GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION

2022

10611

MATHEMATICS

PAPER 1

TIME: 3 hours

MARKS: 150

MATHEMATICS: Paper 1

10611E

10 pages + 1 information sheet

X05

P.T.O.

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, answers should be rounded-off to TWO decimal places, unless stated otherwise.
7. Number the answers correctly according to the numbering system used in the question paper.
8. An INFORMATION SHEET is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $2x(x^2 - 1) = 0$ (2)

1.1.2 $x - 6 + \dfrac{2}{x} = 0$; $x \neq 0$ (correct to TWO decimal places) (4)

1.1.3 $(x-1)(x+4) \geq 6$ (3)

1.1.4 $\sqrt{x-2} + 3 = \dfrac{10}{\sqrt{x-2}}$ (5)

1.2 Solve for x and y:

$x - 2y = 1$ and $2x^2 - xy - 5y - 3y^2 - 2 = 0$ (4)

1.3 Given: $2^{x+1} + 2^x = 3^{y+2} - 3^y$, where x and y are integers.
Determine the value of x and y. (3)

1.4 The equations $x^2 + rx + m = 0$ and $x^2 + mx + r = 0$ have real and EQUAL roots.
Solve for the values of r and m if $r > 0$ and $m > 0$. (6)

[27]

QUESTION 2

2.1 Given the quadratic sequence:

20; 12; 10; 14; …

2.1.1 Determine an expression for the n^{th} term of the pattern in the form $T_n = an^2 + bn + c$. (4)

2.1.2 The FIRST differences form an arithmetic sequence. Determine between which successive terms in the quadratic sequence, the FIRST difference will be 148. (3)

2.1.3 Determine the smallest value of n for which $S_n > 10140$ in the arithmetic sequence. (5)

2.2 If $\sum_{r=1}^{5}(r+b) = 10a$, determine b in terms of a. (3)

[15]

QUESTION 3

Given the following geometric sequence:

$$\frac{24}{x} + 12 + 6x + 3x^2 + \dots$$

3.1 Calculate the sum to infinity of the series. (4)

3.2 Write down the values of x for which this sequence converges. (2)

3.3 For which values of x will the series increase? (2)

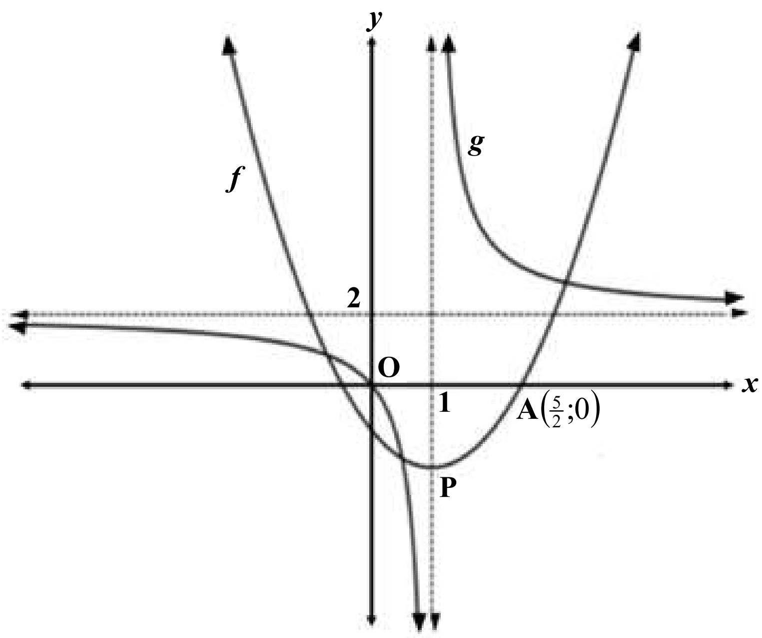
3.4 If $x = 4$, determine the sum of the sequence to 15 terms. (4)

[12]

QUESTION 4

The graphs of $f(x)=(x+p)^2+q$ and $g(x)=\dfrac{a}{x+b}+c$ are drawn in the sketch below and have the following properties:

- $A\left(\dfrac{5}{2};0\right)$ is a point on the graph of f.
- P is the turning point of f and lies on the vertical asymptote of g.
- The vertical asymptote passes through (1;0).
- The horizontal asymptote passes through (0;2).
- Graph g passes through the origin.



