Problems 1-56

1. Graph $y = \left(\frac{1}{2}\right)^x$.

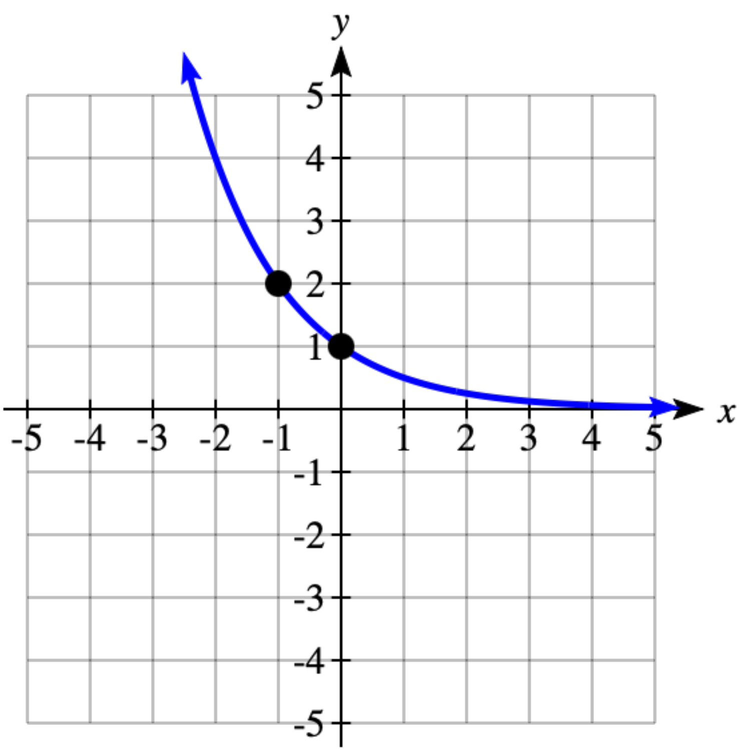
This is an exponential decay function because the base $\left(\dfrac{1}{2}\right)$ is between 0 and 1.

Find key points for the graph.

x	$y = \left(\frac{1}{2}\right)^x$
-2	4
-1	2
0	1

Plot the points and sketch the curve.

The curve approaches zero as x increases and grows as x becomes more negative.



2. Graph $y = \log_4 x$.

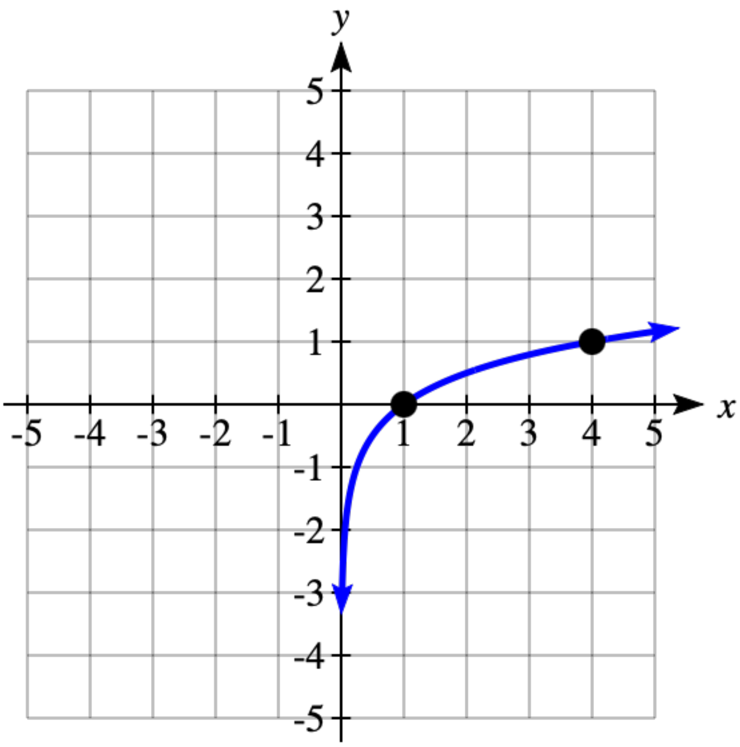
This is a logarithmic function with base 4.

Find key points for the graph.

x	$y = \log_4 x$
1	0
4	1

Plot the points and sketch the curve.

The graph passes through points (1,0), and (4,1) with a vertical asymptote at $x = 0$.



3. Solve $9^x = 243$.

Rewrite each side in terms of the same base.

$9 = 3^2$ and $243 = 3^5$,

$\Rightarrow 9^x = 243 \Rightarrow (3^2)^x = 3^5$.

Simplify the equation using the power of a power property.

$3^{2x} = 3^5$.

Set the exponents equal to each other (since the bases are the same).

$2x = 5$,

$\Rightarrow x = \dfrac{5}{2}$.

$\boxed{x = \left\{\dfrac{5}{2}\right\}}$.

4. Solve $25^{2x+1} = 625^{3x}$.

Express both sides in terms of the same base.
$25 = 5^2 \quad \text{and} \quad 625 = 5^4,$
$\Rightarrow (5^2)^{2x+1} = (5^4)^{3x}.$

Simplify using the power of a power property.
$5^{4x+2} = 5^{12x}.$

Set exponents equal to each other.
$4x + 2 = 12x,$
$\Rightarrow -8x = -2,$
$\Rightarrow x = \dfrac{1}{4}$
$\boxed{x = \left\{\dfrac{1}{4}\right\}}.$

5. The population of a small town in Indiana is declining according to $y = 60000 \cdot 2^{-0.01x}$ where x is the number of years since 1960.

