# Contents

# 1 Basic Maths

## 1. Binomial theorem

$(1 + x)^2 = 1 + 2 \times 1x + x^2$

if $x <<< 1$ then

$(1 + x)^2 = 1 + 2x$

MR* feel

(Carrier + love)$^2$ = Carrier + 2 love

Because carrier >>> love

$$[x+\Delta x]^n = x^n \left[1 + \frac{\Delta x}{x}\right]^n = x^n \left[1 + n\frac{\Delta x}{x}\right]$$

$\Delta x <<<< x$.

- $(1 - x)^n = 1 - nx$
- $(1 - x)^{-n} = 1 + nx$
- $(1 + x)^{-n} = 1 - nx$

## 2. Imp formula

$(a + b)^2 = a^2 + b^2 + 2ab$

$(a - b)^2 = a^2 + b^2 - 2ab$

$a^2 - b^2 = (a + b)(a - b)$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

## 3. AP series

Next term = Previous term + Common difference

$a,\ a + d,\ a+2d,\ a + 3d,\ a + 4d.....$

Ex 2, 5, 8, 11, 14, 17, so on.

d = Common difference

= $n^{th}$ term $- (n-1)^{th}$ term

no. of term

$$T_n = a + (n-1)\,d$$

last term — $1^{st}$ term — Common diff.

no. of terms.

$$S_n = \frac{n}{2}\left[2a + (n-1)\,d\right]$$

NOTE:- n = no. of terms not last term.

## GP series

Next term = Previous term × Common ratio

$a,\ ar,\ ar^2,\ ar^3,\ ar^4$

Ex 16, 8, 4, 2, 1, 1/2, 1/4, so on

$$r\ (\text{Common ratio}) = \frac{n^{th}\ \text{term}}{(n-1)^{th}\ \text{term}}$$

$$\text{Sum} = \frac{a}{1-r},\ \text{valid when } r < 1.$$

Ex- $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, ....$

$$r = \frac{1/4}{1/2} = \frac{1}{2}$$

$$\text{Sum} = \frac{1}{1-\frac{1}{2}} = \frac{1}{1/2} = 2$$

Ex-

$1, -\frac{1}{2}, \frac{1}{4}, \frac{-1}{8}, \frac{1}{16}, \frac{-1}{32}, ....$

$$r = -\frac{1}{2}$$

$$\text{Sum} = \frac{1}{1-(-\frac{1}{2})} = \frac{1}{3/2} = \frac{2}{3}$$

## 4. Quadratic equation

$ax^2 + bx + c = 0$

[redacted]

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a.}$$

Sum of roots $= \frac{-b}{a}$, Products of roots $= \frac{c}{a}$

Q. Find roots of equation $x^2 - 5x + 6 = 0$; find value of a, b & c by comparing with $ax^2 + bx + c = 0$

Ans. $a = 1, b = -5$ & $c = 6$

$$X_1 = \frac{(-5) + \sqrt{(-5)^2 - 4\times1\times6}}{2\times1}$$

$$= \frac{5 + \sqrt{1}}{2} = 3$$

$X_2 = 2$

Q. $x^2 - 4x = 0$

$x^2 = 4x$
$x = 4$ wrong

$x(x - 4) = 0$
$x = 0 ; x = 4$ correct

Q. $x^2 - 4x + 3 = 0$ then find roots.

Ans. $x^2 - 3x - x + 3 = 0$

$x(x-3) - 1(x - 3) = 0$

$(x - 3)(x - 1) = 0$

$x = 3, x = 1$

## 5. Logarithms

$\log y^x = \log x$ on the base y

$\log_e x = 2.303 \log_{10} x$

(a) $\log_a (xy) = \log_a x + \log_a y$

(b) $\log\left(\frac{x}{y}\right) = \log x - \log y$

(c) $\log_y x = \frac{1}{\log_x y}$

(d) $\log_e x^{1/n} = \frac{1}{n} \log_e x$

(e) $\log_e x^n = n \log_e x$

(f) $\log_b a \times \log_a b = 1$

(g) $\log_a a = 1$

$\log_e 1 = 0$

$\log_{10} 2 = 0.30$

$\log_{10} 1 = 0$

$\log_{10} 3 = 0.48 \approx 0.5$

$\log_e(\sin 90°) = 0$

$\log_{10} 5 + \log_{10} 20 = 2$

$$\log_2 3 = \frac{\log_{10} 3}{\log_{10} 2} = \frac{48}{30}$$

+ Concept of Anti-log

$\log e^x = Y$

By taking Anti-log

(convert into concept of power)

$x = e^y$

MR* ka tadka

log → Concept of Power

$$\underset{Base}{2}^{\overset{Power}{3}} = \underset{\uparrow Result}{8} \Rightarrow \log 2^8 = 3$$

Base wahi rahega (Power ⇌ Result interchange hoga)

## 6. Rule of Power

1. **If Power of any non-zero number is zero then result will be one.**

   Ex- $8^0 = 1$

2. Negative Property of exponent (x is non zero number)

$$x^n = \frac{1}{x^{-n}} \Rightarrow \frac{1}{x^n} = x^{-n}$$

$$\frac{1}{10^3} = 10^{-3}$$

3. Product Property of Exponent

$$x^n x^m = x^{n+m}$$

$$10^3 \times 10^4 = 10^7$$

4. Division Property

$$\frac{x^n}{x^m} = x^{n-m} \Rightarrow \frac{10^3}{10^2} = 10^{3-2}$$

5. Power of a Power:

$$(x^n)^m = x^{nm}$$

$$(10^2)^3 = 10^6$$

6. $10^2 + 10^3 = 100 + 1000 = 1100$

7. Fractional exponent

$$(x)^{3/2} = (x^3)^{1/2}$$

8. Multiplication with fraction.

$$0.5 = \frac{1}{2} \qquad 1.33 \times 12 = \frac{4}{3} \times 12 = 16$$

$$0.6 = \frac{6}{10} \qquad 16 \times .25 = \frac{1}{4} \times 16 = 4$$

$$0.4 = \frac{4}{10} \qquad 0.75 \times 16 = \frac{3}{4} \times 16 = 12$$

$$0.66 = \frac{2}{3} \qquad 0.33 \times 15 = \frac{1}{3} \times 15 = 5$$

$$1.33 = \frac{4}{3} \Rightarrow 0.75 = \frac{3}{4} \Rightarrow 0.33 = \frac{1}{3}$$

9. Important property

$$2^{\infty} = \infty \qquad e^{\infty} = \infty$$

$$1^{\infty} = 1 \qquad e^{-\infty} = 0$$

$$4^{-\infty} = 0 \qquad e^0 = 1$$

$$(8)^{2/3} = (8)^{(1/3)\times 2} = (2)^{3\times(1/3)\times 2} = 2^2 = 4$$

$$(32)^{3/5} = (2^5)^{3/5} = 2^3 = 8$$

Important roots

| | |
|---|---|
| $\sqrt{121} = 11$ | $\sqrt{400} = 20$ |
| $\sqrt{144} = 12$ | $\sqrt{900} = 30$ |
| $\sqrt{169} = 13$ | |
| $\sqrt{196} = 14$ | $\sqrt{0.64} = 0.8$ |
| $\sqrt{225} = 15$ | $\sqrt{0.16} = 0.4$ |
| $\sqrt{256} = 16$ | |

## 7. Trigonometry

Angle → Arc = $R\theta$ algebraic function

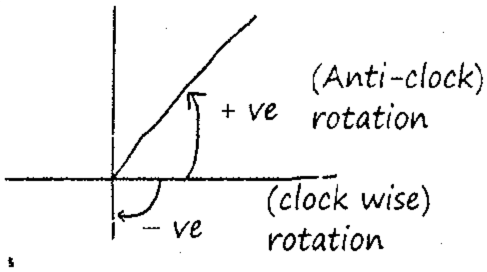
Angle → $\sin\theta/\cos\theta/\tan\theta$ Trigo. function

→ Angle have unit radian. but dimensionless.

→ For algebraic function, we always use S.I. unit radian but for trigonometric function we may use rod/degree.

→ $180° = \pi$ rad

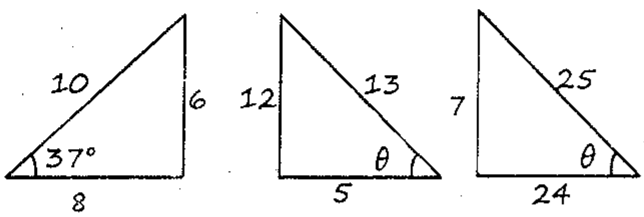
$$1° = \frac{\pi}{180} \text{ rad} \quad 1\text{rad} = \frac{180}{\pi}$$




Q. Total Angle moved by object in $\pi$-rotation?

Ans. – $\theta = \pi(2\pi) = 2\pi^2$ rad.

+ Some Important Triangles



| | 0° | 30° | 45° | 60° | 90° | 120° | 135° | 150° | 180° |
|---|---|---|---|---|---|---|---|---|---|
| Sin θ | 0 | $1/2$ | $1/\sqrt{2}$ | $\sqrt{3}/2$ | 1 | $\sqrt{3}/2$ | $1/\sqrt{2}$ | $1/2$ | 0 |
| Cos θ | 1 | $\sqrt{3}/2$ | $1/\sqrt{2}$ | $1/2$ | 0 | $-1/2$ | $-1/\sqrt{2}$ | $-\sqrt{3}/2$ | −1 |
| Tan θ | 0 | $1/\sqrt{3}$ | 1 | $\sqrt{3}$ | Not define | $-\sqrt{3}$ | −1 | $-1/\sqrt{3}$ | 0 |

$\text{Sin }\theta = \frac{1}{\text{Cosec }\theta}$ $\text{Sec }\theta = \frac{1}{\text{Cos }\theta}$ $\text{Tan }\theta = \frac{1}{\text{Cot }\theta}$

$\text{Sin }(90 + \theta) = \cos\theta$

$\sin(180 - \theta) = \sin\theta$

$\sin(90 - \theta) = \cos\theta$

$\cos(180 - \theta) = -\cos\theta$

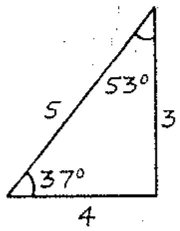
$\cos(90 - \theta) = \sin\theta$

$\cos(90 + \theta) = -\sin\theta$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$




$\text{Sin }\theta = \frac{P}{H}$ $\text{Cos }\theta = \frac{B}{H}$

$\text{Tan }\theta = \frac{P}{B}$ $\text{Sin }37° = \frac{3}{5}$

$\text{Cos }37° = \frac{4}{5}$ $\text{Sin }53° = \frac{4}{5}$ $\text{Cos }53° = \frac{3}{5}$

$\text{Cos }(-60°) = \frac{1}{2} \Rightarrow \text{Sin }(-30°) = -\frac{1}{2}$

$\tan(-135°) = -1$

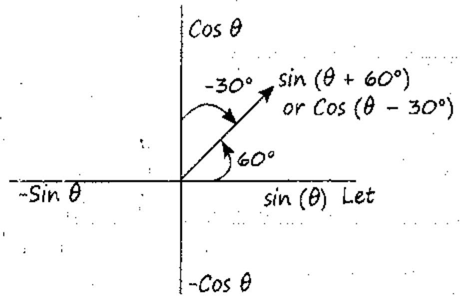
**Unique Relation**

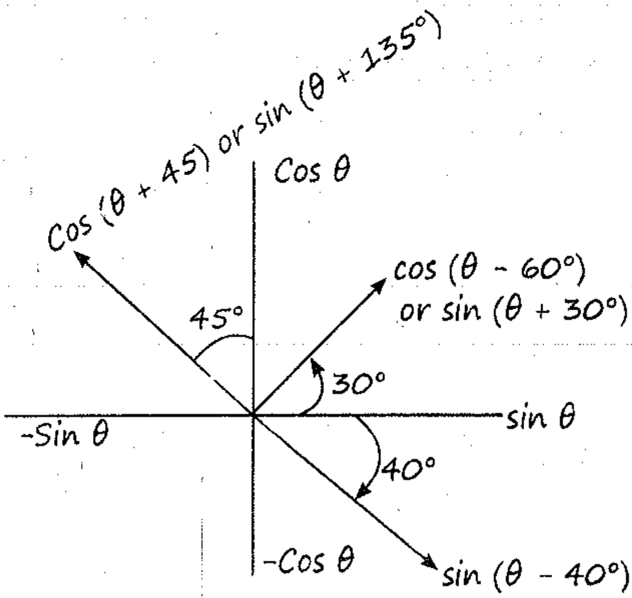
$\text{Sin}^2\theta + \text{Cos}^2\theta = 1 \Rightarrow 1 + \cot^2\theta = \text{Cosec}^2\theta$

$\text{Tan}^2\theta + 1 = \text{Sec}^2\theta$

## 8. Phasor diagram

Vector representation of trigonometric function




| Equation-1 | Equation-2 | Phase difference |
|---|---|---|
| $I = I_0 \sin(\theta + \pi/3)$ | $I = I_0 \sin(\theta - \pi/6)$ | $\phi = 90°$ |
| $I = I_0 \sin(\theta + \pi/3)$ | $I = I_0 \cos(\theta - \pi/6)$ | $\phi = 0°$ |
| $I_1 = I_0 \sin(\theta)$ | $I = I_0 \cos(\theta + \pi/6)$ | $\phi = 2\pi/3$ |
| $I_1 = \sin(\theta - \pi/3)$ | $I = I_0 \cos(\theta + \pi/3)$ | $\phi = \frac{7\pi}{6} = 210°$ |
| $I_1 = \sin(\theta - 60°)$ | $I = I_0 \cos(\theta - 30°)$ | $\phi = \frac{2\pi}{3} = 120°$ |

9. $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(a) $A = B = \theta$

$\sin(A+B) = \sin 2\theta = 2\sin\theta \cos\theta$

$\cos(A+B) = \cos 2\theta = \cos^2\theta - \sin^2\theta$

(b) $2\cos^2\theta = 1 + \cos(2\theta)$.

$2\sin^2\theta = 1 - \cos(2\theta)$

<u>If Angle is Small:-</u>

$\sin\theta \approx \theta \qquad \tan\theta \approx \theta \qquad \cos\theta = 1$

$\sin(2^\circ) = 2^\circ$ (wrong)

$$\sin(2^\circ) = 2 \times \frac{\pi rad}{180^\circ} = \frac{\pi}{90^\circ} rad$$

$\cos(4^\circ) = 1$

$$\tan 3^\circ = \frac{\pi rad}{60}$$

**10.**

| **Trigonometric function** | **Maximum Value** |
|---|---|
| $Y = 3\sin\theta$ | $Y_{max} = 3 \quad Y_{min} = -3$ |
| $Y = 4\sin(5\theta)$ | $Y_{max} = 4 \quad Y_{min} = -4$ |
| $Y = 3\sin\theta + 4\cos\theta$ | $Y_{max} = 5 \quad Y_{min} = -5$ |
| $Y = 3\sin\theta + 4\sin\theta$ | $Y_{max} = 7 \quad Y_{min} = -7$ |
| $Y = 5 - 2\sin\theta$ | $Y_{max} = 7 \quad Y_{min} = 3$ |

Q. Force acting on object $F = \dfrac{4}{3\sin\theta + \cos\theta}$

Then find minimum magnitude of force.

Ans. $$F_{min} = \frac{4}{(3\sin\theta + \cos\theta)_{max}}$$

$$F_{min} = \frac{4}{\sqrt{9+1}} = \frac{4}{\sqrt{10}}$$

**11.** Sum of $1^{st}$ n-natural numbers $= \dfrac{n(n+1)}{2}$

Sum of Squares of $1^{st}$ n-natural numbers $= \dfrac{n(n+1)(2n+1)}{6}$

Sum of Cubes of $1^{st}$ n-natural numbers

$$= \left[\frac{n(n+1)}{2}\right]^2$$

**12. Differentiation** [redacted]

$\dfrac{dy}{dx}$ = The rate of change in y w.r.t.x

= Slope of y-x graph.

$\dfrac{d^2y}{dx^2}$ = Double diff$^n$ of Y w.r.t x

= The rate of change in $\left(\dfrac{dy}{dx}\right)$ w.r.t x

= Slope of Slope

= Change in slope w.r.t x

$$\frac{d \sin x}{dx} = \cos x$$

$$\frac{d \tan x}{dx} = \sec^2 x$$

$$\frac{d \cot x}{dx} = -\operatorname{cosec}^2 x$$

$$\frac{d \log_e x}{dx} = \frac{d \ln x}{dx} = \frac{1}{x}$$

$$\frac{d \cos x}{dx} = -\sin x$$

$$\frac{d \sec x}{dx} = \sec x \tan x$$

$$\frac{d \operatorname{cosec} x}{dx} = -\operatorname{cosec} x \cot x$$

$$\frac{d x^n}{dx} = nx^{n-1}$$

**Rules :-**

1. Addition Rule:-

$$Y = A + B \qquad \frac{dy}{dx} = \frac{dA}{dx} + \frac{dB}{dx}$$

2. Substraction Rule:-

$$Y = A - B \qquad \frac{dy}{dx} = \frac{dA}{dx} - \frac{dB}{dx}$$

3. Multiplication Rule:-

$$Y = A\,B \qquad \frac{dy}{dx} = \frac{A\,dB}{dx} + \frac{B\,dA}{dx}$$

4. Division Rule:-

$$Y = \frac{A}{B} \qquad \frac{dy}{dx} = \frac{B\left(\frac{dA}{dx}\right) - A\left(\frac{dB}{dx}\right)}{B^2}$$

$$\frac{d\sin(90°)}{dx} = 0 \qquad Y = t^2 \text{ find } \frac{dy}{dx}$$

$$\frac{d\,e^x}{dx} = e^x \qquad \frac{dy}{dx} = \frac{dt^2}{dx} \times \frac{dt}{dt}$$

$$\frac{d\,e^2}{dx} = 0 \qquad \frac{dy}{dx} = 2t\,\frac{dt}{dx}$$

The MR*

Outside Inside Rule

$Y = f(z(x)) = y$ is function of $z$ and $z$ is a function of $x$.

$$\frac{dy}{dx} = \begin{pmatrix}\text{differentiation} \\ \text{of outer function} \\ \text{keep inside as it is}\end{pmatrix} \times \begin{pmatrix}\text{diff}^n \text{ of Inner} \\ \text{fu}^n \text{ w.r.t } x\end{pmatrix}$$

Q. $y = \sin(3x)$

$$\frac{dy}{dx} = \cos(3x)\,\frac{d(3x)}{dx}$$

$$= 3\cos(3x)$$

| $Y = e^{(5x)}$ <br> $\frac{dy}{dx} = 5e^{5x}$ | $Y = e^{-4x}$ <br> $\frac{dy}{dx} = -4\,e^{-4x}$ |
|---|---|
| $Y = (x^2+4)^3$ <br> $\frac{dy}{dx} = 3\,(x^2+4)^2\,\frac{d(x^2+4)}{dx}$ <br> $= 3(x^2+4)^2 \times 2x$ | $Y = \sin(4x^2)$ <br> $\frac{dy}{dx} = \cos(4x^2) \times 8x$ |

$$Y = A\sin(wt - kx)$$

$$\frac{dy}{dx} = A\cos(wt - kx) \times (-k)$$

Q. If radius of sphere is increasing $1/\pi$ m/s then find rate of change in volume w.r.t. time when radius is 3m.

Ans. $V = \frac{4}{3}\pi R^3$

$$\frac{dv}{dt} = \frac{4}{3}\pi 3R^2\,\frac{dR}{dt}$$

$$= 4\pi R^2\left(\frac{1}{\pi}\right)$$

$$\left(\frac{dv}{dt}\right) = 4R^2 = 4(3)^2 = 4 \times 9 = 36$$

## 13. Maxima and minima:

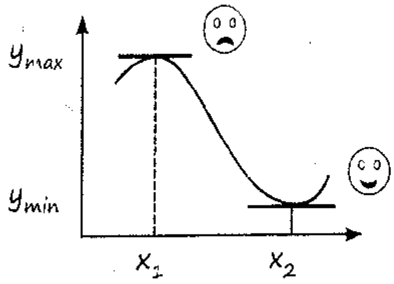
### MR* for maxima/minima

- For location of maxima/minima put $\frac{dy}{dx}$ (slope) = 0 and find value where $x$ will be max$^m$/min$^m$.
- For exact maxima and minima dont check double differentiation. Just put value of $x$ and find $y$.
- Double differentiation check nahi karna just $x$ ki value put kark $y$ nikala jo $y$ jayda wo maximum $y$ ko kam wo minimum $y$.

