4 Motion in 1-D

Observer & Frame of Reference:-

- Observer → who takes observation and from where it takes is called frame of reference.
- Observer always assume to be at rest.
- Nothing is at absolute rest or in absolute motion.
- Agar koi Gadhe pr baitha hai toh

  Gadha:- Frame of Ref. Uske upar joh baitha hoga woh observer!

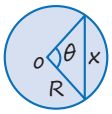
Distance	Displacement
Total Path length	Shortest Path b/w initial and final position
Scalar, Struggle	Always straight line
Can't decrease with time	Vector, success
Always positive	Direction - From initial to final position
Depends on path taken	Can decrease with time
Both have same unit and dimension	May be +ve or –ve
	Does not depends on path

- If we know only initial and final position then we can't calculate distance but can find displacement.
- If initial position (x_1, y_1, z_1) and final position (x_2, y_2, z_2) then displacement $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

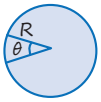
$\Rightarrow$ distance $\geq |$displacement$|$

DISTANCE & DISPLACEMENT ON CIRCULAR PATH

- $\underset{(x)}{Disp^m} = 2R\sin\left(\frac{\theta}{2}\right)$



- Arc = $dist^n$ = $R\theta$

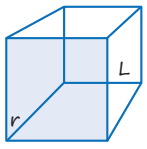


Displacement (2nd floor)	$Disp^m$ must be zero	$Disp^m$ may or may not be zero	If $disp^m$ is zero	If $disp^m$ is not equal to zero then:-
	⇑	⇑	⇓	⇓
Distance (1st floor)	If $dist^n$ is zero	If $dist^n$ is not equal to zero then	$Dist^n$ may or may not be zero	$Dist^n$ must not equal to zero

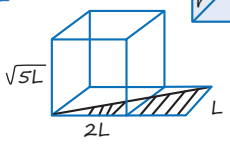
CHALLENGER QUESTION:-

There is a cubical room. One insect is moving from one corner to other body. Diagonal, then find minimum distance

- Can Fly:- Body Diagonal $= \sqrt{3}L$

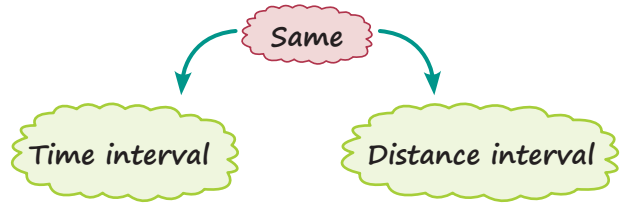


- Can't Fly:-



AVERAGE SPEED (HOW FAST IN AN INTERVAL NOT AT INSTANT):-

$$V_{avg} = \frac{\text{Total } dist^n}{\text{Total time}}$$



$S_{avg} = \frac{V_1 + V_2}{2}$ $\quad$ $S_{avg} = \frac{2V_1 V_2}{V_1 + V_2}$

$t_1 = t_2 = t$ $\quad$ $x_1 = x_2 = x$

$S_{avg} = \frac{V_1 t_1 + V_2 t_2}{t_1 + t_2}$ $\quad$ $S_{avg} = \frac{X_1 + X_2}{\frac{X_1}{V_1} + \frac{X_2}{V_2}}$

SPEED ((How Fast) Scalar, unit: m/s, only magnitude.):-

- Inst. $\quad$ $S_{inst} = \frac{dx}{dt}$
- Average $\quad$ $S_{avg} = \frac{\int S.\,dt}{\int dt}$

$\int \square = \int \frac{\square.\,dt}{\int dt}$

For Uniform motion :- $S_{avg} = S_{inst.}$

VELOCITY (How fast and where):-

Hum kitna Tez bhag rahe hai and kis direction me bhag rhe hai !

