

## GAUTENG DEPARTMENT OF EDUCATION

## PREPARATORY EXAMINATION

# 2021

## 10611

## MATHEMATICS

# PAPER 1

- TIME: 3 hours
- **MARKS: 150**

9 pages + 1 information sheet





#### INSTRUCTIONS AND INFORMATION

- 1. This question paper consists of 11 questions.
- 2. Answer ALL the questions.
- 3. Clearly show ALL calculations, diagrams, graphs, et cetera, that you have used to determine the answers.
- 4. Answers only will NOT necessarily be awarded full marks.
- 5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
- 6. Where necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
- 7. Diagrams are NOT necessarily drawn to scale.
- 8. An information sheet with formulae is included at the end of the question paper.
- 9. Number the answers correctly according to the numbering system used in the question paper.
- 10. Write neatly and legibly.

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- 1.1 Solve for x:
  - $1.1.1 \qquad 3x^2 4x = 0 \tag{2}$
  - 1.1.2  $3x^2 + 10x 4 = 0$  (Correct to TWO decimal places) (3)
  - 1.1.3  $15x 4 < 9x^2$  (3)

1.1.4 
$$\sqrt{x^2 - 5} = 2\sqrt{x}$$
 (4)

1.2 Given: 
$$(3x-y)^2 + (x-5)^2 = 0$$
  
Solve for x and y.

Given: 
$$x-2 = \frac{-4}{x-2} - 4$$
  
If  $y = x - 2$ :

1.3.1	Show that the given equation can be expressed as	$y^2 + 4y + 4 = 0.$	(2)

1.4 Calculate the maximum value of S if:  $S = \frac{6}{x^2 + 2}$  (2)

### **QUESTION 2**

1.3

2.1 The first FOUR terms of a quadratic pattern are 1; 9; 25; 49. Write down the value of the 5<sup>th</sup> term. 2.1.1 (1) 2.1.2 Show that the n<sup>th</sup> term of this sequence can be written as  $T_n = (2n-1)^2$ . (5) 2.1.3 Which term of the sequence will be equal to 10 201? (3) 2.2 The first 24 terms of an arithmetic series are: 35+42+49+...+196. Calculate the sum of ALL the natural numbers from 35 to 196 that are NOT divisible by 7. (5) [14]

(3)

[21]

3.1 Consider the following geometric series:

 $5(3x+1)+5(3x+1)^2+5(3x+1)^3...$ 

- 3.1.1 For which values of x will the series converge? (3)
- 3.1.2 Calculate the sum to infinity of the series if  $x = -\frac{1}{6}$ . (4)
- 3.2 Determine the smallest value of k for which  $\sum_{p=1}^{k} 2^{p-4} > 30$  if k is an integer. (4) [11]

### **QUESTION 4**

4.1 Given:  $f(x) = \frac{2}{x+1} - 3$ 

4.1.1	Write down the coordinates of the <i>y</i> -intercept of $f$ .	(1)
4.1.2	Calculate the coordinates of the $x$ -intercept of $f$ .	(2)
4.1.3	Sketch the graph of $f$ in your ANSWER BOOK clearly showing the asymptotes and the intercepts with the axes.	(3)
4.1.4	One of the axes of symmetry of $f$ is a decreasing function.	
	Determine the equation of this axis of symmetry.	(2)

4.2 The function defined by  $t(x) = \log_a x$  is sketched below.

It is given that:

- Point C is the *x*-intercept of *t*.
- Point A (5; -1) is on *t*.



