

$$\vec{V}_1^{(v)} = \frac{\mu_{sun}}{h_{trans}} e_{trans} \sin(\theta_{venus}) \hat{\boldsymbol{u}}_{\boldsymbol{r}} + \frac{h_{trans}}{r_{venus}} \hat{\boldsymbol{u}}_{\perp} \tag{16}$$

where $V_1^{(v)}$ is the velocity of the spacecraft.

The excess velocity of the spacecraft with respect to Venus can be computed with Venus-Sun components

$$\vec{v}_{\infty,1} = ([V_1^{(v)}]_{\perp} - V_{venus})\hat{\boldsymbol{u}}_{\boldsymbol{V}} + -[V_1^{(v)}]_r \hat{\boldsymbol{u}}_{\boldsymbol{S}} \tag{17}$$

The eccentricity of the flyby can then be found given the periapsis of the flyby. The periapsis of the flyby will be about 14,161 km. The turn angle δ can also be found

$$e_{flyby} = 1 + \frac{r_{p,flyby} v_{\infty}^2}{\mu_{venus}} \tag{18}$$

$$\delta = 2\arcsin(\frac{1}{e_{flyby}}) \tag{19}$$

The angles ϕ_1 and ϕ_2 (for a leading side flyby) can then be found

$$\phi_1 = \arctan(\frac{[v_{\infty,1}]_S}{[v_{\infty,1}]_V}) \tag{20}$$

$$\phi_2 = \phi_1 + \delta \tag{21}$$

The departing hyperbolic excess velocity of the spacecraft is then obtainable

$$\vec{v}_{\infty,2} = (v_{\infty,1}\cos(\phi_2))\hat{u}_V + (v_{\infty,1}\sin(\phi_2))\hat{u}_S \tag{22}$$

The spacecraft velocity in perpendicular and radial components is then

$$\vec{V}_2^{(v)} = ([\vec{v}_{\infty,2}]_V + \vec{V}_{venus})\hat{u}_\perp + (-[v\vec{_{\infty,2}}]_S)\hat{u}_r \tag{23}$$

The flyby analysis is then complete given the spacecraft's heliocentric velocity after the flyby has been found. The flyby saves 3.85562 km/s of Δv.

4.3 Mars Arrival

To find the Δv necessary to circularize around Mars, the arrival velocity of the spacecraft must be found. This is the exact same as the Δv found in §3.3 since the orbits are the same. Thus, the Δv required for the arrival at Mars is 4.86478 km/s.

4.4 Transfers

The transfers for Part 3 are visually the same as Part 2.

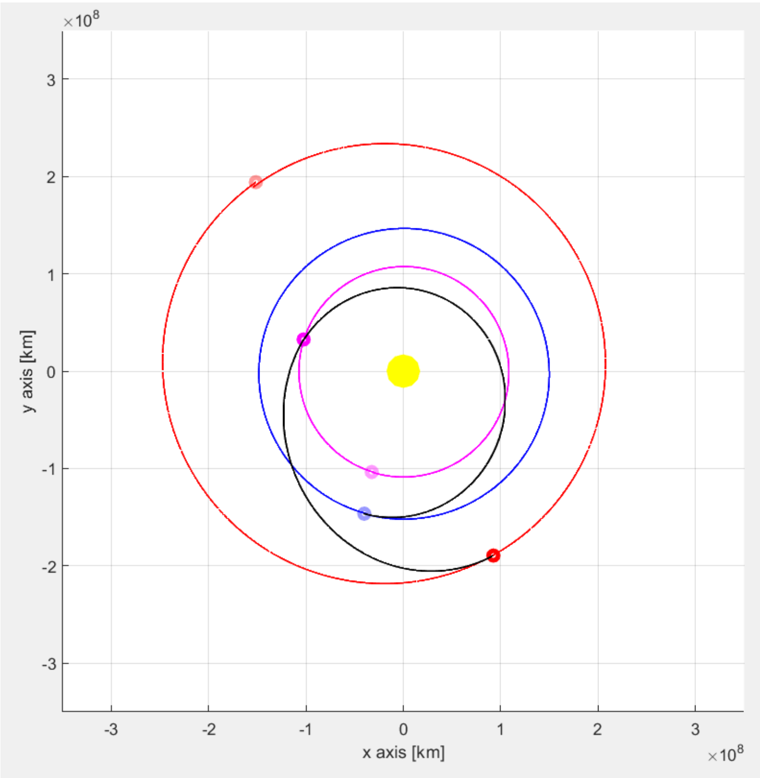


Figure 5: Part 3 Transfer 2D Plot

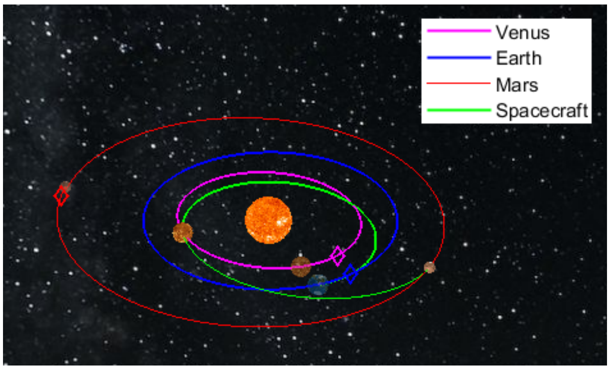


Figure 6: Part 3 Transfer 3D Plot

4.5 Δv Considerations

The total Δv for this mission is 8.1452 km/s. The flyby saved 3.8556 km/s of Δv. This is the cheapest mission in terms of Δv.

5 Conclusion

In doing the analyses, I have concluded that the mission described in Part 3 is the most efficient in terms of Δv. As can be seen in Table 1, the mission uses a total of 8.1452 km/s of Δv. This mission saves Δv by performing a flyby of Venus. The first mission is so expensive because the transfer orbit is very different from the orbits of Earth or Mars. This causes the departure and arrival maneuvers to be extremely expensive. The second mission is less expensive than the first because the maneuvers are not wildly different from the planet's orbital velocities, allowing for a less dramatic change in velocity (and fuel savings). Overall, the analyses show that getting creative with interplanetary trajectories can lead to cost savings. A direct two-impulse trajectory is not always the most efficient transfer for the mission.

Table 1: Total Δv Required for Each Mission

Part	Total Δv [km/s]
1	54.3991
2	12.0009
3	8.1452