



**Linear Learning Company**

# **18 Things You Must Know to Kick Butt on SAT Math**

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## 1. BASIC VOCABULARY

TERM	DEFINITION	EXAMPLE
INTEGER		
NEGATIVE		
POSITIVE		
DISTINCT		
EVEN		
ODD		
DIFFERENCE		
SUM		
PRODUCT		
QUOTIENT		
CONSECUTIVE		
ORDER OF OPERATIONS		
REMAINDER		
PRIME		
FACTOR		
MULTIPLE		
UNION		
INTERSECTION		
VENN DIAGRAM		
MEAN		
MEDIAN		
MODE		
RANGE		
RATIONAL		
IRRATIONAL		
UNDEFINED		

## 2. SOLVING FOR $X$

A number of SAT questions ask you to solve for a single variable in an equation. You can approach these by either solving for the variable, or by plugging-in the answer choice.

### Solving

To solve an equation, you need to isolate the variables. Get the variables on one side of the equal sign and the numbers on the other side. Then, whatever action you perform on one side you must also perform on the other side.

2. If  $10 - 3y = 4$ , then  $y =$

- A)  $-14/3$
- B)  $-2$
- C)  $-1$
- D)  $2$

3. If  $r=2t$  and  $t=3$ , what is the value of  $2r$ ?

- A)  $4$
- B)  $6$
- C)  $8$
- D)  $12$

6

If  $x = \frac{2}{3}y$  and  $y = 18$ , what is the value of  $2x - 3$ ?

- A)  $21$
- B)  $15$
- C)  $12$
- D)  $10$

## PLUGGING IN

Plugging-In turns algebra and geometry problems into arithmetic problems. You can plug in:

- THE ANSWER CHOICES WHEN YOU HAVE VARIABLES IN THE QUESTION.
- THE GIVEN INFORMATION WHEN THE QUESTION INCLUDES AN EQUATION.
- YOUR OWN NUMBERS WHEN YOU HAVE VARIABLES IN THE ANSWER CHOICES.
- YOUR OWN NUMBER WHEN YOU'RE TOLD THAT SOMETHING CHANGES BY FRACTIONS OR PERCENTS AND THERE'S NO STARTING POINT.
- YOUR OWN NUMBERS FOR MISSING ANGLES OR LENGTHS, AS LONG AS YOU FOLLOW THE RULES OF GEOMETRY.

### 3. PLUGGING IN THE ANSWERS

For some plug-in questions you have variables in the question and numbers in the answers. Now, you don't have to make up a number. Instead, you just to plug in the numbers that you are given in the answer choices. And you don't have to check each one. As soon as one works, you're done.

Also, if you start with choice C, you can often reduce the time you spend on the question and only have to do two tests. Here's why: If C works, you win! If C is too big or too small, you can eliminate C as well the other 2 choices that are also too big or too small. (Remember, the answers are always in order, either high-to-low or low-to-high.) Now, with only two choices remaining, you only have to test one.

Here's How to Plug-In the Answers:

- Label your answer column.
- Starting with choice C, work the steps of the problem.
- If C doesn't work, eliminate any other choices that clearly won't work.
- When you've identified the correct answer, STOP!

10. A mechanic buys a set of tools. He gives half of the tools to one co-worker and five of them to another. If the mechanic now has nine tools left, how many tools did he buy originally?

- (A) 7  
(B) 14  
(C) 20  
(D) 28

11. Adam is half as old as Bob and three times as old as Cindy. If the sum of their ages is 40, what is Bob's age?

- A) 6  
B) 12  
C) 18  
D) 24

#### 4. PLUGGING IN THE GIVEN INFORMATION

A favorite trick of the SAT is to give you what appears to be an unusual or complicated formula and then ask a convoluted question. Fortunately, if they give you a formula, they also give you information to plug into the formula in order to answer the question.

$$h = 3a + 28.6$$

A pediatrician uses the model above to estimate the height  $h$  of a boy, in inches, in terms of the boy's age  $a$ , in years, between the ages of 2 and 5. Based on the model, what is the estimated increase, in inches, of a boy's height each year?

- A) 3  
B) 5.7  
C) 9.5  
D) 14.3

$$g(x) = ax^2 + 24$$

For the function  $g$  defined above,  $a$  is a constant and  $g(4) = 8$ . What is the value of  $g(-4)$  ?

- A) 8
- B) 0
- C) -1
- D) -8

## 5. PLUG IN FOR VARIABLES IN THE ANSWER CHOICE (MAKE UP YOUR OWN NUMBERS)

When you see a question that has variables in the answer choice, you can solve the question using your own number for the variable. Solve the problem using your number, then find the answer choice that matches yours. Make sure to check each answer choice to make sure that only one works.

