GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GAUTENG DEPARTMENT OF EDUCATION
PREPARATORY EXAMINATION
2019

10612

MATHEMATICS

PAPER 2

TIME: 3 hours

MARKS: 150

16 pages, 1 information sheet and a 25 page answer book

MATHEMATICS: Paper 2

10612E

X10

P.T.O.

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(Paper 2)

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INSTRUCTIONS AND INFORMATION

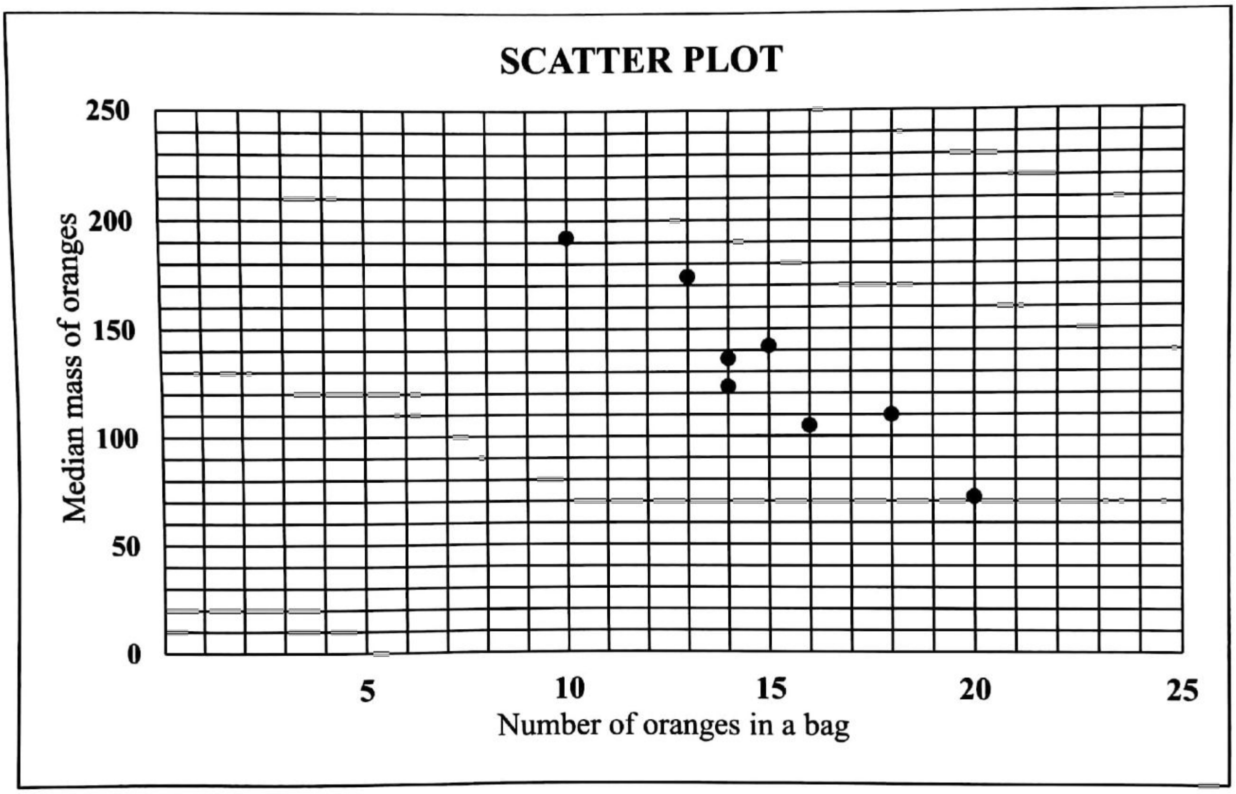
Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you have used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An INFORMATION SHEET with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

A student is investigating the number of oranges in a bag in relation to the median mass of the oranges filled in the same bag. The findings are recorded in the table below.

Number of oranges in a bag	18	16	20	15	14	13	14	10
Median mass of oranges in the same bag (to the nearest gram)	110	105	72	142	123	174	136	192



1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 Write down the correlation coefficient of the data. (1)