Inst. $\quad$ $\vec{V}_{inst.} = \frac{d\vec{x}}{dt}$

Average $\quad$ $V_{avg} = \frac{\int v.dt}{\int dt}$

= Rate of change in position
= Slope of position time graph
= Inst. speed × direction
= How fast × where

On circular path, $V_{avg} = \frac{V\sin(\theta/2)}{(\theta/2)}$

$|\text{Avg speed}| \geq |\text{Avg Velocity}|$

Inst speed = $|\text{Inst Velocity}|$

UNIFORM MOTION:-

- Body moving with constant speed in fixed direction
- Uniform velocity
- Acceleration zero
- Avg. velocity = Inst. velocity
- Must be straight line

NON-UNIFORM MOTION:-

- Velocity non-uniform
- Acceleration non-zero
- Velocity can be change by changing speed only or direction only or both
- In non-uniform speed may constant

  Dimag me set feel ke sath.
- If velocity is uniform then → speed must be uniform.
  - Velocity = Speed + Direction = Constant
- If velocity is variable → Speed may or may not be variable
  - Velocity ko sirf direction change kar ke vary kar sakte hai
- If speed is uniform → then velocity may uniform
  - Direction ka nahi pata.
- If speed is variable → then velocity must be variable
- If avg. velocity is zero then avg. speed may or may not be zero.
- If avg. speed is zero, then avg. velocity must be zero.

ACCELERATION:-

Ye Motion Ka Feel Nai Hai ! Ye velocity me change ka feel hai.

- Accl^n opposite to motion is retardation.
- Negative acceleration does not mean retardation, retardation may be positive or negative.
- Per-sec velocity inject to body or per-sec velocity extract from body ka feel hai.
- Vector → direction of acceleration along change in velocity.

$$\vec{a}_{inst} = \frac{dv}{dt} = \frac{v.dv}{dx} = \frac{d^2x}{dt^2}$$

$$\vec{a}_{avg} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} = \frac{\int_{t_1}^{t_2} \vec{a}_{ins}\, dt}{\int_{t_1}^{t_2} dt}$$

$\frac{d\vec{v}}{dt} = \vec{a}$ = The rate of change in velocity

$\left|\frac{d\vec{v}}{dt}\right| = |\vec{a}|$ = Magnitude of acc^n, $\frac{d|\vec{v}|}{dt}$ = Rate of change in speed.

u → a ← $\theta = 180°$ $\vec{a}.\vec{u}$ = -ve speed↓ retardation Tangential acc^n	u → a → $\theta = 0°$ $\vec{a}.\vec{v}$ = +ve speed↑ Tangential acc^n	u → 90° a ↓ $\theta = 90°$ $\vec{a}.\vec{v} = 0$ at this instant only direction will change normal or centripetal acc^n

MR*

Bade aaram se

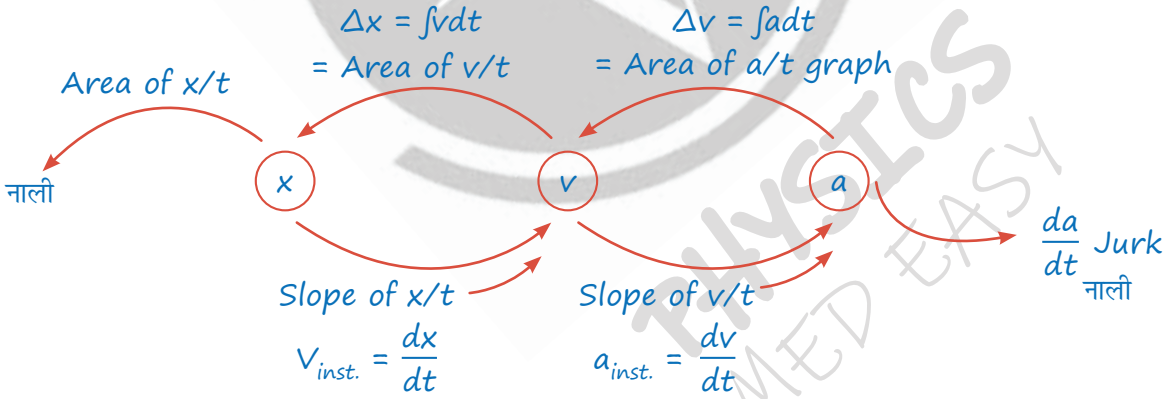
Uniform or constant non-zero acceleration.
Position (x) $\propto t^2$
Velocity (v) $\propto t$
Velocity v $\propto \sqrt{x}$
If acceleration zero then velocity must be non zero constant

