GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATION

2023

10611

MATHEMATICS

(PAPER 1)

TIME: 3 hours

MARKS: 150

MATHEMATICS: Paper 1

10611E

10 pages + 1 information sheet

X05

P.T.O.

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera, that you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round-off answers to TWO decimal places, unless stated otherwise.
7. An INFORMATION SHEET with formulae is included at the end of the question paper.
8. Number the answers correctly according to the numbering system used in the question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(2x+1)^2 - 4 = 0$ (3)

1.1.2 $4x^2 - 11 = -12x$ (Correct to TWO decimal places) (3)

1.1.3 $15x - 4 < 9x^2$ (4)

1.1.4 $\sqrt{2x-2} - \sqrt{7-2x} = 1$ (5)

1.2 Solve the following equations simultaneously:

$a^2b^2 - 2ab - 8 = 0$ and $\log_2(a+5) = 3$ (5)

1.3 If $p = \dfrac{\sqrt{x+2}}{\sqrt{16-x^2}}$, for which values of x will p be real? (4)

[24]

QUESTION 2

2.1 The following is an arithmetic sequence:

$1 - p$; $2p - 3$; $p + 5$; …

2.1.1 Calculate the value of p. (2)

2.1.2 Write down the value of:

(a) The first term of the sequence (1)
(b) The common difference (1)

2.1.3 Explain why NONE of the numbers in this arithmetic sequence are perfect squares. (2)

2.2 The following sequence of numbers forms a quadratic sequence:

–3; –2; –3; –6; –11; …

2.2.1 The FIRST differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)

2.2.2 Calculate the first difference between the 35th and 36th terms of the quadratic sequence. (1)

2.2.3 Determine an expression for the n^{th} term of the quadratic sequence. (4)

2.2.4 Show that the sequence of numbers will NEVER contain a positive term. (2)

[16]

QUESTION 3

3.1 Given: $S_n = 4n^2 + 1$. Determine T_6. (3)

3.2 For which values of x will the following series converge?

$(4x-3) + (4x-3)^2 + (4x-3)^3 + \ldots$ (3)

3.3 Calculate: $\sum_{k=3}^{5}(-1)^k.\frac{2}{k}$ (2)

[8]

QUESTION 4

Consider the graphs of $g(x)=\frac{6}{x+3}-\frac{3}{2}$ and $h(x)=\frac{6}{x-3}+2$.

4.1 Write down the domain of g. (1)

4.2 Write down the range of h. (1)

4.3 If the graph of g is shifted so that it coincides with the graph of h,

4.3.1 how many units must the graph be shifted horizontally? (1)

4.3.2 how many units must the graph be shifted vertically? (1)

4.4 Write down the equations of the asymptotes of g. (2)

4.5 Calculate the x-intercept of g. (1)

4.6 Sketch the graph of g in your ANSWER BOOK.
Show clearly all asymptotes and intercepts with the axes. (3)

4.7 Determine the value of k if $h(x) = -x + k$ is an axis of symmetry of g. (2)

4.8 For which value(s) of x will $\frac{6}{x+3}-\frac{3}{2} > -x+k$? (1)

4.9 The graph of g is reflected in the x-axis.
Write down the new equation in the form $y = \ldots$ (1)

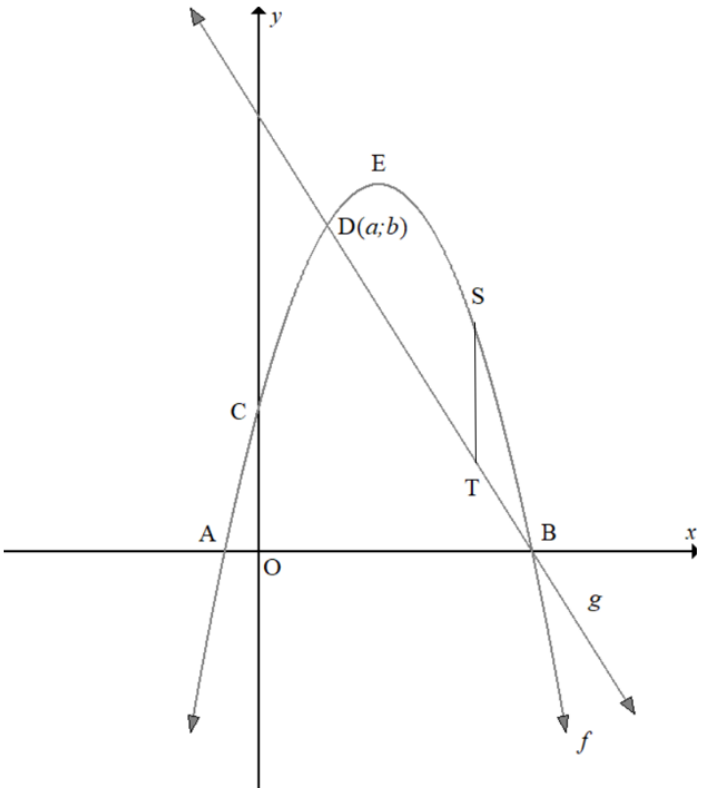
[14]

