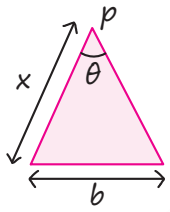
3 Units & Dimensions

MEASUREMENT OF LENGTH:-

Parallax Method } Used to measure large distance

Parallax Angle $= \frac{b}{x}$



$1^{\circ} = 1.745 \times 10^{-2}$ rad

$1' = 2.91 \times 10^{-4}$ rad

$1'' = 4.85 \times 10^{-6}$ rad

For v.small size:-optical, tunneling, electron microscope used:

$1AU = 1.496 \times 10^{11} m$

$1Ly = 9.46 \times 10^{15} m$

$1parsec = 3.08 \times 10^{16} m$

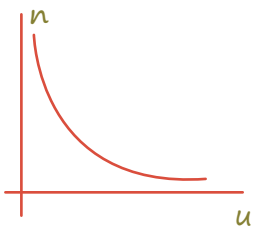
Size of $P^{+} = 10^{-15} m$

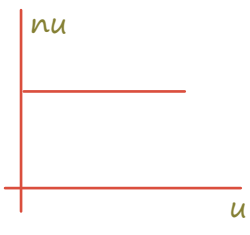
$R_{Earth} = 10^{7} m$

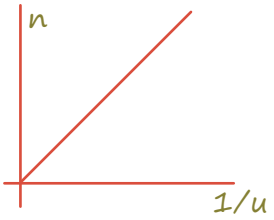
Distance of boundary of Observable Universe $= 10^{26}$ m

- nu = constant, n = measure value of P.Q., u = unit of that P.Q.

$n \propto \frac{1}{u}$







→ Only use to find value of physical quantity in new system of unit, if value is know in one unit.

MEASUREMENT OF MASS & TIME

- 1 amu $= \frac{1}{12}$ Mass of C^{12} atom
- 1 amu $= 1.66 \times 10^{-27}$ kg
- e^{-} mass $= 10^{-30}$ kg
- Earth mass:- 10^{25} kg
- Observable Universe $= 10^{55}$ kg.

Time:-

- Age of universe $= 10^{17}$ s
- Time span of Unstable particle → 10^{-24}s

Q. Convert 18 km/hr in m/s.

Ans. $n_1u_1 = n_2u_2$

18 km/hr $= n_2$ m/s

$\frac{18 \times 10^{3} m}{60 \times 60 s} = n_2$ m/s

$n_2 = 18 \times \frac{5}{9} = 10$

Q. If unit of length is y m in new system of unit then find value of x m^2 area in new system of unit.

Ans. un = cost

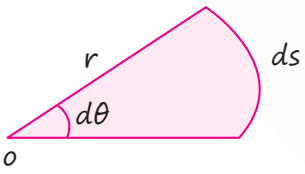
$n_1u_1 = n_2u_2$

$x m^2 = n_2 y^2 m^2$

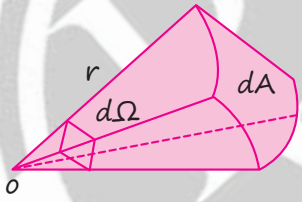
$n_2 = \frac{x}{y^2}$

SI SYSTEM

7 Base/Fundamental Units:-			
No.	Quantity	Unit	Symbol
1.	Length	Meter	m
2.	Mass	Kilogram	kg
3.	Time	Second	S
4.	Temperature	Kelvin	K
5.	Elec. Current	Ampere	A
6.	Luminous int.	Candela	cd
7.	Amt. of Sub^s	Mole	Mol
2 supplementary Units:-			
No.	Quantity	Unit	Symbol
1.	Plane Angle	radian	rad
2.	Solid Angle	Steradian	sr



$$d\theta = \frac{ds}{r}$$



$$d\Omega = \frac{dA}{r^2}\ sr$$

SIGNIFICANT FIGURES:-

1. All non-zero digits are significant.

   eg:- 42.3 → 3 S.F.

   243.4 → 4 S.F.

2. Zero b/n two non-zero digits is significant.

   eg:- 4.03 → 3 S.F.

   243.4 → 4 S.F.

3. Leading Zero or zeros placed to left are never significant.

   eg:- 0.543 → 3 S.F.

   0.006 → 1 S.F.

4. Trailing zeros or zero placed to the right of the number are significant.

   eg:- 4.330 → 4 S.F.

   343.000 → 6 S.F.

5. In exponential expression the numerical postion given the number of S.F.

   eg:- 1.32×10^{-2} → 3 S.F.

   1.32×10^{4} → 3 S.F.

