Monday Afternoon, Part 1

Radio Listening Posts During WWII

(Adapted from Problems 13 and 14, page 38, *Spherical Trigonometry*, Raymond Brink, 1942.)

Imagine the year is 1943, the height of World War II. Radio listening posts at Boston and Norfolk detect an enemy ship sending from a point bearing S $83°15'$E $=$ S $83.25°$E from Boston and N$73°30'$E $=$ N$73.5°$E from Norfolk.

Boston

$$(\lambda_B, \varphi_B) = \text{(latitude,longitude)} = (42°21'\text{N}, 71°4'\text{W}) = (42.35°\text{N}, 71.066666°\text{W})$$

Norfolk

$$(\lambda_N, \varphi_N) = \text{(latitude,longitude)} = (36°50'\text{N}, 76°18'\text{W}) = (36.833333°\text{N}, 76.3°\text{W})$$

Use this data to find the position of the enemy ship and its distances in statute miles from Boston and Norfolk. Take the radius of the earth as $r = 3{,}958.8$ miles.

Also note that

S $83°15'$E means the direction is $83°15'$ east of due south (going away from P, the North Pole)

N$73°30'$E means the direction is $73°30'$ east of due north (going towards P, the North Pole).

Solution

λ_B, the latitude of Boston, equals the angular distance from the equator to Boston.

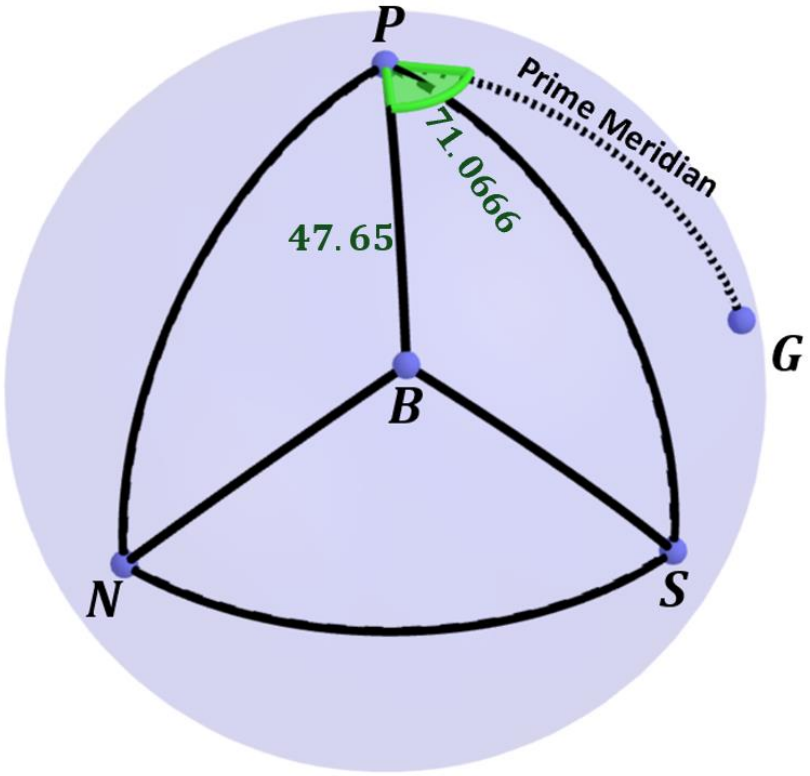
Co-latitude of Boston equals the angular distance from Boston to the North Pole.

Co-latitude of Boston $= \widehat{PB} = 90° - \lambda_B = 90° - 42.35 = 47.65.$

φ_B, the longitude of Boston equals the angular distance from the Prime Meridian to the meridian of Boston.

$\varphi_B =$ spherical $\angle GPB = 71.06666°$

G: Greenwich, England (by rule the Prime Meridian goes through Greenwich)



λ_N, the latitude of Norfolk, equals the angular distance from the equator to Norfolk

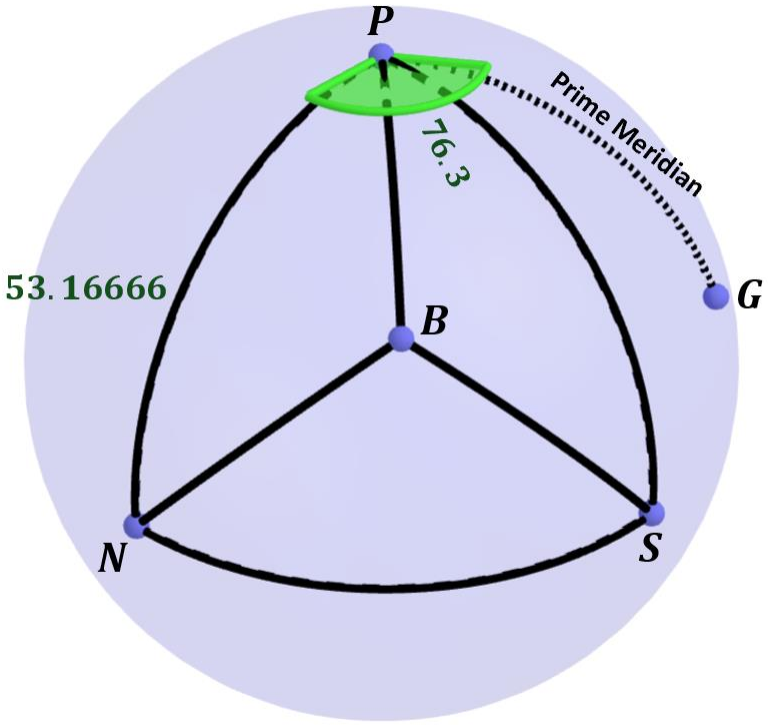
Co-latitude of Norfolk equals the angular distance from Norfolk to the North Pole.