QUESTION 5

The graphs of $f(x) = -\left(x - \frac{7}{2}\right)^2 + \frac{81}{4}$ and $g(x) = -3x + 24$ are sketched below.

The graphs of f and g intersect at points D and B.

Points A and B are the x-intercepts of f.



5.1 Write down the coordinates of E, the turning point of f. (1)

5.2 Determine the average gradient of the curve of f between $x = 1$ and $x = 5$. (4)

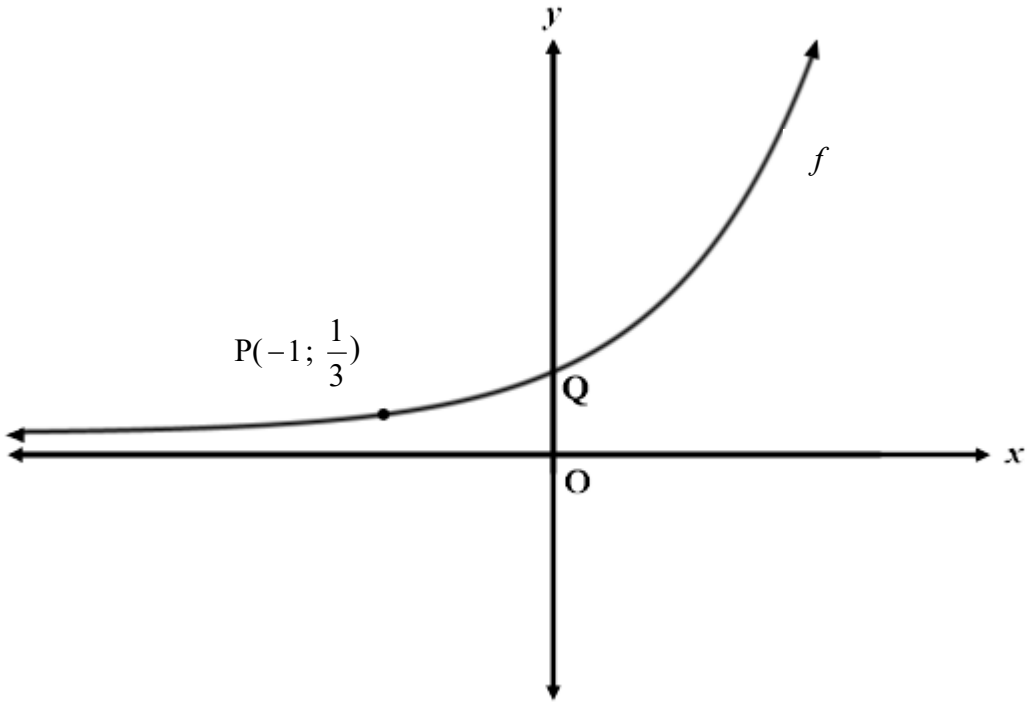
5.3 Calculate the value of a, the x-coordinate of point D. (3)

5.4 Point S(x ; y) is a point on the graph of f, where $a \leq x \leq 8$.
Line ST is drawn parallel to the y-axis with point T on the graph of g.
Determine ST in terms of x. (2)

5.5 Calculate the maximum length of ST. (3)
[13]

QUESTION 6

The graph of $f(x) = 3^x$ is sketched below. P$(-1; \frac{1}{3})$ is a point on f.