ROUNDING OF:-

Addition & Substraction:-

Final result should have same no. of decimal placed as that of original no. with minimum no. of decimal places.

3.1421
0.241
0.09
3.4731

Ans:- 3.47.

Multiplication & Division:-

The no. of S.F. equals the smallest no. of S.F. in any of the original no.

51.028
× 1.31
66.84668

Ans:- 66.8

DIMENSIONAL ANALYSIS:-

Dimension of physical quantity are power to which units of base quantity are raised.

eg:- $[M]^a [L]^b [T]^c [A]^a [K]^C$

-:Applications:-

1. Checking the Correctness of various formulae:-

   eg:- Z = A + B

   [Z] = [A] = [B]

2. Conversion of one system of unit to other.

   $n_1 U_1 = n_2 U_2$

   eg:- $n_1 [M_1^A L_1^B T_1^C] = n_2 [M_2^A L_2^B T_2^C]$

$$n_1 = n_2 \left[\frac{M_2}{M_1}\right]^A \left[\frac{L_2}{L_1}\right]^B \left[\frac{T_2}{T_1}\right]^C$$

- Mass → M
- Length → L
- Time → T
- Velocity → LT^{-1}
- Acceleration → LT^{-2}
- Force → MLT^{-2}
- Energy → ML^2T^{-2}
- Power → ML^2T^{-3}
- Force gradiant → MT^{-2}

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Different physical quantity ka dimension nikalne ke liye force and energy ka dimension yad rakna hai. Avi tension nahi lena aage ke chapter ke sath yad hota jayga.i

3. Formula of force to find dimension of different PQ.

$F = G\frac{m_1m_2}{r^2}$, $F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$

$F = Kx$, $F = \frac{\mu_0 I_1 I_2 d}{4\pi r}$

$F = qE$

$F = qvB$

$F = 6\pi\eta rv$

$F = SI$

Formula of energy to find dimⁿ of different physical quantity	
$E = hf$	$\frac{H}{t} = \frac{kA\Delta T}{l}$
$E = \frac{3}{2}k_BT$	Stress = γ × Strain
$E = \frac{Q^2}{2C}$	$Q = ms\Delta T$
$H = I^2Rt$	$Q = mL$
$E = \frac{1}{2}LI^2$	
$PV = nRT$	

DIMENSIONAL FORMULA

- Pressure=stress=Young's modules=$ML^{-1}T^{-2}$
- Work = Energy = Torque = ML^2T^{-2}
- Power P = ML^2T^{-3}
- Gravitational constant G = $M^{-1}L^3T^{-2}$
- Force constant = Spring constant = MT^{-2}
- Coefficient of viscosity = $ML^{-1}T^{-1}$
- Latent heat L = L^2T^{-2}
- Electric potential = $\frac{P}{I}$ = $ML^2T^{-3}A^{-1}$
- Resistance = $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ = $ML^2T^{-3}A^{-2}$
- Capacitance = $M^{-1}L^{-2}T^4A^2$
- Permittivity ε_0 = $M^{-1}L^{-3}T^4A^2$
- Angular momentum = planck's constant = $M^1L^2T^{-1}$

Time Period:-

$$T\alpha\sqrt{\frac{L}{g}}\alpha\sqrt{\frac{M}{k}}\alpha\sqrt{\frac{R}{g}}$$

$$\frac{L}{R} = RC = \sqrt{LC}$$

MR*

Resistance = R = $\omega L = \frac{1}{\omega C}$

$\omega = \frac{2\pi}{T}$

Time = $\frac{L}{R} = \sqrt{LC} = RC$

Dimensionless Quantities:-

1. Strain
2. Refractive index
3. Relative density
4. Plane Angle
5. Solid Angle
6. Poissons ratio
7. Exponential function
8. Trigonometry function
9. Relative permittivity
10. Pure number
11. Efficiency
12. Current, voltage, power gain
13. Length gratiant
14. Coef. of friction

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Pressure = Stress = Young modulus

= Bulk modulus

$= \frac{1}{2}$ strain × stress = modulus of regitity

$= \frac{B^2}{2\mu_o} = \frac{1}{2}\varepsilon_o E^2$ = energy density $= \frac{nRT}{V}$

dimensionally addition, substraction ko equal le ke solve karte hai.

Kisi be dimensionless function ya quantity ko one likh sakte hai.

