THE LIMIT

LESSON 1

Q) What is The LIMIT ?

A) A LIMIT is The Value a Number gets Closer and Closer to.

Think of it like walking toward a wall.

If you Keep getting Halfway Closer:

$$1,\ 0.5,\ 0.25,\ 0.125,\ \ldots$$

you get Closer and Closer to 0.

$$\lim_{n\to\infty} \frac{1}{2^n} = 0$$

So the limit is 0.

In very few words:

A Limit is the Number Something is APPROACHING.

Q) Why is the Limit 0 for this Term: $\frac{1}{2^n}$ **?**

A) Because (2^n**) keeps getting bigger forever, so** $\frac{1}{2^n}$

Keeps Getting Smaller Toward 0.

Big Bottom → Tiny Value → 0.

Q) For the Term $\frac{1}{n}$ **, What must** n **before its Limit of Zero(0) is Reached ?**

$$\lim_{n \to ?} \frac{1}{n} = 0 , \qquad n = ?$$

What is the Value of n **?**

A) There is no finite value of (n) that makes

$$\boxed{\frac{1}{n} = 0}$$

True.

For every real counting number:

- if $n = 10$, then $\frac{1}{n} = 0.1$
- if $n = 100$, then $\frac{1}{n} = 0.01$
- if $n = 1000$, then $\frac{1}{n} = 0.001$

It gets smaller and smaller, but never actually becomes exactly 0.

What happens is:

$$\lim_{n\to\infty} \frac{1}{n} = 0$$

This means:

As n grows forever, $\frac{1}{n}$ gets closer and closer to 0.

So the answer to:

"What is the Value of (n)?" is: No Exact Number Exists

The "Value" is: $n \to \infty$

which means (n) must grow without end.

Historically, the Concept of a Limit came Before the Formal Concept of the Derivative.

1. Early ideas of LIMITS Came First (Ancient Greece)
Mathematicians like Eudoxus and Archimedes used methods very close to limits through the **Method of EXHAUSTION**.

They would approximate areas and volumes by using polygons with more and more sides:

$$n \to \infty$$

This is Essentially a Limit Idea:

$$\lim_{n \to \infty}$$

Even though they did not use Modern Notation, they were Reasoning about Approaching a Value Arbitrarily Closely.

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2. Derivative ideas appeared later (17th century)
The derivative emerged with calculus through the work of Isaac Newton and Gottfried Wilhelm Leibniz.

They studied **Rates of Change** and tangent lines.

The Derivative is Defined using a Limit:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This shows that, Conceptually the Derivative Depends on the Limit Process.

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3. Formal rigor came even later (19th century)
Mathematicians like Augustin-Louis Cauchy and Karl Weierstrass gave the precise formal definition of limit:

$$\lim_{x \to a} f(x) = L$$

and the Rigorous **ε–δ Definition**, which then made the Derivative Fully Rigorous.