6.1 Write f^{-1} in the form $y = \ldots$ (1)

6.2 Sketch the graphs of $y = f^{-1}(x)$ and $y = f^{-1}(x-2)$ on the same set of axes in your ANSWER BOOK. Clearly indicate ALL intercepts with the axes. (4)

6.3 Use your graphs drawn in QUESTION 6.2 to solve for x if $\log_3(x-2) < 1$. (2)
[7]

QUESTION 7

7.1 A survey conducted in December 2015 determined that 5,7 million South Africans were living with HIV. The researchers used a model of exponential growth $A = P(1+i)^n$ to predict that there will be 6 million people living with HIV in December 2022.
Calculate, as a percentage, the annual rate of increase that the researchers used for the 7 years. (3)

7.2 Shimmy invests R4 000 000 into an account earning interest of 6% per annum, compounded monthly. She withdraws R30 000 per month. Her first withdrawal is exactly one month after she deposited the R4 000 000.

7.2.1 How many withdrawals of R30 000 will Shimmy be able to make? (5)

7.2.2 How many withdrawals will Shimmy be able to make if she changes the amount withdrawn per month to R20 000?
Substantiate your answer. (3)

7.3 Estrid opened a savings account with a single deposit of R1 000 on 1 April 2022. She then makes 18 monthly deposits of R700 at the end of every month. Her first payment is made on 30 April 2022 and her last payment on 30 September 2023. The account earns interest at 15% per annum, compounded monthly.
Determine the amount that should be in her savings account immediately after her last deposit is made (on 30 September 2023). (4)
[15]

QUESTION 8

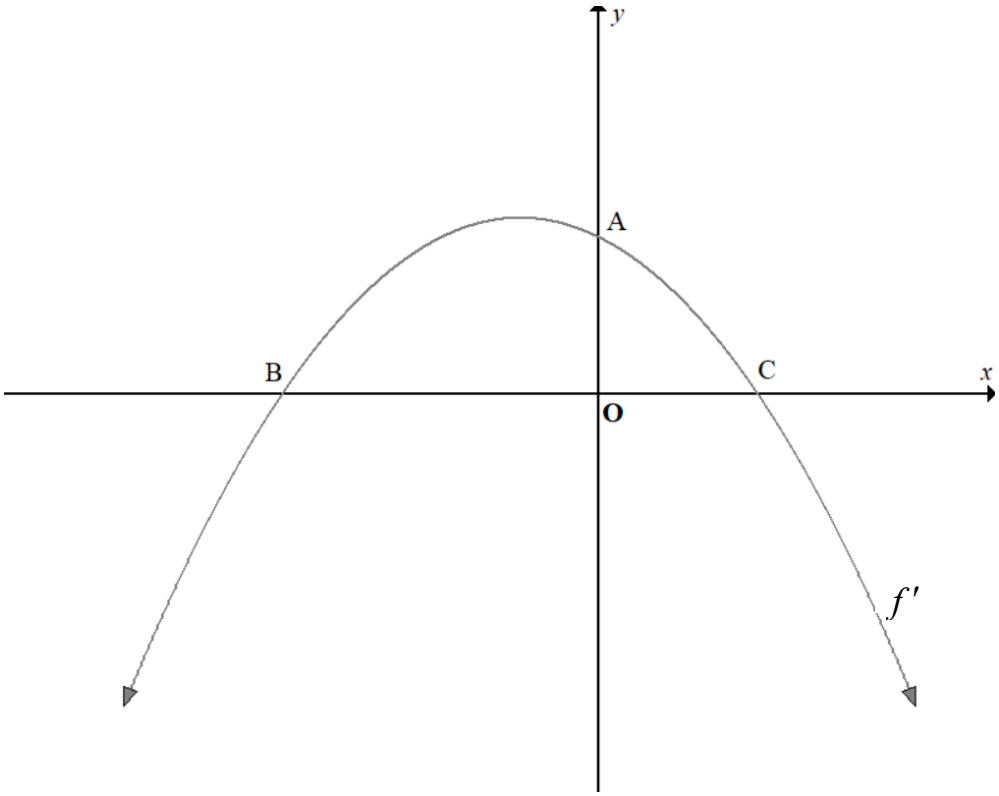
8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 6$. (4)

8.2 Determine the derivative of $f(x) = (2\sqrt{x} - \frac{1}{x})^2$. (5)

8.3 Given: $f(x) = 3x^3 - 3x^2 + 6x - 2$
Determine the interval for which f is concave up. (4)
[13]

QUESTION 9

9.1 Sketched below is the graph of f'. The derivative of $f(x) = -2x^3 - 3x^2 + 12x + 20$.
Points A, B and C are the intercepts of f' with the axes.