Q. If velocity $V = Ax + Bt + C$ find dimension of A, B and C.

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Ans. $V = Ax = Bt = C$

$A = \frac{V}{x} = T^{-1}$

$B = \frac{V}{t} = LT^{-2}$

$C = V = LT^{-1}$

Q. Force $F = \alpha e^{-\beta t}$ then find dimension of α and β.

Ans. $F = \alpha \Rightarrow \alpha = MLT^{-2}$ | $\beta t = 1 \Rightarrow \beta = T^{-1}$

Q. Acceleration $a = \alpha t + \frac{\beta}{t-\delta}$ find dimension of α, β and δ.

Ans. $a = \alpha t = \frac{\beta}{t-\delta}$ ← MR* Ka feel

$\Rightarrow \alpha = \frac{a}{t} = LT^{-3}$ $\Rightarrow \delta = t$

$\Rightarrow a = \frac{\beta}{t}$

$\beta = \alpha t = LT^{-1}$

Q. Fill in the blanks with correct statement, according to given statement

Dimension	(1)	(2)	(c) A physical quantity have dimension	(d) A physical quantity does not have dimension
Unit	(a) A physical quantity have unit	(b) A physical quantity does not have unit	(3)	(4) .....

MR*

Ans.(1) May have dimension/may be dimensionless

(2) Must be dimensionless/does not have dimension

(3) Must have unit

(4) May or may not have unit.

Q. Fill in the blanks with correct statement, according to given statement

Physically correctness	(1)	(2)	(c) Equation is physically wrong	(d) Equation is physically correct
Dimensional correctness	(a) Equation is dimensional wrong	(b) Equation is dimensional correct	(3)	(4) .....

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Ans. (1) Must be physically wrong

(2) May or may not physically correct

(3) May or may be dimensionally correct

(4) Must be dimensionally correct.

$S_{n^{th}} = u + \frac{a}{2}(2n - 1)$

($S_{n^{th}}$ → dimensionally correct because it is displacement in one sec.)

Q. If force, acceleration and time taken as fundamental physical quantity then find dimension of energy?

(a) $F^2A^{-1}T$ (b) FAT^2

(c) $F^{-1}AT^{-2}$ (d) $FA^{-1}T$

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$E(ML^2T^{-2})$ → Mass ka dimension force hi dega ek mass energy me hai to F^1 hona chahiye.

Now L ka square hai ek length force dega ek acceleration hence A^1 hona chahiye.

Q. Planks constant (h), speed of light (c), gravitational constant (G) taken as fundamental quantity then dimension of length in terms of them.

(a) $\frac{\sqrt{hG}}{c^{3/2}}$ (b) $\sqrt{\frac{hc}{G}}$

(c) $\frac{\sqrt{hG}}{c^{5/2}}$ (d) $\frac{\sqrt{Gc}}{h^{3/2}}$

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$M^0T^0l = h^Xc^YG^Z$

We need dimension of length, then mass should be cancell out by arranging h, c and G. c me to mass hai nahi; h → ML^2T^{-3} and G = $M^{-1}L^3T^{-2}$ to h and G ko multiply karne se mass kat jayga. Hence option (b) and (d) wrong ho gaya. Now option (a) and (c) dono me root hai to root laga ke sirf length ka dimension likho phir c se divide kar ke ek length (L^1) sirf rakho.

Q. Dimension of critical velocity V of liquid flowing through the tube are expressed as $\eta^x\delta^yr^z$, where η is coefficient of viscosity, δ is density of liquid and r is radius of the tube then the value of x, y and z are given by.

(a) 1, 1, 1 (b) 1, −1, −1

(c) −1, −1, 1 (d) −1, −1, −1

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Velocity me mass hai nahi to η, δ and r ko arrange kar velocity lena hai hence mass cancell, radius me bhi mass nahi hai, $\delta = ML^{-3}$ and $\eta = ML^{-1}T^{-3}$

δ and η divide karne se mass kat jayga to ek ka power positive ek ka negative hona chahiya.

Q. If energy (E), velocity (V) and time (T) are chosen as the fundamental quantities the dimensional formula of surface tension will be

(a) EV^2T^2 (b) EV^2T^{-2}

(c) $EV^{-2}T^{-2}$ (d) $E^{-2}V^2T^2$

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MR* → Surface tension (MT^{-2}) Ramlal yaha length nahi to length katne ka socho. Sirf (c) me length kat ho raha hai.

Limitation of dimensional analysis:

(1) It is not use to derive dimensionless physicall quantity and constant.

(2) This can not decide weather the give quantity is vector or a scalar.

(3) It can not be use to derive an equation involving more than three physicall quantity.

(4) It can not derive dimensionless function having $\sin\theta$, $\cos\theta$, e^x etc.

(5) Can not use if one quantity depends on two other quantity having same dimension.

(6) It can not derive equation which cantain +ve and −ve terms.