1.3 Draw the least squares regression line on the scatter plot given in your ANSWER BOOK. (2)

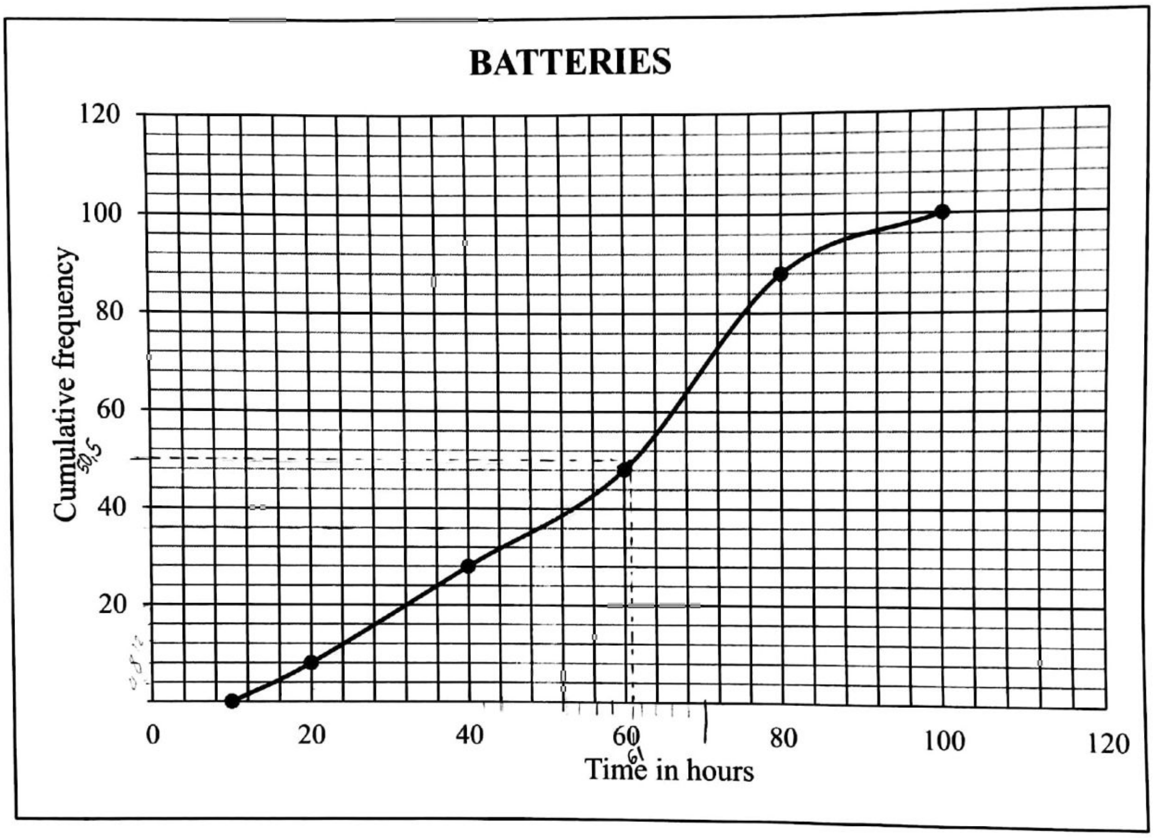
1.4 Comment on the strength of the relationship between the number of oranges in the bag and the median mass of the oranges. (1)

1.5 Determine the possible median mass of oranges in a bag, if there are 12 oranges in that bag. (2)
[9]

QUESTION 2

2.1 Batteries are used in everyday life. The Grade 12 Physical Sciences learners investigated the life span of batteries under constant test conditions.

The ogive (cumulative frequency graph) below shows the lifespan (in hours) of the batteries.



2.1.1 How many batteries were tested for this investigation? (1)

2.1.2 Use the graph to estimate the median time for the life span (in hours) of the batteries. (2)

2.1.3 The minimum lifespan of batteries is 10 hours and the maximum lifespan is 100 hours. Use the cumulative frequency graph to draw a box and whisker diagram in your ANSWER BOOK. (3)

2.1.4 Comment on the skewness of the distribution of the lifespan of the batteries. (1)

2.2 The table below represents values in a data set written in increasing order. None of the values in the data set are repeated.

5	a	19	b	c	d	35

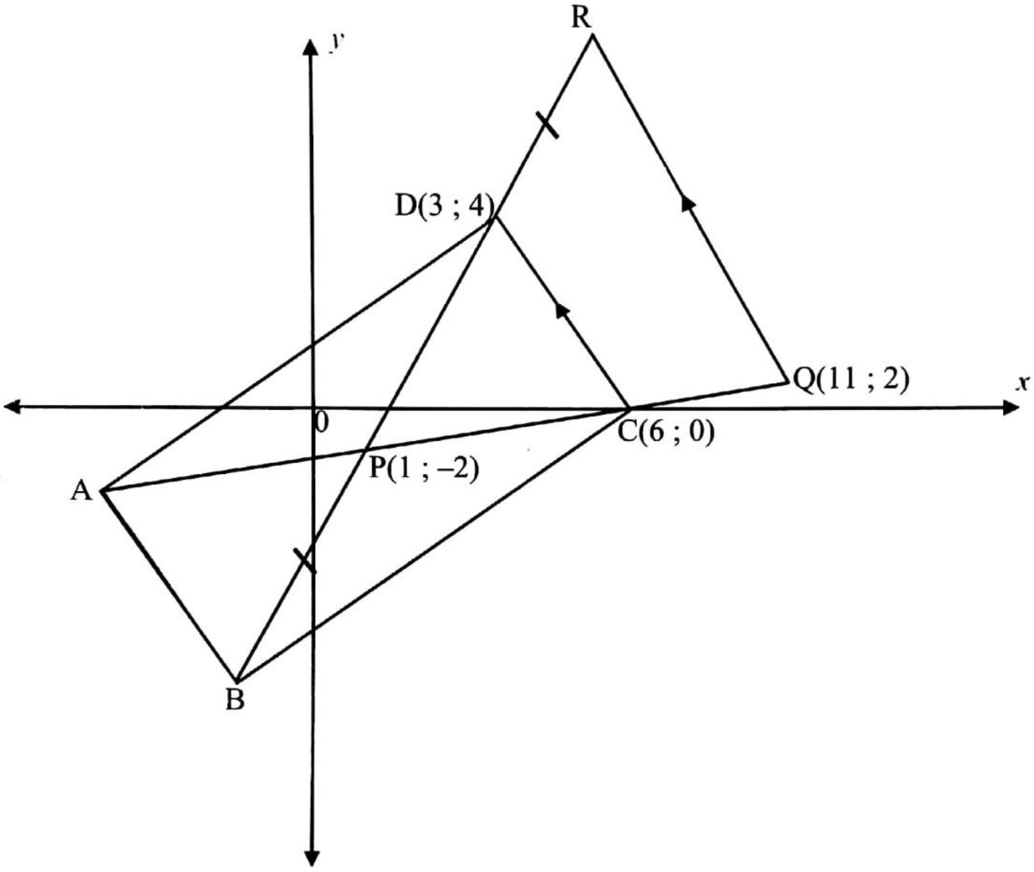
Determine the values of a, b, c, and d if:

- The median is 20.
- The semi interquartile range is 8.
- The upper quartile is twice the lower quartile.
- The mean is 22. (4)

[11]

QUESTION 3

3.1 In the diagram below, A, B, C $(6 ; 0)$ and D $(3 ; 4)$ are the vertices of rectangle ABCD. Diagonals AC and BD bisect each other at $P(1 ; -2)$. AC is produced to $Q(11 ; 2)$ and BD is produced to R such that BP = DR and CD || QR.