(a) What was the population in 1960:

Set $x = 0$.
$$y = 60000 \cdot 2^{-0.01 \cdot 0}$$
$$= \boxed{60000}.$$

(b) What was the population in 1980:

Set $x = 20$.
$$y = 60000 \cdot 2^{-0.01 \cdot 20}$$
$$\approx 60000 \cdot 0.933$$
$$\approx \boxed{52233}.$$

(c) What is the expected population of the town in 2015:

Set $x = 55$.
$$y = 60000 \cdot 2^{-0.01 \cdot 55}$$
$$\approx 60000 \cdot 0.683$$
$$\approx \boxed{40981}.$$

6. Write $4^{-3} = \frac{1}{64}$ in logarithmic form.

Rewrite in logarithmic form:

$$\log_4 \frac{1}{64} = -3.$$

$$\boxed{\log_4 \frac{1}{64} = -3}.$$

7. Write $\log_3 81 = 4$ in exponential form.

Rewrite in exponential form:

$3^4 = 81.$

$\boxed{3^4 = 81}.$

8. Solve $\log_6 x = 3$.

Rewrite in exponential form:

$x = 6^3 = 216.$

$\boxed{x = \{216\}}.$

9. Solve $\log_{16} 2 = x$.

Rewrite in exponential form:

$16^x = 2.$

Express 16 and 2 in terms of base 2: $2^{4x} = 2^1$.

$$4x = 1, \quad x = \frac{1}{4}.$$

$$\boxed{x = \left\{\frac{1}{4}\right\}}.$$

10. Solve $\log_x 225 = 2$.

Rewrite in exponential form:

$x^2 = 225, \quad x = \pm 15.$

Since $x > 0$, we take the positive value.

$\boxed{x = \{15\}}.$

11. Solve $8^x = 21$.

Take the logarithm of both sides.
$\log(8^x) = \log(21)$.

Use the power rule of logarithms.
$x \cdot \log(8) = \log(21)$,
$\Rightarrow x = \dfrac{\log(21)}{\log(8)}$
$= \log_8 21$.
$\boxed{x = \{\log_8 21\}}$.

12. Solve $\log_2(x+1) + \log_2(x-1) = 3$.

Use the product rule for logarithms.
$\log_2((x+1)(x-1)) = 3$.

Rewrite in exponential form.
$(x+1)(x-1) = 2^3$,
$x^2 - 1 = 8$.
Solve for x.
$x^2 = 9$,
$x = \pm 3$.
Since $x - 1 > 0$, $x = 3$ is the only valid solution.
$\boxed{x = \{3\}}$.

For #13-14, use the properties of logarithms to write the expression as a sum or difference of logarithms.

13. Write $\log_2(ac^4)$ as a sum of logarithms.
Apply the product and power rules of logarithms.

$$\begin{aligned}\log_2(ac^4) &= \log_2(a) + \log_2(c^4)\\ &= \log_2(a) + 4\log_2(c).\\ &= \boxed{\log_2(a) + 4\log_2(c)}.\end{aligned}$$

14. Write $\log_6\left(\frac{c\sqrt{d}}{g^3}\right)$ as a sum or difference of logarithms.
Apply the quotient rule, product rule, and power rule of logarithms.

$$\begin{aligned}\log_6\left(\frac{c\sqrt{d}}{g^3}\right) &= \log_6(c) + \log_6(\sqrt{d}) - \log_6(g^3)\\ &= \log_6(c) + \frac{1}{2}\log_6(d) - 3\log_6(g).\\ &= \boxed{\log_6(c) + \frac{1}{2}\log_6(d) - 3\log_6(g)}.\end{aligned}$$

For #15-16, use the properties of logarithms to write a single logarithm.

15. Write $3\log_b(x) - \log_b(y)$ as a single logarithm.
Use the power rule and then the quotient rule.

$$\begin{aligned}3\log_b(x) - \log_b(y) &= \log_b(x^3) - \log_b(y)\\ &= \log_b\left(\frac{x^3}{y}\right).\\ &= \boxed{\log_b\left(\frac{x^3}{y}\right)}.\end{aligned}$$

16. Write $\frac{1}{6}\log_b(k) + 2\log_b(m) - \frac{3}{2}\log_b(n)$ as a single logarithm.
Use the power rule and then the product and quotient rules.

$$\begin{aligned}\frac{1}{6}\log_b(k) + 2\log_b(m) - \frac{3}{2}\log_b(n) &= \log_b\left(k^{\frac{1}{6}}\right) + \log_b(m^2) - \log_b(n^{\frac{3}{2}})\\ &= \log_b\left(\frac{m^2\sqrt[6]{k}}{\sqrt{n^3}}\right).\\ &= \boxed{\log_b\left(\frac{\sqrt[6]{k}m^2}{\sqrt{n^3}}\right)}.\end{aligned}$$

17. Use the change of base rule to express $\log_4 15$ in terms of natural logarithms, and approximate it to the nearest thousandth.
Use the change of base formula.

$$\begin{aligned}\log_4 15 &= \frac{\ln(15)}{\ln(4)} \quad \text{Approximate:}\\ &\approx 1.953.\\ &= \boxed{1.953}.\end{aligned}$$

18. How much money will there be in an account at the end of 4 years if \$1,000 is deposited at 8% compounded quarterly, assuming no withdrawals are made?

Use the compound interest formula $A = P\left(1+\frac{r}{n}\right)^{nt}$.

$$\begin{aligned} &P = 1000, \quad r = 0.08, \quad n = 4, \quad t = 4, \\ &A = 1000\left(1+\frac{0.08}{4}\right)^{4\cdot 4} \\ &\quad = 1000\cdot(1.02)^{16} \\ &\quad \approx 1372.79. \\ &\quad = \boxed{\$1,372.79}. \end{aligned}$$

19. Suppose that a chameleon is growing in length at a continuous rate of 6% annually.

(a) If the chameleon is 2.5 feet in length now, how long will she be in 5 years?
Use the continuous growth formula $A = Pe^{rt} \implies L(t) = L_0e^{rt}$, where L is the future length and L_0 is the current original length.

$$\begin{aligned} &L_0 = 2.5, \quad r = 0.06, \quad t = 5, \\ &L = 2.5e^{0.06\cdot 5} \approx 2.5\cdot 1.3499 \approx 3.375. \\ &\quad = \boxed{3.375 \text{ feet}}. \end{aligned}$$

(b) How long will it take for the chameleon to double in length?

$$\begin{aligned} &\text{Solve } 2L_0 = L_0e^{0.06t} \quad \implies \quad L_0 \text{ cancels out.} \\ &2 = e^{0.06t}, \\ &\ln(2) = 0.06t, \\ &t = \frac{\ln(2)}{0.06} \approx \boxed{11.55 \text{ years}}. \end{aligned}$$

