Surds

Numbers like $\sqrt{4}, \sqrt{7}, \sqrt{5}$ are called **surds.**

We notice that:

- $\sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$
- $\sqrt{4 \times 9} = \sqrt{36} = 6$
- Hence, $\sqrt{4} \times \sqrt{9} = \sqrt{4 \times 9}$
- In general, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- Similarly it can be shown that, $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Some other points to note:

- Surds like $\sqrt{n}$ are irrational unless n is a perfect square.
- Similarly, $\sqrt[3]{m}$ is irrational unless m is a perfect cube.

We normally don't express a fraction with a surd in the denominator. So $\frac{a}{\sqrt{b}}$ will normally be expressed as $\frac{a\sqrt{b}}{b}$, because: $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$. This process is called '**rationalising the denominator**'.

A **common mistake** occurs when we incorrectly write $\sqrt{4} = \pm 2$.
$\sqrt{4}$ means 'the positive square root of 2' and so $\sqrt{4} = 2$.
If $x^2 = 4$, we can conclude that $x = +\sqrt{4}\ or - \sqrt{4}$. Hence $x = +2\ or - 2$.

1. Sort the numbers below into four pairs of equal value:

   $\sqrt{20}$, $\sqrt{12}$, $\sqrt{3}$, $2\sqrt{3}$, 2, $\sqrt{10} \times \sqrt{2}$, $\frac{\sqrt{20}}{\sqrt{5}}$, $\frac{\sqrt{15}}{\sqrt{5}}$

2. Answer true or false:

   a) $\sqrt{28} = 2\sqrt{7}$ b) $\sqrt{4} + \sqrt{4} = \sqrt{8}$ c) If $x^2 = 9, x = 3$ d) $\frac{\sqrt{8}}{2} = \sqrt{2}$
   e) $(\sqrt{2})^4 = 4$ f) $\sqrt{2} + \sqrt{2} = 2\sqrt{2}$ g) $\sqrt{100} = \pm 10$ h) $(1 + \sqrt{3})^2 = 4 + 2\sqrt{3}$

3. Simplify:

   a) $(1 + \sqrt{2})^2$ b) $(2 - \sqrt{3})^2$ c) $(\sqrt{2} + 2)^2$ d) $(\sqrt{18} - \sqrt{2})^2$
   e) $\sqrt{8} \times \sqrt{2}$ f) $\sqrt{300}$ g) $\sqrt{18} \times 3$ h) $\sqrt{20} + \sqrt{45}$
   i) $\sqrt{75} - \sqrt{48}$ j) $\frac{\sqrt{27}}{\sqrt{12}}$ k) $\frac{\sqrt{125}}{\sqrt{20}}$ l) $\sqrt{5} + 4\sqrt{5}$

4. Rationalise the denominators of these fractions:

   a) $\frac{8}{\sqrt{2}}$ b) $\frac{12}{\sqrt{3}}$ c) $\frac{12}{\sqrt{8}}$ d) $\frac{1}{\sqrt{7}}$

5. Rationalise the denominator of $\frac{4}{\sqrt{2}+1}$ by multiplying the numerator and denominator by $(\sqrt{2} - 1)$.

6. Use the same approach from Question 5 to simplify:

   a) $\frac{4}{(\sqrt{3}+1)}$ b) $\frac{1}{(\sqrt{5}+2)}$ c) $\frac{10}{(\sqrt{7}-2)}$