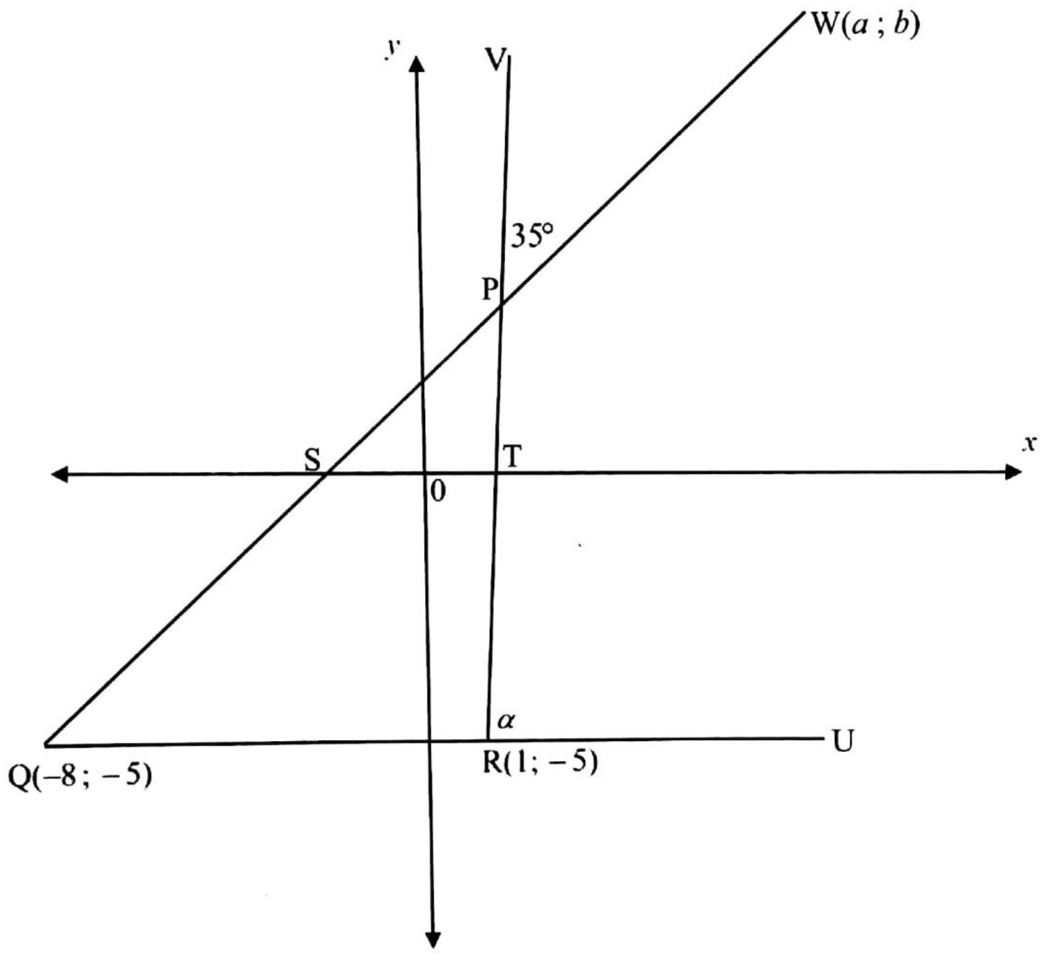
3.1.1 Calculate the coordinates of B. (3)

3.1.2 Determine the gradient of CD. (2)

3.1.3 Show that the equation of QR is $y = -\frac{4}{3}x + \frac{50}{3}$. (2)

3.1.4 If $K(4 ; y)$ is a point in the 4th quadrant such that PK = RQ, calculate the value of y. (6)

3.2 In the diagram below, P, $Q(-8\ ;\ -5)$ and $R(1\ ;\ -5)$ are the vertices of ΔPQR. RP is produced to V and QP is produced to $W(a\ ;\ b)$ such that $V\hat{P}W = 35°$. The equation of QW is $y = x + \frac{2}{3}$. QR is produced to U and $U\hat{R}V = \alpha$. QW and RV intersect the x-axis at S and T respectively.