### WHAT TO PLUG IN

Plug in any number you want, but choose numbers that make the arithmetic as easy as possible. 2, 5, 10, etc. are often good choices.

- Do not plug in numbers that appear in the answer choices or in the question.
- Do not plug in zero or one.
- Do not plug in the same number for 2 different variables.

8. Claire is  $c$  years old, and is 6 years younger than Alan. In terms of  $c$ , how old will Alan be in 3 years?

- A)  $c - 3$
- B)  $c + 3$
- C)  $c + 5$
- D)  $c + 9$

16. If the sum of three consecutive odd integers is  $k$ , then, in terms of  $k$ , what is the greatest of the three integers?

- A)  $(k - 3)/3$
- B)  $k/3$
- C)  $(k + 3)/3$
- D)  $(k + 6)/3$

13. Let the function  $f$  be defined by  $f(x) = 5x$  for all numbers  $x$ . Which of the following is equivalent to  $f(p + r)$ ?

- A)  $5p + r$
- B)  $5p + 5r$
- C)  $10(p + r)$
- D)  $25pr$

If you use a “bad” number, it increases the possibility that more than one answer choice will work. If that happens, eliminate the ones that didn’t work, change your number, and test the remaining answers.

5. If  $y$  is an integer, which of the following must be an odd integer?

- (A)  $3y$
- (B)  $2y + 1$
- (C)  $y + 3$
- (D)  $y/2$

You can even plug-in for geometry questions. Just make sure you follow the rules.

13. In a rectangle, the width of  $y$  is half the length  $x$ . What is the area of the rectangle in terms of  $y$ ?

- (A)  $\frac{1}{2}y$
- (B)  $6y$
- (C)  $2y^2$
- (D)  $y^2$



and here's one with exponents.

$$j = x^2 - 0.49$$

$$k = (x - 0.49)^2$$

$$m = x^3 - 0.49$$

14. If  $x$  is a negative integer, what is the ordering of  $j$ ,  $k$ , and  $m$  from least to greatest?

(A)  $j < k < m$

(B)  $j < m < k$

(C)  $k < j < m$

(D)  $m < j < k$

(E)  $m < k < j$

Sometimes you can plug in, even when there isn't a variable.

If the question asks you to find fractions or percents of something, and doesn't give you a beginning value, *make up your own starting number*. For fraction questions, a good starting number is the product of the denominators. For percent questions, start with 100.

13. The price of a dress is reduced by  $\frac{1}{5}$ . If the new price is then reduced by  $\frac{1}{4}$ . The resulting price is what fractional part of the original price?

A)  $\frac{2}{5}$

B)  $\frac{9}{20}$

C)  $\frac{11}{20}$

D)  $\frac{3}{5}$

You can even plug in on grid-ins.

17. If  $3a = b$ ,  $2b = c$ ,  $3c = d$ , and  $abcd \neq 0$ , what is the value of  $d/a$ ?

## **6. GEOMETRY FACTOIDS**

### **ANGLE FACTOIDS**

- There are  $90^\circ$  in a right angle.
- When two straight lines intersect, angles opposite each other are equal.
- There are  $180^\circ$  in a triangle.
- Two lines are perpendicular when they meet at a  $90^\circ$  angle.
- There are  $180^\circ$  in a straight line.
- Bisect means to cut exactly in half.
- The angles of any four-sided figure add up to  $360^\circ$ .

### **FOUR SIDED FACTOIDS**

#### **In a Square:**

- All 4 sides are equal.
- All 4 angles are equal to  $90^\circ$ .
- Area =  $s^2$
- Perimeter =  $4s$

#### **In a Rectangle:**

- Opposite sides are equal.
- All 4 angles are equal to  $90^\circ$ .
- Area = Length x Width.

#### **In a Quadrilateral:**

- Angles add up to  $360^\circ$ .
- Opposite sides are equal.
- Opposite angles are equal to each other.
- Adjacent angles total  $180^\circ$ .
- Area = Base x Height.
- Height is a perpendicular line from the top to the base.

## **TRIANGLE FACTOIDS**

### **In any triangle:**

- The longest side is opposite the largest angle. The shortest side is opposite the smallest angle.
- Equal sides are opposite equal angle.
- Angles add up to  $180^\circ$ .
- Area =  $\frac{1}{2}$  (base x height).
- The height is a perpendicular distance from the top of the triangle to the base.

### **In an isosceles triangle:**

- Two sides are equal.
- Angles opposite the equal sides are equal.

### **In an equilateral triangle:**

- All three sides are equal.
- All angles are equal to  $60^\circ$ .