20. Determine whether $(-3, 4)$ is a solution to the system:

$$-3x - 2y = 1$$

$$4x + 7y = -16$$

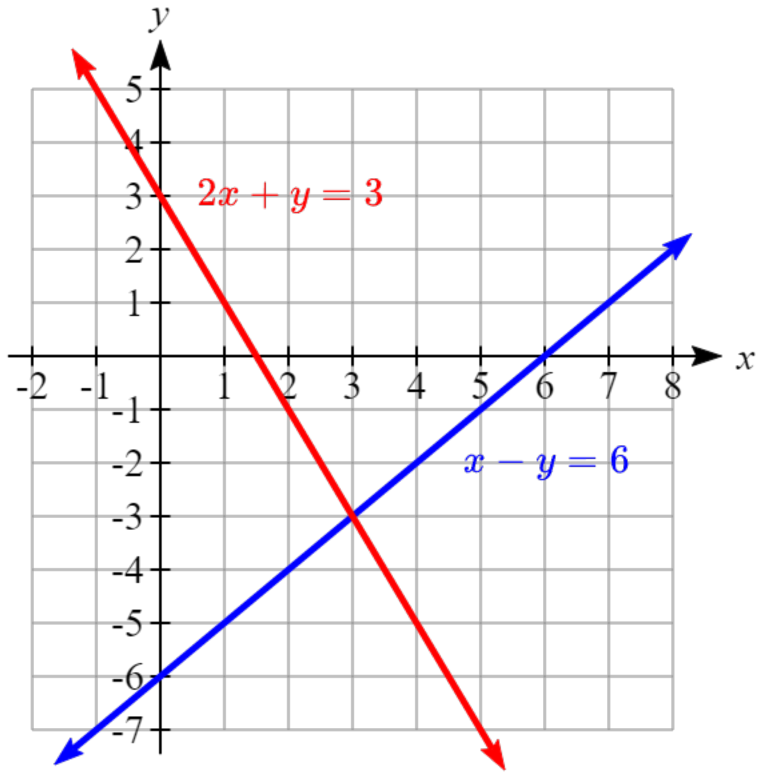
Substitute $x = -3$ and $y = 4$ into each equation.
$-3(-3) - 2(4) = 1$,
$9 - 8 = 1$ True for the first equation.

$4(-3) + 7(4) = -16$,
$-12 + 28 = 16$ Not true for the second equation.

$\boxed{\text{No, } (-3, 4) \text{ is not a solution.}}$

21. Solve the system graphically:

$$2x + y = 3$$
$$x - y = 6$$



The intersection of the two lines is the solution: $\boxed{(3, -3)}$.

For #22-26, Solve using Addition.

22. Solve using addition:

$$3x - y = -15$$
$$4x + 5y = -1$$

Multiply the first equation by 5 to align coefficients of y.
$15x - 5y = -75$.
$4x + 5y = -1$.

Add the equations to eliminate y.
$19x = -76, \quad x = -4$.

Substitute $x = -4$ into the first equation.
$3(-4) - y = -15, \quad -12 - y = -15, \quad y = 3$.

The solution is: $\boxed{(-4, 3)}$.

23. Solve using addition:

$$-x + 2y = -7$$
$$2x - 4y = 14$$

Notice the second equation is a multiple of the first.
$2(-x + 2y) = -7 \times 2,$
$-2x + 4y = -14.$

The equations are equivalent, so there are

$\boxed{\text{Infinitely many solutions: } \left(x,\quad \frac{1}{2}x - \frac{7}{2}\right)}.$

24. Solve using addition:

$$\frac{1}{2}x + \frac{1}{4}y = -3$$
$$\frac{1}{2}x - \frac{3}{4}y = 5$$

Clear the fractions by multiplying both equations by 4.
$2x + y = -12,$
$2x - 3y = 20.$

Subtract the second equation from the first.
$4y = -32, \quad y = -8.$

Substitute $y = -8$ into the first equation.
$2x + (-8) = -12, \quad 2x = -4, \quad x = -2.$

Answer: $\boxed{(-2, -8)}.$

25. Solve the system:

$$x + y + z = 5$$
$$2x - y + z = 6$$
$$x + 2y - z = -3$$

To solve this system of linear equations, we'll use elimination to solve for x, y, and z:
Multiply the first equation by -1 and add it to second equation to eliminate the z

$$\begin{cases} -1(x + y + z = 5) \\ 2x - y + z = 6 \end{cases} \implies \begin{cases} -x - y - z = -5 \\ 2x - y + z = 6 \end{cases} \implies x - 2y = 1$$

Add the second and the third equations:

$$\Longrightarrow \begin{cases} 2x - y + z = 5 \\ x + 2y - z = -3 \end{cases} \quad \Longrightarrow \quad 3x + y = 3$$

$$\begin{cases} x - 2y = 1 \\ 2(3x + y = 3) \end{cases} \quad \Longrightarrow \quad \begin{cases} x - 2y = 1 \\ 6x + 2y = 6 \end{cases} \quad \Longrightarrow \quad 7x = 7 \quad \Longrightarrow \quad x = 1.$$

Back substitute $x = 1$ into $3x + y = 3 \Rightarrow 3 + y = 3 \Rightarrow y = 0$.
Back substitute $x = 1$, and $y = 0$ into the first equation:
$x + y + z = 5 \Longrightarrow 1 + z = 5 \Longrightarrow z = 4$. The solution to the system is:

$$(x, y, z) = \boxed{(1, 0, 4)}.$$

26. Solve the system:

$$2x + y + z = 0$$

$$3x - y + z = 3$$

$$7x - 5y - 3z = 15$$

To solve this system of linear equations, we'll use elimination to solve for x, y, and z:
Multiply the first equation by $-\frac{3}{2}$ and add it to the second equation to eliminate z:

$$\begin{cases} -\frac{3}{2}(2x + y + z = 0) \\ 3x - y + z = 3 \end{cases} \quad \Longrightarrow \quad \begin{cases} -3x - \frac{3}{2}y - \frac{3}{2}z = 0 \\ 3x - y + z = 3 \end{cases}$$

$$\Longrightarrow \quad -\frac{5}{2}y - \frac{1}{2}z = 3 \quad \Longrightarrow \quad -5y - z = 6 \tag{1}$$