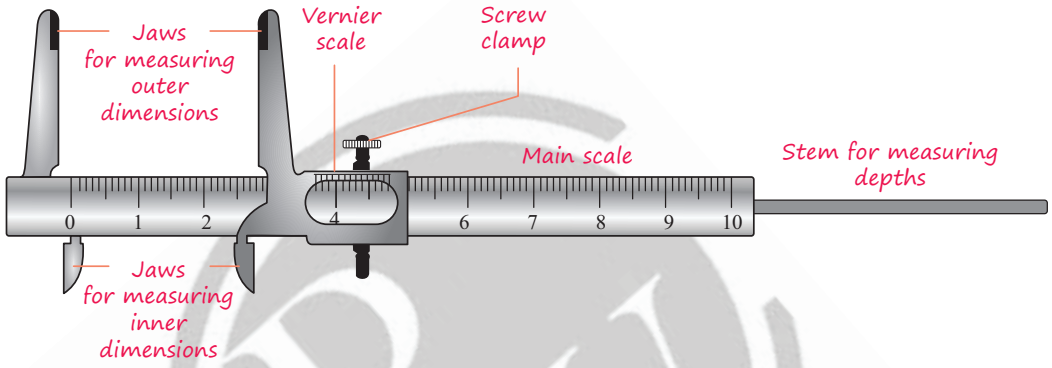
INSTRUMENTS

Least Count:-

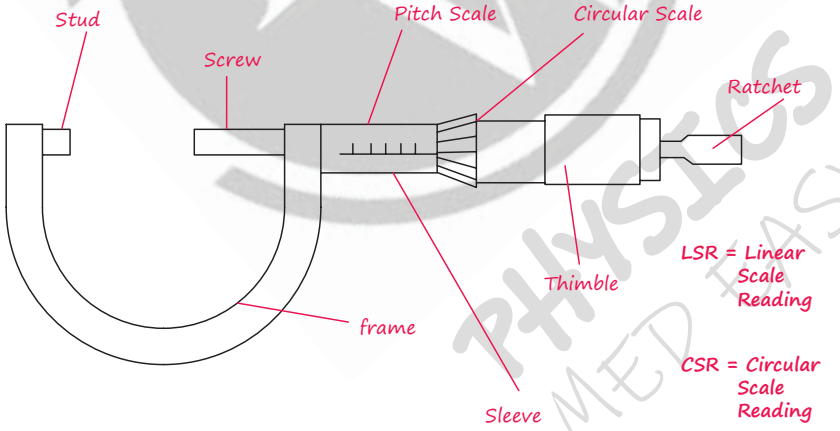
mm Scale	Vernier Scale	Screw Gauge
↓	↓	↓
1mm	0.1mm	0.01mm

Vernier calipers:-

L. C. = 1MSD - 1VSD



Screw gauge:-



$$\text{Pitch} = \frac{\text{MSR}}{\text{no. of rotation}}$$

$$\text{L.C.} = \frac{\text{Pitch}}{\text{Total no. of division on Circular Scale}}$$

$$\text{Total Reading} = 1\ \text{LSR} + \text{CSR} \times \text{LC}$$

If nVsD Coincides with (n-1) MSD then:-

$$(n-1)\ \text{MSD} = n\text{VSD}$$

$$1\ \text{VSD} = \frac{n-1}{n}\ \text{MSD}$$

$$\text{LC} = 1\ \text{MSD} - \frac{n-1}{n}\ \text{MSD} = \frac{1\text{MSD}}{n}$$

$$\text{Total Reading} = 1\ \text{MSR} + \text{coinciding}_{\text{VSR}} \times \text{LC}$$

Measured length	Used instrument
1.56 cm	vernier calliper
6.8 cm	metre scale
8.96 mm	screw gauge
9.812 cm	screw gaye
8.3 mm	vernier calliper

Accuracy: It is the measure of how close the measured value is to the true value. Closness of measured and true value.

Precision: It tells us to what resolution or limit the quantity is measured.