3.2.1 Calculate the size of α. (5)

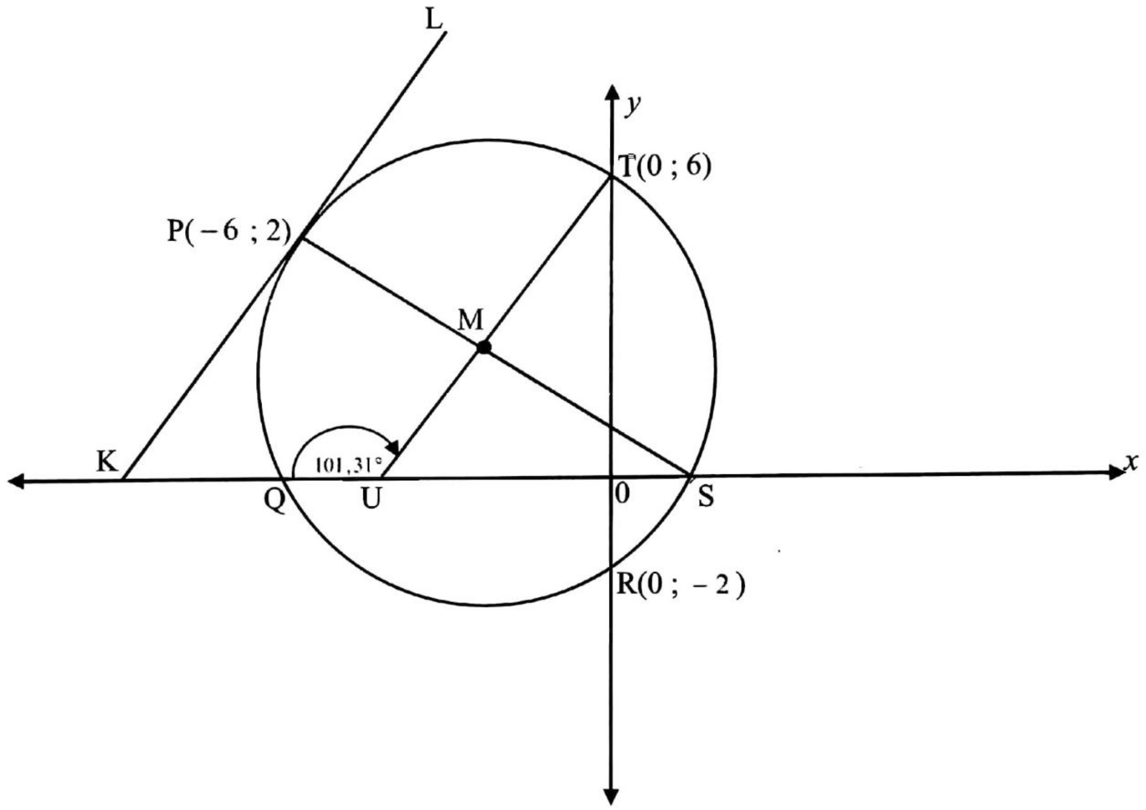
3.2.2 It is further given that $QU \perp WU$ and R is the midpoint of QU. Calculate the area of ΔQWU. (6)

[24]

QUESTION 4

In the diagram below, a circle with centre M, cuts the x-axis at Q and S and the y-axis at T(0 ; 6) and R(0 ; −2). The equation of diameter SMP is $y = \frac{-1}{5}x + \frac{4}{5}$.

KPL is a tangent to the circle at P(−6 ; 2). TM produced cuts the x-axis at U. $Q\hat{U}T = 101{,}31°$.



4.1 Determine the equation of TU. (3)

4.2 Calculate the coordinates of M. (3)

4.3 If the coordinates of M are (−1 ; 1) , determine the equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.4 Prove that KL is parallel to TU. (3)

4.5 Is the point $V\left(-\frac{1}{2} ; 7\right)$ inside the circle? Support your answer with calculations. (3)

[15]

QUESTION 5

5.1 If $\sin 16^\circ = \dfrac{1}{\sqrt{1+k^2}}$, express the following in terms of k, **without the use of a calculator.**

5.1.1 $\tan 16^\circ$ (2)

5.1.2 $\cos 32^\circ$ (3)

5.2 Simplify the following expression.

$$\frac{\cos(90^\circ + x)\,\sin(x - 180^\circ) - \cos^2(180^\circ - x)}{\cos(-2x)}$$ (6)

5.3 Calculate the value of the following, **without the use of a calculator**.

$\cos 75^\circ.\cos 45^\circ - \cos 15^\circ.\cos 45^\circ$ (4)

5.4 Given: $\tan\theta\left(\sin 2\theta + \dfrac{3\cos^2\theta}{\sin\theta}\right) = -2\cos^2\theta + 3\cos\theta + 2$

5.4.1 Prove the identity. (3)

5.4.2 Determine the general solution of:

$$\tan\theta\left(\sin 2\theta + \frac{3\cos^2\theta}{\sin\theta}\right) = 0$$ (4)