Next, multiply the first equation by $-\frac{7}{2}$ and add it to the third equation:

$$\begin{cases} -\frac{7}{2}(2x + y + z = 0) \\ 7x - 5y - 3z = 15 \end{cases} \quad \Longrightarrow \quad \begin{cases} -7x - \frac{7}{2}y - \frac{7}{2}z = 0 \\ 7x - 5y - 3z = 15 \end{cases}$$

$$\Longrightarrow \quad -\frac{17}{2}y - \frac{13}{2}z = 15 \quad \Longrightarrow \quad -17y - 13z = 30 \tag{2}$$

We now have the reduced system of two variables:

$$-5y - z = 6 \tag{1}$$

$$-17y - 13z = 30 \tag{2}$$

To eliminate z, multiply Equation (1) by 13 and Equation (2) by 1, and subtract:

$$\begin{cases} 13(-5y - z = 6) \\ -17y - 13z = 30 \end{cases} \quad \Longrightarrow \quad \begin{cases} -65y - 13z = 78 \\ -17y - 13z = 30 \end{cases}$$

$$\Longrightarrow \quad -65y + 17y = 78 - 30$$

$$\Longrightarrow \quad -48y = 48 \quad \Longrightarrow \quad y = -1 \tag{3}$$

Substitute $y = -1$ into Equation (1):

$$-5(-1) - z = 6 \quad \Longrightarrow \quad 5 - z = 6 \quad \Longrightarrow \quad z = -1 \tag{4}$$

Substitute $y = -1$ and $z = -1$ into the first equation:

$$2x + (-1) + (-1) = 0 \implies 2x - 2 = 0 \implies 2x = 2 \implies x = 1 \tag{5}$$

The solution to the system is:

$$(x, y, z) = \boxed{(1, -1, -1)}$$

For #27-28, solve using Substitution.

27. Solve using substitution:

$$x + 3y = -15$$
$$x - 2y = 20$$

Solve the second equation for x.
$x = 20 + 2y$.

Substitute into the first equation.
$(20 + 2y) + 3y = -15$,
$20 + 5y = -15, \quad y = -7$.

Substitute $y = -7$ into $x = 20 + 2y$.
$x = 20 + 2(-7) = 6$.

Answer: $\boxed{(6, -7)}$.

28. Solve using substitution:

$$2x + y = 19$$
$$y = 10 - x$$

Substitute $y = 10 - x$ into the first equation.
$2x + (10 - x) = 19$,
$x + 10 = 19, \quad x = 9$.

Substitute $x = 9$ into $y = 10 - x$.
$y = 10 - 9 = 1$.

Answer: $\boxed{(9, 1)}$.

For #29-30, solve using Gaussian Elimination.

29. Solve using Gaussian elimination:

$$3x - 4y = -8$$

$$2x + 5y = 33$$

Represent the system as an augmented matrix:

$$\begin{pmatrix} 3 & -4 & | & -8 \\ 2 & 5 & | & 33 \end{pmatrix}.$$

Make the leading entry in Row 1 equal to 1.
Divide Row 1 by 3:

$$R_1 \to \frac{1}{3}R_1$$

$$\begin{pmatrix} 1 & -\frac{4}{3} & | & -\frac{8}{3} \\ 2 & 5 & | & 33 \end{pmatrix}.$$

Eliminate the x-term in Row 2.
To eliminate the x-term in Row 2, subtract $2 \times R_1$ from R_2:

$$R_2 \to R_2 - 2 \times R_1$$

$$\begin{pmatrix} 1 & -\frac{4}{3} & | & -\frac{8}{3} \\ 0 & \frac{23}{3} & | & \frac{115}{3} \end{pmatrix}.$$

Make the leading entry in Row 2 equal to 1.
Multiply Row 2 by $\frac{3}{23}$:

$$R_2 \to \frac{3}{23}R_2$$

$$\begin{pmatrix} 1 & -\frac{4}{3} & | & -\frac{8}{3} \\ 0 & 1 & | & 5 \end{pmatrix}.$$

Eliminate the y-term in Row 1.
To eliminate the y-term in Row 1, add $\frac{4}{3} \times R_2$ to R_1:

$$R_1 \to R_1 + \frac{4}{3} \times R_2$$

$$\begin{pmatrix} 1 & 0 & | & 4 \\ 0 & 1 & | & 5 \end{pmatrix}.$$

From the final augmented matrix:

$$\begin{pmatrix} 1 & 0 & | & 4 \\ 0 & 1 & | & 5 \end{pmatrix},$$

Thus;

$$x = 4, \quad y = 5.$$

$$\boxed{(x, y) = (4, 5)}$$

30. Solve using Gaussian elimination:

$$2x - 3y - 4z = 3$$
$$-4x + 2y - 3z = -13$$
$$-3x - 4y + 2z = 0$$

Write the augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & -3 & -4 & 3 \\ -4 & 2 & -3 & -13 \\ -3 & -4 & 2 & 0 \end{array}\right].$$

Normalize the first row by dividing it by 2:

$$\left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & -2 & \frac{3}{2} \\ -4 & 2 & -3 & -13 \\ -3 & -4 & 2 & 0 \end{array}\right].$$

Eliminate the first entry in rows 2 and 3:

$$R_2 \to R_2 + 4R_1, \quad R_3 \to R_3 + 3R_1.$$

The updated matrix is:

$$\left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & -2 & \frac{3}{2} \\ 0 & -4 & -11 & -7 \\ 0 & -\frac{17}{2} & -4 & \frac{9}{2} \end{array}\right].$$

Normalize the second row by dividing it by -4:

$$\left[\begin{array}{ccc|c} 1 & -\frac{3}{2} & -2 & \frac{3}{2} \\ 0 & 1 & \frac{11}{4} & \frac{7}{4} \\ 0 & -\frac{17}{2} & -4 & \frac{9}{2} \end{array}\right].$$

Eliminate the second entry in rows 1 and 3:

$$R_1 \to R_1 + \frac{3}{2}R_2, \quad R_3 \to R_3 + \frac{17}{2}R_2.$$

The updated matrix is:

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{4} & 2 \\ 0 & 1 & \frac{11}{4} & \frac{7}{4} \\ 0 & 0 & 15 & 15 \end{array}\right].$$

Normalize the third row by dividing it by 15:

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{5}{4} & 2 \\ 0 & 1 & \frac{11}{4} & \frac{7}{4} \\ 0 & 0 & 1 & 1 \end{array}\right].$$

Eliminate the third entry in rows 1 and 2:

$$R_1 \to R_1 - \frac{5}{4}R_3, \quad R_2 \to R_2 - \frac{11}{4}R_3.$$

The final matrix is:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

The solution is:

$$x = 2, \quad y = -1, \quad z = 1 \implies (x, y, z) = \boxed{(2, -1, 1)}$$

For #31-32, evaluate the determinant.