Co-latitude $= \widehat{PN} = 90° -$ latitude $= 90° - 36.833333 = 53.166666°$.

φ_N, the longitude of Norfolk equals the angular distance from the Prime Meridian to the meridian of Norfolk.

$\varphi_N =$ spherical $\angle GPN = 76.3°$

G: Greenwich, England (by rule the Prime Meridian goes through Greenwich)

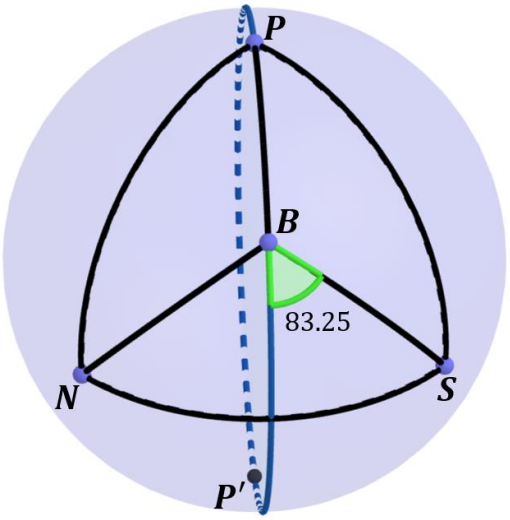


Notice that spherical $\angle BPN$ is the difference between the longitude angles $\angle GPN$ and $\angle GPB$. That is,

$$\angle BPN = \angle GPN - \angle GPB = 76.3 - 71.06666 = 5.23333.$$

A radio listening post at Boston detects an enemy ship sending from a point bearing S $83°15'$E $=$ S $83.25°$E from Boston.

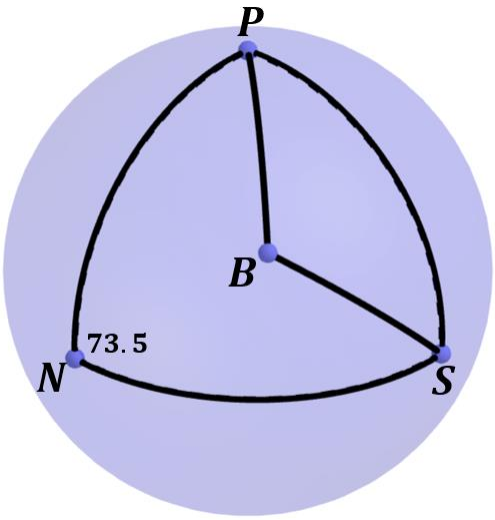
That is, $\angle P'BS = 83.25°$ where P' is the South Pole



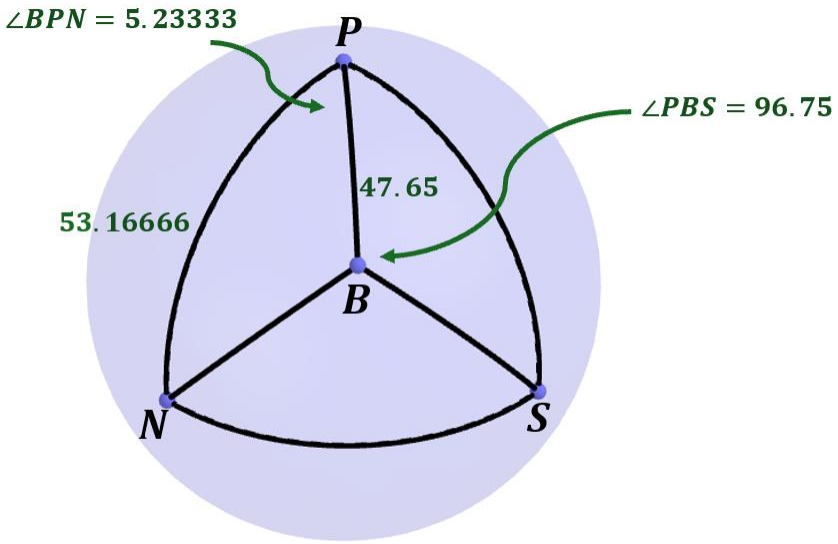
This tells us that $\angle PBS = 180° - \angle P'BS = 180° - 83.25° = 96.75°$.

A radio listening post at Norfolk detects an enemy ship sending from a point bearing N$73°30'$E $=$ N$73.5°$E from Norfolk.

That is, $\angle PNS = 73.5°$.

