GAUTENG DEPARTMENT OF EDUCATION

JOHANNESBURG NORTH DISTRICT

2021

GRADE 12

MATHEMATICS

PAPER 1

PRE-TRIAL EXAM

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MARKS: **150**

TIME: **3 HOURS**

DATE: **13 AUGUST 2021**

This paper consists of 12 printed pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of this question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $4x^2 - 25 = 0$ (3)

1.1.2 $3x^2 + 5x = 4$ (correct to TWO decimal places) (4)

1.1.3 $2^x - 5 \cdot 2^{x+1} = -144$ (3)

1.1.4 $2x^2 + x - 3 > 0$ (3)

1.2 Given: (i) $4^{x+2} \cdot 8^{y+1} = 2^{1-x}$

(ii) $x^2 + y^2 + xy = 7$

1.2.1 Show that for equation (i) above $y = -x - 2$. (3)

1.2.2 Hence solve for x and y simultaneously. (5)

1.3 Prove that the equation $6x^2 + 2gx - 3x - g = 0$ has rational roots for all rational values of g. (4)

[25]

QUESTION 2

Consider the following arithmetic sequence:

$$(x + 5); (37 - x); (x + 13); \ldots$$

2.1 Determine the value of x. (3)

2.2 Determine the general term of the sequence in the form: $T_n = \ldots$ (3)

2.3 The sum of the first three terms of a geometric sequence is 91, and its common ratio is 3, determine the first term of the sequence. (3)

2.4 In a convergent series, $S_2 = 90$ and $S_\infty = \frac{375}{4}$. Determine the first term and its common ratio. (6)

2.5 An entrepreneur decides to monitor the share price of a company over a five day period. The entrepreneur observes that the share price follows a quadratic pattern. The share prices over a 5 day period are shown below:

Day	Amount (R)
1	32 699
2	32 896
3	33 091
4	33 284
5	33 475

2.5.1 Show that the pattern is quadratic. (2)

2.5.2 Determine the nth term of the first difference. (2)

2.5.3 Determine the nth term of the quadratic pattern. ‘ (4)

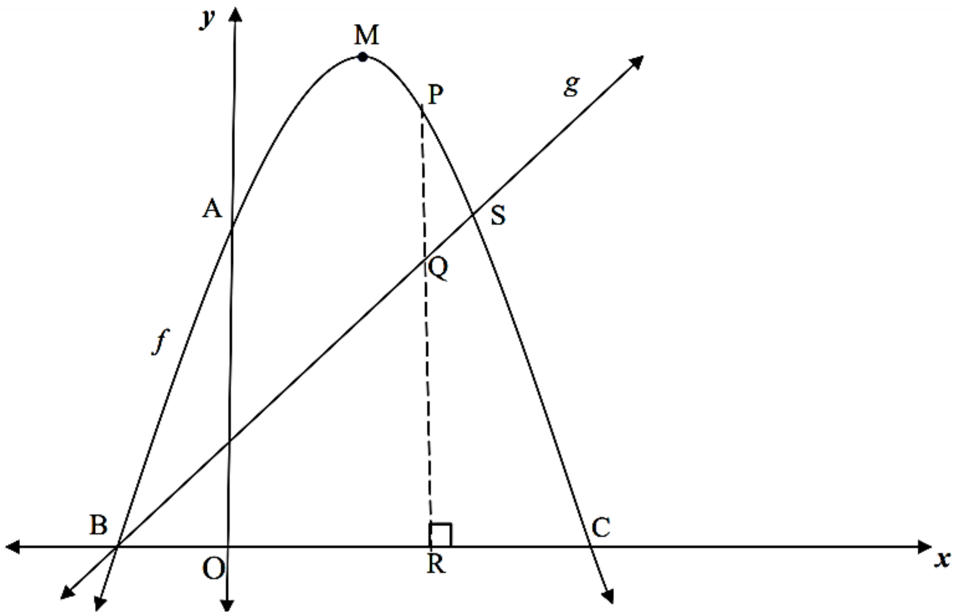
2.5.4 After how many days, will the share price be at a maximum? (3)

[26]

QUESTION 3

The diagram below shows the graphs of $f(x) = -x^2 + 5x + 6$ and $g(x) = x + 1$.