31. Evaluate the determinant:

$$\begin{vmatrix} 2 & -3 \\ -5 & 8 \end{vmatrix}$$

Use the formula for the 2×2 determinant $ad - bc$.

$$\begin{aligned} &= (2)(8) - (-3)(-5), \\ &= 16 - 15 = 1. \\ &= \boxed{1}. \end{aligned}$$

32. Evaluate the determinant:

$$\begin{vmatrix} 2 & -1 & 3 \\ 0 & -3 & 6 \\ 1 & 4 & -2 \end{vmatrix}$$

Use cofactor expansion along the first row.

$$= 2\begin{vmatrix} -3 & 6 \\ 4 & -2 \end{vmatrix} - (-1)\begin{vmatrix} 0 & 6 \\ 1 & -2 \end{vmatrix} + 3\begin{vmatrix} 0 & -3 \\ 1 & 4 \end{vmatrix}.$$

Evaluate each 2x2 determinant:

$$\begin{aligned} &= 2((-3)(-2) - (6)(4)) + 1((0)(-2) - (6)(1)) + 3((0)(4) - (-3)(1)) \\ &= 2(6 - 24) + 1(0 - 6) + 3(0 + 3) \\ &= 2(-18) - 6 + 9 = -36 - 6 + 9 = -33. \end{aligned}$$

Answer: $\boxed{-33}$.

For #33-34, solve the system using Cramer's Rule.

33. Solve the system using Cramer's Rule:

$$2x + 3y = -1$$

$$5x - 4y = 9$$

Calculate the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} 2 & 3 \\ 5 & -4 \end{vmatrix} = (2)(-4) - (3)(5) = -8 - 15 = -23.$$

Calculate D_x by replacing the x-column with constants.

$$D_x = \begin{vmatrix} -1 & 3 \\ 9 & -4 \end{vmatrix} = (-1)(-4) - (3)(9) = 4 - 27 = -23.$$

Calculate D_y by replacing the y-column with constants.

$$D_y = \begin{vmatrix} 2 & -1 \\ 5 & 9 \end{vmatrix} = (2)(9) - (-1)(5) = 18 + 5 = 23.$$

Solve for x and y using $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$.

$$x = \frac{-23}{-23} = 1, \quad y = \frac{23}{-23} = -1.$$

Answer: $\boxed{(1, -1)}$.

34. Solve the system using Cramer's Rule:

$$x - 2y + 3z = 6$$

$$2x + y - 3z = -5$$

$$x - y - z = -4$$

To solve the system of equations using Cramer's Rule: we rewrite it in matrix form:

$$\begin{pmatrix} 1 & -2 & 3 \\ 2 & 1 & -3 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -5 \\ -4 \end{pmatrix}$$

Using Cramer's Rule, the solution for each variable x, y, and z is given by:

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

Calculate D The determinant D of the matrix A is:

$$D = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -3 \\ 1 & -1 & -1 \end{vmatrix}$$

Expanding along the first row:

$$D = 1 \cdot \begin{vmatrix} 1 & -3 \\ -1 & -1 \end{vmatrix} - (-2) \cdot \begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$$

Calculating each minor: $\begin{vmatrix} 1 & -3 \\ -1 & -1 \end{vmatrix} = -4, \quad \begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix} = 1, \quad \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$

Thus,

$$D = 1(-4) + 2(1) + 3(-3) = -4 + 2 - 9 = -11$$

Calculate D_x, D_y, and D_z.
Determinant D_x:
Replace the first column of A with the constants from the right-hand side:

$$D_x = \begin{vmatrix} 6 & -2 & 3 \\ -5 & 1 & -3 \\ -4 & -1 & -1 \end{vmatrix}$$

Then,

$$D_x = 6 \cdot \begin{vmatrix} 1 & -3 \\ -1 & -1 \end{vmatrix} - (-2) \cdot \begin{vmatrix} -5 & -3 \\ -4 & -1 \end{vmatrix} + 3 \cdot \begin{vmatrix} -5 & 1 \\ -4 & -1 \end{vmatrix}$$

Calculating each minor:
$\begin{vmatrix} 1 & -3 \\ -1 & -1 \end{vmatrix} = -4, \quad \begin{vmatrix} -5 & -3 \\ -4 & -1 \end{vmatrix} = -7, \quad \begin{vmatrix} -5 & 1 \\ -4 & -1 \end{vmatrix} = 9$
Thus,

$$D_x = 6(-4) + 2(-7) + 3(9) = -24 - 14 + 27 = -11$$

Determinant D_y:
Replace the second column of A with the constants:

$$D_y = \begin{vmatrix} 1 & 6 & 3 \\ 2 & -5 & -3 \\ 1 & -4 & -1 \end{vmatrix}$$

Then,

$$D_y = 1 \cdot \begin{vmatrix} -5 & -3 \\ -4 & -1 \end{vmatrix} - 6 \cdot \begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & -5 \\ 1 & -4 \end{vmatrix}$$