MR*

approach to solve question

$\vec{a} = 0$	$\vec{a}$ = non zero constant	$\vec{a}$ = variable
$\vec{v}$ = constant uniform motion $S = vt$	Equation of motion applicable	Eq^n of motion is not applicable do integration or differentiation

MR*



Q. Which of the following is correct for velocity and acceleration?

(a) Velocity increasing, acceleration decreasing

(b) Velocity decreasing, acceleration increasing

(c) Both increasing

(d) Both decreasing

(e) All of these

Ans. (e)

Q. If position $x = at^2 - bt^3$. Find the time when acceleration is zero ?

Ans. $x = at^2 - bt^3$

$v = \frac{dx}{dt} = 2at - 3bt^2$

$a = 2a - 6bt = 0$

$2a = 6bt$

$t = \frac{a}{3b}$

Q. If velocity $v \propto \sqrt{x}$ then which of the following function is correct for position time relation.

(a) $x \propto t$

(b) $x \propto t^2$

(c) $x \propto \sqrt{t}$

(d) $x \propto t^{3/2}$

Ans. MR* question me acc^n constant then option me acceleration constant option (b)

Q. If acceleration $a = \beta t^{3/2}$ then find velocity after time t if intial velocity is u.

Ans. Equation of motion is not valid

$a = \frac{dv}{dt} = \beta t^{3/2}$

$\int_u^v dv = \beta \int t^{3/2}\, dt$

$v - u = \frac{\beta t^{5/2}}{5/2}$

Q. If acceleration of object $a = \beta x^2$ then find velocity after x displacement, if initial velocity was zero.

Ans. $a = v\frac{dv}{dx} = \beta x^2$

$\int_0^v v dv = \int_0^x \beta x^2 dx$

$\frac{v^2}{2} = \frac{\beta x^3}{3}$

$V = \sqrt{\frac{2\beta x^3}{3}}$

MR SPECIAL *

Majduri se duri MR hai jaruri

Position Ke Formula Mein Time ke dono term ko dekho agar dono term +ve/+ve ya −ve/−ve sign rakhta hai toh woh U-turn Nai lenge ya dist^n = |disp^m| agar sign +ve/-ve Rahi toh U-turn lenge aur distance ≠ | disp^m|

yaad rahe 1-D mein U-turn keliye rukhna hoga (v = 0) ∴ dist^n, ≠ disp^n

Note: To calculate dist^n, disp^m from x-t eq^n :

Ex. $x = t^2 - 4t + 8$ then take v-t graph, plot it using "v" eq^n which we"ll get by differentiating "x" eq^n & then put time given from t_1 & t_2 & see graph calculate dist^n/ disp^m.

- Moving Frame se body ko drop/release karne pr frame ka velocity share hojata hai but accl^n nai!

MOTION WITH CONSTANT ACCELERATION :-

$\vec{v} = \vec{u} + \vec{a}t$

$v^2 - u^2 = 2\vec{a}\vec{s}$

$\vec{s} = \vec{x}_f - \vec{x}_c = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{V}_{Avg} = \frac{\vec{v} + \vec{u}}{2}$ $\quad S = \frac{\vec{u} + \vec{v}}{2} \times t$

$S_{nth} = u + \frac{1}{2}(2n - 1)$

$$\frac{S_{n^{th}}}{S_n} = \frac{2n-1}{n^2} = \frac{2}{n} - \frac{1}{n^2}$$

Q. Object starts from rest and constant acceleration attained velocity 32 m/s in 10 sec. then find displacement in next 10 sec.

Ans. $S = \frac{u + v}{2} \times t \Rightarrow S = \frac{0 + 32}{2} \times 10$

$S = 160$ m in 1^{st} 10 sec.