**Maxima**

$$\frac{dy}{dx} = 0 \qquad \frac{d^2y}{dx^2} = -ve$$

Slope ↓




**Minima**

$$\frac{dy}{dx} = 0 \quad \frac{d^2y}{dx^2} = +ve$$

Slope ↑

## 14. Integration:

→ Area under the curve → Inverse of differentiation

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad \text{Not valid for } n = -1$$

Addition Rule:

$$\int (u+v).\,dx = \int u.dx + \int v.dx$$

$$\int \sin x\,dx = -\cos x + c.$$

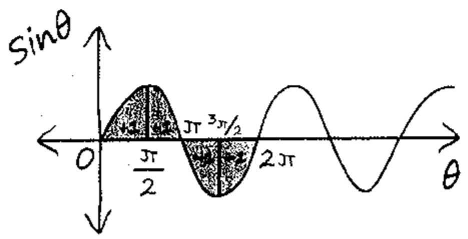
$$\int \cos x\,dx = \sin x + c.$$

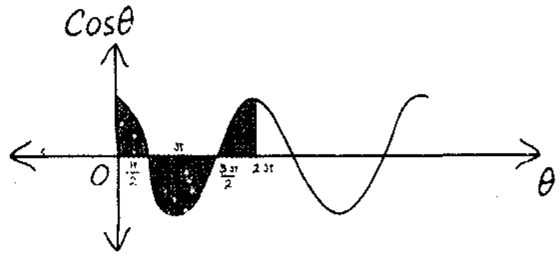
$$\int e^x dx = e^x + c$$

$$\int {}^1\!/\!_x\,dx = \ln x + c.$$

$$\int \sec^2 dx = \tan x + c$$

$$\int e^{3x} dx = \frac{e^{3x}}{3} + c.$$




Chain Rule → MR*

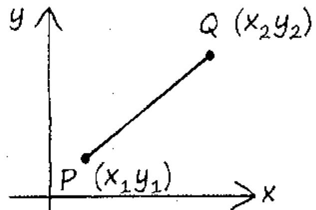
Applicable when power of x is one

$$\int y\,dx = \frac{\text{Integration of outer function keep inside as it is.}}{\text{Coefficient of } (x)}$$

$$\int (2x+3)^4\,dx = \frac{(2x+3)^5}{5[2]} + C$$

$$\int \sin(3x-4)\,dx = \frac{-\cos(3x-4)}{3} + C$$

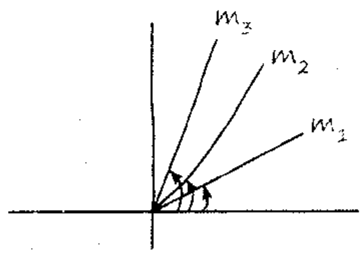
## 15. Co-ordinate geometry and graph:



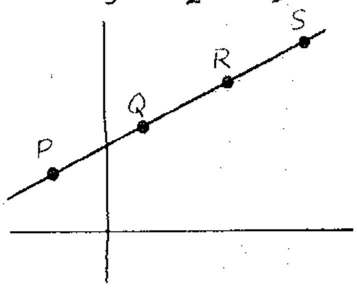

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\tan\theta = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

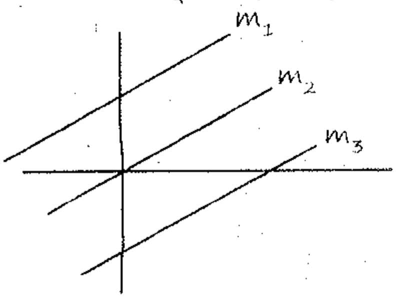
- Slope of straight line remains same at all the point
- If $0° \le \theta < 90°$ then slope is positive
- If $90° < \theta \le 180°$ then slope is negative
- If $\theta = 90°$ then slope is infinite
- If $\theta = 0°$ then slope is zero
- If straight line parallel to x-axis then slope zero



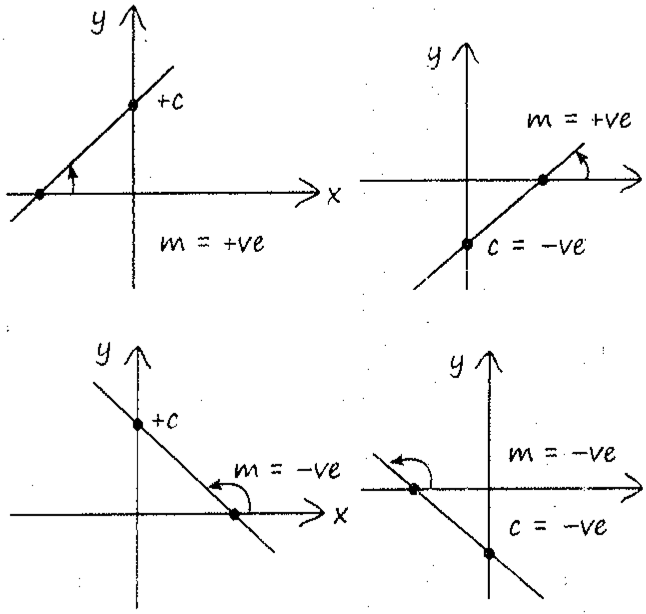
$m_3 > m_2 > m_1$




$m_P = m_Q = m_R = m_S$

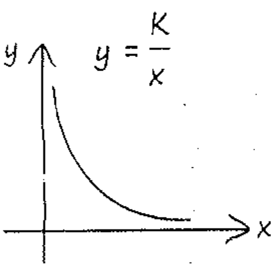



$m_1 = m_2 = m_3$

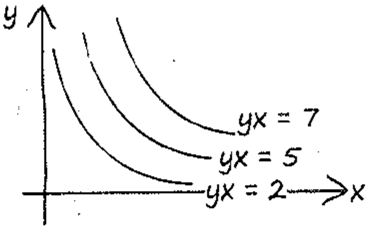



If two straight line perpendicular to each other then product of their slope is −1.

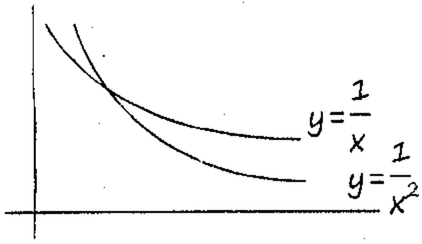
## 16. Rectangular Hyperbola:

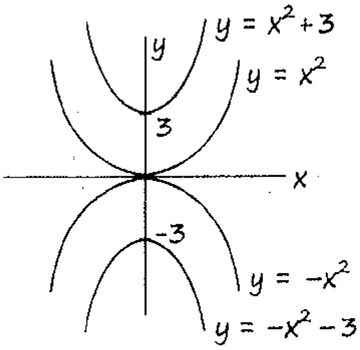


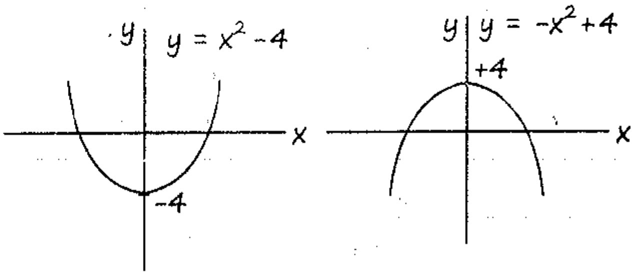

K is value Jitna Jayda graph utna upar shift hoga.

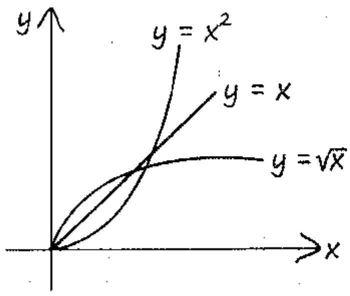



x ka pawer jitna jayda graph utna niche jayga.

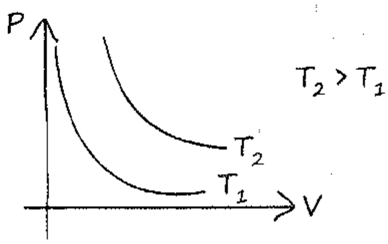


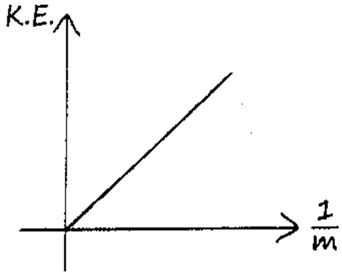



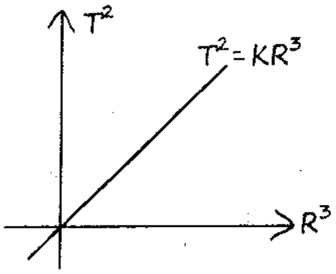

+ graph for $PV = nRT$

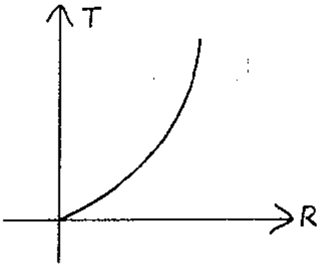


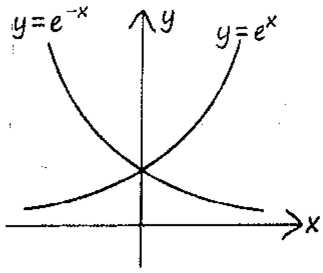
+ K.E. = $\frac{P^2}{2m}$ graph b/w K.E. and $\frac{1}{m}$ for constant momentum.




MR* → Jisko x- & y-axis pe plot krenge uska power dekhte hai.




## 17. Equation of Circle

$(x - x_o)^2 + (y - y_o)^2 = R^2$

R is radius & centre is at $(x_o, y_o)$

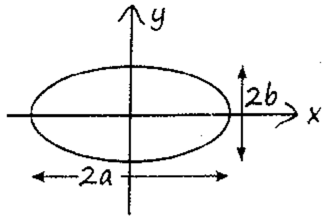
$x^2 + y^2 = 5^2$ centre at (0, 0) R = 5

$(x + 4)^2 + (y - 3)^2 = 49$ centre at (−4, 3)

R = 7

## 18. Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$




MR* For Slope

हँसता हुआ रामलाल — Slope always increasing

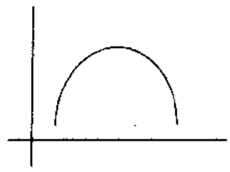
रोता हुआ रामलाल — Slope always decreasing

For magnitude of slope → Now we are talking about value of slope, we will ignore +ve & −ve only consider magnitude.

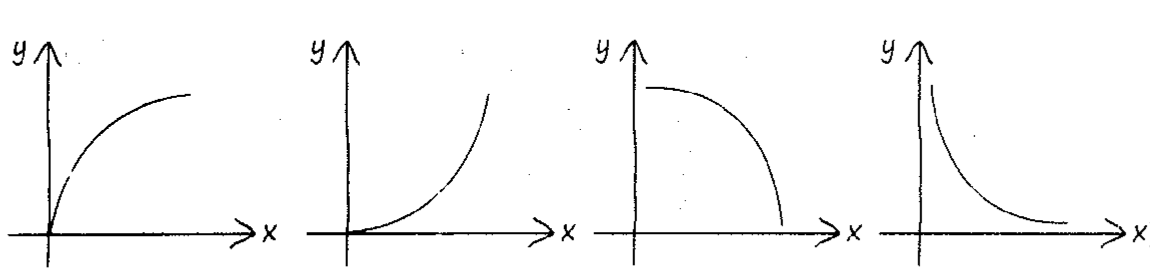
MR* → Locate where slope is zero

+ Starting me zero then increasing magnitute of slope.
+ Last me zero then decreasing magnitude of slope and becomes zero.

Slope → Increasing
magnitute of slope $1^{st}$ decreasing then increasing



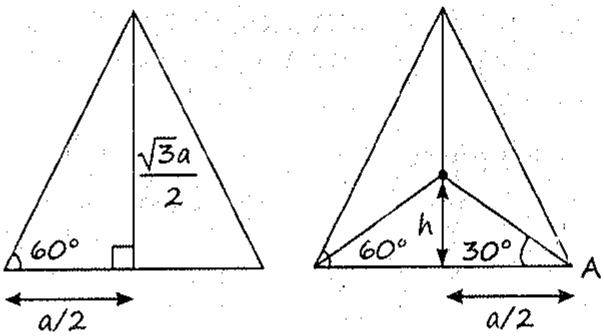
Slope → Decreasing
magnitute of slope $1^{st}$ decreasing then increasing




| | | | | |
|---|---|---|---|---|
| **Slope** | → decreasing | increasing | decreasing | increasing |
| **Magnitute of slope** | → decreasing | increasing | increasing | decreasing |

## 19. Some Basic Geometry Shapes:

Equibtral Triangle of side (a)




$$\tan 30° = \frac{h}{a/2}$$

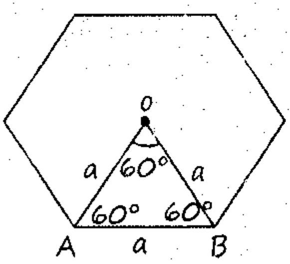
$$h = \frac{a}{2\sqrt{3}}$$

distance from centre to corner

$$= \frac{\sqrt{3}a}{2} \times \frac{2}{3} = \frac{a}{\sqrt{3}}$$

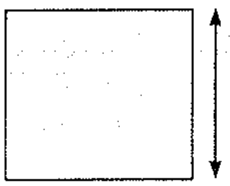
$$\text{Area} = \frac{\sqrt{3}a^2}{4}$$

Hexagonal of side 'a'



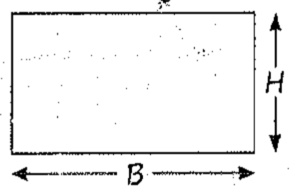

centre to corner
dist$^n$ = a

Square



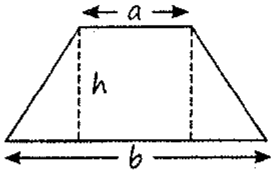
Area = $l^2$
Perimetre = $4l$

Rectangle

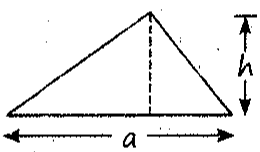



Area = BH
Perimetre = 2(H+B)

Trapezoid

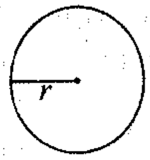



$$\text{Area} = \frac{1}{2}(a+b)h$$

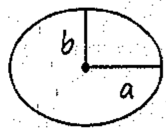



$$\text{Area} = \frac{1}{2}ah$$

Circle

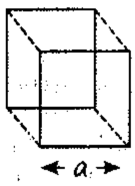



Circumference = $2\pi r$
Area = $\pi r^2$



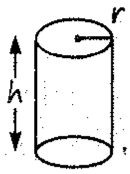

Area = $\pi ab$
Circumference = $\pi r$

Cube



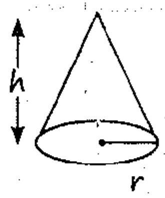

Area = $6a^2$
Volume = $a^3$

Cylinder



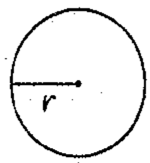

Area = $2\pi r^2 + 2\pi rh$
Volume = $\pi r^2 h$

Cone




$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

Sphere




Area = $4\pi r^2$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

## 20. Average of a varying quantity

If $y = f(t)$ then

$$\langle y \rangle_{Avg} = \frac{\int_{t_1}^{t_2} y\,dt}{\int_{t_1}^{t_2} dt} = \frac{\int_{t_1}^{t_2} y\,dt}{t_2 - t_1}$$

Y may be any physical quantity.

MR* if y is varying linearly then $y_{Avg} = \frac{y_i + y_f}{2}$

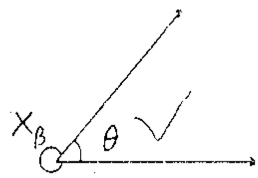
MR* If x+y = constant then xy will be maximum for $x = y = \frac{C}{2}$

If sum of two number is constant then product of these two number will be maximum, only when both number are equal.

