1. You play a casino game in Las Vegas with these rules:
   - You roll a six-sided fair die.
   - If you roll a non-6, you win the value of the roll in dollars and the game ends.
   - If you roll a 6, on top of winning $6 you get to play again with the same rules.

   What is the expected value of your winnings?

   Hint: Expected Value is an anticipated average value that describes the long-term average level of a random variable based on its probability distribution. The expected value is often calculated by multiplying each of the possible outcomes by the likelihood each outcome will occur and then summing all of those values.

2. The sum of the squares of three natural numbers is divisible by 9. Prove that we can choose two of these numbers such that their difference is divisible by 9.

   Hint: Find all possible remainders given by perfect squares when divided by 9. Is it true that the sum of some triple of them must be divisible by 9? What do you notice?

3. Six equilateral triangles, all with side length 1 are lined up in a row. What is the total length of the highlighted segments (in yellow)?
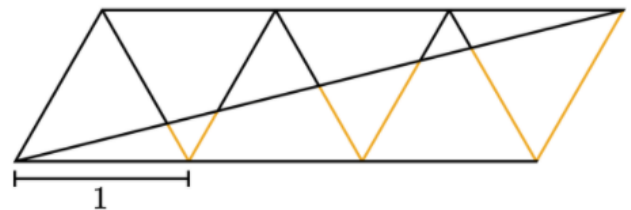
4. Vedant and Advika are playing a game in which they take turns removing cards from a deck of 16 cards. These are the rules:
   - If there are N cards left in the deck on a player's turn, they must remove between 1 and $\frac{N}{2}$ cards (inclusively). For example, if there are 10 cards left, a player can remove 1, 2, 3, 4 or 5 cards.
   - The player who is left with exactly 1 card in the deck on their turn loses.

   Vedant goes first. Assuming both players play perfectly, who will win this game?

5. In the number fitment puzzle, the top row, bottom row, and diagonals have the same prime sum. The seven numbers that need to be filled in are: 5, 7, 11, 13, 17, 19, 23

   What is the sum of the middle column?