Hence in next 10 sec.

it is $3 \times 160 = 480$ m

Q. If velocity of object $V = \sqrt{25 - 8x}$ then find velocity and acceleration.

Ans. Acceleration is constant then compare velocity with 3rd equation of motion

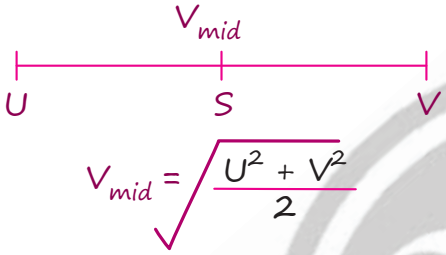
$v^2 = 25 - 8x$ or $v^2 = u^2 + 2ax$

$u^2 = 25$ $\quad -8x = 2ax$

$u = 5$ m/s $\quad 2a = -8$

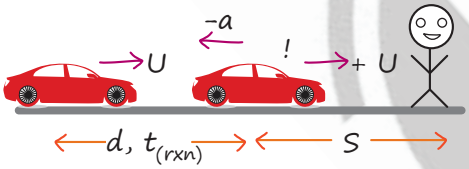
$a = -4$ m/s^2

Note:-



$$V_{mid} = \sqrt{\frac{U^2 + V^2}{2}}$$

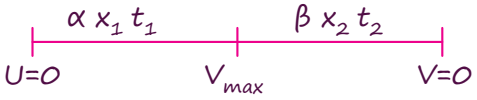
Stopping Distance:-



$d = Ut_{rxn}$ (Reaction time)

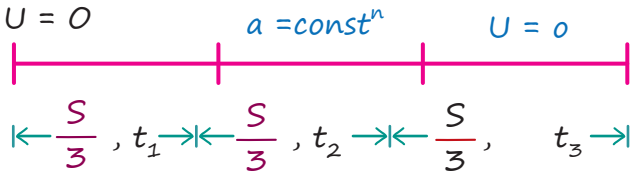
$$S = \frac{U^2}{2a}$$

Rest To Rest Motion:-



$\alpha t_1 = \beta t_2$ $\quad V_{max} = \left[\frac{\alpha\beta}{\alpha+\beta}\right] T$

$\alpha x_1 = \beta x_2$ $\quad s = \frac{1}{2}\left[\frac{\alpha\beta}{\alpha+\beta}\right] T^2$



Ratio of time for equal dist^n interval:-

$$t_1 : t_2 : t_3 = 1 : \sqrt{2}-1 : \sqrt{3}-\sqrt{2}$$

Ratio of disp^m for equal time interval:-

$$S_{1st} : S_{2nd} : S_{3rd} = 1 : 3 : 5$$
$$S_{1s} : S_{2s} : S_{3s} = 1 : 4 : 9$$

$S_t : S_{next\ t} = 1 : 3$ or $x : 3x$

Ratio of displacement in time t and next same time intraval t, where motion starts from rest and constant acceleration

$S_t : S_{2t} = 1 : 4$ or $x : 4x$

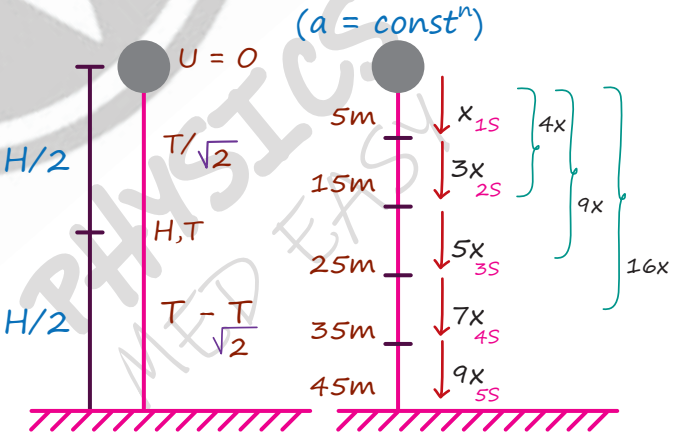
displacement in time t total time (2t)