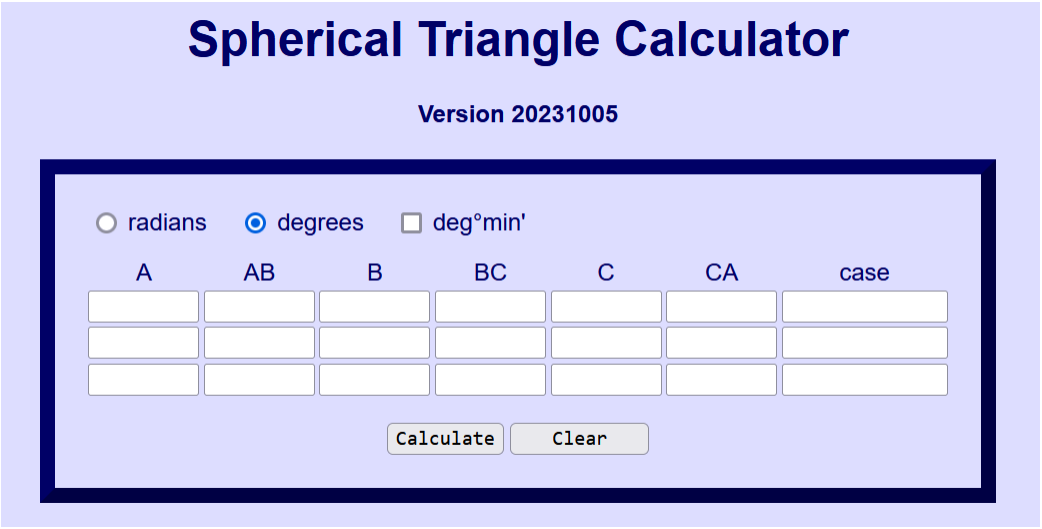


We have enough information to solve spherical ΔPBN. We have three pieces of information about ΔPBN. Namely, we know $\angle BPN = 5.2333°, \widehat{PN} = 53.16666°$ and $\widehat{PB} = 47.65°$.



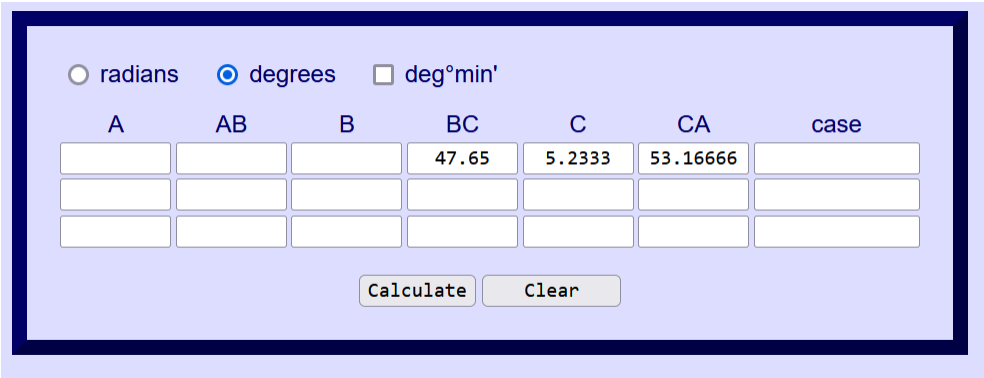
We *could* use what we learned and practice so far to find all the missing pieces *by hand*. But it is a lot of work and we have a lot more triangles to solve in this problem. So I will pull out a computer program to solve this triangle for us.

https://emf.neocities.org/tr/spherical



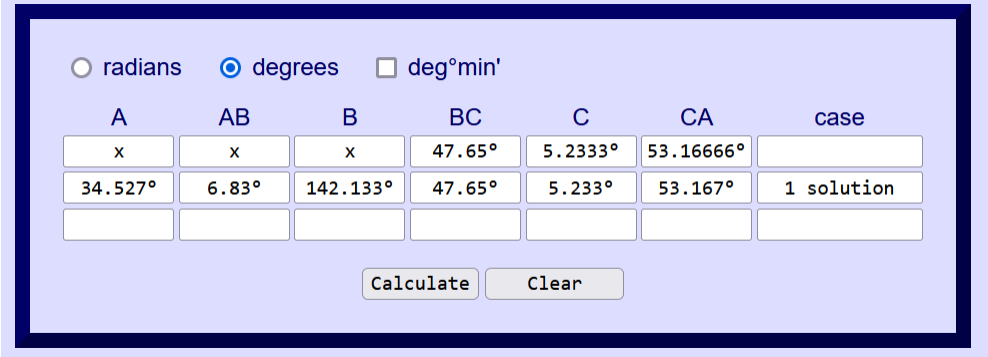
Let's think of our N their A, B their B, and P their C. Then our data would be labeled on this computer program as

$$BC = 47.65, CA = 53.1666, C = 5.23333$$

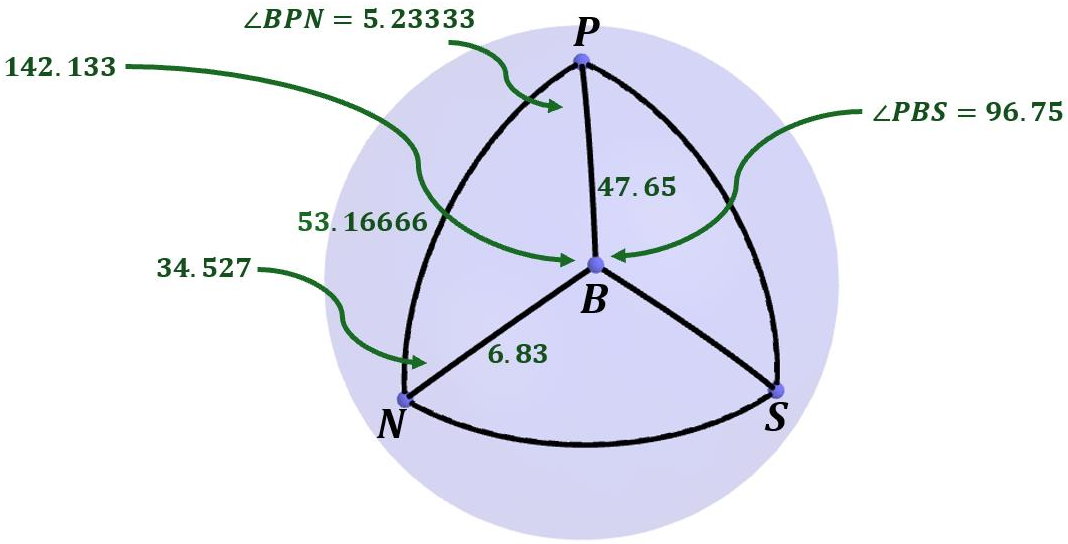


Notice that I've entered the data in degrees with decimals (as to opposed to (degrees, minutes)). So I set the mode to "degrees" in the top line.