Calculating each minor:
$\begin{vmatrix} -5 & -3 \\ -4 & -1 \end{vmatrix} = -7, \quad \begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix} = 1, \quad \begin{vmatrix} 2 & -5 \\ 1 & -4 \end{vmatrix} = -3$
Thus,

$$D_y = 1(-7) - 6(1) + 3(-3) = -7 - 6 - 9 = -22$$

Determinant D_z:
Replace the third column of A with the constants:

$$D_z = \begin{vmatrix} 1 & -2 & 6 \\ 2 & 1 & -5 \\ 1 & -1 & -4 \end{vmatrix}$$

Then,

$$D_z = 1 \cdot \begin{vmatrix} 1 & -5 \\ -1 & -4 \end{vmatrix} - (-2) \cdot \begin{vmatrix} 2 & -5 \\ 1 & -4 \end{vmatrix} + 6 \cdot \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$$

Calculating each minor:
$\begin{vmatrix} 1 & -5 \\ -1 & -4 \end{vmatrix} = -9, \quad \begin{vmatrix} 2 & -5 \\ 1 & -4 \end{vmatrix} = -3, \quad \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$
Thus,

$$D_z = 1(-9) + 2(-3) + 6(-3) = -9 - 6 - 18 = -33$$

Compute x, y, and z.
Now we can use Cramer's Rule:

$$x = \frac{D_x}{D} = \frac{-11}{-11} = 1$$

$$y = \frac{D_y}{D} = \frac{-22}{-11} = 2$$

$$z = \frac{D_z}{D} = \frac{-33}{-11} = 3$$

The solution to the system is: $(x, y, z) = \boxed{(1, 2, 3)}$.

For #35-38 Solve using any method.

35. Solve the system:

$$x - 2y + z = 0$$
$$2x + 3z = -1$$
$$y + 2z = -2$$

Express y in terms of z from the third equation:

$$y + 2z = -2 \Rightarrow y = -2z - 2$$

Substitute $y = -2z - 2$ into the first equation:

$$x - 2(-2z - 2) + z = 0$$
$$x + 4z + 4 + z = 0$$
$$x + 5z = -4$$

So, $x = -5z - 4$. Substitute $x = -5z - 4$ into the second equation:

$$2(-5z - 4) + 3z = -1$$
$$-10z - 8 + 3z = -1$$
$$-7z = 7$$
$$z = -1$$

Substitute $z = -1$ back to find y and x: - For y:

$$y = -2(-1) - 2 = 2 - 2 = 0$$

- For x:

$$x = -5(-1) - 4 = 5 - 4 = 1$$

Solution:

$$(x, y, z) = (1, 0, -1)$$

$$\text{Answer: } \boxed{(x, y, z) = (1, 0, -1)}.$$

36. Solve the system:

$$x^2 + y^2 = 4$$
$$4x^2 - y^2 = 16$$

Add the equations:

$$(x^2 + y^2) + (4x^2 - y^2) = 4 + 16$$
$$5x^2 = 20 \Rightarrow x^2 = 4$$
$$x = 2 \quad \text{or} \quad x = -2$$

Substitute into $x^2 + y^2 = 4$ to find y: - For $x = 2$ or $x = -2$:

$$4 + y^2 = 4 \Rightarrow y^2 = 0 \Rightarrow y = 0$$

Solution set:

$$(x, y) = (2, 0) \quad \text{and} \quad (x, y) = (-2, 0)$$

$$\text{Answer: } \boxed{\{(2, 0), (-2, 0)\}}.$$

37. Solve the system:

$$3x - y = 6$$
$$xy = 9$$

From $3x - y = 6$, solve for y:

$$y = 3x - 6$$

Substitute $y = 3x - 6$ into $xy = 9$:

$$x(3x - 6) = 9$$
$$3x^2 - 6x = 9$$
$$3x^2 - 6x - 9 = 0$$
$$x^2 - 2x - 3 = 0$$

Factor the quadratic equation:

$$(x - 3)(x + 1) = 0$$
$$x = 3 \quad \text{or} \quad x = -1$$

Find y for each x: - For $x = 3$:

$$y = 3(3) - 6 = 3$$

Solution: $(x, y) = (3, 3)$. - For $x = -1$:

$$y = 3(-1) - 6 = -9$$

Solution: $(x, y) = (-1, -9)$.

$$(x, y) = (3, 3) \quad \text{and} \quad (x, y) = (-1, -9)$$

$(x, y) = (3, 3)$ and $(x, y) = (-1, -9)$.

Answer: $\boxed{\{(3, 3), (-1, -9)\}}$.

38. Solve the system:

$$y = 2x - 1$$
$$x^2 + y^2 = 2$$

Substitute $y = 2x - 1$ into $x^2 + y^2 = 2$:

$$x^2 + (2x - 1)^2 = 2$$
$$x^2 + 4x^2 - 4x + 1 = 2 \Rightarrow 5x^2 - 4x - 1 = 0$$

Solve the quadratic equation $5x^2 - 4x - 1 = 0$ using the quadratic formula:

$$x = \frac{4 \pm \sqrt{36}}{10} = \frac{4 \pm 6}{10}$$
$$x = 1 \quad \text{or} \quad x = -\frac{1}{5}$$

Find y for each x:
- For $x = 1$: $y = 2(1) - 1 = 1$
- For $x = -\frac{1}{5}$: $y = 2\left(-\frac{1}{5}\right) - 1 = -\frac{7}{5}$

$$(x, y) = \left(-\frac{1}{5}, -\frac{7}{5}\right) \text{ and } (x, y) = (1, 1).$$

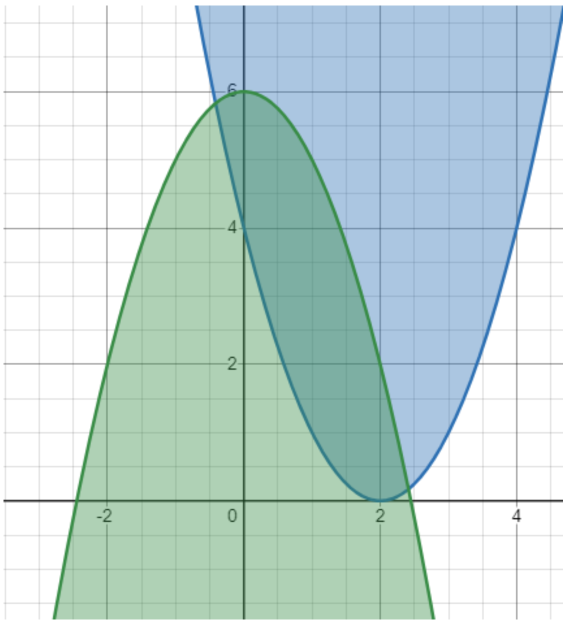
$$\text{Answer: } \boxed{\left\{\left(-\frac{1}{5}, -\frac{7}{5}\right), (1, 1)\right\}}.$$

For #39 – 40, graph the solution.