Q. Object starts from and constant acceleration moves 80 m in 7 sec. then find displacement in next 7 sec.

Ans. Displacement in next 7 sec = 3x

= 3 x 80 = 240 m

MOTION UNDER GRAVITY:-



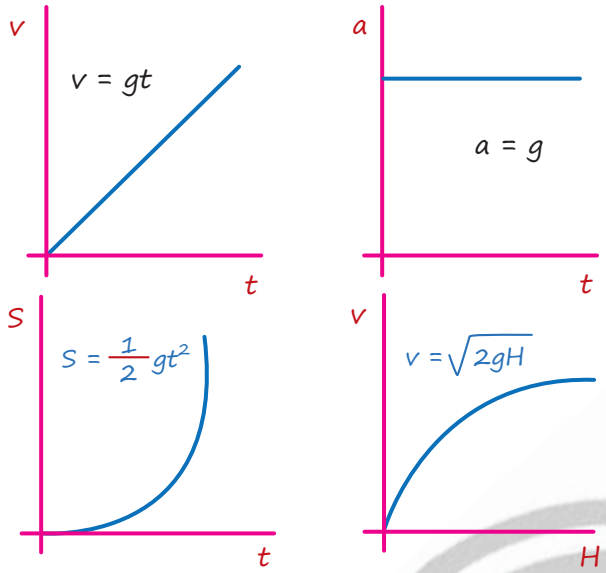
$S_{1st} : S_{2nd} : S_{3rd} = 1 : 3 : 5$

$S_{1s} : S_{2s} : S_{3s} = x : 4x : 9x$

Note:-

1. Time of Fight (T_F) = $\sqrt{\frac{2H}{g}}$
2. Velocity at ground :- $v = \sqrt{2gH}$

Graphs:-

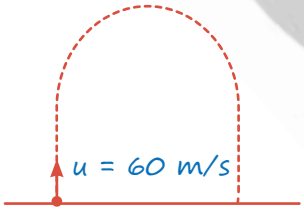


MOTION UNDER GRAVITY FROM GROUND TO GROUND:-

- Non-uniform motion (velocity = variable) with constant acceleration (g).
- At maximum height velocity zero and a = g.

$H_{max} = \frac{U^2}{2g}$ $\quad T_f = \frac{2U}{g}$

$T_{up} = \frac{U}{g}$ $\quad T_{down} = \frac{U}{g}$



1. Total time of flight $T = \frac{2u}{g} = \frac{2\times60}{10}$ = 12 sec.
2. Maximum Height $H = \frac{u^2}{2g}$ = 180 m
3. Velocity at t = 7 sec.
   v = u + gt
   = 60 − 10×7 = −10 m/s
4. Displacement in 8 sec.
   $S = ut + \frac{1}{2}at^2$ = 160 m
5. Distance in 8 sec.
   at t = 6 sec. body comes to at rest and takes u-turn hence calculate distance 0 to 6 sec. then 6 to 8 sec
   S = 180 + 20 = 200 m
6. Distance in 9th sec. downward journey ka 3rd sec = 25 m. Use ratio.
7. Distance in last sec of upward journey = distance in 1st sec of downward journey = 5 m (always)

Q. A stone with weight W is thrown vertically upward into the air with initial velocity v_o. If a constant force, due to air drag acts on the stone throughout the flight & if the maximum height attain by stone is h and velocity when it strikes to the ground is u. Which one is correct?

(a) $h = v_o^2\left(1+\frac{f}{W}\right)/2g,\ v = v_o$

(b) $h = v_o^2/2g\left(1+\frac{f}{W}\right),\ v$ = zero

(c) $h = v_o^2/2g\left(1+\frac{f}{W}\right),\ v = v_o\sqrt{\frac{W-f}{W+f}}$

(d) $h = v_o^2/2g\left(1+\frac{f}{W}\right),\ v = v_o\sqrt{\frac{W+f}{W-f}}$

Ans.

MR*

If f = 0 then

$H = \frac{v^2}{2g}$ and $v = v_o$

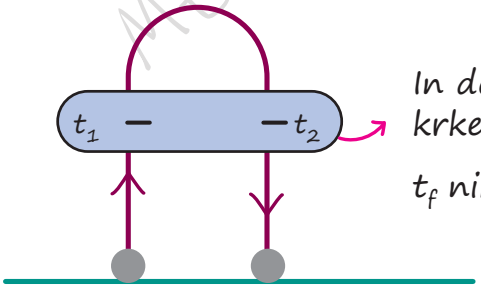
The MR*

If f ≠ 0

$H < \frac{v^2}{2g}$ and $v < v_o$

Kam karne ke liye niche +ve hoga.