The graph of f intersects the x-axis at B and C and the y-axis at A. The graph of g intersects the graph of f at B and S. PQR is perpendicular to the x-axis with points P and Q on f and g respectively. M is the turning point of f.



3.1	Write down the coordinates of A.	(1)
3.2	S is the reflection of A about the axis of symmetry of f. Determine the coordinates of S.	(2)
3.3	Calculate the coordinates of B and C.	(3)
3.4	If PQ = 5 units, calculate the length of OR.	(5)
3.5	Calculate the:	
	3.5.1 Coordinates of M.	(4)
	3.5.2 Maximum length of PQ between B and S.	(4)

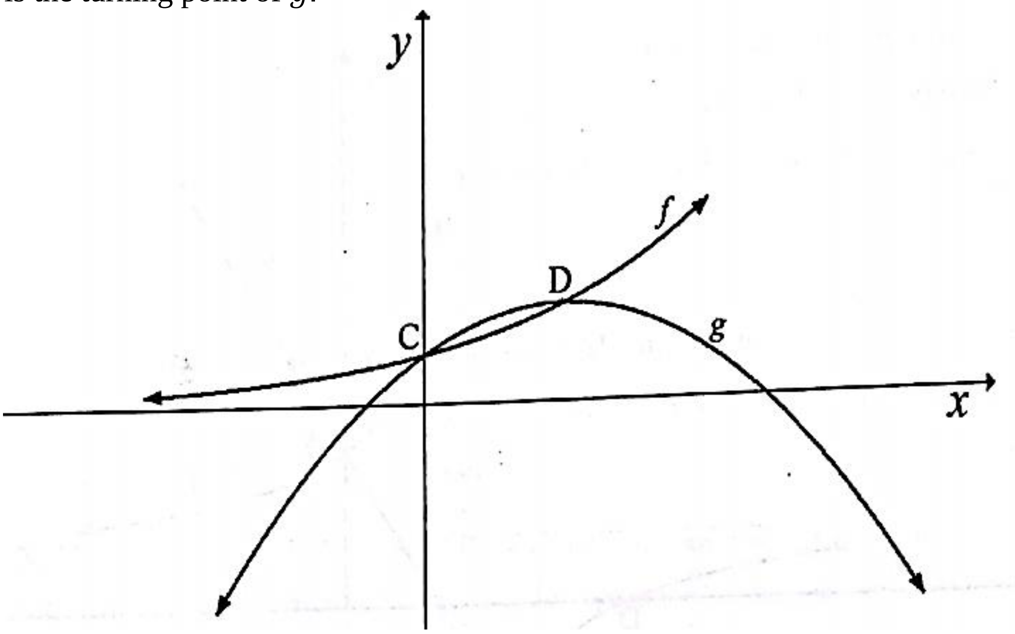
[19]

QUESTION 4

Sketched below are the graphs of $f(x) = 2^x$ and $g(x) = -(x-1)^2 + q$, where q is a constant. The graphs of f and g intersect at C and D.

C is the y-intercept of both f and g.

D is the turning point of g.



4.1 Show that q = 2. (2)

4.2 Write down the coordinates of the turning points of g. (2)

4.3 Determine the value(s) of t for $g(x) = t$ if the roots are equal. (1)

4.4 Write down $f^{-1}(x)$ in the form $y = \ldots$ (2)

4.5 Sketch the graph of f^{-1} on a system of axes. Indicate the x-intercept and the coordinates of one other point on your graph. (3)

4.6 Write down the equation of h if $h(x) = g(x+1) - 2$ (2)

4.7 How can the domain of h be restricted so that h^{-1} is called a function. (1)

[13]

QUESTION 5

5.1 Tebogo bought a car for R180 000. The value of the depreciated at 15% p.a. according to the reducing balance method. The book value of Sandile's car is currently R79 866,96.