## **CIRCLE FACTOIDS**

There are  $360^\circ$  in a circle.

### **Radius:**

- The distance from the center to any point on the edge of the circle.
- All radii in a circle are equal.

### **Diameter:**

- The straight line distance from one point on the circle to another, passing through the center.
- The longest line in the circle.
- Equal to twice the radius.

### **Chord:**

- Any line segment from one point on the circle to another.
- The diameter is the longest chord.

### **Circumference:**

- The distance around the outside of the circle.

- Formula for the circumference is diameter times  $\pi$ , or  $d\pi$ . (traditionally measured as  $2\pi r$ ).

#### **Arc:**

- Any part of the circumference. A “minor arc” is the shortest distance between 2 points on the circle. A “major arc” is longer distance between the 2 points.
- The measure of the arc to the circumference is proportional to the measure of the interior angle to  $360^\circ$ .

#### **Area:**

- The amount of space within the boundaries of the circle.
- The formula for the area is radius squared times  $\pi$ , or  $r^2\pi$ . (traditionally measured as  $\pi r^2$ ).

#### **Equation of a Circle:**

For any circle, given a point on the circle (x,y) and the center of the circle (h,k) the equation is:

$$(x-h)^2 + (y-k)^2 = \text{radius}^2$$

#### **RDCA Chart**

The RDCA (Radius, Diameter, Circumference, Area) Chart is an easy way to quickly determine the four major measures of a circle.

Remember that: Diameter=2r   Circumference= $d\pi$    Area= $r^2\pi$ . When you’ve got one measure, the others are easy to fill in.

R	D	C	A
4	8	$8\pi$	$16\pi$
		$12\pi$	
	20		
			$81\pi$
3			

#### **Ray:**

- A ray extends infinitely in one direction from an endpoint.
- The degree measure of a ray is  $180^\circ$ .

**Tangent:**

- Tangent means touching at one point. For example, a line, line segment, or ray can intersect the circumference of a circle at only one point.
- A tangent line is always perpendicular to a radius.
- Circles can also be tangent. If one circle touches the other from inside that's called "internally tangent". If one circle is not inside the other, that's called "externally tangent".

**Transformational Factoids:****Translation**

- A translation moves a figure without rotation or reflection.

**Rotation:**

- Rotation means turning an object around a point which is called *the center of rotation*.

**Reflection:**

- To reflect an object means to create its mirror image across a *line of reflection*.
- Lines reflected across the X-axis have slopes that are negatives of each other. They also have Y-intercepts that are negatives of each other.
- Lines reflected across the Y-axis have the same Y-intercept as the original line, but slopes that are negatives of each other.

**Symmetry:**

- A figure has reflective symmetry if it looks the same after a reflection. Such a figure reflects across a *line of symmetry*.
- A figure has rotational symmetry if it can be rotated and still look the same. Such a figure is rotated around a *point of symmetry*.

## 7. TRIANGLES

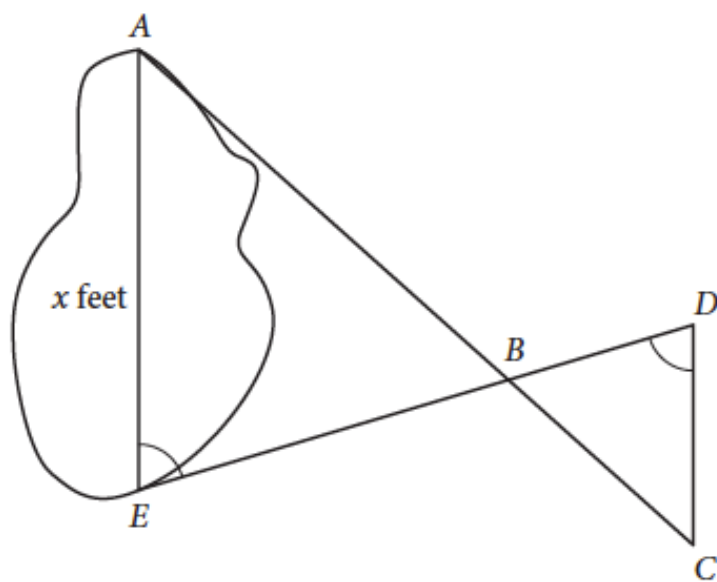
**SIMILAR TRIANGLES**

Triangles are similar if the measure of all three angles is the same. Similar means same shape, different size.

The corresponding sides of two similar triangles are proportional in length.