5 Solve for a and b:

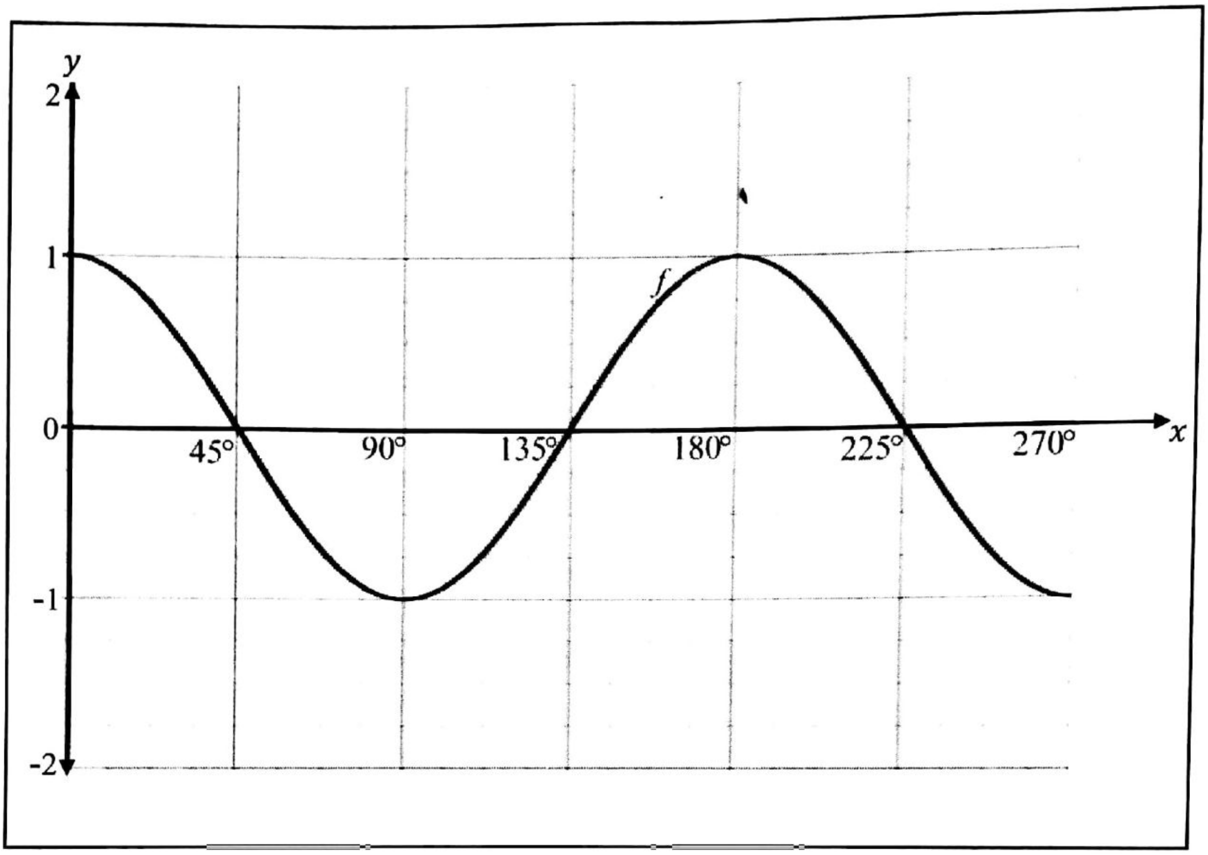
$\cos(a+b) = -\dfrac{\sqrt{2}}{2}$ if $a+b \in [0^\circ\ ;\ 180^\circ]$

$\cos(a-2b) = \dfrac{1}{2}$ if $a-2b \in [0^\circ\ ;\ 180^\circ]$ (4)

[26]

QUESTION 6

In the diagram below, the graph of $f(x) = \cos 2x$ is drawn for the interval $x \in [0°;\ 270°]$.



6.1 Draw the graph of $g(x) = -\frac{1}{2}\tan x$ for the interval $x \in [0°\ ;\ 270°]$ on the grid provided in the ANSWER BOOK. Show all intercepts with the axes and asymptotes. (4)

6.2 Write down the range of $h(x) = 3 - f(x)$. (1)

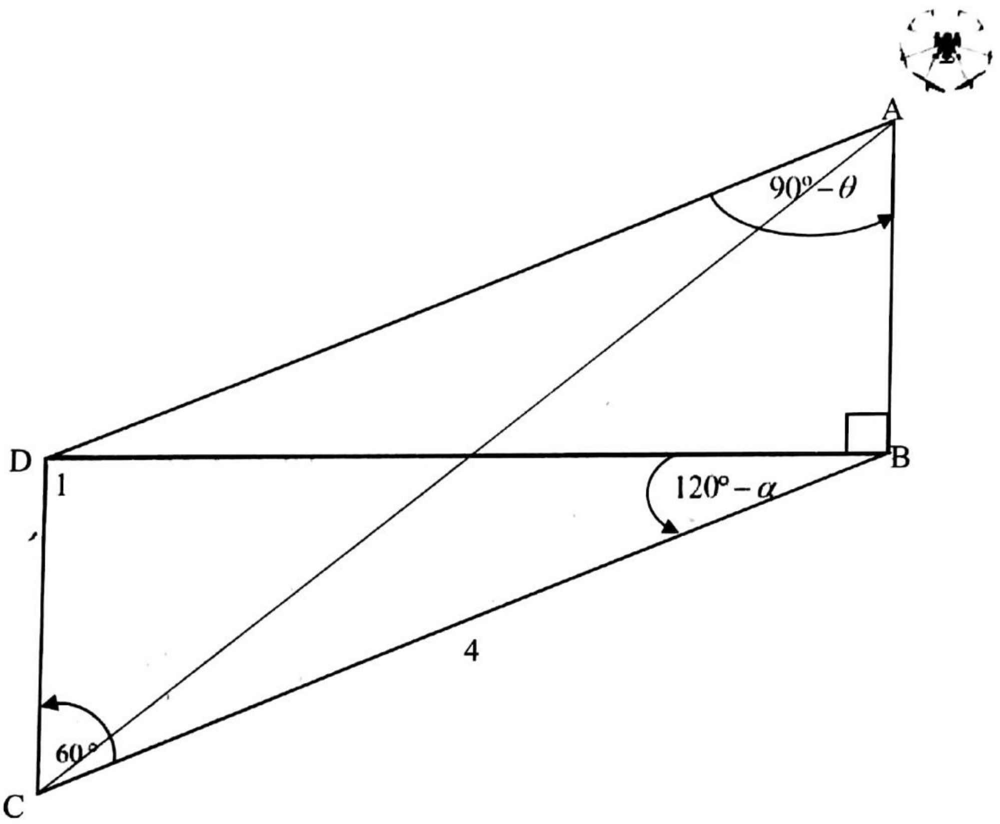
6.3 Use the graph to determine the value(s) of x in the interval $x \in [135°\ ;\ 270°]$ for which $\frac{f(x)}{g(x)} \geq 0$. (2)

[7]

QUESTION 7

In the diagram below, B, C and D are in the same horizontal plane. A drone positioned at A, captures the images of two objects at B and C. B is directly below the drone and C is 4 units away from B.

$D\hat{C}B = 60^\circ$; $D\hat{B}C = 120^\circ - \alpha$ and $D\hat{A}B = 90^\circ - \theta$.