4.2.1	Write down the coordinates of point C.	
4.2.2	Write down the range of the graph of $t$ .	(1)
4.2.3	Calculate the value of <i>a</i> .	(2)
4.2.4	The graph of $h = t(x)$ is transformed by a reflection of t about the line $y = x$ .	
	Determine the equation of $h$ in the form $y = \dots$ .	(2)
4.2.5	Write down the domain of $h$ .	(1)
4.2.6	Write down the equation of the asymptote of $h$ .	(1) [ <b>16</b> ]

Given:  $f(x) = -ax^2 + bx + 6$ 

5.1	The gradient of the tangent to the graph of $f$ at the point $\left(-1; \frac{7}{2}\right)$ is 3.			
	Show that $a = \frac{1}{2}$ and $b = 2$ .	(5)		
5.2	Calculate the x-intercepts of $f$ .	(2)		
5.3	Calculate the coordinates of the turning point of $f$ .	(3)		
5.4	Sketch the graph of $f$ in your ANSWER BOOK. Clearly indicate ALL intercepts with the axes and the turning point.	(2)		
5.5	Use the graph to determine the values of x for which $f(x) > 6$ .	(1)		
5.6	Sketch the graph of $g(x) = -x - 1$ on the same set of axes as the graph of $f$ in QUESTION 5.4. Clearly indicate ALL intercepts with the axes.	(2)		
5.7	Write down the values of x for which $f(x).g(x) \le 0$ .	(2) [ <b>17</b> ]		

6.1	Janet in The inv	Janet invests a lump sum of R5 000 into a savings account for exactly 2 years. The investment earns interest at 10% p.a. compounded quarterly.			
	6.1.1	Wha	it is the quarterly interest rate for Janet's investment?	(1)	
	6.1.2	Calc	sulate the amount in Janet's savings account at the end of 2 years.	(3)	
6.2	Sihle in at a rat R10 00 investr	nherits e of 14 00 from nent.	R800 000. He invests his entire inheritance in a fund which earns interest 4% p.a., compounded monthly. At the end of each month, Sihle withdraws in the fund. His first withdrawal is exactly one month after his initial		
	6.2.1	How	many withdrawals of R10 000 will Sihle be able to make from his fund?	(5)	
	6.2.2	Exac rema	ctly four years after his initial deposit, Sihle decides to withdraw all the aining money from his account to use as a deposit towards a house.		
		(a)	What is the value of Sihle's deposit to the closest Rand?	(4)	
		(b)	Sihle's deposit is exactly 30% of the purchase price of the house.		
			What is the purchase price of the house to the nearest Rand?	(1) [ <b>14</b> ]	

### **QUESTION 7**

7.1

- 7.1.1 Determine f'(x) from FIRST PRINCIPLES. (4)
  - 7.1.2 Hence, calculate the gradient of the tangent of f at x = 2. (2)

7.2 Determine 
$$f'(x)$$
 if  $f(x) = \sqrt[3]{x^2} + \frac{1}{4x^4}$ . (3)

7.3 Given:  $h(x) = ax^2$ ; x > 0

Given:  $g(x) = 1 - 4x^2$ 

Determine the value of *a* if it is given that  $h^{-1}(8) = h'(4)$ . (6)

[15]

Consider the graph of  $g(x) = x^3 - 3x^2$  and  $h(x) = -\frac{2}{3}x - \frac{4}{3}$ .

8.1	Determi Show al	ine whether the graph of $h$ intersects the graph of $g$ at its point of inflection. Il necessary calculations.	(6)
8.2	Determine the stationary points of $y = g'(x)$ . Classify the stationary point(s) as either MAXIMUM or MINIMUM.		
8.3 Hence, or otherwise, determine:			
	8.3.1	The value(s) for x for which $g$ is concave down	(1)
	8.3.2	The gradient of the tangent to $g$ at its point of inflection	(2)
	8.3.3	A student claims that the gradient of $g$ at ANY point will never be less than $-3$ . Is the student correct? Explain.	(2)
8.4	Determine new gra	ine the value of k, if the graph of g is shifted so that the values of x for which the aph $p(x) = (x+k)^3 - 3(x+k)^2$ decreases, is between $-3$ and $-1$ .	(4) [ <b>19</b> ]

#### **QUESTION 9**

The bracelet below is beaded by using 10 plastic spheres and 10 plastic cylinders.

When completed, the bracelet will be painted. (Ignore the holes in the spheres and cylinders.)

