Applying the Normal Distribution

Ayak David Chol

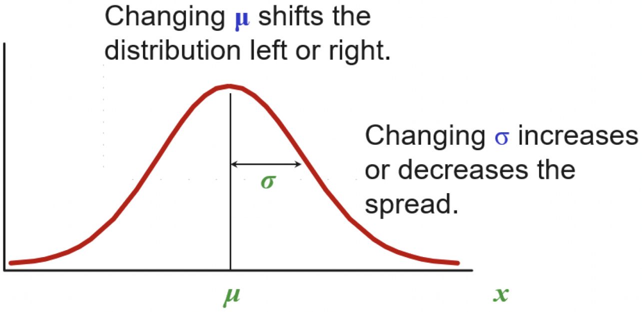
July 17, 2025

Objectives

By the end of this lesson, you will be able to:

- **Describe** properties of the normal distribution
- **Standardize** values using **z-scores**
- Use the **standard normal table** to find probabilities
- Apply the normal distribution to solve **real-world problems**

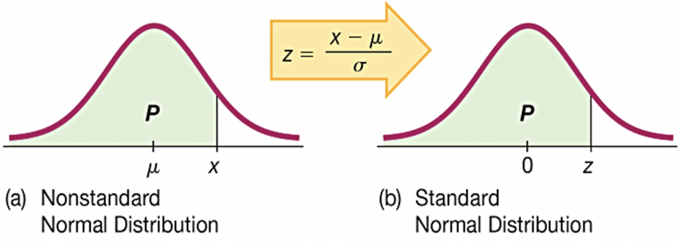
The Normal Distribution



Key Characteristics

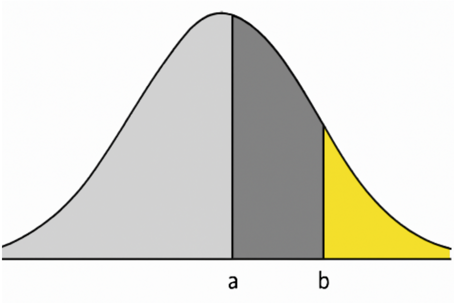
- Bell-shaped and symmetric about the mean (μ)
- Characterized by:
  - Mean (μ)
  - Standard deviation (σ)
- Symmetric when **mean = median = mode**

Standardizing Normal Distribution



- Converts any normal variable to the standard normal distribution making $\mu = 0$ and $\sigma = 1$.
- This is good for computing normal probabilities by transforming **X** units into **Z** units $\Rightarrow z = \dfrac{x - \mu}{\sigma}$.

Finding Probabilities



General Formula

$$P(a < X < b) = P(Z < z_b) - P(Z < z_a)$$

- Use the **z-table** or technology (e.g., calculators, software) to find probabilities.

Example 1

Problem

SAT Math scores are normally distributed with $\mu = 500$ and $\sigma = 100$.

What proportion of students score below 650?

Example 1

Problem

SAT Math scores are normally distributed with $\mu = 500$ and $\sigma = 100$.

What proportion of students score below 650?

Solution

1. Compute the z-score:

$$z = \frac{x - \mu}{\sigma} = \frac{650 - 500}{100} = 1.5$$

Example 1

Problem

SAT Math scores are normally distributed with $\mu = 500$ and $\sigma = 100$.

What proportion of students score below 650?

Solution

1. Compute the z-score:

$$z = \frac{x - \mu}{\sigma} = \frac{650 - 500}{100} = 1.5$$

2. Look up $z = 1.5$ in the standard normal table.

Example 1 Cont ⋯

Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279
1.5	**0.9332**	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406

Solution

$P(Z < 1.5) = 0.9332$

Interpretation

About **93.32%** of students score below 650.

Example 2

Problem

IQ scores are normally distributed with $\mu = 100$, $\sigma = 15$.

What percentage of people have an IQ above 130?

Example 2

Problem

IQ scores are normally distributed with $\mu = 100$, $\sigma = 15$.

What percentage of people have an IQ above 130?

Solution

$$z = \frac{x - \mu}{\sigma} = \frac{130 - 100}{15} = 2.0$$

Example 2

Problem

IQ scores are normally distributed with $\mu = 100$, $\sigma = 15$.

What percentage of people have an IQ above 130?

Solution

$$z = \frac{x - \mu}{\sigma} = \frac{130 - 100}{15} = 2.0$$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
2.0	**0.9772**	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803

Example 2

Problem

IQ scores are normally distributed with $\mu = 100$, $\sigma = 15$.

What percentage of people have an IQ above 130?

Solution

$$z = \frac{x - \mu}{\sigma} = \frac{130 - 100}{15} = 2.0$$

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
2.0	**0.9772**	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803

$P(Z > 2.0) = 1 - P(Z < 2.0) = 1 - 0.9772 = 0.0228$

Example 2

Problem

IQ scores are normally distributed with $\mu = 100$, $\sigma = 15$.

What percentage of people have an IQ above 130?

Solution

$$z = \frac{x - \mu}{\sigma} = \frac{130 - 100}{15} = 2.0$$

Z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**
2.0	**0.9772**	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803

$P(Z > 2.0) = 1 - P(Z < 2.0) = 1 - 0.9772 = 0.0228$

Answer

2.28% have an IQ above 130.

Summary

Key Points in this lesson:

- The **normal distribution** models real-world variables.
- **Z-scores** allow comparison across different normal distributions.
- Use **z-tables** or software to compute probabilities.