4.1 Show that the equation of g can be written as $y=\dfrac{2}{x-1}+2$. (3)

4.2 Calculate the coordinates of P. (3)

4.3 Write down the equation of the vertical asymptote of p if $p(x) = g(x - 1)$. (1)

4.4 Determine the equation of the axis of symmetry of g in the form $y = mx + c$, if $m < 0$. (2)

4.5 Write down the equation of k, if k is the reflection of g about the x-axis. (2)

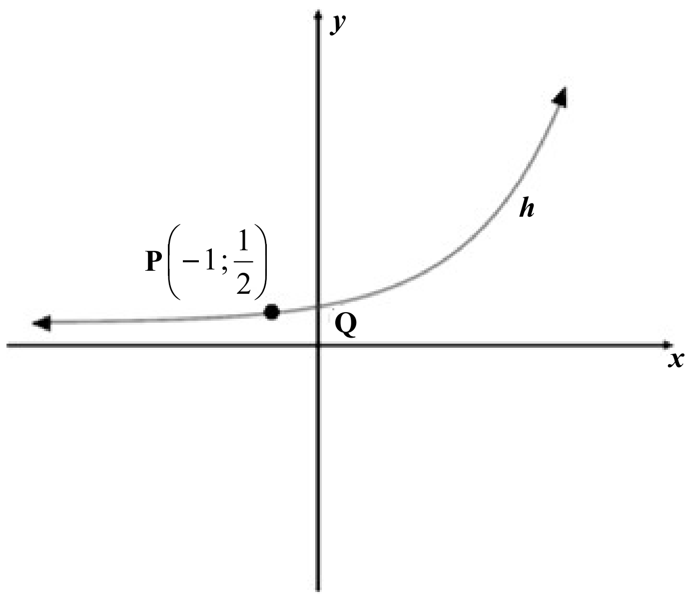
4.6 For which values of x is $f'(x).g(x) > 0$? (2)

4.7 For which values of k will the equation $g(x) = x + k$ have two real roots of opposite signs? (1)

[14]

QUESTION 5

The graph of $h(x) = a^x$, where $a > 0$, is sketched below. $P\left(-1\,;\,\frac{1}{2}\right)$ is a point on h.



5.1 Write down the coordinates of Q. (1)

5.2 Determine the value of a. (2)

5.3 Write down the equation for h^{-1} in the form $y = \ldots$ (2)

5.4 Sketch the graph of h^{-1} in your ANSWER BOOK. Clearly show all intercepts with the axes. (2)

5.5 Write down the domain of h^{-1}. (1)

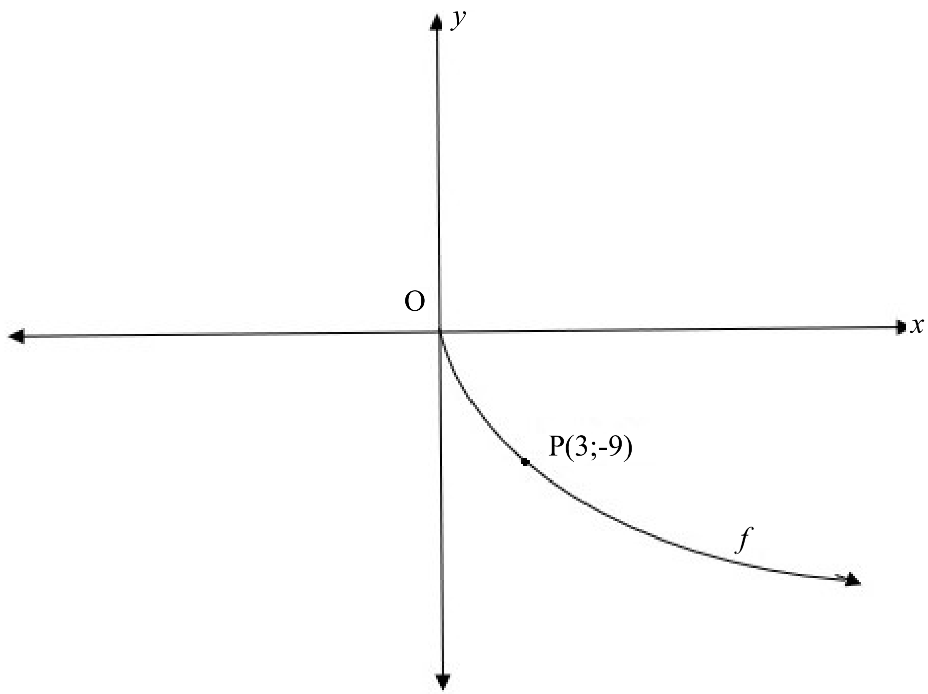
5.6 Hence or otherwise, determine the value(s) of x for which $\log_2 x > 1$. (1)

5.7 If $g(x) = (100).3^x$, calculate the value of x for which $h(x) = g(x)$. (3)

[12]

QUESTION 6

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below. The point P(3 ; –9) lies on the graph of *f*.



6.1 Use the graph to determine the values of x for which $f(x) \geq -9$. (2)

6.2 Write down the equation of f^{-1} in the form $y = \ldots$
Indicate ALL restrictions. (4)

6.3 Sketch the graph of f^{-1} in your ANSWER BOOK. Indicate all intercept(s) with the axes and the coordinates of ONE other point on your sketch. (3)

6.4 Describe the transformation from *f* to *g* if $g(x) = \sqrt{27x}$ where $x \geq 0$. (1)
[10]

QUESTION 7

7.1 At what annual percentage interest rate, compounded quarterly, should a lump sum be invested in order for it to double in 6 years ? (5)

7.2 Simon buys furniture to the value of R10 000. He borrows the money on 1 February 2020 from a financial institution that charges interest at a rate of 9,5% p.a. compounded monthly. He agrees to pay monthly installments of R450. The agreement of the loan allows him to pay equal monthly installments from the 1 August 2020.