9.1.1 Write down the coordinates of A. (1)

9.1.2 Determine the coordinates of B and C. (3)

9.1.3 Which points on the graph of f will have exactly the SAME x-values as B and C? (1)

9.1.4 For which values of x will f be increasing? (3)

9.1.5 Determine the y-coordinate of the point of inflection of f. (4)

9.2 The tangent at P$(3\ ;\ -10)$, to the curve is given by $y = -x^2 - 1$, intersects the x-axis at point R. Line PT is drawn perpendicular to the x-axis with T on the x-axis.
Determine the length of RT. (5)
[17]

QUESTION 10

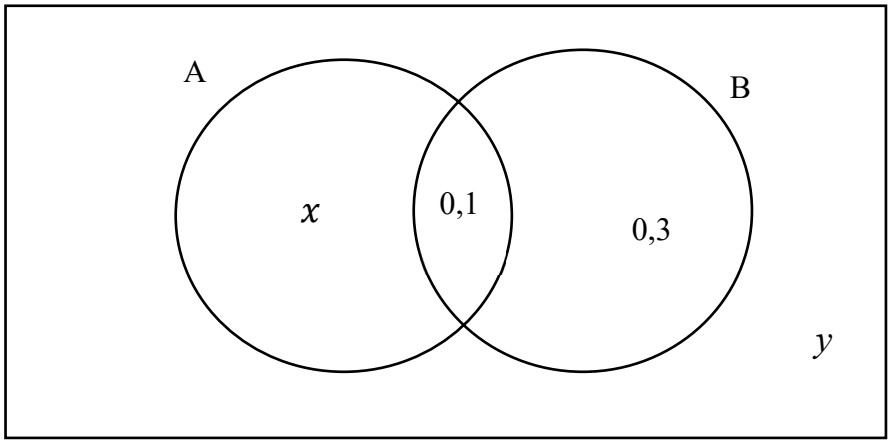
The profit, $W(x)$, earned by a company to manufacture x bicycles per week, is given by:

$$W(x) = -\frac{x^3}{150} + 3x^2 - 250x - 2700.$$

What should the weekly bicycle production be to maximise profit? **[8]**

QUESTION 11

11.1 Machine A and machine B are two different coin-pressing machines that operate at the same time. The probability that machine A ONLY presses a R5 coin, is x and the probability that machine B ONLY presses a R5 coin, is 0,3.
The probability that both the machines press R5 coins at the same time is 0,1.



11.1.1 If A and B are independent events, determine the values of x and y. (4)

11.1.2 Determine the probability that exactly one of the machines is pressing a R5 coin. (1)

11.2 Wilson takes a driver's test. The probability that he will succeed on his first attempt is $\frac{3}{7}$.

For each attempt that he redoes the test, the probability of passing increases to $\frac{3}{5}$.

11.2.1 What is the probability that Wilson will succeed after 2 attempts? (2)

11.2.2 Determine the probability that Wilson will succeed after 3 attempts. (2)
[9]

QUESTION 12

12.1 When Marge turned eight, her friends Emily, Klara, Cory, Liza, Shirley and Penny were invited to her birthday party. Marge and her friends sat in a row and played a game. In how many ways can they be seated if:

12.1.1 They sit in alphabetical order? (1)

12.1.2 Emily and Klara do NOT want to sit next to each other? (3)

12.2 The probability that a certain rugby team has all its players fit to play is 70%. The probability that they will win a game if all their players are fit is 90%. When they are not fit the probability of them winning becomes 45%. Calculate the probability of them winning the FIRST game. (2)

[6]

TOTAL: 150

END

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1};\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r};\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

area $\Delta\mathrm{ABC} = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $\quad \mathrm{P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)}$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$