# 2 Vector

| Scalar Quantity | Vector Quantity |
|---|---|
| ○ Having Magnitude only | ○ Having Magnitude, direction and follow triangle law of vector addition. |
| ○ Follow simple algebric addition | ○ Can be changed by changing magnitude only, or changing dir$^n$ only or changing both. |
| ○ Can be changed only by changing its value | |
| Ex-Speed, time, Mass, Volume, density current, etc. | Ex-Force, Velocity, current density, torque etc |

1. In vector +ve and –ve indicate direction only. Ex- +5N and −5N, same magnitude of force in opposite direction.
2. Angle between vector – When two vectors are placed head to head or tail to tail then smaller angle between vector is called angle between vector.



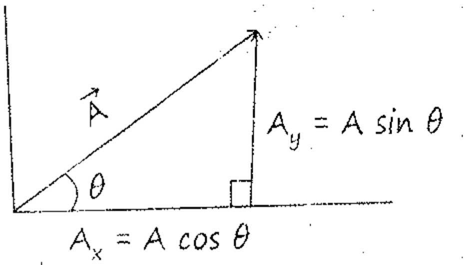

3. Vector can be shifted parallel to itself by keeping magnitude cmd direction fixed.
4. Rotation of vector not allowed it will change meaning of vector.
5. If Angel between $\vec{A}$ and $\vec{B}$ vector is $\theta$ then angle between $\vec{A}$ and $-\vec{B}$ is $(180-\theta)$.

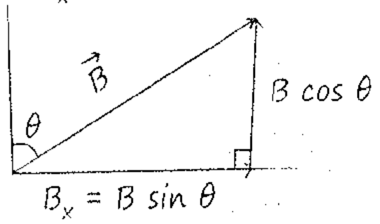
## Type of Vectors

| Type | Magnitude | Direction\Angle |
|---|---|---|
| Equal Vector | Same | Same ($\theta = 0$) |
| Parallels Vector | May or May not same | Same ($\theta = 0$) |
| Opposite Vector or Negative Vectors | Same | Opposite $\theta = 180°$ |
| Antiparalles Vector | May or May not same | $\theta = 180°$ opposite |
| Orthogonal | May same | $\theta = 90°$ |
| Zero/Null Vector | Zero | any direction |
| Unit Vectors | One | $\hat{A} = \frac{\vec{A}}{A}$ |

- All equal vectors are parallel but all parallels are not equal.
- All opposite (Negative) Vectors are Antiparallel but all antiparallel are not Opposite Vector

## Component of Vector (effect of Vector)




## Magnitude of Vectors:

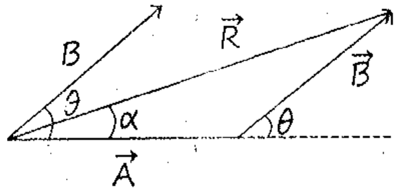
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

If vector is making an angle $\alpha$, $\beta$ and $\gamma$ from x, y and z-axis respectively then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ; $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$$\cos\alpha = \frac{Ax}{A} \qquad \cos\beta = \frac{Ay}{A} \qquad \cos\gamma = \frac{Az}{A}$$

## Triangle Law of Vector addition




$\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$

resultant must be in the plane of $\vec{A}$ and $\vec{B}$.

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

| If $\theta = 0°$ | $\theta = 90°$ | $\theta = 180°$ |
|---|---|---|
| $R_{max} = A + B$ | $R = \sqrt{A^2 + B^2}$ | $R_{min} = A - B$ |

$A - B \le R \le A + B$

If $|\vec{A}| = |\vec{B}| = A$ and Angle b/w them $\theta$

$|\vec{R}| = 2A\cos(\theta/2) \qquad D = 2A\sin(\theta/2)$

| $\theta = 0°$ | $\theta = 60°$ | $\theta = 90°$ | $\theta = 120°$ | $\theta = 180°$ |
|---|---|---|---|---|
| $R = 2A$ | $R = \sqrt{3}A$ | $R = \sqrt{2}A$ | $R = A$ | $R = 0$ |
| $D = 0$ | $D = A$ | $D = \sqrt{2}A$ | $D = \sqrt{3}A$ | $D = 2A$ |

## Vector Subtraction

Angle B/w $\vec{A}$ & $\vec{B}$ is $\theta$ then $\vec{D} = \vec{A} - \vec{B}$

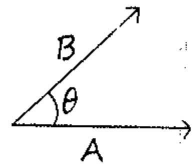
$$|\vec{D}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

| $\vartheta = 0°$ | $\theta = 90°$ | $\theta = 180°$ |
|---|---|---|
| $D_{min} = A - B$ | $D = \sqrt{A^2 + B^2}$ | $D = A + B$ |

$A - B \le D \le A + B$

1. Magnitude of Vector addition and subtraction same at 90°.
2. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ Commutative
3. $n(\vec{A} + \vec{B}) = n\vec{A} + n\vec{B}$ distributive
4. $\vec{A} - \vec{B} \ne \vec{B} - \vec{A}$
5. $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ then $\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$
6. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ is 120°
7. If $|\vec{A}| + |\vec{B}| = |\vec{A} + \vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ is zero.
8. If $\vec{A} + \vec{B} = \sqrt{A^2 + B^2}$ then angle between $\vec{A}$ and $\vec{B}$ is 90°.
9. If $|\vec{A} + \vec{B}| = |\vec{B} - \vec{A}|$ then angle between $\vec{A}$ and $\vec{B}$ is 90°.

## Scalar Product (Dot Product)




$\vec{A}\cdot\vec{B} = A(B\cos\theta) = A(\text{Component of } B \text{ along } A)$

$= (A\cos\theta)\,B = B(\text{Component of } A \text{ along } B)$

Component of B along A $= \dfrac{\vec{A}\cdot\vec{B}}{A}$

Component of A along B $= \dfrac{\vec{A}\cdot\vec{B}}{B}$

- Result of dot product is always scalar.

$\hat{i}\cdot\hat{i} = 1 \qquad \hat{j}\cdot\hat{j} = 1 \qquad \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{k} = 0 \qquad \hat{j}\cdot\hat{i} = 0 \qquad \hat{k}\cdot\hat{j} = 0$

$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_xB_x + A_yB_y + A_zB_z$

## Application of dot Product

(i) To Find Angle B/W vectors

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

(ii) To check unit vector

If $\vec{A}$ is a unit vector then $\vec{A}\cdot\vec{A} = 1$

(iii) To check perpendicular vector (orthogonal)

If $\vec{A}\cdot\vec{B} = AB\cos 90° = 0$

$\vec{A}\cdot\vec{B} = 0 \quad (\vec{A} \perp r\ \vec{B})$

(iv) To find component of one vector along other.

$\vec{A}\cdot\vec{B} = A(B\cos\theta)$

$B\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{A} = \text{Comp}^n \text{ of } B \text{ along } A$

**Cross-Product : [Vector Product]**

$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

$\hat{n}$ is direction of $\vec{A}\times\vec{B}$ which is perpendicular to $\vec{A}$ & $\vec{B}$.

$(\vec{A}\times\vec{B})\cdot\vec{A} = 0 \qquad (\vec{A}\times\vec{B})\cdot\vec{B} = 0$

$B\sin\theta = \dfrac{\vec{A}\times\vec{B}}{A}$ = component of B perpendicular of A

$\vec{R} = \vec{A}\times\vec{B}$

Place your finger of right hand along $\vec{A}$ and slap $\vec{B}$ then thumb will represent $\vec{R}$.

$\hat{i}\times\hat{i} = 0 = \hat{j}\times\hat{j} = \hat{k}\times\hat{k}$

$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{j}\times\hat{i} = -\hat{k}$

$\hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{j} = -\hat{i}$

$\hat{k}\times\hat{i} = \hat{j} \qquad \hat{i}\times\hat{k} = -\hat{j}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$= i\,(A_yB_z - A_zB_y)$

$- J(A_xB_z - A_zB_x)$

$+ K(A_xB_y - A_yB_x)$

- Unit vector does not have any unit only have direction and magnitude one.
- Minimum no. of vectors whose resultant can be zero is '2'.
- Minimum no of unequal vectors whose resultant can be zero is 3.
- The resultant of 3 Non-coplaner vectors can't be zero.
- Minimum no of Non-coplaner, vectors whose resultant can be zero is 4.

Q. If $|\vec{A}\times\vec{B}| = \sqrt{3}\,\vec{A}\cdot\vec{B}$ then angle between $\vec{A}$ and $\vec{B}$ is?

Sol$^n$. $AB\sin\theta = \sqrt{3}\,AB\cos\theta$

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

- Division of vector with vector is not possible
- Division of magnitude of vector is possible
- Vector can be divided by scalar.
- If vector multiplied by positive scalar then magnitude change direction remains same.
- If vector multiplied by negative scalar then magnitude change direction becomes opposite.

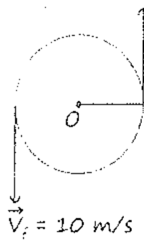
○ <u>Scalar triple Product :</u>

$R = (\vec{A} \times \vec{B}) \cdot \vec{C}$ Result R will be scalar and R will be zero if any of these two vector becomes parallel.

Q. Ramlal is moving with velocity 6m/s along east and pinky with 6 m/s at 30° east of north then relative of pinky w.r.t Ramlal.

Sol. $\vec{V}_{PR} = \vec{V}_P - \vec{V}_R$ same vector ka subtraction at 60° $|\vec{V}_{PR}| = 6$ m/s

Q.




Change in speed = 0 magnitude of change in velocity = 20 m/s

Q. If $\vec{A} = 0.6\hat{i} + \beta\hat{j}$ is a unit vector then find value of β.

Sol$^n$ $|\vec{A}| = 1$ if A is unit vector

$$\sqrt{(0.6)^2 + \beta^2} = 1$$

$$\beta^2 + 0.36 = 1$$

$$\beta = \sqrt{0.64} = 0.8$$

Q. Two force 10N and 6N acting then find resultant of these two force may be?

Sol$^n$ $10 - 6 \le R \le 10 + 6$

R will be 4N to 16N

Q. The angle which a vector $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ makes with x, y and z-axis

$$\cos\alpha = \frac{Ax}{A} \qquad \cos\beta = \frac{Ay}{A} \qquad \cos\gamma = \frac{Az}{Z}$$

$$\alpha = 60° \qquad \beta = 60° \qquad \gamma = 45°$$

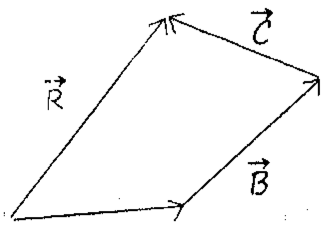
Q. In which of the following combination of three force resultant will be zero.

(a) 3N, 7N, 8N
(b) 2N, 5N, 1N
(c) 3N, 12N, 7N
(d) 4N, 5N, 10N

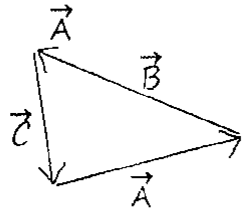
Sol$^n$ Sum of two smaller must be greater or equal to ($3^{rd}$).

○ Polygon Law of vector addition

Start tail of next vector from head of previous vector and so on.




$\vec{A} + \vec{B} + \vec{C} = \vec{R}$




$\vec{A} + \vec{B} + \vec{C} = 0$

○ Angle between $(\vec{A} \times \vec{B})$ and $(\vec{A} + \vec{B})$ is zero

Q. Force acting on object $\vec{F} = 5\hat{i} + 3\hat{j} - 7\hat{k}$ position vector $\vec{r} = 2i + 2j - k$ then find torque ?? (NEET 2022)

$$\vec{\tau} = r \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 5 & 3 & -7 \end{vmatrix}$$

$$\hat{i}(-14 - (-3)) - \hat{j}(-14 - (-5)) + \hat{k}(6 - 10)$$

$$-11\hat{i} + 9\hat{j} - 4\hat{k}$$

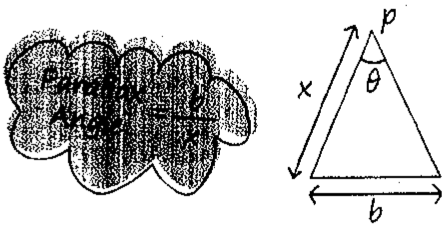
**MR***

"अपनी पढाई छोड़ जो तेरे पीछे चला आयेगा।
वो खुद का ना हो सका, तेरा क्या हो जायेगा।।"

# 3 Units & Dimensions

## MEASUREMENT OF LENGTH:-

Parallax Method } Used to measure large distance




$1^\circ = 1.745 \times 10^{-2}$ rad

$1' = 2.91 \times 10^{-4}$ rad

$1'' = 4.85 \times 10^{-6}$ rad

**For v.small size:-optical, tunneling, electron microscope used:**

$1AU = 1.496 \times 10^{11} m$

$1Ly = 9.46 \times 10^{15} m$

$1 parsec = 3.08 \times 10^{16} m$

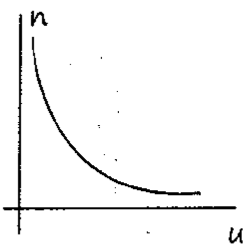
Size of $P^+ = 10^{-15} m$

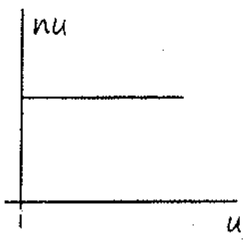
$R_{Earth} = 10^7 m$

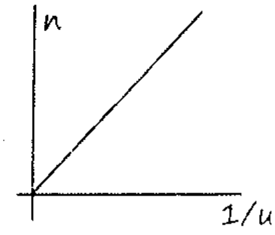
Distance of boundary of Observable Universe $= 10^{26}$ m

- $nu$ = constant, $n$ = measure value of P.Q., $u$ = unit of that P.Q.

$$n \propto \frac{1}{u}$$




→ Only use to find value of physical quantity in new system of unit, if value is know in one unit.

## MEASUREMENT OF MASS & TIME

- 1 amu $= \frac{1}{12}$ Mass of $C^{12}$ atom
- 1 amu $= 1.66 \times 10^{-27}$ kg
- $e^-$ mass $= 10^{-30}$ kg
- Earth mass:- $10^{25}$ kg
- Observable Universe $=10^{55}$ kg.

**Time:-**

- Age of universe $= 10^{17}$ s
- Time span of Unstable particle → $10^{-24}$s

Q. Convert 18 km/hr in m/s.

Ans. $n_1u_1 = n_2u_2$

18 km/hr = $n_2$ m/s

$$\frac{18 \times 10^3 m}{60 \times 60 s} = n_2 \text{ m/s}$$

$$n_2 = 18 \times \frac{5}{9} = 10$$

Q. If unit of length is y m in new system of unit then find value of x $m^2$ area in new system of unit.

Ans. un = cost

$$n_1u_1 = n_2u_2$$

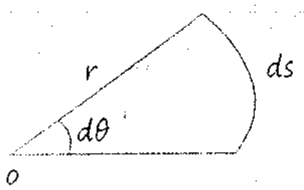
$$xm^2 = n_2y^2m^2$$

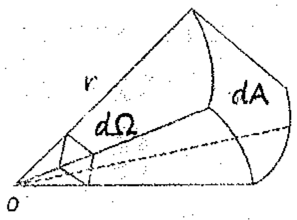
$$n_2 = \frac{x}{y^2}$$

## SI SYSTEM

| 7 Base/Fundamental Units:- | | | |
|---|---|---|---|
| No. | Quantity | Unit | Symbol |
| 1. | Length | Meter | m |
| 2. | Mass | Kilogram | kg |
| 3. | Time | Second | S |
| 4. | Temperature | Kelvin | K |
| 5. | Elec. Current | Ampere | A |
| 6. | Luminous int. | Candela | cd |
| 7. | Amt. of Sub$^s$ | Mole | Mol |

| 2 supplementary Units:- | | | |
|---|---|---|---|
| No. | Quantity | Unit | Symbol |
| 1. | Plane Angle | radian | rad |
| 2. | Solid Angle | Steradian | sr |




$$d\theta = \frac{ds}{r} \qquad d\Omega = \frac{dA}{r^2}\ sr$$

## SIGNIFICANT FIGURES:-

1. All non-zero digits are significant.

   eg:- 42.3 → 3 S.F.

   243.4 → 4 S.F.

2. Zero b/n two non-zero digits is significant.

   eg:- 4.03 → 3 S.F.

   243.4 → 4 S.F.

3. Leading Zero or zeros placed to left are never significant.

   eg:- 0.543 → 3 S.F.

   0.006 → 1 S.F.

4. Trailing zeros or zero placed to the right of the number are significant.

   eg:- 4.330 → 4 S.F.

   343.000 → 6 S.F.

5. In exponential expression the numerical postion given the number of S.F.

   eg:- $1.32 \times 10^{-2}$ → 3 S.F.

   $1.32 \times 10^{4}$ → 3 S.F.