And presto!

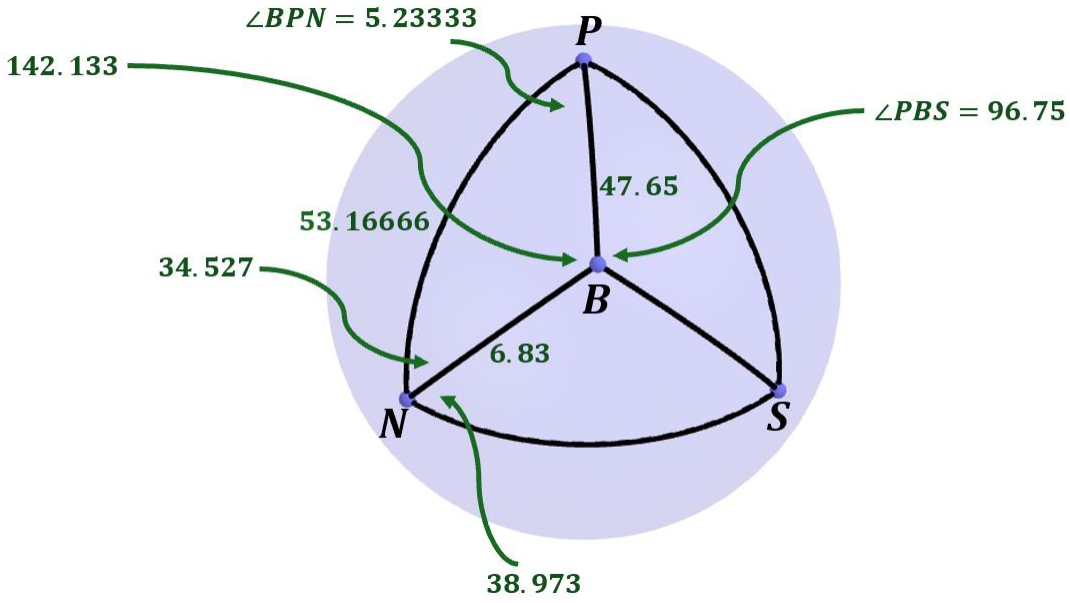


Updating our diagram we know have the following.



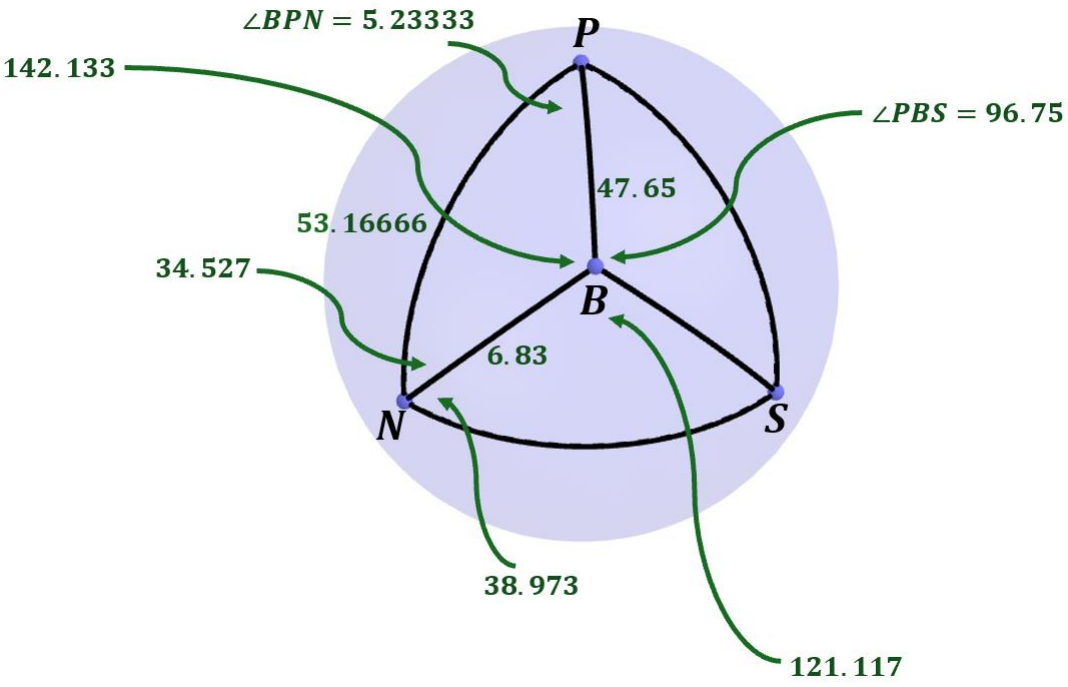
But remember that we also know that $\angle PNS = 73.5°$. Therefore,

$$\angle BNS = \angle PNS - \angle PNB = 73.5 - 34.527 = 38.973°.$$



We can also find $\angle NBS$ now because

$$\begin{aligned}\angle NBS &= 360° - \angle PBN - \angle PBS \\ &= 360° - 142.133° - 96.75° \\ &= 121.117°.\end{aligned}$$



Now we have three pieces of information on spherical triangle ΔSBN. Namely, $\angle SNB$, $\angle NBS$ and $\overset{\frown}{NB}$.