MR TABLE



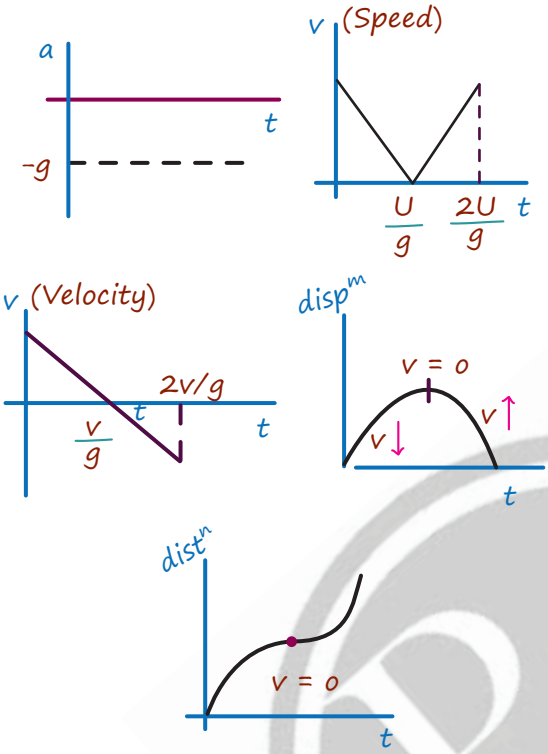
In dono ko add krke

t_f nikal sakte !

Object is at same height at t_1 and t_2.

That height $h = \frac{1}{2}gt_1t_2$

- Ball is projected up with speed "U" graphs:-

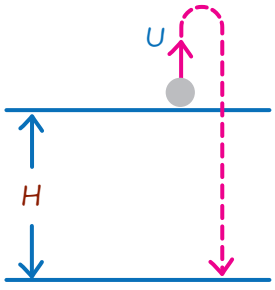


- If air friction in not ignored then:-

$$\frac{t_{up}}{t_{down}} = \sqrt{\frac{g-a}{g+a}}$$

  - $t_{up} < t_{down}$
  - $V_{projection} > V_{collision}$

MOTION UNDER GRAVITY FROM HEIGHT TO GROUND:-



$$t = \frac{U}{g} + \sqrt{\frac{U^2}{g^2} + \frac{2H}{g}}$$

MR*

If u = 0 then it is like drop from height H then

$$t = \sqrt{\frac{2H}{g}}$$

MR*

If H = 0 then it is like ground to ground motion

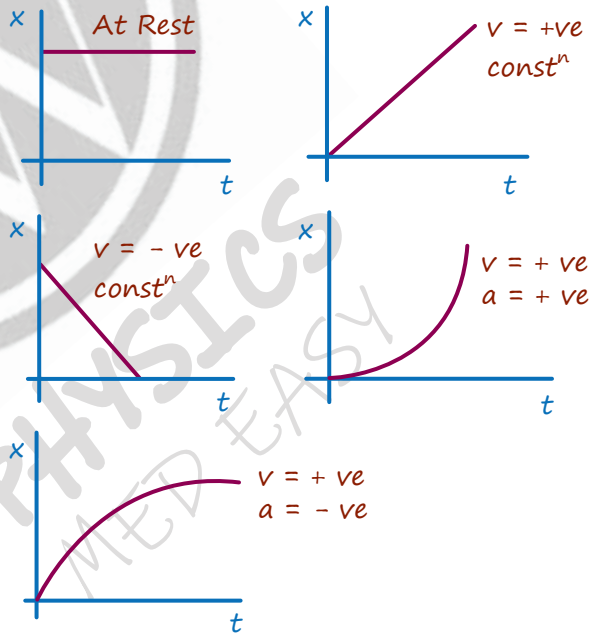
$$t = \frac{2u}{g}$$

Q. Ball is projected up with speed "u" from height H. Then time of fight T_1. With same speed "u" it is projected downward then time of fight is T_2. find time of fight "T" when object is dropped from same height.