The following formulae are given:

$$V = \pi r^2 h \qquad \qquad V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2 \qquad \qquad S = 2\pi r^2 + 2\pi h$$

The following essential properties of the different beads are:

- The radii, *r*, of the spheres and the cylinders are exactly the same.
- The height of each cylinder is *h*.

9.1	If the volume of a cylinder is $6 \text{ cm}^3$ , write <i>h</i> in terms of <i>r</i> .	(1)
9.2	Show that the total surface area (S) of all the painted surfaces on the bracelet is equal to $S(r) = 60\pi r^2 + \frac{120}{r}$ .	(4)
9.3	Determine the value of $r$ so that the LEAST amount of paint will be used.	(3)

[8]

Events A, B and C occur as follows, where A and B are independent events:

•	P(A) =	0.38
•	I(A) -	0,50

- P(B) = 0,42
- $P(A \cap B) = 0,1596$
- P(C) = 0,28

10.1	Are A and B mutually exclusive events? Motivate your answer.	(2)
10.2	By using an appropriate formula, show that the value of $P(A \cup B) = 0,64$ .	(1)
10.3	Calculate the number of people in the sample space.	(2)
10.4	Determine $n(C')$ .	(2) [ <b>7</b> ]

### **QUESTION 11**

11.1 Each of the digits: 1;1;2;3;4;7 is written on a separate card. The cards are then placed next to each other to create a 6 digit number.

	11.1.1	How many numbers start and end with the same digit?	(1)
	11.1.2	Find the probability that the number is 112347 or 743211.	(4)
11.2	<i>n</i> people (numbered 1; 2; 3; 4; 5; 6;; <i>n</i> ) are arranged randomly in a line.		
	Eind the	number of more in terms of a that needed 1 and needed 2 and	

Find the number of ways, in terms of n, that person 1 and person 2 are standing next to each other. (You do not need to simplify your answer.) (3) [8]

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#### **INFORMATION SHEET**

 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ A = P(1+ni) A = P(1-ni)  $A = P(1-i)^n$   $A = P(1+i)^n$  $\sum_{n=1}^{n} 1 = n \qquad \sum_{n=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d$  $S_n = \frac{n}{2} (2a + (n-1)d) \quad T_n = ar^{n-1} S_n = \frac{a(r^n - 1)}{r-1} \quad ; \qquad r \neq 1 \quad S_\infty = \frac{a}{1-r} ; -1 < r < 1$  $F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$  $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ y = mx + c  $y - y_1 = m(x - x_1)$   $m = \frac{y_2 - y_1}{x_2 - x_1}$   $m = \tan \theta$   $(x - a)^2 + (y - b)^2 = r^2$ In  $\triangle ABC$ :  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$   $a^2 = b^2 + c^2 - 2bc \cdot \cos A$   $area \Delta ABC = \frac{1}{2}ab \cdot \sin C$  $\sin(\alpha + \beta) = \sin \alpha . \cos \beta + \cos \alpha . \sin \beta \qquad \qquad \sin(\alpha - \beta) = \sin \alpha . \cos \beta - \cos \alpha . \sin \beta$  $\cos(\alpha + \beta) = \cos \alpha . \cos \beta - \sin \alpha . \sin \beta \qquad \qquad \cos(\alpha - \beta) = \cos \alpha . \cos \beta + \sin \alpha . \sin \beta$  $\int \cos^2 \alpha - \sin^2 \alpha$  $\cos 2\alpha = \left\{ 1 - 2\sin^2 \alpha \right\}$  $\sin 2\alpha = 2\sin \alpha . \cos \alpha$  $2\cos^2\alpha - 1$ 

 $(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$ 

$$\overline{x} = \frac{\sum fx}{n} \qquad \qquad \sigma^2 = \frac{\sum_{i=1}^n (x_i - \overline{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad \qquad P(A \text{ of } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\widehat{y} = a + bx \qquad \qquad b = \frac{\sum (x - \overline{x})(y - \overline{y})}{\sum (x - \overline{x})^2}$$