3.17

17

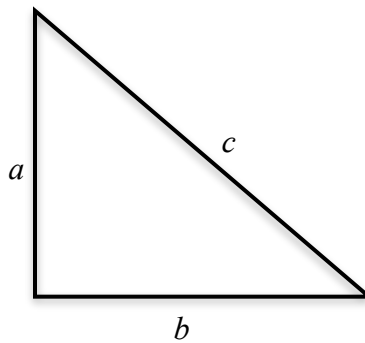


A summer camp counselor wants to find a length,  $x$ , in feet, across a lake as represented in the sketch above. The lengths represented by  $AB$ ,  $EB$ ,  $BD$ , and  $CD$  on the sketch were determined to be 1800 feet, 1400 feet, 700 feet, and 800 feet, respectively. Segments  $AC$  and  $DE$  intersect at  $B$ , and  $\angle AEB$  and  $\angle CDB$  have the same measure. What is the value of  $x$  ?

## RIGHT TRIANGLES

If you know two sides of a right triangle you can find the third side by using the **Pythagorean theorem**. (If you forget it, it's in the formula box that is at the beginning of every SAT math section).

Pythagorean Theorem:  $a^2 + b^2 = c^2$  where  $c$  is the hypotenuse



### PYTHAGORIAN TRIPLES:

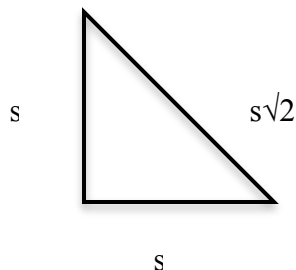
The SAT likes to test pythagorean triples. These are relationships between the sides that remain constant, and are as follows:

3:4:5          6:8:10          5:12:13

### SPECIAL RIGHT TRIANGLES

All right triangle questions can be solved using the Pythagorean theorem. However, by knowing the “triples” shown above, you answer some questions quite a bit faster. This is also true for the two “special” right triangles that have relationships that don’t change.

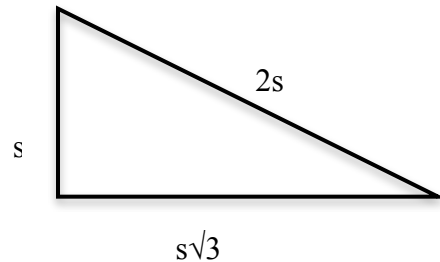
#### 45-45-90



This special right triangle is  $\frac{1}{2}$  of a square.

If the length of a side is 4, what is the length of the hypotenuse? \_\_\_\_\_

**30-60-90**

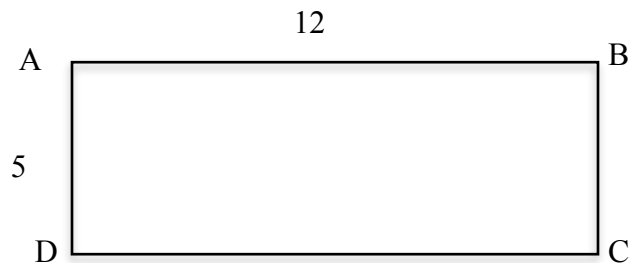


The 30-60-90 right triangle is  $\frac{1}{2}$  of an equilateral triangle.

If the length of the side is 4, what are the lengths of the other two sides?

10. In rectangle to the right, what is the sum of the diagonals AC and BD?

- A) 34
- B) 26
- C) 13
- D) 12

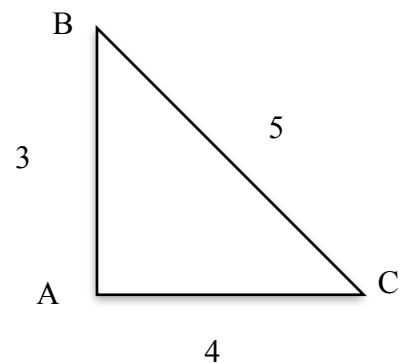


**SOH-CAH-TOA**

( SINE = OPPOSITE/HYPOTENUSE)

(COSINE = ADJACENT/HYPOTENUSE)

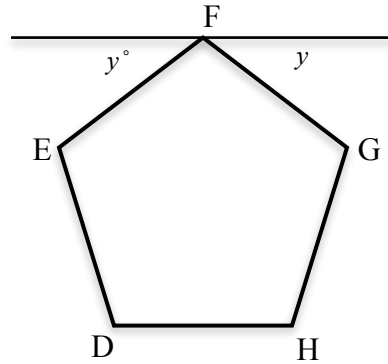
(TANGENT = OPPOSITE/ ADJACENT)





### FOUR OR MORE SIDES

The sum of the angles in a triangle is  $180^\circ$ . Each additional side adds another  $180^\circ$ .