## ROUNDING OF:-

Addition & Substraction:-

Final result should have same no. of decimal placed as that of original no. with minimum no. of decimal places.

$$\left.\begin{array}{r} 3.1421 \\ 0.241 \\ 0.09 \\ \hline 3.4731 \end{array}\right\} \text{Ans:- } 3.47.$$

Multiplication & Division:-

The no. of S.F. equals the smallest no. of S.F. in any of the original no.

$$\left.\begin{array}{r} 51.028 \\ \times\ 1.31 \\ \hline 66.84668 \end{array}\right\} \text{Ans:- } 66.8$$

## DIMENSIONAL ANALYSIS:-

Dimension of physical quantity are power to which units of base quantity are raised.

eg:- $[M]^a\,[L]^b\,[T]^c\,[A]^a\,[K]^C$

-:Applications:-

1. Checking the Correctness of various formulae:-

   eg:- $Z = A + B$

   $[Z] = [A] = [B]$

2. Conversion of one system of unit to other.

   $n_1\,U_1 = n_2\,U_2$

   eg:- $n_1\,[M_1^A L_1^B T_1^C] = n_2[M_2^A L_2^B T_2^C]$

$$n_1 = n_2 \left[\frac{M_2}{M_1}\right]^A \left[\frac{L_2}{L_1}\right]^B \left[\frac{T_2}{T_1}\right]^C$$

- Mass → $M$
- Length → $L$
- Time → $T$
- Velocity → $LT^{-1}$
- Acceleration → $LT^{-2}$
- Force → $MLT^{-2}$
- Energy → $ML^2T^{-2}$
- Power → $ML^2T^{-3}$
- Force gradiant → $MT^{-2}$

MR*

Different physical quantity ka dimension nikalne ke liye force and energy ka dimension yad rakna hai. Avi tension nahi lena aage ke chapter ke sath yad hota jayga.i

**3. Formula of force to find dimension of different PQ.**

$$F = G\frac{m_1 m_2}{r^2},\ F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$F = Kx,\ F = \frac{\mu_0 I_1 I_2 d}{4\pi r}$$

$$F = qE$$

$$F = qvB$$

$$F = 6\pi\eta r v$$

$$F = Sl$$

**Formula of energy to find dim$^n$ of different physical quantity**

$$\frac{H}{t} = \frac{kA\Delta T}{l}$$

$$\text{Stress} = Y \times \text{Strain}$$

$$Q = ms\Delta T$$

$$Q = mL$$

## DIMENSIONAL FORMULA

- Pressure = stress = Young's modules = $ML^{-1}T^{-2}$
- Work = Energy = Torque = $ML^2T^{-2}$
- Power $P = ML^2T^{-3}$
- Gravitational constant $G = M^{-1}L^3T^{-2}$
- Force constant = Spring constant = $MT^{-2}$
- Coefficient of viscosity = $ML^{-1}T^{-1}$
- Latent heat $L = L^2T^{-2}$
- Electric potential = $\frac{P}{I} = ML^2T^{-3}A^{-1}$
- Resistance = $\sqrt{\frac{\mu_0}{\varepsilon_0}} = ML^2T^{-3}A^{-2}$
- Capacitance = $M^{-1}L^{-2}T^4A^2$
- Permittivity $\varepsilon_0 = M^{-1}L^{-3}T^4A^2$
- Angular momentum = planck's constant = $M^1L^2T^{-1}$

**Time Period:-**

$$T \propto \sqrt{\frac{L}{g}} \propto \sqrt{\frac{M}{k}} \propto \sqrt{\frac{R}{g}}$$

$$\frac{L}{R} = RC = \sqrt{LC}$$

MR*

$$\text{Resistance} = R = \omega L = \frac{1}{\omega C}$$

$$\omega = \frac{2\pi}{T}$$

$$\text{Time} = \frac{L}{R} = \sqrt{LC} = RC$$

**Dimensionless Quantities:-**

1. Strain
2. Refractive index
3. Relative density
4. Plane Angle
5. Solid Angle
6. Poissons ratio
7. Exponential function
8. Trigonometry function
9. Relative permittivity
10. Pure number
11. Efficiency
12. Current, voltage, power gain
13. Length gratiant
14. Coef. of friction

MR*

Pressure = Stress = Young modulus

= Bulk modulus

$= \frac{1}{2}$ strain × stress = modulus of regitity

$= \frac{B^2}{2\mu_0} = \frac{1}{2}\varepsilon_0 E^2$ = energy density $= \frac{nRT}{V}$

dimensionally addition, substraction ko equal le ke solve karte hai.

Kisi be dimensionless function ya quantity ko one likh sakte hai.

Q. If velocity $V = Ax + Bt + C$ find dimension of A, B and C.

MR*

Ans. $V = Ax = Bt = C$

$A = \frac{V}{x} = T^{-1}$

$B = \frac{V}{t} = LT^{-2}$

$C = V = LT^{-1}$

Q. Force $F = \alpha e^{-\beta t}$ then find dimension of $\alpha$ and $\beta$.

Ans. $F = \alpha$, $\alpha = MLT^{-2}$ | $\beta t = 1$, $\beta = T^{-1}$

Q. Acceleration $a = \alpha t + \frac{\beta}{t-\delta}$ find dimension of $\alpha$, $\beta$ and $\delta$.

Ans. $a = \alpha t = \frac{\beta}{t = \delta}$ MR* Ka feel

$\Rightarrow \boxed{\alpha = \frac{a}{t} = LT^{-3}}$ $\Rightarrow \boxed{\delta = t}$

$\boxed{\Rightarrow a = \frac{\beta}{t} \quad \beta = \alpha t = LT^{-1}}$

Q. Fill in the blanks with correct statement, according to given statement

| Dimension | (1) .............. | (2) .............. | (c) A physical quantity have dimension | (d) A physical quantity does not have dimension |
|---|---|---|---|---|
| Unit | (a) A physical quantity have unit | (b) A physical quantity does not have unit | (3) .............. | (4) .............. |

MR*

Ans.(1) May have dimension/may be dimensionless

(2) Must be dimensionless/does not have dimension

(3) Must have unit

(4) May or may not have unit.

Q. Fill in the blanks with correct statement, according to given statement

| Physically correctness | (1) .............. | (2) .............. | (c) Equation is physically wrong | (d) Equation is physically correct |
|---|---|---|---|---|
| Dimensional correctness | (a) Equation is dimensional wrong | (b) Equation is dimensional correct | (3) .............. | (4) .............. |

MR*

Ans. (1) Must be physically wrong

(2) May or may not physically correct

(3) May or may be dimensionally correct

(4) Must be dimensionally correct.

$$S_{nth} = u + \frac{a}{2}(2n - 1)$$

($S_{nth} \rightarrow$ dimensionally correct because it is displacement in one sec.)

Q. If force, acceleration and time taken as fundamental physical quantity then find dimension of energy?

(a) $F^2A^{-1}T$ (b) $FAT^2$

(c) $F^{-1}AT^{-2}$ (d) $FA^{-1}T$

MR*

$E(ML^2T^{-2}) \rightarrow$ Mass ka dimension force hi dega ek mass energy me hai to $F^1$ hona chahiye.

Now L ka square hai ek length force dega ek acceleration hence $A^1$ hona chahiye.

Q. Planks constant (h), speed of light (c), gravitational constant (G) taken as fundamental quantity then dimension of length in terms of them.

(a) $\frac{\sqrt{hG}}{c^{3/2}}$ (b) $\sqrt{\frac{hc}{G}}$

(c) $\frac{\sqrt{hG}}{c^{5/2}}$ (d) $\frac{\sqrt{Gc}}{h^{3/2}}$

MR*

$M^0T^0l = h^xc^YG^Z$

We need dimension of length, then mass should be cancell out by arranging h, c and G. c me to mass hai nahi; $h \rightarrow ML^2T^{-3}$ and $G = M^{-1}L^3T^{-2}$ to h and G ko multiply karne se mass kat jayga. Hence option (b) and (d) wrong ho gaya. Now option (a) and (c) dono me root hai to root laga ke sirf length ka dimension likho phir c se divide kar ke ek length ($L^1$) sirf rakho.

Q. Dimension of critical velocity V of liquid flowing through the tube are expressed as $\eta^x\delta^y r^z$, where $\eta$ is coefficient of viscosity, $\delta$ is density of liquid and r is radius of the tube then the value of x, y and z are given by.

(a) 1, 1, 1 (b) 1, −1, −1

(c) −1, −1, 1 (d) −1, −1, −1

MR*

Velocity me mass hai nahi to $\eta$, $\delta$ and r ko arrange kar velocity lena hai hence mass cancell, radius me bhi mass nahi hai, $\delta = ML^{-3}$ and $\eta = ML^{-1}T^{-3}$

$\delta$ and $\eta$ divide karne se mass kat jayga to ek ka power positive ek ka negative hona chahiya.

Q. If energy (E), velocity (V) and time (T) are chosen as the fundamental quantities the dimensional formula of surface tension will be

(a) $EV^2T^2$ (b) $EV^2T^{-2}$

(c) $EV^{-2}T^{-2}$ (d) $E^{-2}V^2T^2$

MR*

MR* → Surface tension ($MT^{-2}$) Ramlal yaha length nahi to length katne ka socho. Sirf (c) me length kat ho raha hai.

**Limitation of dimensional analysis:**

(1) It is not use to derive dimensionless physicall quantity and constant.

(2) This can not decide weather the give quantity is vector or a scalar.

(3) It can not be use to derive an equation involving more than three physicall quantity.

(4) It can not derive dimensionless function having $\sin\theta$, $\cos\theta$, $e^x$ etc.

(5) Can not use if one quantity depends on two other quantity having same dimension.

(6) It can not derive equation which cantain +ve and −ve terms.

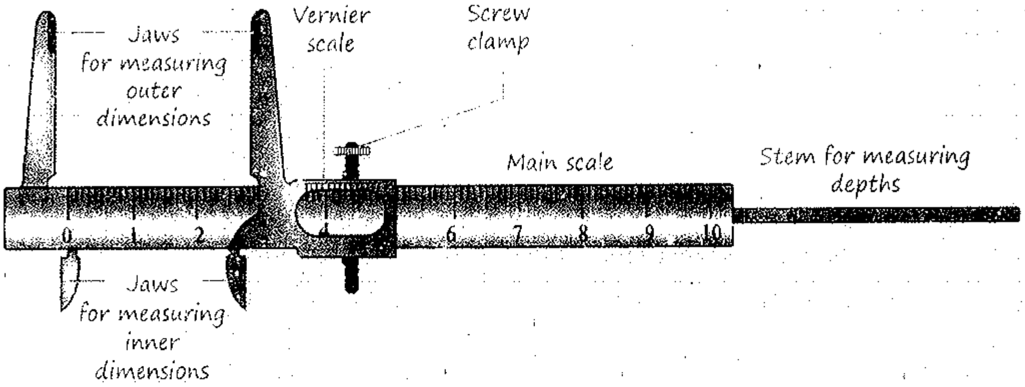
## INSTRUMENTS

**Least Count:-**

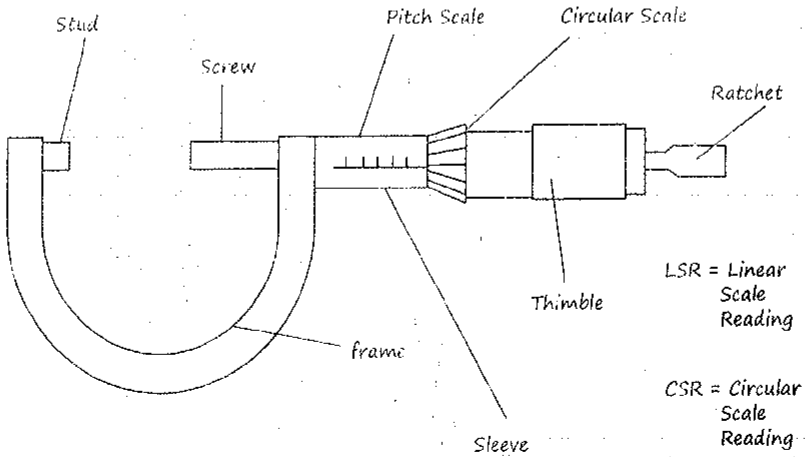
| mm Scale | Vernier Scale | Screw Gauge |
|---|---|---|
| ↓ | ↓ | ↓ |
| 1mm | 0.1mm | 0.01mm |

**Vernier calipers:-**

L. C. = 1MSD - 1VSD




**Screw gauge:-**




$$\text{Pitch} = \frac{\text{MSR}}{\text{no. of rotation}}$$

$$\text{L.C.} = \frac{\text{Pitch}}{\text{Total no. of division on Circular Scale}}$$

**Total Reading = 1 LSR + CSR × LC**

If nVsD Coincides with (n-1) MSD then:-

$$(n-1)\ \text{MSD} = n\text{VSD}$$

$$1\ \text{VSD} = \frac{n-1}{n}\ \text{MSD}$$

$$\boxed{LC = 1\ \text{MSD} - \frac{n-1}{n}\ \text{MSD} = \frac{1\text{MSD}}{n}}$$

**Total Reading = 1 MSR + coinciding VSR × LC**

| Measured length | Used instrument |
|---|---|
| [illegible] | vernier calliper |
| [illegible] | metre scale |
| [illegible] | screw gauge |
| [illegible] | screw gage |
| [illegible] | vernier calliper |

Accuracy: It is the measure of how close the measured value is to the true value. Closness of measured and true value.

Precision: It tells us to what resolution or limit the quantity is measured.

Q. If true value of length is 6.57 m then which of the following reading is most accurate and most precise.

(a) 6.52 m  (b) 6.61 m

(c) 6.513 m  (d) 6.68 m

Ans. Most accurate (b), most precise (c)

## ERROR IN MEASUREMENT:-

Difference between true value & measured value of a quantity

- **Systematic Errors**
  Errors which tend to occur only in one direction, either positive or negative
  - **Instrumental**
    Due to inbuilt defect of measuring instrument
  - **Experimental**
    Limitation in experimental technique
  - **Personal**
    Due to individual bias, Lack of proper setting of apparatus
- **Random Errors**
  Irregular and at random in magnitude & direction

**Absolute Error:-** $\Delta a = |a_i - a_{mean}|$

$$a_{mean} = \frac{a_1 + a_2 \ldots + an}{n}$$

- Always positive
- Unit and dimension same as physical quantity
- Least count error can be taken as absolute error
- It can not tell about accuracy of measurment

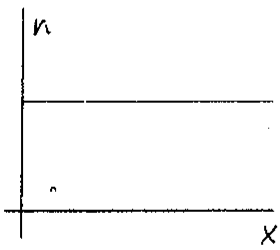
**Relative Error:-** $\frac{\Delta a_{mean}}{a_{mean}}$

$$\Delta a_{mean} = \frac{\Delta a_1 + \Delta a_2 \ldots + \Delta a_n}{n}$$

$$\text{Percentage Error} = \frac{\Delta a_{mean}}{a_{mean}} \times 100$$

- Unit and dimension less
- It tell about accuracy of measurment
- Random error can be decreas by decreasing no of obserbation.

$nx = cost^n$, $n$ = no. of obserbation, $x$ = Random error




- In 5 reading random error is 3% and systematic error is 4%. It we increased no. of observation to 30 then random error 1/2% and systematic error remains 4%.

**General Rule:-**

$$Z = \frac{A^p B^q}{C^r},$$

Then max. frac$^n$ relative error in Z will be:-

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

Combination of errors:-

| Operations | Formula Z | Absolute error $\Delta Z$ | Relative error $\Delta Z/Z$ | Percentage error $100 \times \Delta Z/Z$ |
|---|---|---|---|---|
| Sum | $A + B$ | $\Delta A + \Delta B$ | $\frac{\Delta A + \Delta B}{A + B}$ | $\frac{\Delta A + \Delta B}{A + B} \times 100$ |
| Difference | $A - B$ | $\Delta A + \Delta B$ | $\frac{\Delta A + \Delta B}{A - B}$ | $\frac{\Delta A + \Delta B}{A - B} \times 100$ |
| Multiplication | $A \times B$ | $A\Delta B + B\Delta A$ | $\frac{\Delta A}{A} + \frac{\Delta B}{B}$ | $\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$ |
| Division | $\frac{A}{B}$ | $\frac{B\Delta A + A\Delta B}{B^2}$ | $\frac{\Delta A}{A} + \frac{\Delta B}{B}$ | $\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right) \times 100$ |
| Power | $A^n$ | $nA^{n-1}\,\Delta A$ | $n\frac{\Delta A}{A}$ | $n\frac{\Delta A}{A} \times 100$ |
| Root | $A^{1/n}$ | $\frac{1}{n}A^{((1/n)-1)}\Delta A$ | $\frac{1}{n}\frac{\Delta A}{A}$ | $\frac{1}{n}\frac{\Delta A}{A} \times 100$ |

MR*

- Addition/Substration me pahle absolute error nikalenge phir relative.
- Power/multiplication/division me pahle relative error nikalenge phir absolute.

Example: $y = 3A^2$ ← Power hai to direct relative error likho, constant ko remove karo, power ko aage multiply kar do.

$$\frac{\Delta y}{y} = 2 \times \frac{\Delta A}{A}$$

Example:

$$y = \frac{2A^4\sqrt{B}}{C^3}$$

$$\frac{\Delta y}{y} = 4 \times \frac{\Delta A}{A} + \frac{1}{2} \times \frac{\Delta B}{B} + 3 \times \frac{\Delta C}{C}$$

Q. In an experiment the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half-a-degree (= 0.5°) then the least count of the instrument is:

(a) one minute (b) half minute

(c) one degree (d) half degree

Ans. MSD = 0.5°

30 VSD = 29 MSD

Least Count

= Length of 1 main scale division / No. of divisions of Vernier scale

= 0.5° / 30

= 0.5 × 60min / 30

= 1 min

Q. A vernier callipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier callipers, the least count

(a) 0.02 mm (b) 0.05 mm

(c) 0.1 mm (d) 0.2 mm

Ans. MSD = 1 mm

20 VSD = 16 MSD

⇒ VSD = 16/20 MSD = 0.8 MSD = 0.8 mm

Least Count = MSD−VSD = 1−0.8 = 0.2 mm

Q. If the error in the measurement of area of sphere is 3% then find percentage error in measurement of volume of sphere

Ans. $A = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^2$

$$\frac{\Delta A}{A} = 2\frac{\Delta r}{r} \qquad ...(1)$$

$$\frac{\Delta V}{V} = 3\frac{\Delta r}{r} \qquad ...(2)$$

$$(ii)/(i)\ \frac{\Delta V}{V} \Big/ \frac{\Delta A}{A} = \frac{3}{2}$$

$$100 \times \frac{\Delta V}{V} = \frac{3}{2}\frac{\Delta A}{A} \times 100 = \frac{3}{2} \times 3 = 4.5\%$$

Q. If $T = 2\pi\sqrt{\frac{\ell}{g}}$ then find percentage error in measurement of acceleration due to gravity.

Ans. Ignore constant

$$T^2 = \frac{\ell}{g},\ g = \frac{\ell}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}$$

MR

"खुल जायेंगे सभी रास्ते,
तू रुकावटों से लड़ तो सही।
सब होगा हासिल,
तू अपनी जिद्द पर अड़ तो सही।।"

# 4 Motion in 1-D

## Observer & Frame of Reference:-

- Observer → who takes observation and from where it takes is called frame of reference.
- Observer always assume to be at rest.
- Nothing is at absolute rest or in absolute motion.
- Agar koi Gadhe pr baitha hai toh

  Gadha:- Frame of Ref. Uske upar joh baitha hoga woh observer!

| Distance | Displacement |
|---|---|
| Total Path length | Shortest Path b/w initial and final position |
| Scalar, Struggle | Always straight line |
| Can't decrease with time | Vector, success |
| Always positive | Direction - From initial to final position |
| Depends on path taken | Can decrease with time |
| Both have same unit and dimension | May be +ve or −ve |
| | Does not depends on path |

- If we know only initial and final position then we can't calculate distance but can find displacement.
- If initial position $(x_1, y_1, z_1)$ and final position $(x_2, y_2, z_2)$ then displacement