7.1 Determine $\hat{D}_1$ in terms of α . (2)

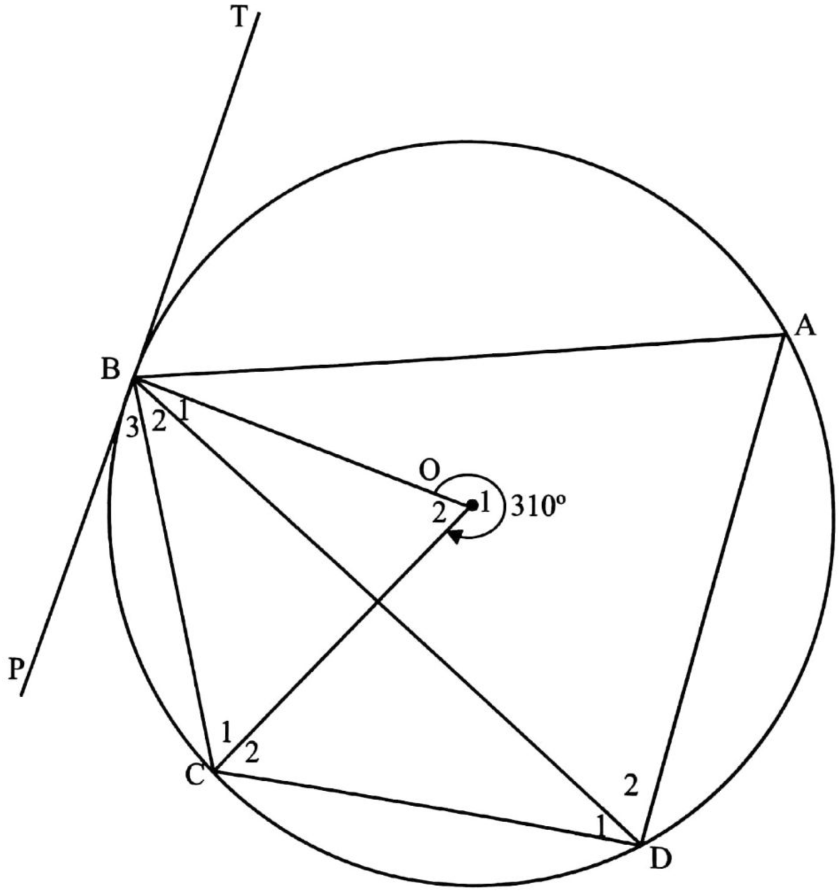
7.2 **Without the use of a calculator**, determine BD in terms of α . (3)

7.3 Show that $AB = \dfrac{2\sqrt{3}\tan\theta}{\sin\alpha}$. (3)

[8]

QUESTION 8

In the diagram below, A, B, C and D are points on a circle having centre O.
PBT is a tangent to the circle at B.
Reflex $B\hat{O}C = \hat{O}_1 = 310^\circ$ as shown in the diagram below.



Calculate, giving reasons, the size of:

8.1 $\hat{D}_1$ (3)

8.2 $\hat{B}_3$ (2)

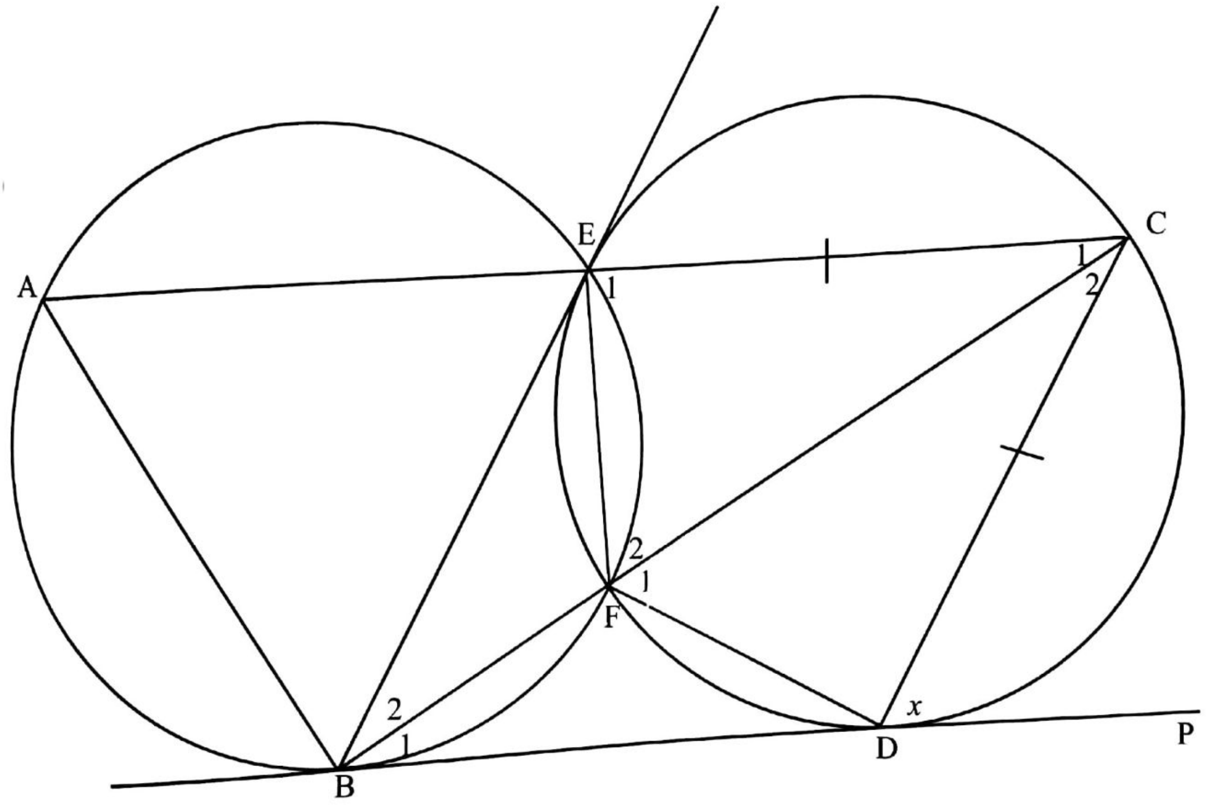
8.3 $\hat{B}_1$, if it is given that $\hat{A} = 60^\circ$. (4)