7.2.1 Calculate the total amount owing to the financial institution on 1 July 2020. (2)

7.2.2 How many months will it take Simon to repay the loan? (4)

7.2.3 What is the balance of the loan immediately after Simon has made the 25^{th} payment? (2)

[13]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 + 2x$. (4)

8.2 Determine: $\frac{dy}{dx}$ if $y = 4\sqrt{x} - \frac{8}{\sqrt{x}} + \pi x^3$. (4)

8.3 The graph of $g(x) = ax^2 + \frac{b}{x} = 96$ has a minimum value at $x = 4$.

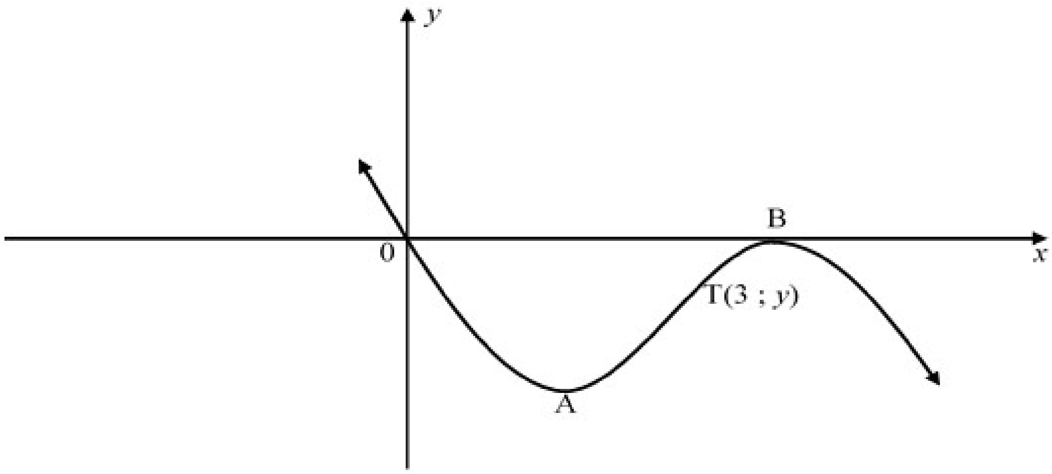
Calculate the values of a and b. (6)

[14]

QUESTION 9

The graph of $h(x) = -x^3 + ax^2 + bx + c$ is sketched below.

- A and B(6 ; 0) are the turning points of *h*.
- T(3 ; *y*) is a point on *h*.
- The graph of *h* passes through the origin.



9.1 Show that $a = 12$ and $b = -36$. (3)

9.2 Calculate the coordinates of A. (3)

9.3 Calculate the value of y. (1)

9.4 Is the graph of *h* concave up or concave down at point T?
Show ALL your calculations. (3)

9.5 Determine the coordinates of the point of inflection. (2)

[12]

QUESTION 10

A hotel has 72 rooms to let to clients. The daily rent per room is R500. If the rent is increased by R100 per day, 2 rooms can be left vacant daily.
The rent is Rx per room per day, where $x > 500$.

10.1 Show that the total daily income (I) is given by $I = 82x - \frac{x^2}{50}$. (5)

10.2 Calculate the maximum daily income per room. (2)

[7]

QUESTION 11

11.1 Events A and B are mutually exclusive.
It is given that:

- P(B) = 2P(A)
- P(A or B) = 0,57

Calculate P(B). (3)

11.2 A box of 40 calculators is sent to a store by a supplier. The owner of the store is not aware that 5 of the calculators are defective. Two calculators are selected at random from the box, the first one not being replaced before the second one is selected.

11.2.1 What is the probability that the first calculator chosen is NOT defective? (1)

11.2.2 What is the probability that if two calculators are selected, ONE calculator is defective and the other is not? (3)

11.2.3 What is the probability that if two calculators are selected, BOTH are defective? (3)

11.3 Four different Economics books and three different Life Sciences books are required to be placed on a shelf.

11.3.1 If you decide to place any book in any position, in how many different ways can you arrange the books on the shelf? (1)

11.3.2 If two particular books must be placed next to each other, in how many different ways can you arrange the books on the shelf? (1)

11.3.3 If all the Economics books must be placed next to one another and all the Life Sciences books must be placed next to one another, in how many different ways can you arrange the books on the shelf? (2)

[14]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f^{/}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}\ ;\frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m\left(x - x_1\right)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$\left(x-a\right)^2 + \left(y-b\right)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta\mathrm{ABC} = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad \mathrm{P(A\ or\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\ and\ B)}$

$\hat{y} = a + bx$ $\quad b = \frac{\sum\left(x - \bar{x}\right)\left(y - \bar{y}\right)}{\sum\left(x - \bar{x}\right)^2}$