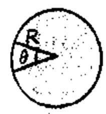
  $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

$\Rightarrow$ distance $\geq |\text{displacement}|$

## DISTANCE & DISPLACEMENT ON CIRCULAR PATH

- $\underset{(x)}{\text{Disp}^m} = 2R\sin\left(\frac{\theta}{2}\right)$


- Arc = $\text{dist}^n = R\theta$



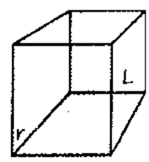

| Displacement (2nd floor) | $\text{Disp}^m$ must be zero | $\text{Disp}^m$ may or may not be zero | If $\text{disp}^m$ is zero | If $\text{disp}^m$ is not equal to zero then:- |
|---|---|---|---|---|
| Distance (1st floor) | If $\text{dist}^n$ is zero | If $\text{dist}^n$ is not equal to zero then | $\text{Dist}^n$ may or may not be zero | $\text{Dist}^n$ must not equal to zero |

## CHALLENGER QUESTION:-

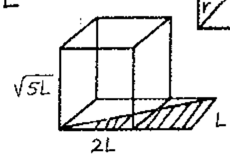
There is a cubicol room. One insect is moving from one corner to other body. Diagonal, then find minimum distance

- Can Fly:- Body Diagonal

  $= \sqrt{3}L$




- Can't Fly:-




## AVERAGE SPEED (HOW FAST IN AN INTERVAL NOT AT INSTANT):-

$$V_{avg} = \frac{\text{Total dist}^n}{\text{Total time}}$$

Same

| Time interval | Distance interval |
|---|---|
| $S_{avg} = \dfrac{V_1 + V_2}{2}$ | $S_{avg} = \dfrac{2V_1 V_2}{V_1 + V_2}$ |
| $(t_1 = t_2 = t$ | $x_1 = x_2 = x)$ |
| $S_{avg} = \dfrac{V_1 t_1 + V_2 t_2}{t_1 + t_2}$ | $S_{avg} = \dfrac{X_1 + X_2}{\dfrac{X_1}{V_1} + \dfrac{X_2}{V_2}}$ |

**SPEED ((How Fast) Scalar, unit: m/s, only magnitude.):-**

- Inst.
- Average

$$S_{inst} = \frac{dx}{dt} \qquad S_{avg} = \frac{\int S.\,dt}{\int dt}$$

$$\int \square = \frac{\int \square .\, dt}{\int dt}$$

For Uniform motion :- $S_{avg} = S_{inst.}$

**VELOCITY (How fast and where):-**
Hum kitna Tez bhag rahe hai and kis direction me bhag rhe hai !

Inst. Average

$$\vec{V}_{inst.} = \frac{d\vec{x}}{dt} \qquad V_{avg} = \frac{\int v.dt}{\int dt}$$

= Rate of change in position
= Slope of position time graph
= Inst. speed × direction
= How fast × where

On circular path, $V_{avg} = \dfrac{V\sin(\theta/2)}{(\theta/2)}$

$|\text{Avg speed}| \geq |\text{Avg Velocity}|$

Inst speed = $|\text{Inst Velocity}|$

## UNIFORM MOTION:-

- Body moving with constant speed in fixed direction
- Uniform velocity
- Acceleration zero
- Avg. velocity = Inst. velocity
- Must be straight line

## NON-UNIFORM MOTION:-

- Velocity non-uniform
- Acceleration non-zero
- Velocity can be change by changing speed only or direction only or both
- In non-uniform speed may constant

  Dimag me set feel ke sath.
- If velocity is uniform then → speed must be uniform.
  - Velocity = Speed + Direction = Constant
- If velocity is variable → Speed may or may not be variable
  - Velocity ko sirf direction change kar ke vary kar sakte hai
- If speed is uniform → then velocity may uniform
  - Direction ka nahi pata.
- If speed is variable → then velocity must be variable
- If avg. velocity is zero then avg. speed may or may not be zero.
- If avg. speed is zero, then avg. velocity must be zero.

## ACCELERATION:-

Ye Motion Ka Feel Nai Hai ! Ye velocity me change ka feel hai.

- Acclⁿ opposite to motion is retardation.
- Negative acceleration does not mean retardation, retardation may be positive or negative.
- Per-sec velocity inject to body or per-sec velocity extract from body ka feel hai.
- Vector → direction of acceleration along change in velocity.

$$\vec{a}_{inst} = \frac{dv}{dt} = \frac{v.dv}{dx} = \frac{d^2x}{dt^2}$$

$$\vec{a}_{avg} = \frac{\vec{V_2} - \vec{V_1}}{\Delta t} = \frac{\int_{t_1}^{t_2} \vec{a}_{ins}\, dt}{\int dt}$$

$\frac{d\vec{v}}{dt} = \vec{a}$ = The rate of change in velocity

$\left|\frac{d\vec{v}}{dt}\right| = |\vec{a}|$ = Magnitude of $acc^n$, $\frac{d|\vec{v}|}{dt}$ = Rate of change in speed.

| | | |
|---|---|---|
| $\bullet\!\longrightarrow u$<br>$\longleftarrow a$<br>$\theta = 180°$<br>$\vec{a}.\vec{u}$ = −ve<br>speed↓<br>retardation<br>Tangential $acc^n$ | $\bullet\!\longrightarrow u$<br>$\longrightarrow a$<br>$\theta = 0°$<br>$\vec{a}.\vec{v}$ = +ve<br>speed↑<br>Tangential $acc^n$ | $\longrightarrow u$, $\downarrow a$, $90°$<br>$\theta = 90°$<br>$\vec{a}.\vec{v} = 0$<br>at this instant only direction will change normal or centripetal $acc^n$ |

MR*

**Bade aaram se**

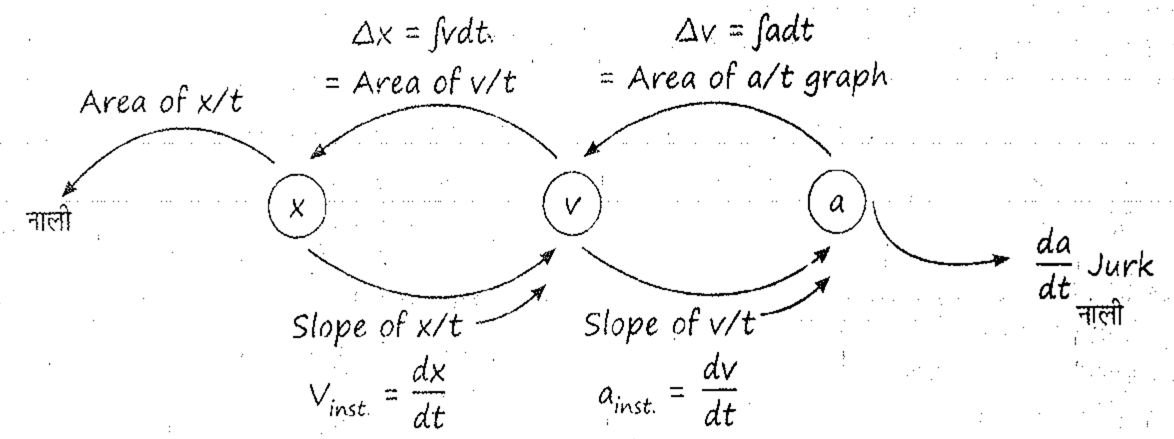
Uniform or constant non-zero acceleration.
Position (x) $\propto t^2$
Velocity (v) $\propto$ t
Velocity v $\propto \sqrt{x}$
If acceleration zero then velocity must be non zero constant

MR*

approach to solve question

| $\vec{a} = 0$ | $\vec{a}$ = non zero constant | $\vec{a}$ = variable |
|---|---|---|
| $\vec{v}$ = constant uniform motion S = vt | Equation of motion applicable | $Eq^n$ of motion is not applicable do integration or differentiation |

MR*




Q. Which of the following is correct for velocity and acceleration?

(a) Velocity increasing, acceleration decreasing

(b) Velocity decreasing, acceleration increasing

(c) Both increasing

(d) Both decreasing

(e) All of these

Ans. (e)

Q. If position $x = at^2 - bt^3$. Find the time when acceleration is zero ?

Ans. $x = at^2 - bt^3$

$$v = \frac{dx}{dt} = 2at - 3bt^2$$

$a = 2a - 6bt = 0$

$2a = 6bt$

$t = \dfrac{a}{3b}$

Q. If velocity $v \propto \sqrt{x}$ then which of the following function is correct for position time relation.

(a) $x \propto t$

(b) $x \propto t^2$

(c) $x \propto \sqrt{t}$

(d) $x \propto t^{3/2}$

Ans. MR* question me $acc^n$ constant then option me acceleration constant option (b)

Q. If acceleration $a = \beta t^{3/2}$ then find velocity after time t if intial velocity is u.

Ans. Equation of motion is not valid

$$a = \frac{dv}{dt} = \beta t^{3/2}$$

$$\int_u^v dv = \beta \int t^{3/2}\, dt$$

$$v - u = \frac{\beta t^{5/2}}{5/2}$$

Q. If acceleration of object $a = \beta x^2$ then find velocity after x displacement, if initial velocity was zero.

Ans. $a = v\dfrac{dv}{dx} = \beta x^2$

$$\int_0^v v\,dv = \int_0^x \beta x^2 dx$$

$$\frac{v^2}{2} = \frac{\beta x^3}{3}$$

$$V = \sqrt{\frac{2\beta x^3}{3}}$$

## MR SPECIAL *

### Majduri se duri MR hai jaruri

Position Ke Formula Mein Time ke dono term ko dekho agar dono term +ve/+ve ya −ve/−ve sign rakhta hai toh woh U-turn Nai lenge ya $dist^n = |disp^m|$ agar sign +ve/-ve Rahi toh U-turn lenge aur distance $\neq |disp^m|$

# yaad rahe 1-D mein U-turn keliye rukhna hoga ($v = 0$) $\therefore$ $dist^n \neq disp^n$

Note: To calculate $dist^n$, $disp^m$ from x-t $eq^n$ :

Ex. $x = t^2 - 4t + 8$ then take v-t graph, plot it using "v" $eq^n$ which we'll get by differentiating "x" $eq^n$ & then put time given from $t_1$ & $t_2$ & see graph calculate $dist^n$/ $disp^m$.

- Moving Frame se body ko drop/release karne pr frame ka velocity share hojata hai but $accl^n$ nai!

## MOTION WITH CONSTANT ACCELERATION :-

$$\vec{v} = \vec{u} + \vec{a}t$$

$$v^2 - u^2 = 2\vec{a}\vec{s}$$

$$\vec{s} = \vec{x}_f - \vec{x}_c = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}_{Avg} = \frac{\vec{v} + \vec{u}}{2} \qquad S = \frac{\vec{u} + \vec{v}}{2} \times t$$

$$S_{nth} = u + \frac{1}{2}(2n - 1)$$

Q. Object starts from rest and constant acceleration attained velocity 32 m/s in 10 sec. then find displacement in next 10 sec.

Ans. $S = \dfrac{u + v}{2} \times t \Rightarrow S = \dfrac{0 + 32}{2} \times 10$

$S = 160$ m in $1^{st}$ 10 sec.

Hence in next 10 sec.

it is $3 \times 160 = 480$ m

Q. If velocity of object $V = \sqrt{25 - 8x}$ then find velocity and acceleration.

Ans. Acceleration is constant then compare velocity with $3^{rd}$ equation of motion

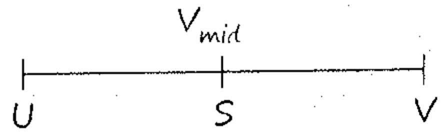
$v^2 = 25 - 8x$ or $v^2 = u^2 + 2ax$

$u^2 = 25$ $\quad -8x = 2ax$

$u = 5\ m/s$ $\quad 2a = -8$

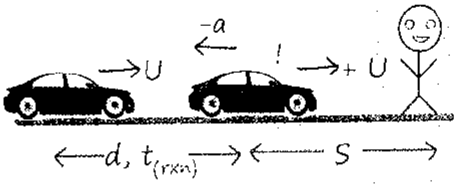
$a = -4\ m/s^2$

**Note:-**




$$V_{mid} = \sqrt{\frac{U^2 + V^2}{2}}$$

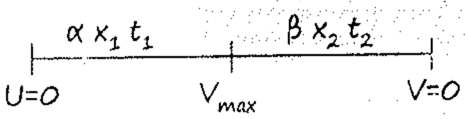
**Stopping Distance:-**




$\boxed{d = Ut_{rx^n}}$ $\quad \boxed{S = \frac{U^2}{2a}}$

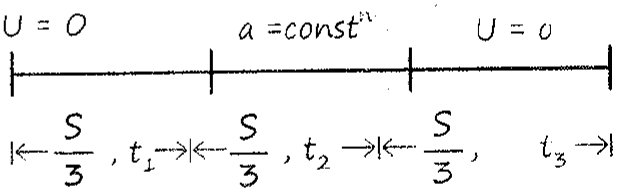
Reaction time

**Rest To Rest Motion:-**




$\alpha\ t_1 = \beta t_2$ $\quad V_{max} = \left[\frac{\alpha\ \beta}{\alpha + \beta}\right] T$

$\alpha\ x_1 = \beta x_2$ $\quad s = \frac{1}{2}\left[\frac{\alpha\ \beta}{\alpha + \beta}\right] T^2$




Ratio of time for equal dist$^n$ interval:-

$t_1 : t_2 : t_3 = 1 : \sqrt{2}-1 : \sqrt{3}-\sqrt{2}$

Ratio of disp$^m$ for equal time interval:-

$S_{1st} : S_{2nd} : S_{3rd} = 1 : 3 : 5$

$S_{1s} : S_{2s} : S_{3s} = 1 : 4 : 9$

$S_t : S_{next\ t} = 1 : 3$ or $x : 3x$

Ratio of displacement in time t and next same time intraval t, where motion starts from rest and constant acceleration

$S_t : S_{2t} = 1 : 4$ or $x : 4x$

displacement in time t total time (2t)

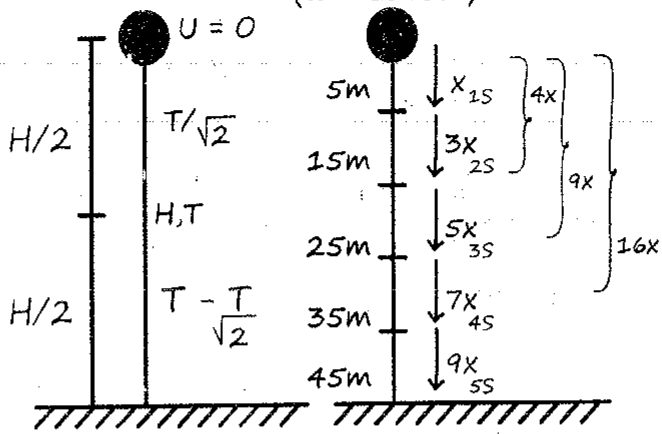
Q. Object starts from and constant acceleration moves 80 m in 7 sec. then find displacement in next 7 sec.

Ans. Displacement in next 7 sec = 3x

$= 3 \times 80 = 240$ m

**MOTION UNDER GRAVITY:-**

(a = const$^n$)



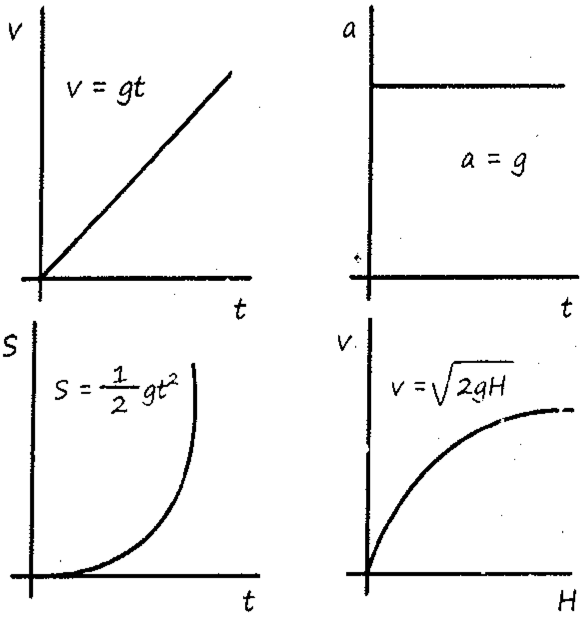

$S_{1st} : S_{2nd} : S_{3rd} = 1 : 3 : 5$

$S_{1s} : S_{2s} : S_{3s} = x : 4x : 9x$

**Note:-**

1. Time of Fight $(T_F) = \sqrt{\frac{2H}{g}}$
2. Velocity at ground :- $v = \sqrt{2gH}$

## Graphs:-

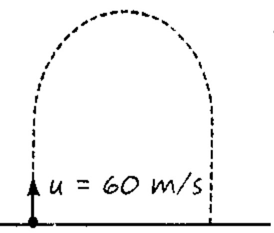



## MOTION UNDER GRAVITY FROM GROUND TO GROUND:-

- Non-uniform motion (velocity = variable) with constant acceleration (g).
- At maximum height velocity zero and a = g.

$$H_{max} = \frac{U^2}{2g} \qquad T_f = \frac{2U}{g}$$

$$T_{up} = \frac{U}{g} \qquad T_{down} = \frac{U}{g}$$




1. Total time of flight $T = \frac{2u}{g} = \frac{2 \times 60}{10}$ = 12 sec.
2. Maximum Height $H = \frac{u^2}{2g} = 180$ m
3. Velocity at t = 7 sec.
   $v = u + gt$
   $= 60 - 10 \times 7 = -10$ m/s
4. Displacement in 8 sec.
   $S = ut + \frac{1}{2}at^2 = 160$ m
5. Distance in 8 sec.
   at t = 6 sec. body comes to at rest and takes u-turn hence calculate distance 0 to 6 sec. then 6 to 8 sec
   $S = 180 + 20 = 200$ m
6. Distance in $9^{th}$ sec. downward journey ka $3^{rd}$ sec = 25 m. Use ratio.
7. Distance in last sec of upward journey = distance in $1^{st}$ sec of downward journey = 5 m (always)

Q. A stone with weight W is thrown vertically upward into the air with initial velocity $v_0$. If a constant force, due to air drag acts on the stone throughout the flight & if the maximum height attain by stone is h and velocity when it strikes to the ground is u. Which one is correct?