15. In the figure above, pentagon DEFGH has equal sides and equal angles. What is the value of  $y$ ?

- A) 30
- B) 36
- C) 45
- D) 108

16. A six-sided polygon has five equal angles. If the measure of one of the angles is  $100^\circ$ , what is the degree measure of each of the other five angles?

- A) 88
- B) 100
- C) 124
- D) 540

## **8. ARITHMETIC**

**BASICS (+, -, X, ÷)**

**COMBINING TERMS**

**MANIPULATING FRACTIONS**

## 9. THE COORDINATE PLANE: SLOPE AND EQUATION OF A LINE

### SLOPE

Slope is one of the properties that defines a line. Slope is written as a fraction. Often referred to as “rise/run”, the slope of a line, given any two points on the line, is:

$$Y_1 - Y_2 / X_1 - X_2$$

The change in the Y values divided by the change in the X values.

3.12

A line in the  $xy$ -plane passes through the origin and has a slope of  $\frac{1}{7}$ . Which of the following points lies on the line?

- A) (0, 7)
- B) (1, 7)
- C) (7, 7)
- D) (14, 2)

- Lines that lean to the right have a positive slope. Lines that lean to the left have a negative slope, and lines that are horizontal have a slope of zero. Vertical lines have a slope that cannot be determined.
- A line that bisects a quadrant has a slope of  $45^\circ$  if it bisects quadrants I or III the slope is positive  $45^\circ$ . If it bisects quadrants II or IV it has a slope of negative  $45^\circ$ .

### EQUATION OF A LINE

The equation of a line is:

$$y = mx + b$$

Where:

- $y$  is the  $y$  coordinate of a point on the line.

- **x** is the x-coordinate of that same point.
- **m** is the slope of the line.
- **b** is the y-intercept of the line.

The same equation can be used for everything from production models and cost functions to boat rentals,

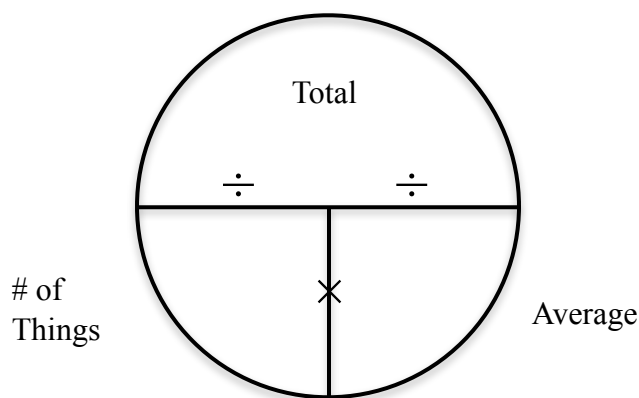
## 10. AVERAGE AND RATES

### AVERAGE (ARITHMETIC MEAN)

SAT problems that ask you for the average will almost always include a trick or twist. Rather than trying to do everything at once, take it one piece at a time. All we need to know are 2 out of the three things in the pie, and every SAT average problem either gives us two things, or shows the way to get two things. Then we just do the math.

$$\text{Total} \div \# \text{ of Things} = \text{Average} \quad \text{Total} \div \text{Average} = \# \text{ of Things}$$

$$\# \text{ Things} \times \text{Average} = \text{Total}$$



4. The average (arithmetic mean) of 25, 30, and  $x$  is 40. What is the value of  $x$ .

- (A) 55
- (B) 60
- (C) 65
- (D) 70

16. If the average (arithmetic mean) of six numbers is 28, and the average of two of these numbers is 18, what is the average of the other four numbers?

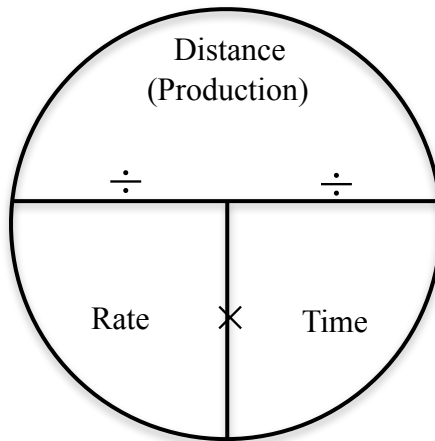
- A) 30
- B) 31
- C) 32
- D) 33

### RATES

The term rate can apply to either a “rate of speed”, i.e. miles per hour, or a “rate of production”, i.e. paperclips per minute. The calculations for rate are:

Rate  $\times$  Time = Distance or Rate  $\times$  Time = Units Produced

Note that we can use the Rate Pie to organize this information.



5. An automobile traveling 55 miles per hour will travel 1,100 miles in how many hours?

- (A) 2
- (B) 5
- (C) 20
- (D) 22

15. A radio advertisement script contains 480 words. The advertisement needs to be exactly one minute in length. What is the average rate in words per second at which the announcer must read the script?