39. Graph the solution:

$$y \geq (x-2)^2$$
$$y \leq 6 - x^2$$

The solution region is the intersection of $y \geq (x-2)^2$ and $y \leq 6 - x^2$.
Sketch the region bounded by the parabola opening upwards and the parabola opening downwards.



40. Graph the solution:

$$3x - 4y < 12$$
$$2x + 5y \geq 10$$

Rewrite Each Inequality in Slope-Intercept Form

Rewrite $3x - 4y < 12$:

$$-4y < -3x + 12 \implies y > \frac{3}{4}x - 3$$

The boundary line is $y = \frac{3}{4}x - 3$ (dashed because the inequality is strict).

Rewrite $2x + 5y \geq 10$:

$$5y \geq -2x + 10 \implies y \geq -\frac{2}{5}x + 2$$

The boundary line is $y = -\frac{2}{5}x + 2$ (solid because the inequality is inclusive).

Plot the Boundary Lines

- For $y = \frac{3}{4}x - 3$:
- Find two points by substituting $x = 0$ and $x = 4$:

$$\text{If } x = 0,\ y = -3 \quad (\text{Point: } (0, -3))$$
$$\text{If } x = 4,\ y = 0 \quad (\text{Point: } (4, 0))$$

- Plot the dashed line through these points. - For $y = -\frac{2}{5}x + 2$:
- Find two points by substituting $x = 0$ and $x = 5$:

$$\text{If } x = 0,\ y = 2 \quad (\text{Point: } (0, 2))$$
$$\text{If } x = 5,\ y = 0 \quad (\text{Point: } (5, 0))$$

- Plot the solid line through these points.

Shade the Regions

- For $y > \frac{3}{4}x - 3$: Shade above the dashed line because y is greater.
- For $y \geq -\frac{2}{5}x + 2$: Shade above or on the solid line because y is greater than or equal to the boundary.

Identify the Solution Region

The solution is the region where the shaded areas from both inequalities overlap.

Test a Point in the Overlapping Region

Choose a point within the overlap, such as $(0, 3)$, and verify it satisfies both inequalities:

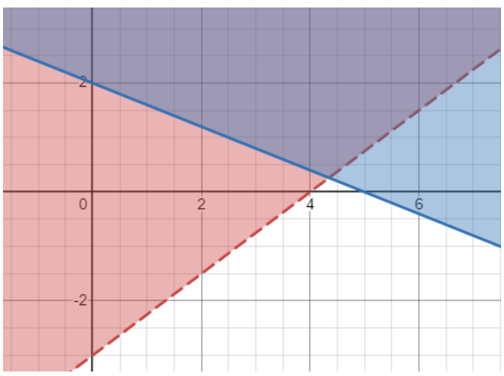
- For $3x - 4y < 12$:

$$3(0) - 4(3) = -12 \quad (\text{true, since } -12 < 12).$$

- For $2x + 5y \geq 10$:

$$2(0) + 5(3) = 15 \quad (\text{true, since } 15 \geq 10).$$

Thus, the region in purple where the two inequalities overlap is the solution.



41. Kelly has 36 coins in her purse, only nickels and quarters. She has eight more quarters than nickels. If she has a total of \$6.20, how many nickels and quarters does she have?

Let n = number of nickels and q = number of quarters.

$n + q = 36,$

$q = n + 8,$

$0.05n + 0.25q = 6.20.$

Substitute $q = n + 8$ into the total coin equation.

$n + (n + 8) = 36,$

$2n + 8 = 36, \quad n = 14, \quad q = 22.$

Answer: $\boxed{\text{14 nickels, 22 quarters}}$.

42. On the opening night of the school musical, a total of 800 tickets were sold. An advanced purchase ticket costs \$14.50 and a ticket purchased at the door costs \$22.00. In all, \$16,640 was taken in. How many tickets were sold at the door?

Let a = number of advanced tickets and d = number of door tickets.

$a + d = 800,$

$14.5a + 22d = 16640.$

Solve $a = 800 - d$ and substitute.

$14.5(800 - d) + 22d = 16640,$

$11600 - 14.5d + 22d = 16640, \quad 7.5d = 5040, \quad d = 672.$

$a = 800 - 672 = 128.$

$\boxed{\text{128 advanced tickets, 672 door tickets}}$.

43. Seattle Star blends whole bean coffee worth \$0.93 per pound with half bean coffee worth \$1.20 per pound to get 30 pounds of a coffee blend worth \$1.02 per pound. How many pounds of each type of coffee does she use?

Let x = pounds of \$0.93 coffee and $(30 - x)$ = pounds of \$1.20 coffee.

$0.93x + 1.20(30 - x) = 1.02 \times 30.$

Expand and solve.

$0.93x + 36 - 1.20x = 30.6,$

$-0.27x = -5.4, \quad x = 20.$

Answer: $\boxed{\text{20 lbs. of \$0.93 coffee, 10 lbs. of \$1.20 coffee}}$.

44. Dave has \$40,000 to invest. He invests part of it at 5%, one-fourth of this amount at 6%, and the rest of the money at 7%. His total annual interest income is \$2,530. Find the amount invested at each rate.

Let x = amount at 5%, $\frac{1}{4}x$ = amount at 6%, $40000 - x - \frac{1}{4}x$ = amount at 7%.

$0.05x + 0.06\left(\frac{1}{4}x\right) + 0.07(40000 - x - \frac{1}{4}x) = 2530.$

Solving for x :

$x = 12000, \quad \frac{1}{4}x = 3000, \quad 40000 - 15000 = 25000.$

Answer: $\boxed{\$12,000 \text{ at } 5\%,\ \$3,000 \text{ at } 6\%,\ \$25,000 \text{ at } 7\%}$.

For #45-47, find the first 5 terms of the sequence.