(a) $h = v_0^2\left(1 + \frac{f}{W}\right) / 2g,\ v = v_0$

(b) $h = v_0^2 / 2g\left(1 + \frac{f}{W}\right),\ v = \text{zero}$

(c) $h = v_0^2/2g\left(1 + \frac{f}{W}\right),\ v = v_0\sqrt{\frac{W-f}{W+f}}$

(d) $h = v_0^2/2g\left(1 + \frac{f}{W}\right),\ v = v_0\sqrt{\frac{W+f}{W-f}}$

Ans.

MR*

If f = 0 then

$H = \frac{v^2}{2g}$ and $v = v_0$

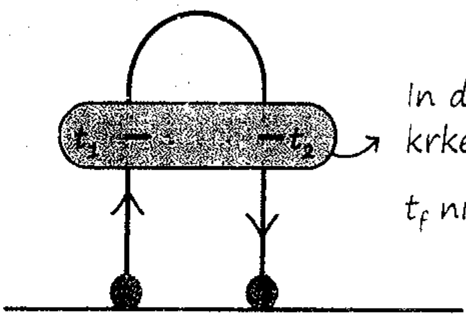
The MR*

If $f \neq 0$

$H < \frac{v^2}{2g}$ and $v < v_0$

Kam karne ke liye niche +ve hoga.

### MR TABLE

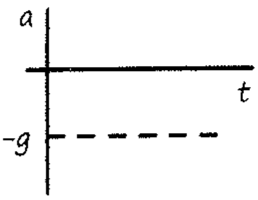


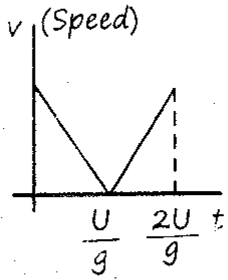

In dono ko add krke $t_f$ nikal sakte !

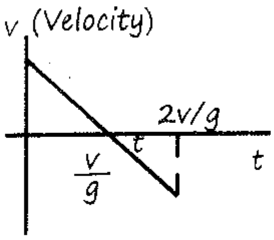
Object is at same height at $t_1$ and $t_2$.

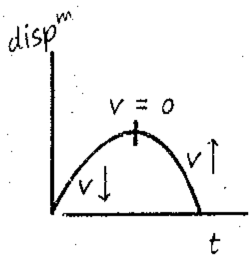
That height $\boxed{h = \frac{1}{2}gt_1t_2}$

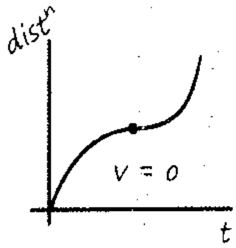
- Ball is projected up with speed "U" graphs:-



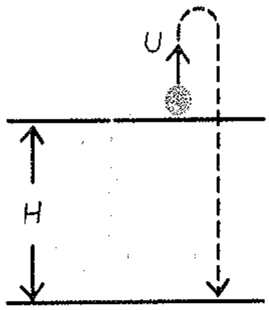




- If air friction in not ignored then:-

$$\frac{t_{up}}{t_{down}} = \sqrt{\frac{g - a}{g + a}}$$

  - $t_{up} < t_{down}$
  - $V_{projection} > V_{collision}$

## MOTION UNDER GRAVITY FROM HEIGHT TO GROUND:-




$$\boxed{t = \frac{U}{g} + \sqrt{\frac{U^2}{g^2} + \frac{2H}{g}}}$$

MR*

If $u = 0$ then it is like drop from height H then

$$t = \sqrt{\frac{2H}{g}}$$

MR*

If $H = 0$ then it is like ground to ground motion

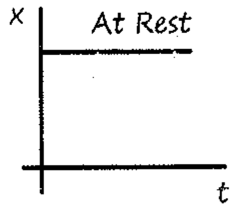
$$t = \frac{2u}{g}$$

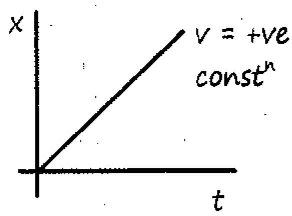
Q. Ball is projected up with speed "u" from height H. Then time of fight $T_1$. With same speed "u" it is projected downward then time of fight is $T_2$. find time of fight "T" when object is dropped from same height.

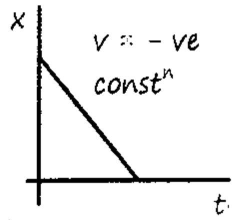
[redacted]

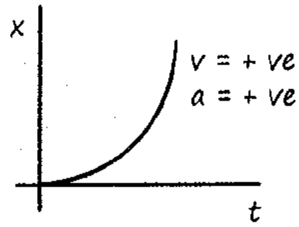
## Graph:-

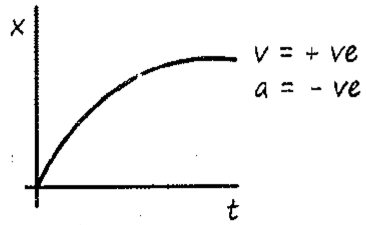
### 1. Position - Time Graph:-



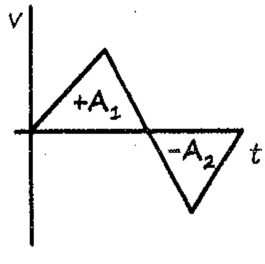



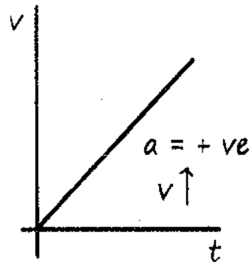




### 2. Velocity - Time Graph:-

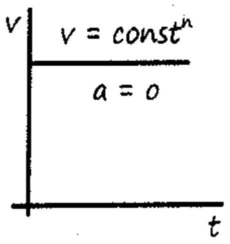


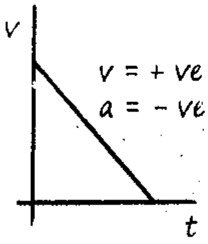



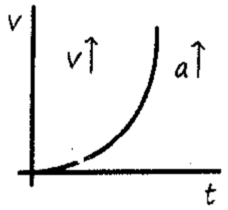
Area = displacement = $A_1 - A_2$

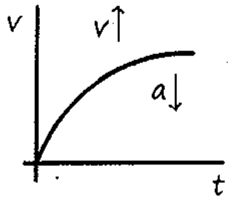
slope = acceleration

but distance = $A_1 + A_2$

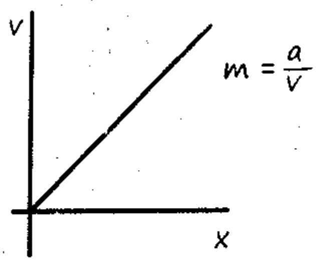


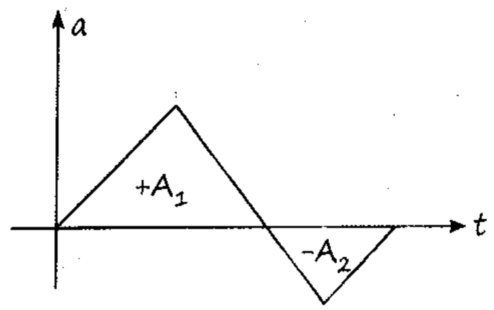




### 3. Velocity - Position Graph:-

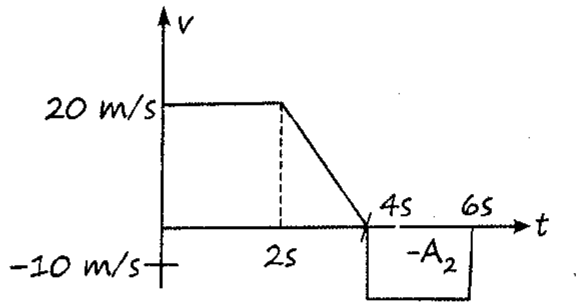



### 4. Acceleration time graph:-




Slope = नाली (Jurk)

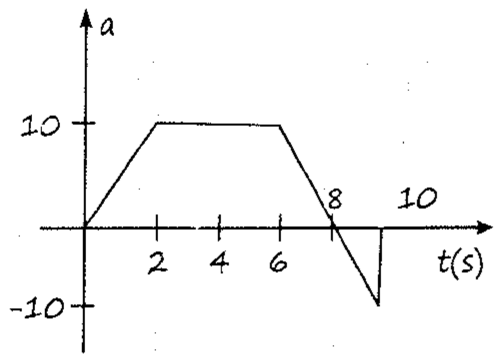
Area = Change in velocity = $A_1 - A_2$




Distance = total area = 40 + 20 + 20 = 80 m

Displacement = 40 + 20 – 20 = 40 m

Q. if initial velocity of object is 10 m/s then find velocity at 10 sec




Ans. Area = change in velocity

$$\vec{V}_f - \vec{V}_i = \frac{1}{2} \times 12 \times 10 - \frac{1}{2} \times 10 \times 2$$

$$V_f - V_i = 60 - 10 = 50$$

$$V_f = 50 + V_i = 50 + 10 = 60 \text{ m/s}$$

## Relative Motion in 1-D

- Observer khud ko hamesa rest me assume karta hai, or uska pas jo bhi velocity, acceleration hota hai, ulta kar ke jisko dekhta hai usme chipka deta hai.

  $\vec{x}_{AB}$ = Position of A w.r.t. B = $\vec{x}_A - \vec{x}_B$

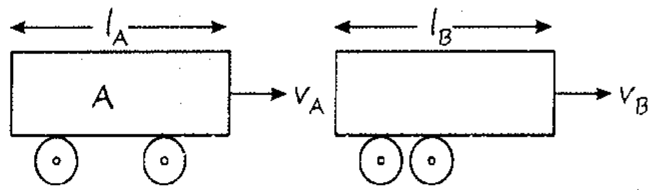
  $\vec{x}_{BA}$ = Position of B w.r.t. A = $\vec{x}_B - \vec{x}_A$

  differentiation w.r.t. time

  $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$ $\quad\quad$ $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

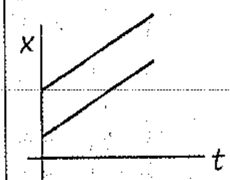
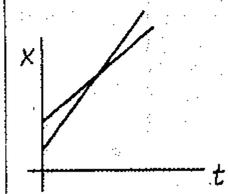
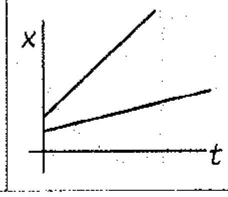
  $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$ $\quad\quad$ $\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$

  $\vec{V}_{AB} = -\vec{V}_{BA}$ $\quad\quad$ $\vec{a}_{AB} = -\vec{a}_{BA}$


- Time taken to overtake = $\dfrac{l_A + l_B}{V_A - V_B}$

- If they are moving opposite to each other $= \dfrac{l_A + l_B}{V_A + V_B}$

| | | | |
|---|---|---|---|
| If $V_A = V_B$ | $V_{AB} = 0$ | $x_{AB}$ = $cost^n$ |  |
| If $V_A > V_B$ | $V_{AB}$ = +ve<br>$V_{BA}$ = −ve | $\vec{x}_{AB}$ = decrease then increase |  |
| If $V_A < V_B$ | $V_{AB}$ = −ve<br>$V_{BA}$ = +ve | $\vec{x}_{AB}$ = Increasing |  |

## Motion of Object on the Moving Surface

1. Man is running on the surface of train with $V_M$ in the direction of train ($V_T$)

   $V_{MG} = V_T + V_M$

   If man is running in opposite direction then, $V_{MG} = V_T - V_M$

2. River is flowing with $V_R$ and man is swimming with $V_M$ in downstream then $V_{MG}$ = Velocity of man w.r.t ground or effective velocity of man = $V_M + V_R$

   In upstream $V_{MG} = V_R - V_M$

3. Same as above in stair case.

- Motion under gravity of one object w.r.t other which is also in motion under gravity is uniform relative motion.

  $a_{AB} = 0 \qquad V_{AB} = Cost^n$

  $S_{AB}$ = Increasing or decreasing linear

  Time of collision = $\frac{S_{AB}}{V_{AB}}$

Q. A ball is drop from 80 m height and another ball is projected with speed 40 m/s then they will collide.

Ans. $V_{relative} = 40$ m/s

$a_{relative} = 0 \qquad t = \frac{80}{40} = 2$ sec

$S_{relative} = 80$

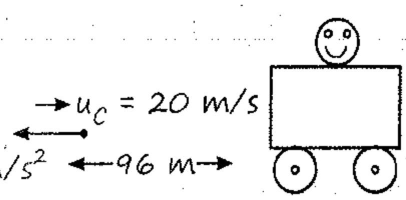
Q. A ball thrown downward with speed 20 m/s and 30 m/s simultaneously, then find relative velocity and separation b/w them after 4 sec

Ans. $a_{AB} = 0$ $V_{BA}$ = 10 m/s (const w.r.t bus)

$S_{BA} = V_{BA}t = 10 \times 4 = 40$ m

Q. A bus starts from rest moving with an acceleration of 2 m/s². A cyclist, 96 m behind the bus starts simultaneously towards the bus at 20 m/s. After what time will he be able to overtake the bus:-

Ans.




$S = ut + \frac{1}{2}at^2$ (cyclist w.r.t bus)

$96 = 20t - \frac{1}{2}2t^2$

$t^2 - 20t + 96 = 0 \quad t = 12s$ and $t = 8$ sec

at 8 sec cyclist overtake bus and at 12 sec bus will again cross cyclist.

MR*

"पछतावा अतीत नहीं बदल सकता और चिंता भविष्य नहीं सँवार सकती। इसलिए वर्तमान का आनंद लेना ही, जीवन का सच्चा सुख है।"

# 5 Motion in a Plane

2-D Motion $\Rightarrow [1\text{-}D]_{x\text{-axis}} + [1\text{-}D]_{y\text{-axis}}$

MR.* feel

- Disp$^m$, velocity and acceleration along x-axis Independent upon disp$^m$, velocity and acceleration of y-axis

- Vector → Component of vector (तोड़ना) ; Magnitude of vector (जोड़ना)

**1> Velocity :-**

$$\vec{V} = V_x\hat{i} + V_y\hat{j} \qquad |\vec{V}| = \sqrt{V_x^2 + V_y^2}$$

**2> Accl$^n$ :-**

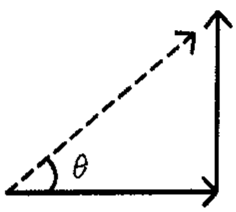
$$\vec{a} = a_x\hat{i} + a_y\hat{j} \qquad |\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

**3> Disp$^n$ :-**

$\vec{r} = \vec{r_2} - \vec{r_1} \qquad \vec{r} = [x_2 - x_1]\hat{i} + [y_2 - y_1]\hat{j}$,

here, $\vec{r} = \Delta x\hat{i} + \Delta y\hat{j}$ is position vector.

**4> Dir$^n$ of motion :-**



$$\tan\theta = \frac{V_y}{V_x}$$

**5> Eq$^n$ of Motion in a plane:-**

$a$ = const$^n$

$$\vec{V_x} = \vec{U_x} + \vec{a_x}\,t \qquad \vec{X} = \vec{U_x}t + \frac{1}{2}\vec{a_x}t^2$$

$$\vec{V_y} = \vec{U_y} + \vec{a_y}t \qquad \vec{y} = \vec{U_y}t + \frac{1}{2}\vec{a_y}t^2$$

- x-axis and y-axis ka motion independent hota hai. Acceleration along x only change velocity of x-axis

$$\vec{V_x} = \frac{d\vec{x}}{dt} \qquad \vec{V_y} = \frac{d\vec{y}}{dt}$$

$$\vec{a_x} = \frac{d\vec{V_x}}{dt} \qquad \vec{a_y} = \frac{d\vec{V_y}}{dt}$$

- Only time is same in both co-ordinate

Q. Initial velocity of object $\vec{u} = 3\hat{i} + 4\hat{j}$ and acceleration $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$ then find velocity after $t = 10$ sec.

Sol. $\vec{V_x} = U_x + a_x t = 3 + 0.4 \times 10 = 7\hat{i}$

$\vec{V_y} = U_y + a_y t = 4 + 0.3 \times 10 = 7\hat{j}$

$\vec{V} = 7\hat{i} + 7\hat{j}$

$|V| = 7\sqrt{2}$

Q. If initial velocity of object $\vec{u} = 3\hat{i} + 4\hat{j}$ after some time $\vec{V} = 4\hat{i} + 3\hat{j}$ then find.

(i) Change in Magnitude of velocity.

(ii) Magnitude of change in velocity.

Sol. (i) $\Delta|\vec{V}| = |\vec{V_f}| - |\vec{V_i}| = 5 - 5 = 0$

(ii) $\Delta\vec{V} = \vec{V_f} - \vec{V_i} = 4i + 3j - 3i - 4j$

$\Delta\vec{V} = \hat{i} - \hat{j}$

$|\Delta\vec{V}| = \sqrt{2}$

Q. x and y of the particle are $x = 5t - 2t^2$ and $y = 10t$, acceleration of particle at $t = 2s$.

Sol.

$$V_x = \frac{dx}{dt} = 5 - 4t$$

$$V_y = \frac{dy}{dt} = 10$$

$|\vec{a}| = 4m/s^2 \qquad a_x = \frac{dV_x}{dt} = -4$

$a_y = 0$

Q. Initial velocity of bus is 5m/s east after 2sec its velocity becomes 5m/s north then find acceleration.

Sol. $\vec{a} = \frac{\vec{V_f} - \vec{V_i}}{\Delta t} = \frac{5\hat{j} - 5\hat{i}}{2}$

$|\vec{a}| \quad \frac{5\sqrt{2}}{2} = \frac{5}{\sqrt{2}}$ (North-west)

## 6> Equation of Trajectory:-

Relation between $x^{th}$ and $y^{th}$ co-ordinate which are derived with the help of time.