17. Machine A produces pencils at a constant rate of 3 pencils per minute, and machine B produces pencils at a constant rate of 6 pencils per minute. If the two machines work together, then in how many minutes will they produce 1,269 pencils? (This is a grid in question)

## 11. EXPONENTS AND ROOTS

Exponents just mean multiplication. Instead of writing  $5 \times 5 \times 5 \times 5$  we write  $5^4$ .

**When in doubt, expand it out.**

You can use the exponent rules to multiply or divide expressions with exponents. Just make sure the bases are the same.

To Multiply Exponents: Add Them Together

To Divide Exponents: Subtract Bottom from Top

When You Have Parentheses: Multiply Them

Try These examples:

1.  $2^2 \times 2^3 =$

2.  $x^6/x^2 =$

3.  $(x^2)^3 =$

8. If  $(x^a)^6 = x^{18}$  and  $x^b/x^2 = x^8$ , then  $b - a = ?$

A) -2

B) 4

C) 7

D) 13

18. If  $y^6 < y^3$ , which of the following could be the value of  $y$ ?

(A)  $-1/3$

(B)  $1/3$

(C) 0

(D) 1

- If you square a positive fraction less than 1, it gets smaller.
- A negative number raised to an even power becomes positive.
- A negative number raised to an odd power stays negative.

SPECIAL RULES
$1^{1,425} =$
$924^0 =$
$0^5 =$
$3^{-2} =$
$16^{1/2} =$

The exponent applies to all parts in parentheses, so:

$$(2x)^3 = 2^3 \times (x)^3 = 8x^3$$

## ROOTS

- If  $x^2 = 9$ , then  $x = 3$  or  $x = -3$ . However, for the SAT, the square root of a number is always the positive root!
- You can add and subtract square roots when the number under the root sign is the same. i.e.  $3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$  and  $8\sqrt{5} - 2\sqrt{5} = 6\sqrt{5}$
- Square roots can be multiplied or divided. Just put everything under the root sign.  
 $\sqrt{3} \times \sqrt{12} = \sqrt{3 \times 12} = \sqrt{36} = 6$      $\sqrt{48}/\sqrt{3} = \sqrt{48/3} = \sqrt{16} = 4$
- If you take the square root of a fraction less than 1, it gets bigger.

Try these:

$$6. \frac{3\sqrt{3} \times 2\sqrt{8}}{\sqrt{12}} =$$

- (A)  $1/6$
- (B)  $3\sqrt{2}$
- (C)  $6$
- (D)  $6\sqrt{2}$

17. If  $\frac{\sqrt{3}}{n} = \frac{n}{\sqrt{12}}$ , which of the following could be the value of  $n$ ?

- (A)  $\sqrt{6}$
- (B) 3
- (C)  $2\sqrt{6}$
- (D) 6

## 12. FUNCTIONS BASICS

On the SAT, functions are shown more than one way. Rather than  $f(x)$ , you might see a weird symbol. It's just there to make easy math look difficult. Just plug in for the variables, and do the arithmetic.

$$\diamond y = 2(y-1) \qquad x \$ y = (x + 2y)/y$$

Try this one:

If  $a \# b = (a + b)/2$ , then  $3 \# 4 =$  \_\_\_\_\_

If the question uses  $f(x)$ , plug in the  $x$  value in the given equation. If you have to solve for the  $x$  value, plug in the answer choices.

10. If  $f(x) = x^2 + 8x + 2$ , then  $f(3) =$

- (A) 11
- (B) 29
- (C) 33
- (D) 35



18. If  $f(x) = x^3 - 4x + 1$ , for what value of  $x$  does  $f(x) = 16$ ?

- (A) 1
- (B) 3
- (C) 4
- (D) 5

### 13. PROPORTIONS (DIRECT AND INVERSE VARIATION)

A car travels at a rate of 50 miles per hour. How long will it take to travel 300 miles.

This is a DIRECT variation (or direct proportion). The variables move in the same direction. As one increases, so does the other.

To solve: Set up an equation with two fractions, then cross multiply.

The relationship you know = The relationship you want to find.

Just make sure you put the same variables on the top and bottom.

Miles/Hour = Miles/Hour

$50/1 = 300/x$  so:  $50x=300$   $x=6$  hours.

3. On a certain map, Washington D.C. and Boston are 4 inches apart. If Washington and Boston are actually 500 miles apart, and if the map is drawn to scale, then 1 inch on the map actually represents how many miles?