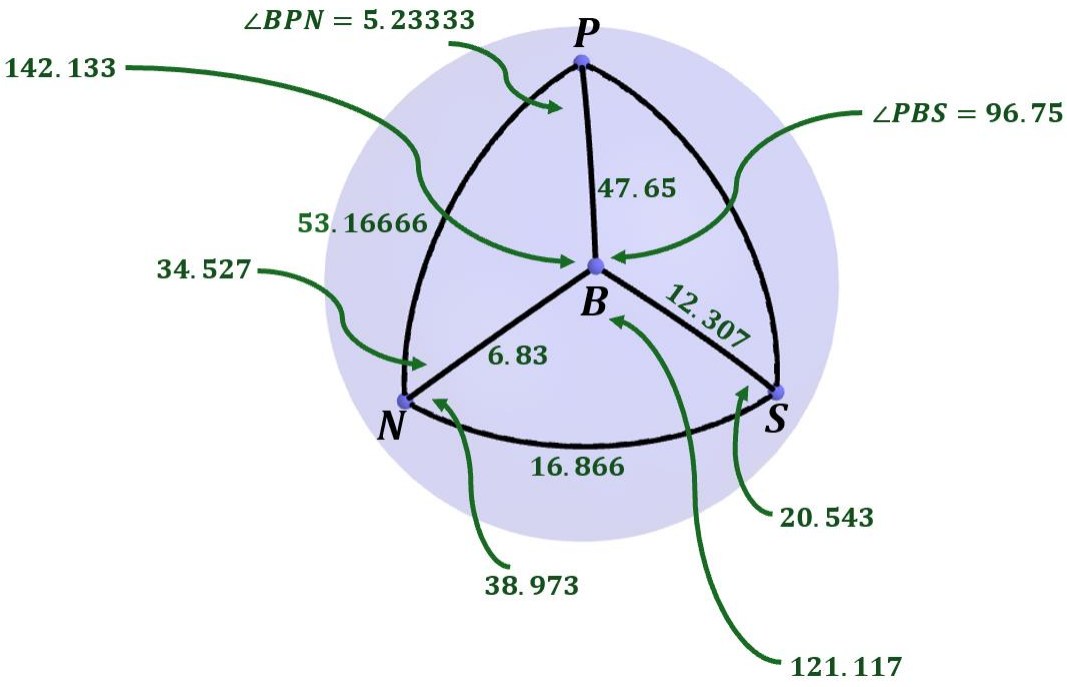
This time we will let our N be their A, B be their B and S be their C and we can use that online spherical triangle solver once again.

radians degrees deg°min'

A	AB	B	BC	C	CA	case
38.973°	6.83°	121.117°	x	x	x	
38.973°	6.83°	121.117°	12.307°	20.543°	16.866°	1 solution

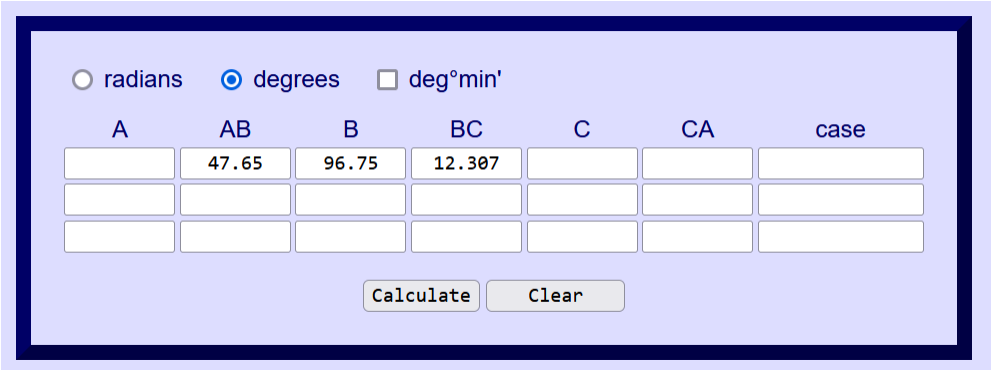
Calculate Clear

Updating our figure with this new information we now have the following.



Now, finally, we have the information we need to solve the spherical triangle ΔSBP we which allow us to read off the latitude and longitude of the enemy ship at S.

This time we take our P as their A, B as their B and S as their C.