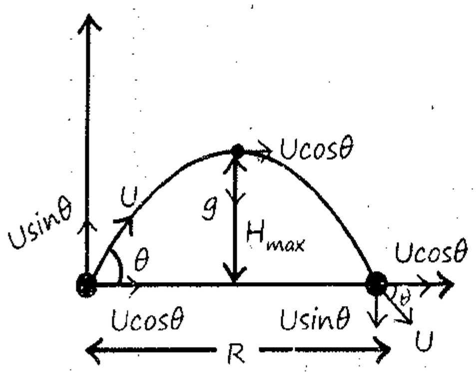
| Position Vector | Equation of Trajectory |
|---|---|
| (1) $\vec{r} = A\sin(\omega t)\hat{i} + A\cos(\omega t)\hat{j}$ | $x^2 + y^2 = A^2$ (Circle) |
| (2) $\vec{r} = A\sin(\omega t)\hat{i} + B\cos(\omega t)\hat{j}$ | $x^2/A^2 + y^2/B^2 = 1$ (Ellipse) |
| (3) $\vec{r} = A\sin(\omega t)\hat{i} + B\sin(\omega t)\hat{j}$ | (Straight line) $Y = (B/A)\,x$ |
| (4) $\vec{r} = 2t\,\hat{i} + 4t^2\,\hat{j}$ | $Y = x^2$ (Parabola) |

- Motion starts from rest and constant acceleration then path → straight line
- If Angle between velocity and $acc^n$ is always 90° then path → circle
- If Angle between initial velocity and acceleration is other than 0° or 180° then path → Parabolic.

## 7> Projectile motion:- Non uniform motion with uniform acceleration

θ = with Horizontal

θ = with Vertical
sab change except range
sinθ ⌒ cosθ



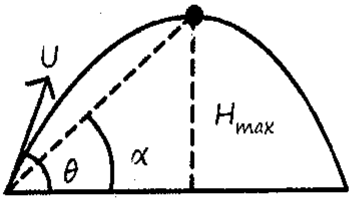

$$T_f = \frac{2u\sin\theta}{g} = \frac{2U_y}{g} \qquad R = U_x T_f = U\cos\theta\,\frac{2U\sin\theta}{g}$$

$$R = \frac{U^2\sin 2\theta}{g} = \frac{2U_xU_y}{g} \qquad H_{max} = \frac{U^2\sin^2\theta}{2g} = \frac{U_y^{\,2}}{2g}$$

| x-axis | y-axis |
|---|---|
| At $t = 0$, $\vec{U}_x = U\cos\theta\,\hat{i}$ | $\vec{U}_y = U\sin\theta\,\hat{j}$ |
| At 't', $\vec{V}_x = U\cos\theta\,\hat{i}$ | $V_y = (u\sin\theta - gt)\,\hat{j}$, $\vec{a}_y = -g\,\hat{j}$ |
| $\vec{a}_x = 0$ | $y = U\sin\theta\, t - \frac{1}{2}gt^2$ |
| $x = U_xt = U\cos\theta\, t$ | $\lvert\Delta\vec{P}\rvert = (2mU\sin\theta)$ |
| $(V_{avg}) = U\cos\theta$ | |

"H" = same = Vertical velocity same = $T_f$ same
V collision with ground = $U\cos\theta\,\hat{i} - U\sin\theta\,\hat{j}$

8> **Elevation angle of Max. Point from point of projection:-**




$\frac{R}{2}$

$$\tan\alpha = \frac{\tan\theta}{2}$$

9> **Relation b/n Range & $H_{max}$ ?**

$$H = \frac{R\tan\theta}{4}$$

10> **Speed at any point:-**

$$\tan\alpha = \frac{U\sin\theta - gt}{U\cos\theta}$$

$$\vec{V} = U\cos\theta\,\hat{i} + U\sin\theta - gt\,\hat{j}$$

$$|\vec{V}| = \sqrt{(U\cos\theta)^2 + (U\sin\theta - gt)^2}$$

11> **Speed at point $\frac{H_{max}}{2}$ ?**

$$V = \sqrt{(U\cos\theta)^2 + \left[\frac{U\sin\theta}{\sqrt{2}}\right]^2}$$

12> **Condition of Max. horizontal Range:-**

$$R_{max} = \frac{U^2}{g}$$

$\theta = 45^\circ$ iske upar $R\downarrow$

$$H = \frac{U^2}{4g}$$

13> **Complementary Angle:-**

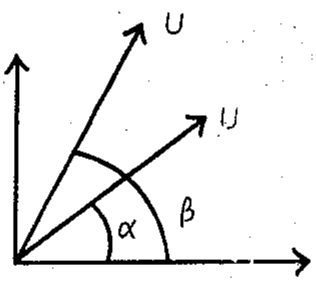
$$\alpha + \beta = 90^\circ \; ; \; R_1 = R_2$$

IIT

$$\frac{H_1}{H_2} = \tan^2\alpha$$

$$\Rightarrow \frac{T_1}{T_2} = \tan\alpha$$

$$H_1 H_2 = \frac{R^2}{16}$$




14> **Eq$^n$ of Trajectory in Projectile Motion:-**

$$y = x\tan\theta\left[1 - \frac{x}{R}\right]$$

$$R = \frac{2U^2\cos\theta.\sin\theta}{g}$$

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{U^2\cos^2\theta}$$

15> **Time at which particle moving $\perp^{er}$ to initial velocity:-**

$$t = \frac{U}{g\sin\theta}$$

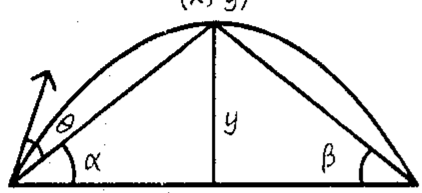
Note

$(v \perp U)$

*Only valid when $\theta \geq 45^\circ$. For $\theta < 45^\circ$ kabhi $\perp^{er}$ nai hoga!*

16>

$(x, y)$




$x_1$ $x_2$

$$\tan\theta = \tan\alpha + \tan\beta, \quad \tan\theta = y\left[\frac{1}{x_1} + \frac{1}{x_2}\right]$$

- Ball is projected with same speed at 42° and 47° then Range $R_1$ and $R_2$ respectively then $R_1 < R_2$

  hint :- Angle 45° ke jitna pas range utna jyada.

Q. Equation of trajectory $y = \sqrt{3}x - \frac{gx^2}{\sqrt{3}}$ then find range and angle of projectile.

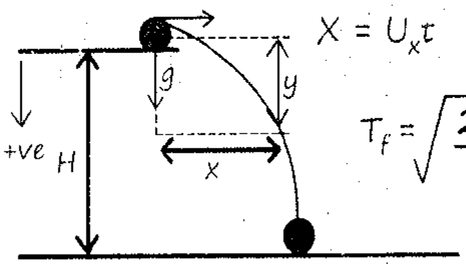
Sol. $Y = x\tan\theta\left[1 - \frac{x}{R}\right]$ $\quad \tan\theta = \sqrt{3}$, $\theta = 60^\circ$

When $y=0$ then $x=R$

$$0 = \sqrt{3}x - \frac{gx^2}{\sqrt{3}} \Rightarrow \sqrt{3}x = \frac{gx^2}{\sqrt{3}}$$

$$\Rightarrow x = R = \frac{3}{10}\text{ m.}$$

## 17> Horizontal Projectile motion from some height:-




$X = U_x t \quad y = \frac{1}{2} g t^2 \quad |V|_{ground} = \sqrt{U^2 + (2gh)}$

$T_f = \sqrt{\frac{2H}{g}} \qquad \vec{V} = U\hat{i} + gt\hat{j}$

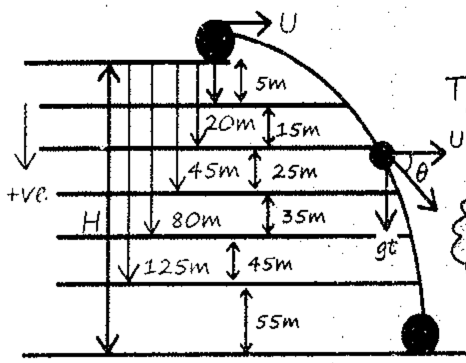
- Eq$^n$ of Trajectory:-

$$y = \frac{1}{2} g \left[\frac{x}{U}\right]^2$$

MR* "Raste me jitne sangharsh ho, manzil utna hi khubsoorat hota hai."

- Speed at any time "t" :-

$$|\vec{V}| = \sqrt{U^2 + (gt)^2}$$




$T_f = \sqrt{\frac{2H}{g}}$

$\tan\theta = \frac{gt}{U}$ angle made by projectile with horizontal

$R = U_x T_f$

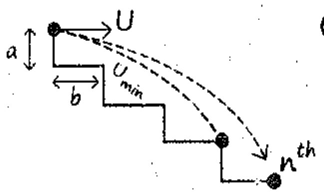
$R = U_x \sqrt{\frac{2H}{g}}$

MR* "Make yourself better than yesterday."

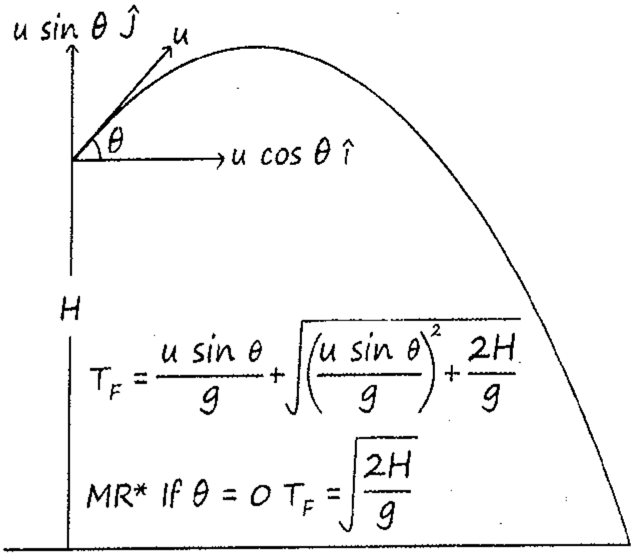
- Velocity of ball so that it'll fall on $n^{th}$ step:-

$$U_{min} = \sqrt{\frac{(n-1)\, b^2 g}{2a}}$$

$$U = \sqrt{\frac{n b^2 g}{2a}}$$



## 18> Projectile Motion from some height at angle $\theta$,




$$T_F = \frac{u \sin\theta}{g} + \sqrt{\left(\frac{u \sin\theta}{g}\right)^2 + \frac{2H}{g}}$$

MR* If $\theta = 0$ $T_F = \sqrt{\frac{2H}{g}}$

MR* If $H = 0$ $\quad T_F = \frac{2u \sin\theta}{g}$

Q. Ball is projected in Horizontal direction with speed u then find time when distance moved in horizontal and vertical direction is same.

Sol. $x = ut \qquad y = \frac{1}{2} g t^2$

$$ut = \frac{1}{2} g t^2$$

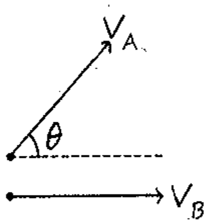
$$t = \frac{2u}{g}$$

## 19> Relative Motion in a Plane :

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

If Angle between $V_A$ and $V_B$ is $\theta$ then.

$$|\vec{V}_{AB}| = \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos\theta}$$




- Relation between $V_A$ and $V_B$ so that A always moving along y-axis.

$V_A \cos\theta = V_B$

Q. Bus is moving in east with 30m/s and car in north with speed 40m/s then velocity of Car w.r.t bus.

Sol. $\vec{V}_{CB} = \vec{V}_C - \vec{V}_B = 40\hat{j} - 30\hat{i}$

$|\vec{V}_{CB}| = 50 m/s$

$= 37°$ east of north

## 20> River man Problem:-

$V_{MR}$ = Velocity of Man w.r.t River

= Velocity of Man w.r.t still water

= Velocity by which man can swim

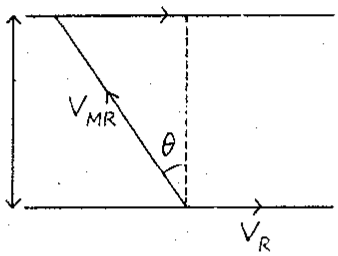
MR** → River apne dam pe cross kiya jata hai

→ Ramlal puri jan apne jan ki taraf lagayga river minimum time me cross karne ke liye. (Man will swim perpendicular to flow of river.)

$$t_{min} = \frac{D}{V_{MR}}$$

$$\text{Drift along river} = V_R \times \frac{D}{V_{MR}}$$

Shortest Path (Just want to reach opposite end)




$[V_{MR} \sin\theta = V_R]$

for shortest path

$$\text{time} = \frac{D}{V_{MR}\cos\theta}$$

$$= \frac{D}{\sqrt{V_M^2 - V_R^2}}$$

Q. A swimmer swimming across a river flowing at a velocity of 4m/s swims at the velocity of 2 m/s. Calculate the actual velocity of the swimmer and the angle.

Sol. The actual velocity of the swimmer can be found out as follows:

$V_{actual} = \sqrt{2^2 + 4^2} = 4.47 m/s$

The angle is calculated as follows:

$\tan\theta = \frac{2}{4}$

$\theta = \tan^{-1}\frac{2}{4} = 26.57°$

Q. A boat takes 2 hours to travel 8 km and back in still water lake. With water velocity of 4 km/h, the time taken for going upstream of 8 km and coming back is

Sol. Velocity of boat = $\frac{8+8}{2} = 8$ km/h

Velocity of water = 4 km/h

$\Rightarrow t = \frac{8}{8-4} + \frac{8}{8+4}$

$= \frac{8}{3} h = 160$ minutes

Q. The speed of a swimmer in still water is 20 m/s. The speed of river water is 10 m/s and is flowing due east. If he is standing on the south bank and wishes to cross the river along the shortest path the angle at which he should make his strokes w.r.t north is given by:

Sol. $|\vec{V}_{SR}| = 20 m/s$

$|\vec{V}_{RG}| = 10 m/s$

$\vec{V}_{SG} = \vec{V}_{SR} + \vec{V}_{RG}$

So,

$\sin\theta = \frac{|\vec{V}_{RG}|}{|\vec{V}_{SR}|}$

$\sin\theta = \frac{10}{20}$

$\Rightarrow \sin\theta = \frac{1}{2}$

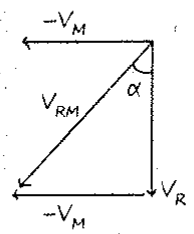
$\Rightarrow \theta = 30°$ west

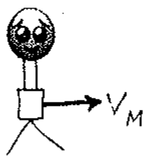
### 21> Rain Man Problem:-

Case 1 : Rain is Falling vertically with $\vec{V}_R$ and Man is running horizontally with $\vec{V}_M$ then velocity of rain relative to man

$\vec{V}_{RM} = \vec{V}_R - \vec{V}_M$

$|\vec{V}_{RM}| = \sqrt{V_R^2 + V_M^2}$

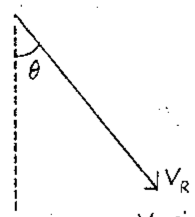



$\tan\alpha = \frac{V_M}{V_R}$

direction of umbrella from vertical

Case 2 : Rain is falling vertically at an angle θ then find velocity of man so that rain appears to fall vertically downward.




$V_R \sin\theta = V_M$

Condition of Collision : Position of A and B are $\vec{r}_A$ and $\vec{r}_B$ moving with velocity $\vec{V}_A$ and $\vec{V}_B$ then find condition of collision

$$\frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|} = -\frac{\vec{V}_A - \vec{V}_B}{|\vec{V}_A - \vec{V}_B|} \Leftrightarrow \hat{r}_{AB} = -\hat{V}_{AB}$$

Direction of relative velocity opposite to relative position. Hence relative velocity perpendicular to line joining of particle is zero.

Q. Rain is falling vertically with a speed of $30 ms^{-1}$. A woman rides a bicycle with a speed of $10 ms^{-1}$ in the north to south direction. What is the direction in which she should hold her umbrella?

Sol. Here, $v_c$ = Velocity of the cyclist

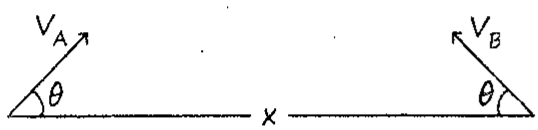
$v_r$ = Velocity of falling rain

In order to protect herself from the rain, the woman must hold her umbrella in the direction of the relative velocity (v) of the rain with respect to the woman.

$\tan\theta = v_c/v_r = 10/30$

$\theta = 18°$

MR** → Component of their velocity perpendicular to line joining will be same.



Condition of collision = VA sin $\theta$ = VB sin $\theta$

$$t = \frac{x}{V_A \cos\theta + V_B \cos\theta}$$

# Two object moving perpendicular to each other with same speed V; having initial separation d then

$$d_{min} = \frac{d}{\sqrt{2}}$$

$$time = \frac{d}{2V}$$

n-person is standing on the cornor of n-side polygon starts moving towards each other with same speed, then time when they will meet.

$$t = \frac{d}{V - V\cos\left(\frac{2\pi}{n}\right)}$$

| | |
|---|---|
| $n = 2$ | $t = \frac{d}{2V}$ |
| $n = 3$ (Triangle) | $n = \frac{2d}{3V}$ |
| $n = 4$ (Square) | $n = d/V$ |
| $n = 6$ (Hexagon) | $t = \frac{2d}{V}$ |

'Main kisi se behtar banu kya fark padta hai...., Main kisi ka behtar karu usse se bahut fark padta hai.'

# 6 Newton's Laws of Motion

## STATE OF A BODY

We define physical state of a body with the help of velocity.

1> State of rest ($\vec{v} = 0$)

2> State of uniform motion ($\vec{v}$ = const)

## NEWTON'S $I^{ST}$ LAW:-

Law of inertia : No external net force required to keep the body in same physical state, Net force required to change physical state.

Constant velocity

$\uparrow$

Fnet = 0 $\searrow$ Rest

$\hookrightarrow$ a = 0

Inertia → Property of object, not a physical quantity, does not have unit and dimension. Inertia $\propto$ Mass

## READING OF SPRING:-

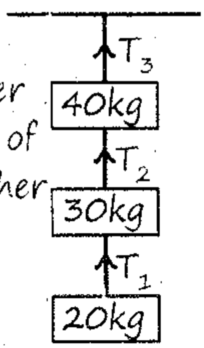
For ideal spring replace it by string & find tension that will be equivalent to spring force.

**Ideal Spring:-** Force is same at all Points!

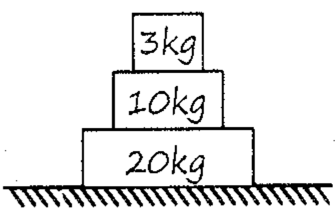
Q. $T_1$ = 200N

$T_2$ = 500N Make F.B.D. of (20+30) together

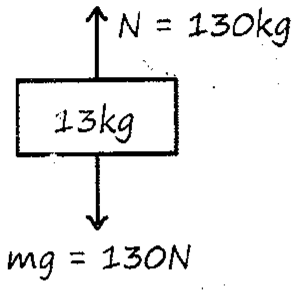
$T_3$ = 900N make F.B.D. of (40+30+20) together




Q.