Q. If true value of length is 6.57 m then which of the following reading is most accurate and most precise.

(a) 6.52 m (b) 6.61 m

(c) 6.513 m (d) 6.68 m

Ans. Most accurate (b), most precise (c)

ERROR IN MEASUREMENT:-

Difference between true value & measured value of a quantity

- **Systematic Errors** — Errors which tend to occur only in one direction, either positive or negative
  - **Instrumental** — Due to inbuilt defect of measuring instrument
  - **Experimental** — Limitation in experimental technique
  - **Personal** — Due to individual bias, Lack of proper setting of apparatus
- **Random Errors** — Irregular and at random in magnitude & direction

Absolute Error:- $\Delta a = |a_i - a_{mean}|$

$$a_{mean} = \frac{a_1 + a_2 \ldots + an}{n}$$

- Always positive
- Unit and dimension same as physical quantity
- Least count error can be taken as absolute error
- It can not tell about accuracy of measurment

Relative Error:- $\frac{\Delta a_{mean}}{a_{mean}}$

$$\Delta a_{mean} = \frac{\Delta a_1 + \Delta a_2 \ldots + \Delta a_n}{n}$$

$$\text{Percentage Error} = \frac{\Delta a_{mean}}{a_{mean}} \times 100$$

- Unit and dimension less
- It tell about accuracy of measurment
- Random error can be decreas by decreasing no of obserbation.

$nx = cost^n$, n = no. of obserbation, x = Random error

- In 5 reading random error is 3% and systematic error is 4%. It we increased no. of observation to 30 then random error 1/2% and systematic error remains 4%.

General Rule:-

$$Z = \frac{A^p B^q}{C^r},$$

Then max. frac^n relative error in Z will be:-

$$\frac{\Delta Z}{Z} = P\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

Combination of errors:-

Operations	Formula Z	Absolute error ΔZ	Relative error $\Delta Z/Z$	Percentage error $100 \times \Delta Z/Z$
Sum	$A + B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A + B}$	$\frac{\Delta A + \Delta B}{A + B} \times 100$
Difference	$A - B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{A - B}$	$\frac{\Delta A + \Delta B}{A - B} \times 100$
Multiplication	$A \times B$	$A\Delta B + B\Delta A$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A + A\Delta B}{B^2}$	$\frac{\Delta A}{A} + \frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$
Power	A^n	$nA^{n-1}\Delta A$	$n\frac{\Delta A}{A}$	$n\frac{\Delta A}{A} \times 100$
Root	$A^{1/n}$	$\frac{1}{n}A^{((1/n)-1)}\Delta A$	$\frac{1}{n}\frac{\Delta A}{A}$	$\frac{1}{n}\frac{\Delta A}{A} \times 100$

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- Addition/Substration me pahle absolute error nikalenge phir relative.
- Power/multiplication/division me pahle relative error nikalenge phir absolute.

Example: $y = 3A^2$ ← Power hai to direct relative error likho, constant ko remove karo, power ko aage multiply kar do.

$$\frac{\Delta y}{y} = 2 \times \frac{\Delta A}{A}$$

Example:

$$y = \frac{2A^4\sqrt{B}}{C^3}$$

$$\frac{\Delta y}{y} = 4 \times \frac{\Delta A}{A} + \frac{1}{2} \times \frac{\Delta B}{B} + 3 \times \frac{\Delta C}{C}$$

Q. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°) then the least count of the instrument is:

(a) one minute (b) half minute

(c) one degree (d) half degree

Ans. MSD = 0.5°

30 VSD = 29 MSD

Least Count

= Length of 1 main scale division / No. of divisions of Vernier scale

= 0.5° / 30

= 0.5 × 60min / 30

= 1 min

Q. A vernier callipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier callipers, the least count

(a) 0.02 mm (b) 0.05 mm

(c) 0.1 mm (d) 0.2 mm

Ans. MSD = 1 mm

20 VSD = 16 MSD

⇒ VSD = 16/20 MSD = 0.8 MSD = 0.8 mm

Least Count = MSD−VSD = 1−0.8 = 0.2 mm

Q. If the error in the measurement of area of sphere is 3% then find percentage error in measurement of volume of sphere

Ans. $A = 4\pi r^2$ $\quad V = \frac{4}{3}\pi r^2$

$$\frac{\Delta A}{A} = 2\frac{\Delta r}{r} \quad ...(1)$$

$$\frac{\Delta V}{V} = 3\frac{\Delta r}{r} \quad ...(2)$$

(ii)/(i) $\frac{\Delta V}{V} \Big/ \frac{\Delta A}{A} = \frac{3}{2}$

$$100 \times \frac{\Delta V}{V} = \frac{3}{2}\frac{\Delta A}{A} \times 100 = \frac{3}{2} \times 3 = 4.5\%$$

Q. If $T = 2\pi\sqrt{\frac{\ell}{g}}$ then find percentage error in measurement of acceleration due to gravity.

Ans. Ignore constant

$$T^2 = \frac{\ell}{g},\ g = \frac{\ell}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}$$

MR*

खुल जायेंगे सभी रास्ते,
तू रुकावटों से लड़ तो सही।
सब होगा हासिल,
तू अपनी जिद्द पर अड़ तो सही॥