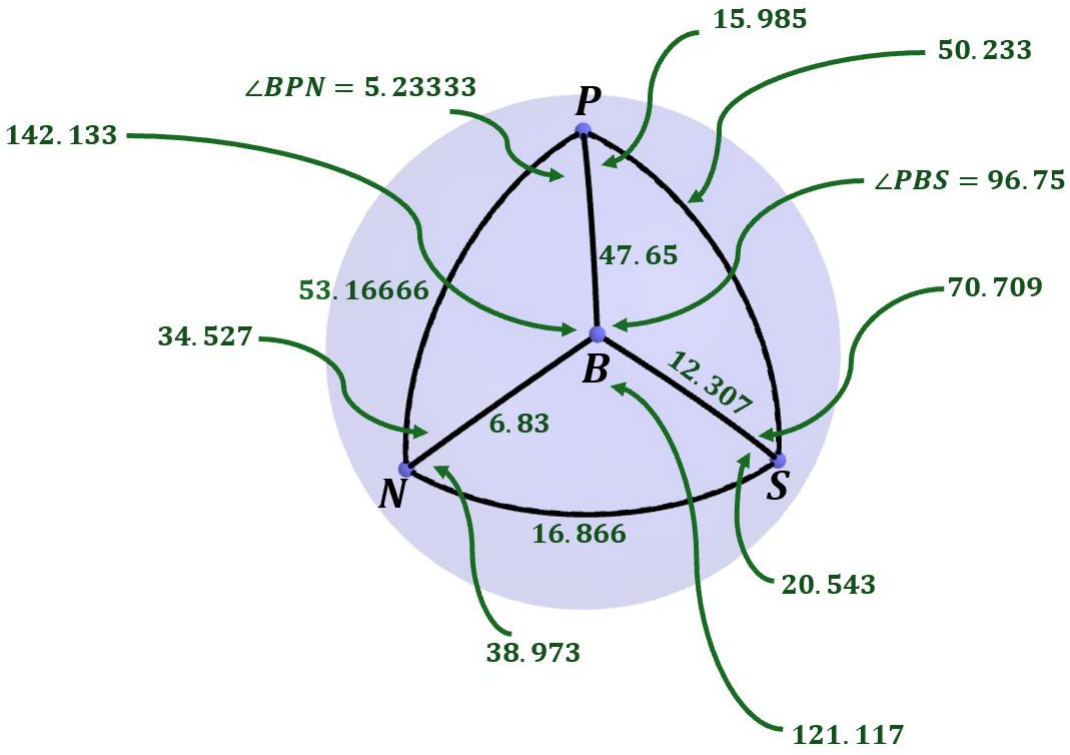
Calculating we find

radians degrees deg°min'

A	AB	B	BC	C	CA	case
x	47.65°	96.75°	12.307°	x	x	
15.985°	47.65°	96.75°	12.307°	72.709°	50.233°	1 solution

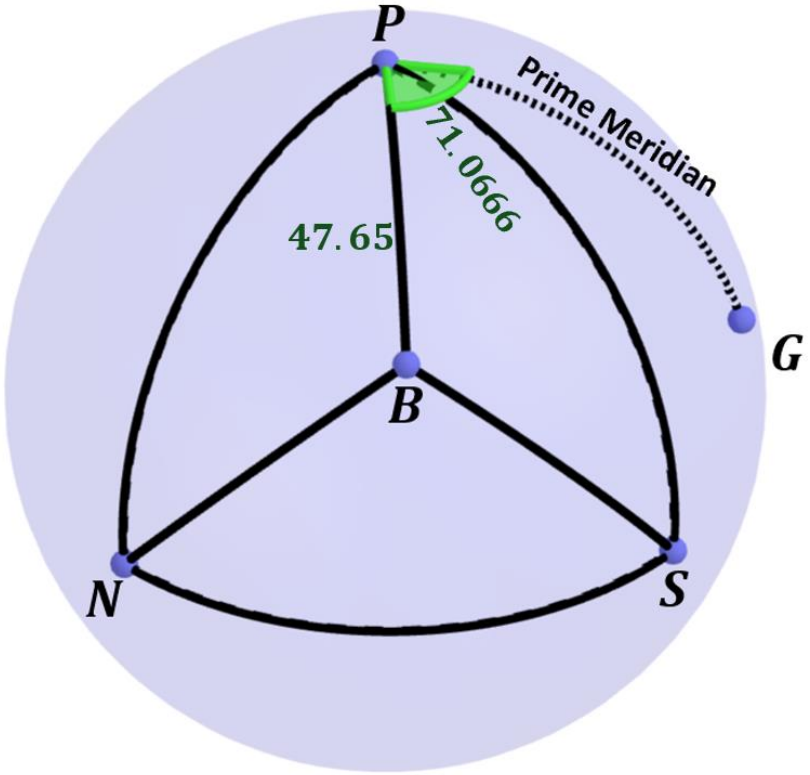
Calculate Clear

Updating our figure for a final time we have our final results.



The co-latitude of the ship at S equals $\widehat{PS} = 50.233°$. So $\lambda_S = 90° - 50.233° = 39.767°$.

The longitude of the ship we need to look back to the figure showing the longitude of Boston.



In this diagram, the longitude of the ship is the spherical angle $\angle GPS$. We see that

$$\angle GPS = \angle GPB - \angle SPB = 71.0666 - 15.985 = 55.0816°.$$

The $\widehat{NS}$, angular distance in degrees from the ship to Norfolk, is $\widehat{NS} = 16.866°$. To find the physical distance from the ship to Norfolk we have to convert this to miles.

$$\widehat{NS} \cdot \frac{\pi}{180°} \cdot r = 16.866° \cdot \frac{\pi}{180°} \cdot 3958.8 = 1165.382 \text{ miles.}$$

The $\widehat{NS}$, angular distance in degrees from the ship to Boston, is $\widehat{BS} = 12.307°$. To find the physical distance from the ship to Boston we have to convert this to miles.

$$\widehat{BS} \cdot \frac{\pi}{180°} \cdot r = 12.307° \cdot \frac{\pi}{180°} \cdot 3958.8 = 850.3410 \text{ miles.}$$