[9]

QUESTION 9

9.1 Complete the statement so that it is TRUE.
Angles subtended by a chord of a circle, on the same side of a chord, are … (1)

9.2 In the diagram below, ABFE and EFDC are cyclic quadrilaterals in two equal circles that intersect at E and F. BFC and AEC are straight lines. BD is a common tangent to the circles at B and D respectively. EC = CD.
Let $C\hat{D}P = x$



Prove, giving reasons, that:

9.2.1 $\hat{F}_1 = \hat{F}_2$. (3)

9.2.2 ABDC is a cyclic quadrilateral. (3)

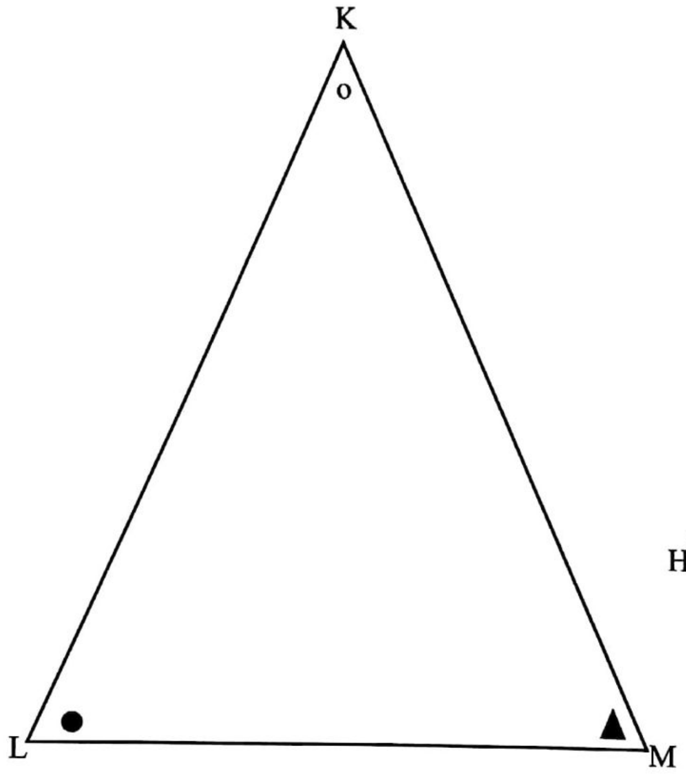
9.2.3 BE || CD. (2)

9.2.4 FC is a diameter of circle FDCE if it is given that EBDC is a rhombus. (5)

[14]