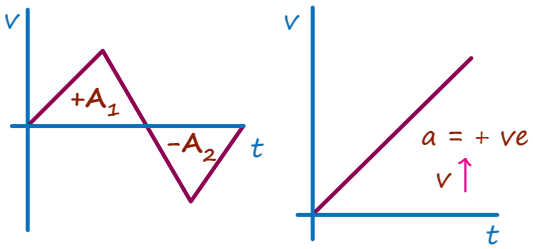
$$t = \sqrt{T_1 T_2}$$

Graph:-

1. Position - Time Graph:-



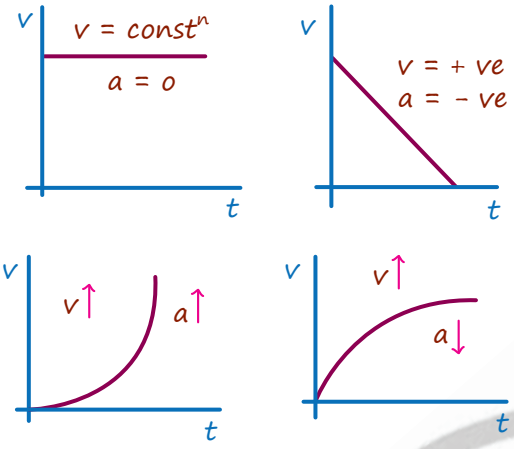
2. Velocity - Time Graph:-



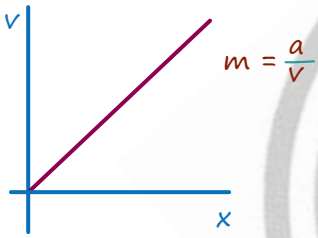
Area = displacement = $A_1 - A_2$

slope = acceleration

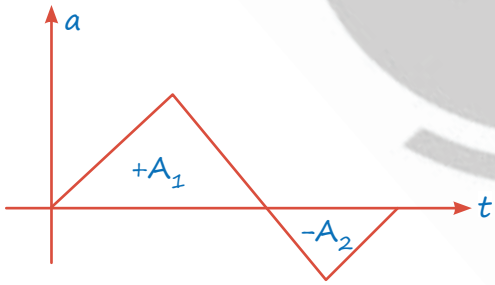
but distance = $A_1 + A_2$



3. Velocity - Position Graph:-

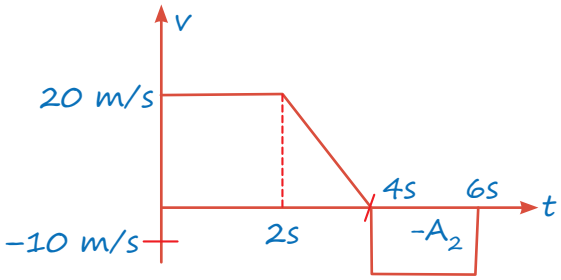


4. Acceleration time graph:-



Slope = नाली (Jurk)

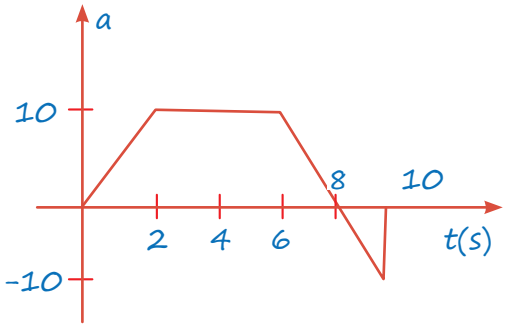
Area = Change in velocity = $A_1 - A_2$



Distance = total area = 40 + 20 + 20 = 80 m

Displacement = 40 + 20 − 20 = 40 m

Q. If initial velocity of object is 10 m/s then find velocity at 10 sec



Ans. Area = change in velocity

$$\vec{V}_f - \vec{V}_i = \frac{1}{2} \times 12 \times 10 - \frac{1}{2} \times 10 \times 2$$

$$V_f - V_i = 60 - 10 = 50$$

$$V_f = 50 + V_i = 50 + 10 = 60 \text{ m/s}$$

Relative Motion in 1-D

- Observer khud ko hamesa rest me assume karta hai, or uska pas jo bhi velocity, acceleration hota hai, ulta kar ke jisko dekhta hai usme chipka deta hai.