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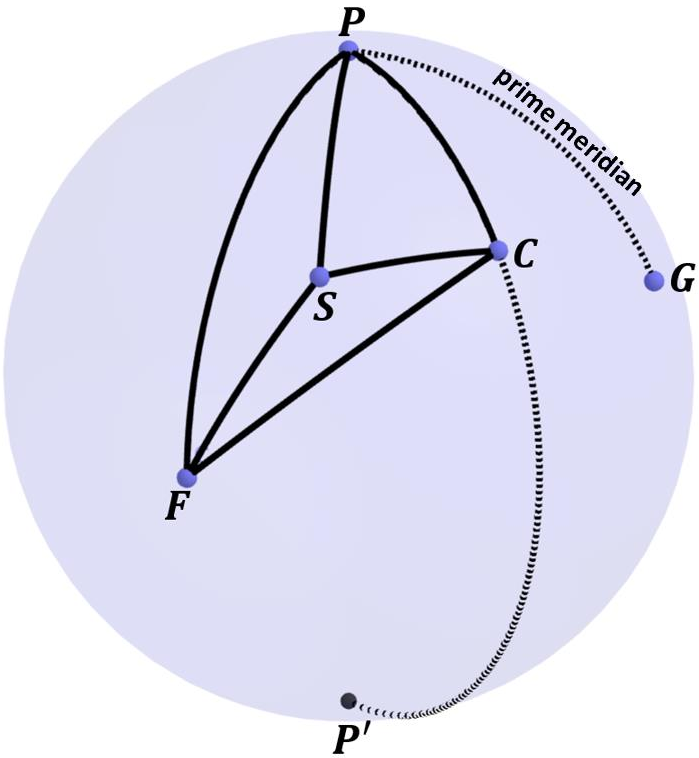
Operation Pastorius

Operation Pastorius was a failed German sabotage mission on America soil staged in June of 1942. Nazi Germany landed at least eight saboteurs by submarine. Four saboteurs who landed in New York were captured within a few weeks and another four who landed in Florida were captured soon afterwards. But the concern about additional spies kept the radio listening stations on high alert.

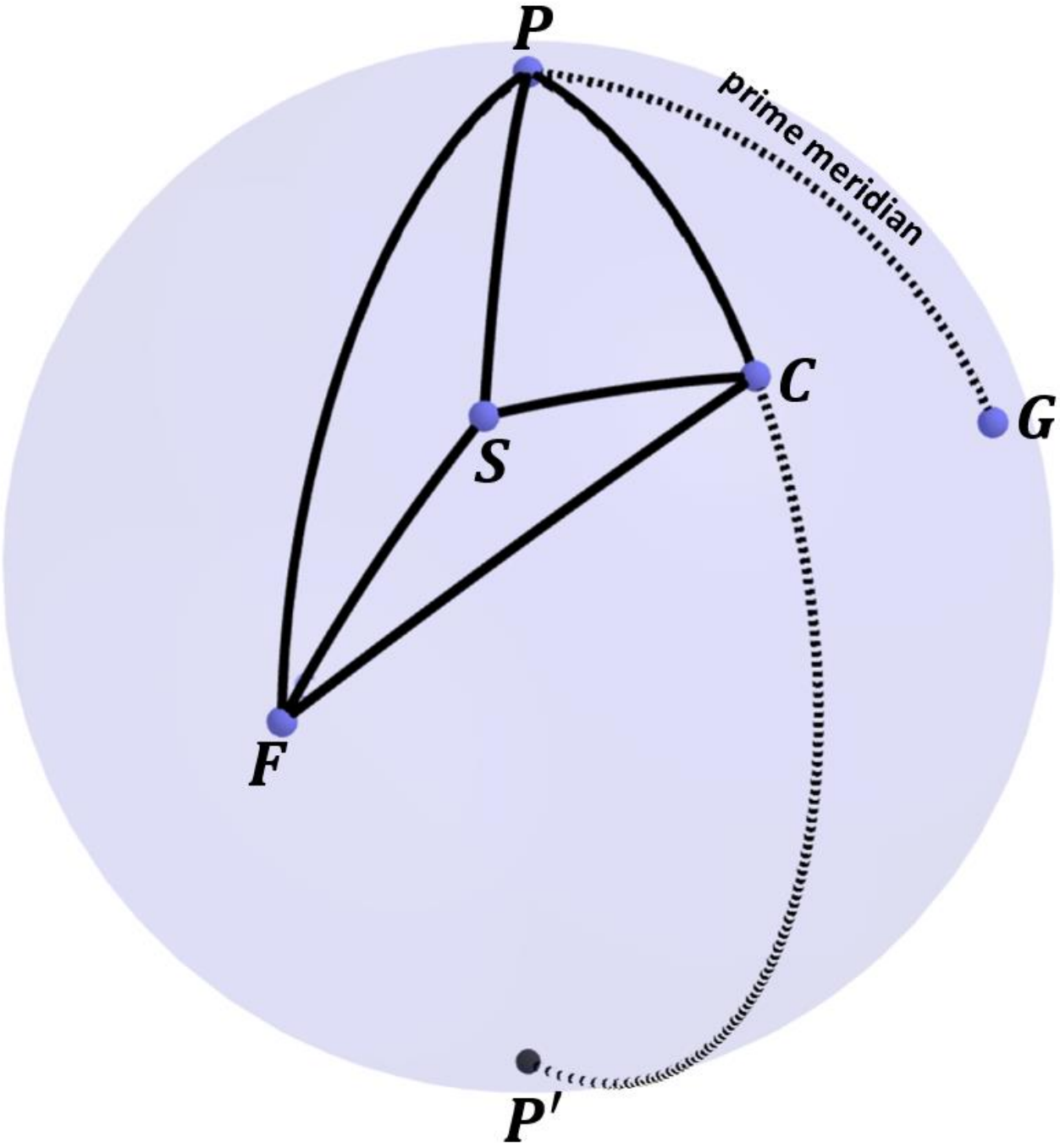
To test the abilities of their radio message interceptors the U.S. government ran a contest for all their radio intercept stations. US government agents began broadcasting signals at various times from a hidden location and the contest was to see which station would be the first to successfully pinpoint the signal.