- (A) 125
- (B) 150
- (C) 250
- (D) 375

14. In 1976, Elvis consumed three times his body weight in peanut butter and banana sandwiches. If Elvis's body weight in 1976 was 250 pounds, and if peanut butter and banana sandwiches weigh four ounces, then how many such sandwiches did Elvis consume during 1976? (1 lb. = 16 ounces)

- (A) 750
- (B) 1,000
- (C) 1,500
- (D) 3,000

SAT might refer to a proportion by telling you that 2 things *vary directly* or are *directly proportional*.

If  $x$  and  $y$  vary directly, then  $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

Some questions might ask about *inverse variation* or *inverse proportion*. When two things vary inversely with each other, it means that as one gets bigger, the other gets smaller. The product of the two remains constant.

If  $x$  and  $y$  vary indirectly, then:  $x_1y_1 = x_2y_2$

12. The number of Elvis sightings per year in any city varies inversely with that city's distance from Graceland. Last year, there were 88 Elvis sightings in Washington D.C. If Washington D.C. is 875 miles from Graceland, then how many Elvis sightings were there last year in Philadelphia, which is 1000 miles from Graceland?

- (A) 12
- (B) 77
- (C) 82
- (D) 88

#### 14. PERCENTS

Percents are also fractions. (Percent means "out of 100"). For example:

$$20\% = 20/100 = 1/5 \quad 250\% = 250/100 = 25/10 = 5/2 \quad 0.5\% = .5/100$$

Translate percent problems into fraction problems using this table:

ENGLISH	MATH EQUIVALENT
% (percent)	$\div 100$
Of	X
What	The variable: i.e. $x, y, z$
is, are were, did, does	=

Translate this to a fraction problem:

1) 40 percent of 35 is equal to 25 percent of what number?

2) What percent of 5 is 6?

13. At Joe's farm there are 20 cows, 17 pigs, and the 13 horses. What percent of the animals on the farm are not pigs?

- A) 20%
- B) 33%
- C) 34%
- D) 66%

16. On a two-part exam, a student answered 40 questions in the 1st part correctly and 60% of the questions in the 2nd part correctly. If the student answered a total of 70 questions correctly, how many questions were there in the 2nd part of the exam? (Grid-In)

## PERCENT CHANGE

**For percent change questions, the approach is always the same.**

**Difference/Original**

## 15. RATIOS and PROBABILITIES

### RATIOS

A ratio is like a recipe: it tells you how many parts of each “ingredient” go into a “mixture”.

*To make punch, makes 2 parts grape juice with 3 parts orange juice.*

The relationship (or ratio) of grape juice to orange juice is 2 to 3. It doesn't matter whether you mix the punch in an 8-ounce glass or a 2 quart pitcher. The relationship remains constant.

### Ratio Box

In a club of 35 members, there is a 2 to 3 ratio of boys to girls among the members.

BOYS	GIRLS	WHOLE
2	3	
		35

1. Put the ratios in the first row. Add them together. This gives you the number of students in a “group”.
2. Divide “total students” by “students in a group”. This give you your multiplier.
3. Use the same multiplier for each column and multiply down.

You can now answer any question about this class.

What is the total number of boys? \_\_\_\_\_

What is the total number of girls? \_\_\_\_\_

Girls make up what fractional part of the group? \_\_\_\_\_

Boys make up what percent of the group? \_\_\_\_\_

16. If a bed of 56 flowers consists of tulips, roses, and daffodils and the ratio of tulips to roses to daffodils is 3:5:6, then how many of the flowers are roses?

- (A) 5
- (B) 12
- (C) 20
- (D) 24

## PROBABILITY

Probability is the likelihood that something will happen. The probability of an event can never be less than 0 or greater than 1. A probability of 0 means that something is impossible. A probability of 1 means that something has a 100% chance of happening.

$$\text{Probability} = \frac{\text{\# of outcomes fulfilling the requirements}}{\text{Total number of possible outcomes}}$$

9. Stephen owns 20 van Halen T-shirts: 6 of them are long-sleeved, 11 of them are short-sleeved, and the 3 of them are sleeveless. If he chooses a shirt at random, what is the probability that the shirt will not be long-sleeved?

- (A) 3/20
- B) 9/20
- C) 11/20
- D) 7/10

16. Twenty cards are numbered from 1 to 20 and one card is chosen at random. What is the probability that the number on the chosen card is prime? (Grid-in)

## 16. SIMULTANEOUS EQUATIONS

simultaneous equations on the SAT are much easier than they look-just stack them then add or subtract.

11. If  $4x - 5y = 15$  and  $2x - y = 9$ , then  $6x - 6y = ?$

- A) 5
- B) 9
- C) 15
- D) 24

Sometimes you can't get the expression that you need by simply adding or subtracting the equations. Try multiplying or dividing the resulting equation by a number to get what the question is asking for.