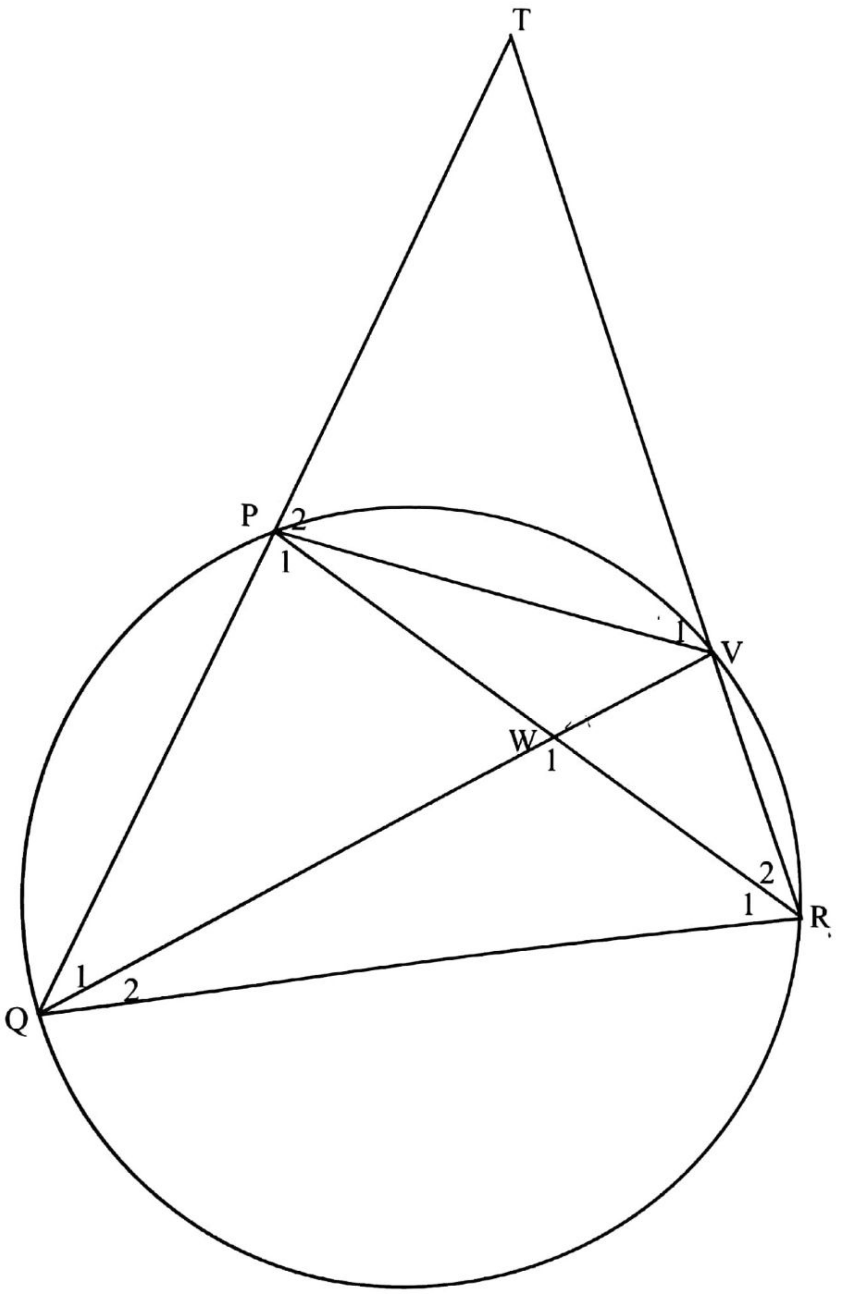
QUESTION 10

10.1 In the diagram below, ΔKLM and ΔGHI are drawn such that $\hat{K} = \hat{G}$, $\hat{L} = \hat{H}$ and $\hat{M} = \hat{I}$.
Prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e. prove that $\frac{KL}{GH} = \frac{KM}{GI}$.



(5)

10.2 In the diagram below, ΔPQR is an equilateral triangle inscribed in a circle. V is a point on the circle. QP produced meets RV produced at T. PR and QV intersect at W.



Prove, giving reasons, that:

10.2.1 $\hat{W}_1 = T\hat{R}Q$ (3)

10.2.2 $\Delta TQR \,|||\, \Delta QRW$ (3)

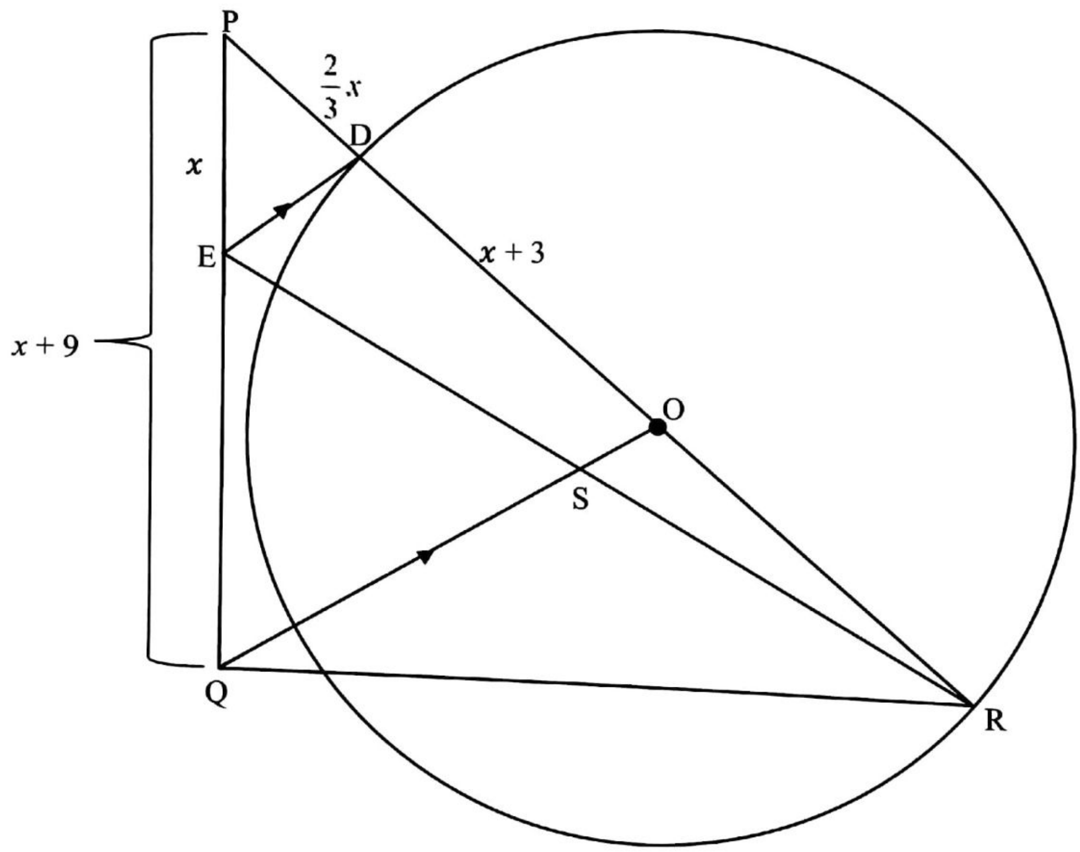
10.2.3 $\dfrac{PT}{QW} = \dfrac{PV}{WR}$ (6)

[17]

QUESTION 11

In the diagram below, the circle with centre O is drawn. OQ is drawn parallel to a tangent to the circle at D. ER is drawn with S on OQ.
RD is produced to P and PQ is joined.

PE = x units, PQ = $x + 9$ units, PD = $\frac{2}{3}x$ units and DO = $x + 3$ units.



11.1 Calculate the length of RO. (4)

11.2 If OS = 1,4 units and S is the midpoint of ER, determine the length of DE. (2)

11.3 If the area of $\Delta PED = 2{,}7$ units^2, find the area of ΔPER. (4)
[10]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r};\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2};\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

InΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}\ ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$