5.1.1 How many years ago did Sandile buy the car? (3)

5.1.2 At exactly the same time that Tebogo bought the car, Bianca deposited R49 000 into a savings account at an interest rate of 10% p.a., compounded quarterly. Has Bianca accumulated enough money in her savings account to buy Tebogo's car now? (3)

5.2 Exactly 10 months ago, a bank granted Anita a loan of R800 000 at an interest rate of 10,25% p.a. compounded monthly.

The bank stipulated that the loan:

- Must be repaid over 20 years.
- Must be repaid by means of monthly repayments of R7 853,15, starting one month after the loan was granted.

5.2.1 How much did Anita owe immediately after making her 6^{th} repayment ? (4)

5.2.2 Due to financial difficulties as a result of Covid 19, Anita missed the 7^{th}, 8^{th} and 9^{th} payments. She was able to make payments from the end of the 10^{th} month onwards. Calculate Anita's increased monthly repayment in order to settle the loan in the original 20 years as stipulated by the bank. (5)

[15]

QUESTION 6

6.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2$. (4)

6.2 Determine $\frac{dy}{dx}$ if $y = 7x^4 - 5\sqrt{x} - \frac{3}{x}$. (4)

6.3 It is given that $g(x) = ax^3 - 24x + b$ has a local minimum turning point at $(-2; 17)$. Determine the values of a and b. (5)

[13]

QUESTION 7

7.1 Given: $f(x) = -2x^3 + 5x^2 + 4x - 3$

7.1.1 Calculate the coordinates of the x-intercepts of f if $f(3) = 0$.

Show ALL calculations. (4)

7.1.2 Calculate the x-values of the stationary points of f. (4)

7.1.3 For which values of x is f concave up? (2)

7.2 The function g, is defined by $g(x) = ax^3 + bx^2 + cx + d$ has the following properties:

- $g(-2) = g(4) = 0$
- The graph of $g'(x)$ is concave up.
- The graph of $g'(x)$ has x-intercepts at $x = 0$ and $x = 4$ and a turning point at $x = 2$.

7.2.1 Use this information to draw a neat sketch of g without actually solving for a, b, c and d. Clearly show all x-intercepts, x-values of the turning points and then x-value of inflection on your sketch. (4)

7.2.2 For which values of x will $g(x).g''(x) > 0$? (3)

[17]

QUESTION 8

A car speeds along a 1 kilometre in 25 seconds. It distance (in metres) from the start after *t* seconds is given by: $s(t) = t^2 + 15t$.

8.1 Determine an expression for the speed of the car (the rate of change of distance with time) after t seconds. (2)

8.2 Determine the speed of the car as it crosses the finish line. (2)

8.3 Write down an expression for the acceleration of the car (the rate of change of speed with time) after *t* seconds. (1)

8.4 Hence or otherwise calculate the acceleration of the car after 5 seconds. (1)

8.5 Calculate the speed of the car when it is 250m down the track from its starting position. (4)

[10]

QUESTION 9

At Radley Private School, a survey was carried out to determine the number of Grade 12 learners who take Mathematics (M), Physical Sciences (P) and Accounting (A). The following information was collected:

- 135 learners took part in the survey
- 5 learners take Mathematics and Accounting but not Physical Sciences
- 12 learners take Mathematics and Physical Sciences but not Accounting
- 24 learners take Physical Sciences and Accounting but not Mathematics
- y learners take Physical Sciences only
- x learners take all the three subjects
- y learners take Accounting only
- $2y + 3$ learners take Mathematics only
- 60 learners take Accounting
- The number of learners who take Mathematics is equal to the number of learners who take Physical Sciences

9.1 Represent the above information in a Venn diagram. (4)

9.2 Determine the values of x and y. (4)

9.3 Calculate the probability that a learner chosen at random does Mathematics or both Physical Sciences and Accounting. (4)

[12]

TOTAL : 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ M$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\quad$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\sin\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$