12. If  $7x + 35y = 13$  and  $23x - 5y = 67$ , then what is the value of  $3x + 3y$  ?

- A)  $3/8$
- B)  $9/5$
- C)  $8$
- D)  $30$

For many questions, you won't actually have to solve for the variable. Pay attention to what the question asks you to find.

3. If  $3(5p - 6q) = 12$  then  $5p - 6q = ?$

- A)  $30$
- B)  $15$
- C)  $12$
- D)  $4$

## 17. QUADRATIC EQUATIONS

**A rectangular lot has a width that is 4 units shorter than it's length, and an area of 160 square units. Can you create an equation that expresses this?**

**Quadratic Identities:**

$$X^2 + 2XY + Y^2 = (X + Y)(X + Y) \qquad X^2 - Y^2 = (X + Y)(X - Y)$$

$$X^2 - 2XY + Y^2 = (X - Y)(X - Y)$$

**A quadratic expression is any expression that has a “squared” variable. They come in 5 different flavors (or levels of difficulty).**

### Level 0

$$X^2 = 100 \qquad W^2 = 36 \qquad Y^2 = 17$$

### Level 1

$$(X + 3)^2 = 25 \qquad (Y - 4)^2 = 49$$

### Level 2

$$X^2 + 4X + 4 = 25 \qquad Y^2 - 20Y + 100 = 7 \qquad Z^2 - 10Z + 25 = 169$$

**Level 3**

$$X^2 + 8X + 15 = 80$$

$$Y^2 - 6Y + 11 = 27$$

$$W^2 + 90 = 22W - 31$$

**Level 4**

$$X^2 + 3X + 1 = 5$$

$$Y^2 - 5Y + 6 = 2$$

**Level 5**

$$3X^2 + 5X + 1 = 9$$

**SOME EXTRA STUFF****TRANSLATING ENGLISH TO MATH**

if the question doesn't give you an equation, you may have to make your own. Translate the words in the question into an equation.

3. If  $\frac{1}{4}$  of a certain number is 15 less than the number, what is the number?

- (A) -20
- (B) 5
- (C) 15
- (D) 20

**COMBINATIONS**

Sometimes the SAT will ask you to count how many different ways it's possible to arrange a group of items. In most cases, there will be too many possible arrangements to write them out. So, you have to be systematic.

**Combination with No Restrictions**

12. Four chefs are available to cook four different meals. If each chef is to cook one of the meals, in how many different ways could the four chefs be assigned to the four meals?

- (A) 4
- (B) 8
- (C) 16
- (D) 24

---

How many chefs are available to cook meal #1? Once you've assigned a chef, how many are left to cook meal #2? How many for meal #3? How many for meal #4 ?

---

Meal 1    X    Meal 2    X    Meal 3    X    Meal 4    =    Total Combinations

\_\_\_\_\_

Notice that, by restricting spaces, we can increase the difficulty.

16. Six children, one boy and five girls, must stand in a line. If the boy cannot stand first or last, how many different ways could the children be arranged?

- (A)    720
- (B)    480
- (C)    360
- (D)    240

---

Use the same technique as above, but fill the restricted spaces first.



PAGE #	ANSWERS
3	2-D , 3-D, 6-A
5	8-D, 11-D, A
6	A, 8-D
7	16-D, 13-B, 5-B, 13-C
8	14-D, 13-D, 17-18
13	17-1600
15	10-B
16	15-B, 16-C
18	3.12-D
19	4-C
20	16-D, 5-C
21	15-8, 17-141, $1-2^5$ , $2-X^4$ , $3-X^6$ , 8-D, 18-A
22	6-D
23	17-D, $3\sqrt[4]{3.5}$ , 10-D
24	18-B, 3-A, 14-D
25	12-B
26	13-D, 16-50
27	16-C
28	9-B, $16-2/5$ , 11-D
29	12-C, 3-D LEVEL 0: $X=10$ OR $-10$ $W=6$ OR $-6$ , $Y=\sqrt{17}$ OR $-\sqrt{17}$ LEVEL 1: $X=2$ OR $-8$ , $Y=11$ OR $-3$ LEVEL 2: $X=3$ OR $-7$ , $Y=10+\sqrt{7}$ OR $10-\sqrt{7}$ $Z=18$ OR $-8$
30	LEVEL 3: $X=5$ OR $-13$ , $Y=8$ OR $-2$ , $W=11$ LEVEL 4: $X=1$ OR $-4$ , $Y=4$ OR $1$ LEVEL 5: $X=1$ OR $-16/6$ , 3-D, 12-D
31	16-B