$\vec{x}_{AB}$ = Position of A w.r.t. B = $\vec{x}_A - \vec{x}_B$

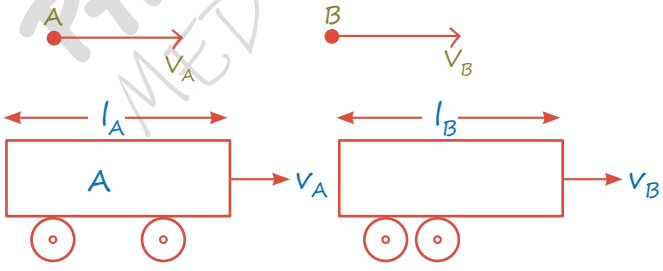
$\vec{x}_{BA}$ = Position of B w.r.t. A = $\vec{x}_B - \vec{x}_A$

differentiation w.r.t. time

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B \qquad \vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

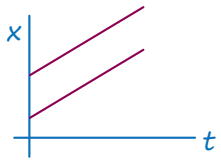
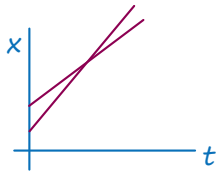
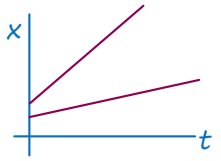
$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B \qquad \vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$

$$\vec{V}_{AB} = -\vec{V}_{BA} \qquad \vec{a}_{AB} = -\vec{a}_{BA}$$



- Time taken to overtake = $\frac{l_A + l_B}{V_A - V_B}$

- If they are moving opposite to each other = $\frac{l_A + l_B}{V_A + V_B}$

If $V_A = V_B$	$V_{AB} = 0$	x_{AB} = cost^n	
If $V_A > V_B$	V_{AB} = +ve V_{BA} = –ve	$\vec{x}_{AB}$ = decrease then increase	
If $V_A < V_B$	V_{AB} = –ve V_{BA} = +ve	$\vec{x}_{AB}$ = Increasing	

Motion of Object on the Moving Surface

1. Man is running on the surface of train with V_M in the direction of train (V_T)

   $V_{MG} = V_T + V_M$.

   If man is running in opposite direction then, $V_{MG} = V_T - V_M$

2. River is flowing with V_R and man is swimming with V_M in downstream then V_{MG} = Velocity of man w.r.t ground or effective velocity of man = $V_M + V_R$

   In upstream $V_{MG} = V_R - V_M$

3. Same as above in stair case.

- Motion under gravity of one object w.r.t other which is also in motion under gravity is uniform relative motion.

  $a_{AB} = 0$ V_{AB} = Cost^n

  S_{AB} = Increasing or decreasing linear

  Time of collision = $\frac{S_{AB}}{V_{AB}}$

Q. A ball is drop from 80 m height and another ball is projected with speed 40 m/s then they will collide.

Ans. $V_{relative} = 40$ m/s

$a_{relative} = 0$ $\quad t = \frac{80}{40} = 2$ sec

$S_{relative} = 80$

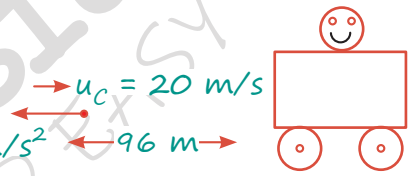
Q. A ball thrown downward with speed 20 m/s and 30 m/s simultaneously, then find relative velocity and separation b/w them after 4 sec

Ans. $a_{AB} = 0$ $V_{BA} = 10$ m/s (const w.r.t bus)

$S_{BA} = V_{BA}t = 10 \times 4 = 40$ m

Q. A bus starts from rest moving with an acceleration of 2 m/s². A cyclist, 96 m behind the bus starts simultaneously towards the bus at 20 m/s. After what time will he be able to overtake the bus:-

Ans.



$S = ut + \frac{1}{2}at^2$ (cyclist w.r.t bus)

$96 = 20t - \frac{1}{2}2t^2$

$t^2 - 20t + 96 = 0$ $\quad t = 12$s and $t = 8$ sec

at 8 sec cyclist overtake bus and at 12 sec bus will again cross cyclist.

MR*

पछतावा अतीत नहीं बदल सकता और
चिंता भविष्य नहीं सँवार सकती।
इसलिए वर्तमान का आनंद लेना ही,
जीवन का सच्चा सुख है।