The winner was Darby House (Chopmist Hill Listening Station) in Scituate RI. The coordinates for this mysterious listening post were $(41.81990349230438\text{ N}, 71.64534100323873\text{ W})$. Darby House picked up signals from a point bearing $S\ 56.21°\ W$. Once they picked up this signal they notified all the other listening posts and the first post to confirm the signal was the listening post at Cape Fear, NC $(35.4250°N, 78.8150°W)$ and they picked up the signal from a bearing $N21.807E$.

Your job is to find the coordinates of this fake spy signal at S. Use the spherical triangle solver at https://emf.neocities.org/tr/spherical to do all the lengthy calculations.



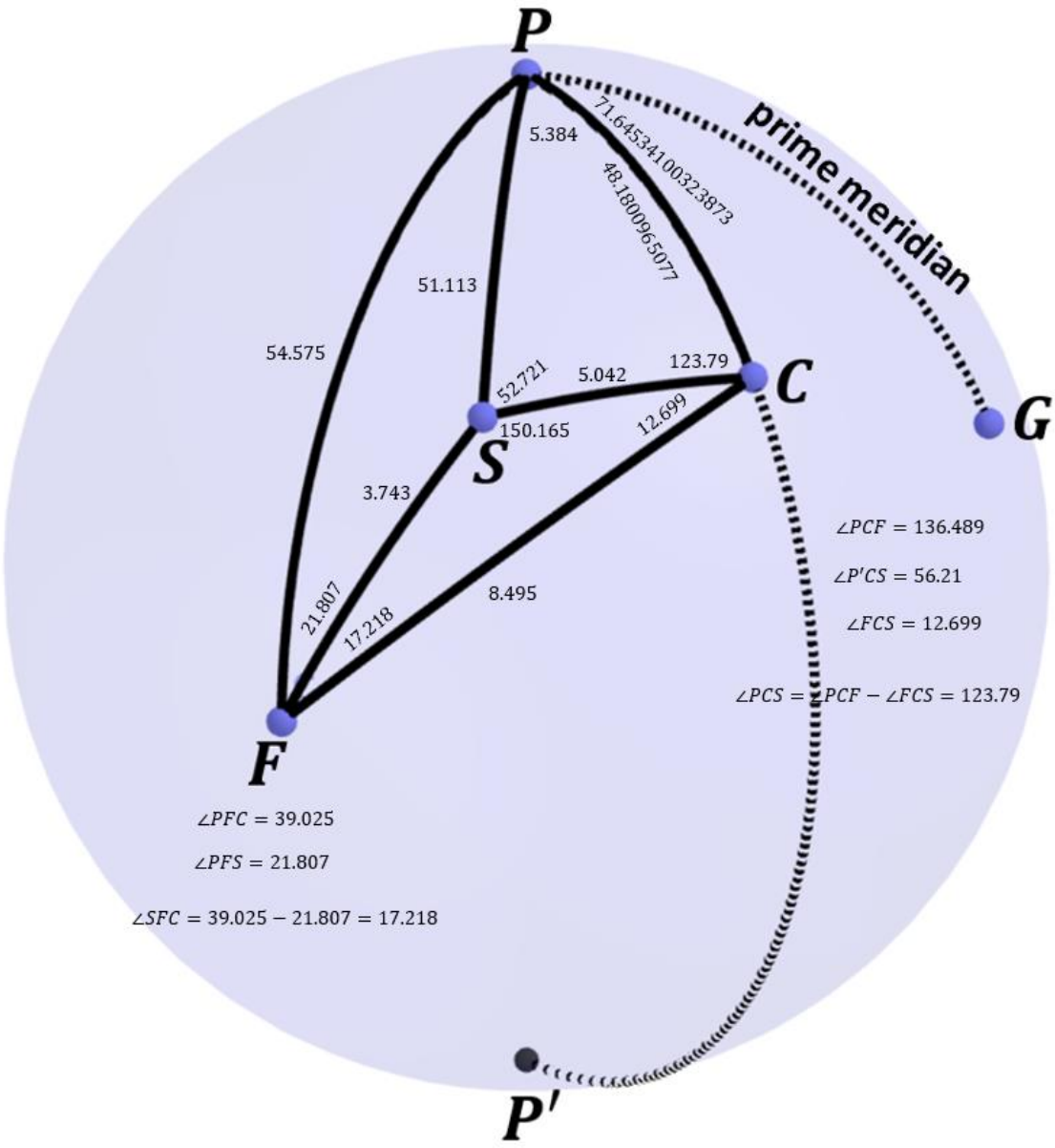
P: North Pole	C: Chopmist Hill, RI
P': South Pole	F: Cape Fear, NC
G: Greenwich, England	S: Spy Signal



(This figure has the right orientation of Cape Fear, the Spy Signal and Chopmist Hill but the scale has been altered to make everything fit on the page and be readable.)

Darby House (Chopmist Hill Listening Station), 183 Darby Rd, Scituate, RI.

Solution


P
prime meridian
71.64534100323873
5.384
48.18009650 77
51.113
54.575
52.721
5.042
123.79
C
G
150.165
S
12.699
3.743
21.807
17.218
8.495
∠PCF = 136.489
∠P′CS = 56.21
∠FCS = 12.699
∠PCS = ∠PCF − ∠FCS = 123.79
F
∠PFC = 39.025
∠PFS = 21.807
∠SFC = 39.025 − 21.807 = 17.218
P′

Latitude of Spy Signal

$$90° - 51.113° = 38.887°$$

Longitude of Spy Site

$$71.64534100323873 + 5.384 = 77.029341°$$

$$(38.887N, 77.029341W)$$