45. Find the first 5 terms of $a_n = (-1)^n(3n - 2)$.

Calculate each term.
$a_1 = (-1)^1(3 \cdot 1 - 2) = -1,$
$a_2 = (-1)^2(3 \cdot 2 - 2) = 4,$
$a_3 = (-1)^3(3 \cdot 3 - 2) = -7,$
$a_4 = (-1)^4(3 \cdot 4 - 2) = 10,$
$a_5 = (-1)^5(3 \cdot 5 - 2) = -13.$
The first five terms are: $\boxed{-1, 4, -7, 10, -13}$.

46. Arithmetic sequence: $a_4 = -6$, $d = 3$.

Find a_1 using $a_n = a_1 + (n - 1)d$.
$a_4 = a_1 + 3d, \quad -6 = a_1 + 3 \cdot 3, \quad a_1 = -15.$
First five terms:
$a_1 = -15, \quad a_2 = -12, \quad a_3 = -9, \quad a_4 = -6, \quad a_5 = -3.$
The first five terms are: $\boxed{-15, -12, -9, -6, -3}$.

47. Geometric sequence: $a_4 = 6$, $r = \frac{1}{3}$.

Find a_1 using $a_n = a_1 \cdot r^{n-1}$.
$a_4 = a_1 \cdot \left(\frac{1}{3}\right)^3, \quad 6 = a_1 \cdot \frac{1}{27}, \quad a_1 = 162.$
First five terms:
$a_1 = 162, \quad a_2 = 54, \quad a_3 = 18, \quad a_4 = 6, \quad a_5 = 2.$
The first five terms are: $\boxed{162, 54, 18, 6, 2}$.

48. Find the 100^{th} term of the sequence $9, 0, -9, -18, \ldots$.
This is an arithmetic sequence with $a_1 = 9$ and $d = -9$

$$\begin{aligned} a_{100} &= a_1 + (100-1)d \\ &= 9 + 99(-9) \\ &= 9 - 891 = -882. \\ &= \boxed{-882}. \end{aligned}$$

49. Find the 8^{th} term of the sequence $2, -3, \frac{9}{2}, -\frac{27}{4}, \ldots$.

This is a geometric sequence with $a_1 = 2$ and $r = -\frac{3}{2}$.

$$\begin{aligned} a_8 &= a_1 \cdot r^7 \\ &= 2 \cdot \left(-\frac{3}{2}\right)^7 \\ &= -\frac{2187}{64}. \\ &= \boxed{-\frac{2187}{64}}. \end{aligned}$$

For #50 − 51, find S_5, (the sum of the first five terms)for the sequence.

50. Find S_5 for the arithmetic sequence with $a_1 = -7$ and $d = 3$. Use the sum formula

$$\begin{aligned} S_n &= \frac{n}{2}(2a_1 + (n-1)d). \\ S_5 &= \frac{5}{2}(2(-7) + 4 \cdot 3) \\ &= \frac{5}{2}(-14 + 12) = \frac{5}{2} \cdot (-2) = -5. \\ &= \boxed{-5}. \end{aligned}$$

51. Find S_5 for the geometric sequence with $a_1 = 8$ and $r = \frac{1}{3}$.
Use the formula for the sum of the first n terms of a geometric sequence:

$$S_n = a_1 \frac{1 - r^n}{1 - r}$$

.

$$S_5 = 8 \cdot \frac{1 - \left(\frac{1}{3}\right)^5}{1 - \frac{1}{3}} = 8 \cdot \frac{1 - \frac{1}{243}}{\frac{2}{3}} = 8 \cdot \frac{\frac{242}{243}}{\frac{2}{3}} = 8 \cdot \frac{242}{243} \cdot \frac{3}{2} = \boxed{\frac{968}{81}}.$$

For #52 -56, find each sum that exists.

52. Find $\sum_{n=1}^{7}(3n-2)$.

Identify that this is an arithmetic sequence with $a_1 = 1$ and $d = 3$.

$$\begin{aligned} S_7 &= \frac{7}{2}\cdot(2\cdot 1+(7-1)\cdot 3) \\ &= \frac{7}{2}\cdot(2+18) \\ &= \frac{7}{2}\cdot 20 = 70. \\ &= \boxed{70}. \end{aligned}$$

53. Find $\sum_{m=1}^{150} -2m$.

This is an arithmetic sequence with $a_1 = -2, \quad d = -2, \quad n = 150$.
Use the formula for the sum of an arithmetic sequence:

$$\begin{aligned} S_n &= \frac{n}{2}\,(2a+(n-1)d) \\ S_{150} &= \frac{150}{2}\Big(2\cdot(-2)+(150-1)(-2)\Big). \\ &= \frac{150}{2}\cdot(-302). \\ &= 75\cdot(-302) \\ &= -22650. \\ &= \boxed{-22650}. \end{aligned}$$

54. Find $\sum_{N=1}^{\infty} 36\left(-\frac{1}{2}\right)^N$ if the sum exists.

This is an infinite geometric series with $a = 36\cdot\left(-\frac{1}{2}\right) = -18$ and $r = -\frac{1}{2}$. The sum exists if $|r| < 1$.

$$\begin{aligned} S &= \frac{a}{1-r} \\ &= \frac{-18}{1-\left(-\frac{1}{2}\right)} \\ &= \frac{-18}{1.5} = -12. \\ &= \boxed{-12}. \end{aligned}$$

55. Find $\sum_{i=1}^{\infty} 13\left(\frac{11}{9}\right)^i$ if the sum exists.

This is an infinite geometric series with $r = \frac{11}{9}$.

Since $|r| > 1$, the sum does not exist.

Thus, the sum: $\boxed{\text{Does not exist}}$.

56. Find $\sum_{n=1}^{6} 2n^2$.

To solve this, we need to find the sum of the terms $2n^2$ for $n = 1, 2, \dots, 6$.
Write out the terms:

$$\begin{aligned}\sum_{n=1}^{6} 2n^2 &= 2\cdot 1^2 + 2\cdot 2^2 + 2\cdot 3^2 + 2\cdot 4^2 + 2\cdot 5^2 + 2\cdot 6^2. \\ &= 2 + 8 + 18 + 32 + 50 + 72. \\ &= \boxed{182}.\end{aligned}$$