Contact force b/w 10kg. and 20kg. Make F.B.D. of (3+10) kg.




Q. A uniform rope of mass m and length L is fixed at one end and vertically from rigid support then tension in rope at distance x from rigid support.

(a) $mg\dfrac{L}{L+x}$ (b) $mg\dfrac{L+x}{L}$

(c) $mg\dfrac{x}{L}$ (d) $mg\left(\dfrac{L}{x}\right)$

(e) $\dfrac{mg}{L}(L-x)$

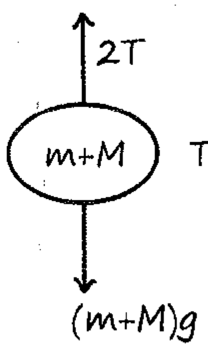
## MR*

If x=0 T=mg and If x=L then T=0

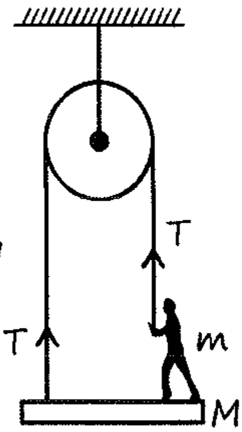
Q. A man of mass m is standing on a plank of mass M. A light string Passing over a fixed smooth pulley connect man and Plank. find tension force exterted by man on string to keep block at rest.

## MR*

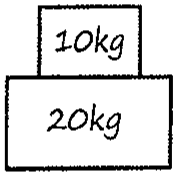
F.B.D. of Man + Plank




$$T = \left(\frac{m+M}{2}\right)g$$

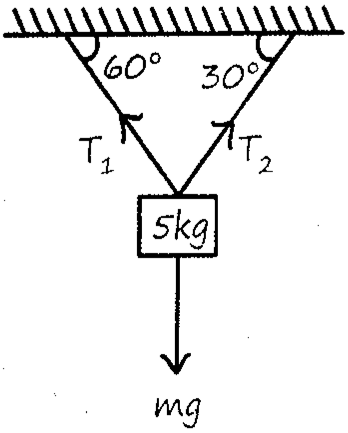


Q. Release from rest then falling down find Normal reation b/w them

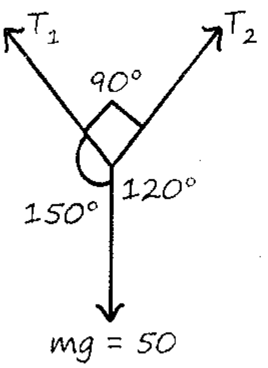



Ans. N=0

Q. Find Tension in wire?



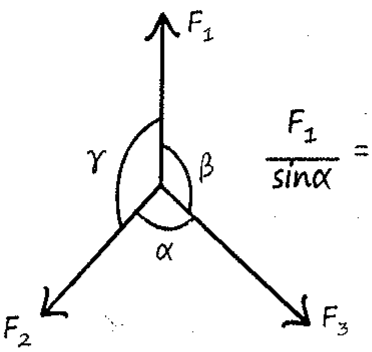

**Sol.**




$$\frac{T_1}{\sin 120^\circ} = \frac{T_2}{\sin 150^\circ} = \frac{50}{\sin 90^\circ}$$

$$T_1 = \frac{50\sqrt{3}}{2} \qquad T_2 = 25\ N$$

## LAMIS THEOREM:-




$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

## NEWTON'S 2ND LAW:-

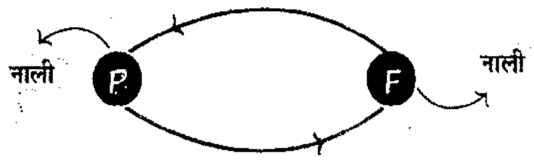
<u>Momentum</u> → Quantity of motion contained in a body.

$$\vec{P} = M\vec{V}$$

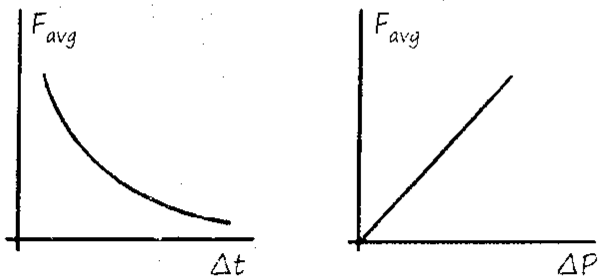
→ Vector, Kg m/sec, parallel to velocity

→ frame dependent.

$$\Delta P = \int F.dt \text{ Area}$$




$$\vec{F} = \frac{d\vec{P}}{dt} = \text{Slope}$$




$$\vec{F}_{avg} = \frac{\vec{\Delta P}}{\Delta t}$$

$$\vec{F} = \frac{m.d\vec{v}}{dt} + \vec{V}\frac{dm}{dt}$$

m = const$^n$

$$\boxed{\vec{F} = m\vec{a}}$$

V = const$^n$ (variable mass)

$$\boxed{\vec{F} = \vec{v}\frac{dm}{dt}}$$

Ex:- Rocket Propulsion

## BALL REBOUNDS WITH SAME SPEED FIND CHANGE IN MOMENTUM:-

$$\boxed{\vec{\Delta P} = 2mV_\perp}$$

$V_\perp$ = Velocity $\perp^{er}$ to Surface

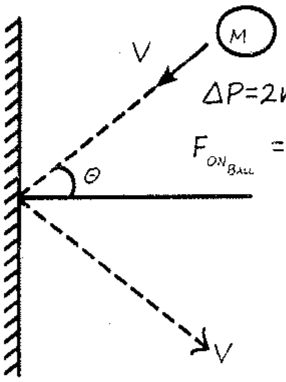
Ball change direction perpendicular with same speed then, $|\vec{\Delta P}| = \sqrt{2}\ mv$

## BULLET PLATE QUESTION:-

$$\vec{F} = \frac{2nmu}{\Delta t} \qquad \vec{P} = 2nmu.$$

## IMPULES [Change in momentum]

$$\vec{I} = \int \vec{F}.\,\vec{dt} = \Delta\vec{P} = \overrightarrow{F_{avg}}.\,\Delta t$$




$$\Delta P = 2mv\cos\theta$$

$$F_{ON_{BALL}} = \frac{\Delta P}{\Delta T} = \frac{2mv\cos\theta}{\Delta T}$$

## NEWTON'S 3RD LAW:-

[Action-Reaction]

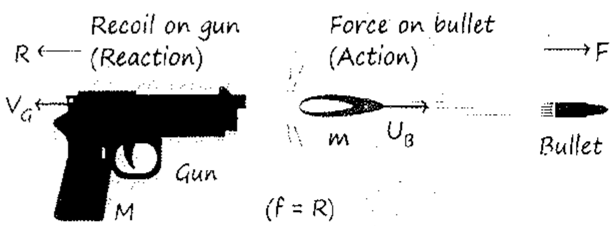
- Action-Reaction pair acts on two diff. body but they should be of same nature.

## CONSERVATION OF MOMENTUM:-

$$F = \frac{dp}{dt} \qquad (2^{nd}\ \text{law})$$

$$F_{ext} = 0 \qquad P = const^n$$

- Gun-Bullet System:-




$$\vec{V}_{gun} = -\frac{m\overrightarrow{U_B}}{M}$$

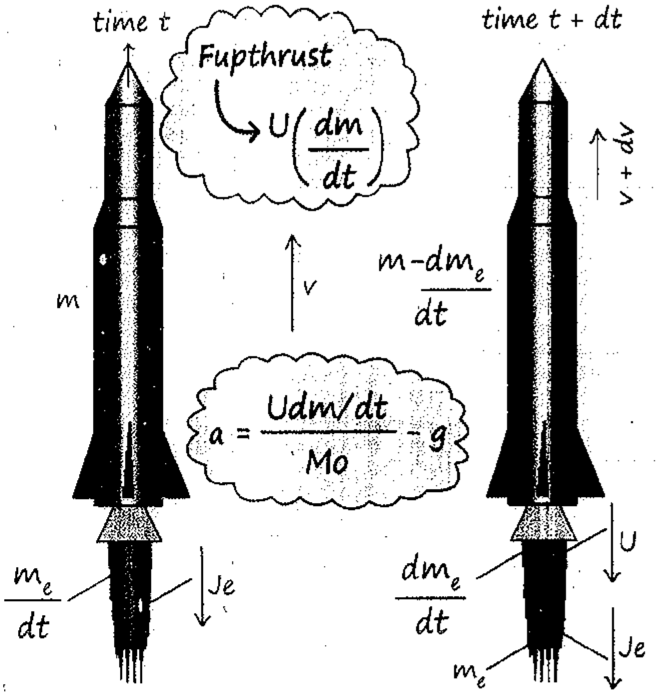
$$[F_N]_{bullet} = nmU_B = \frac{NmU_B}{\Delta t}$$

$\rightarrow F_{gun}$

$$n = \frac{N}{\Delta t} = \text{No. of bullet fired per sec.}$$

$$|\overrightarrow{P_{gun}}| = |\overrightarrow{P_{bullet}}| \qquad \frac{(KE)_{gun}}{(KE)_{bullet}} = \frac{m}{M}$$ [redacted]

## Rocket Problem:-

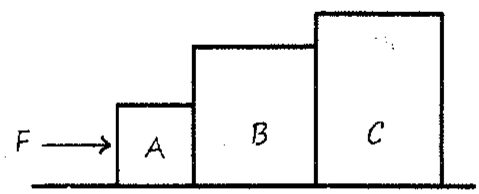



$$a_t = \frac{U.\frac{dm}{dt}}{M_0 - \left(\frac{dm}{dt}\right)t} - g$$

$$Mt = M_0 - \frac{dm(t)}{dt}$$ Remaining Mass

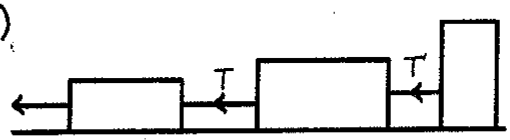
## CONNECTED BODY DYNAMICS:-

(i)



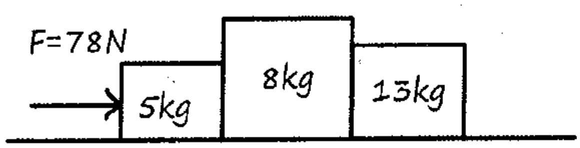
**N = Front Mass x Common Accl$^n$**

(ii)




**Tension = Backside Mass x Common Accl$^n$**

Q. Find acceleration and normal reaction.




**MR***

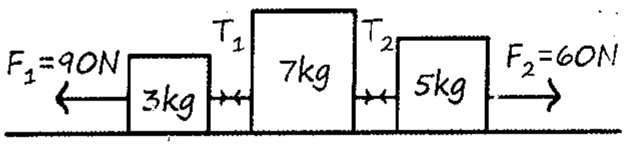
Normal on 13 kg

$N_1 = 13kg \times a = 13 \times 3 = 39N$

Normal on 8 kg by 5 kg

$N_2 = (8+13)a = 21 \times 3 = 63N$

Q.




**Sol.**

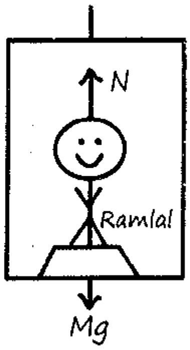
**MR***

$$a = \frac{\text{Net Pulling force}}{\text{Net Mass}} = \frac{90-60}{15} = 2m/s^2$$

$90 - T_1 = 3 \times 2 \qquad T_1 = 90 - 6 = 84N$

For 7kg. $T_1 - T_2 = 7 \times 2$; $84 - 14 = T_2$
$T_2 = 70N$

## LIFT SAWAAL:-




(1) $N = M(g + a)$ = Accl$^n$ up.

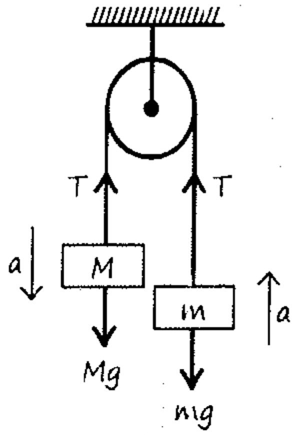
(2) $N = M(g - a)$ = Accl$^n$ down

(3) $N = Mg$ = Accl$^n$ = 0
V = const$^n$

## PULLEY BLOCK SYSTEM-I:-

$$a_{system} = \frac{Mg - mg}{M + m}$$

$$T = \frac{2M_1M_2}{M_1 + M_2}g$$




**MR***

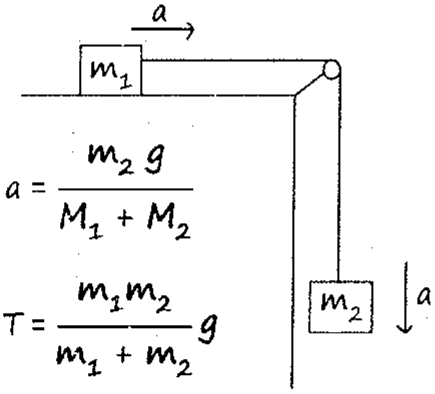
If $M = m \rightarrow a = 0$

If $M = 0$ or $m = 0 \rightarrow a = g$

$\Rightarrow$ Think MR* for tension ☺

| | | |
|---|---|---|
| If $m_1 = 0$<br>$T = 0$ | $m_2 = 0$<br>$T = 0$ | check also dimension of acc$^n$ and tension |

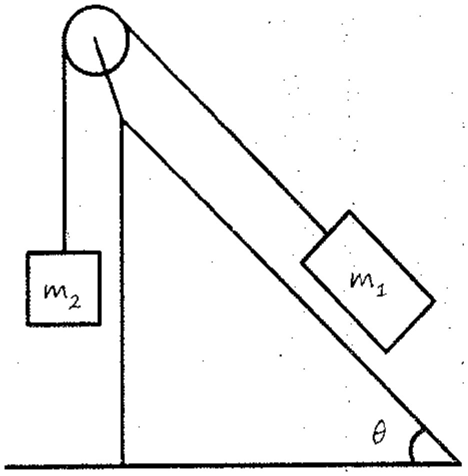
## PULLEY BLOCK SYSTEM-II:-




$$a = \frac{m_2 g}{M_1 + M_2}$$

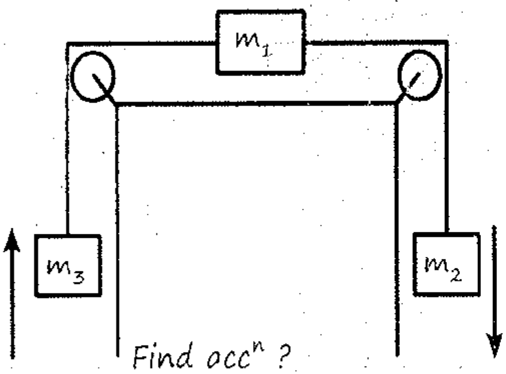
$$T = \frac{m_1 m_2}{m_1 + m_2} g$$

Think MR* for 'a' and Tension




acc$^n$ of $m_1$

$$a = \frac{m_1 g\sin\theta - m_2 g}{m_1 + m_2}$$




Find acc$^n$ ?

$$a = \frac{(m_2 - m_3)g}{m_1 + m_2 + m_3}$$

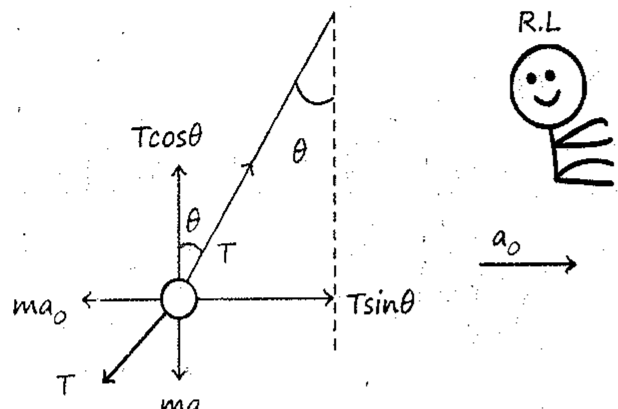
## PSEUDO FORCE CONCEPT:-

- Used to valid Laws of Motion in non-inertial frame.

$$\vec{F}_{pseudo} = -m\vec{a}_{frame}$$

- Apparent weight (Normal) and weight change nai hoga inertial/non inertial frame ho
- Pendulum in Car

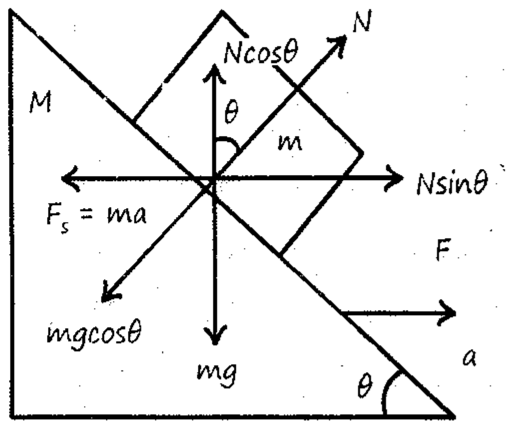
$$\sin\theta = \frac{a}{\sqrt{a^2+g^2}}$$




$\theta = \tan^{-1}(a_0/g)$

$$T = m\sqrt{a^2+g^2}$$

- Accl$^n$ of incline plane so that block over it does not slip:-

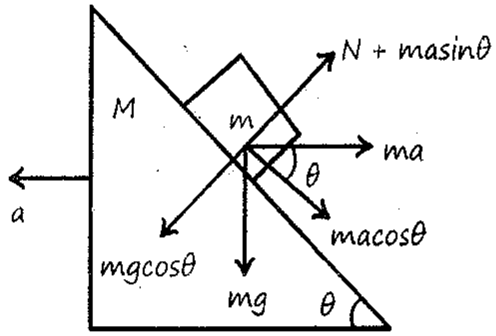



$$N = \frac{Mg}{\cos\theta}$$

$(a = g\tan\theta)$

$\Rightarrow f = (M+m)g\tan\theta$

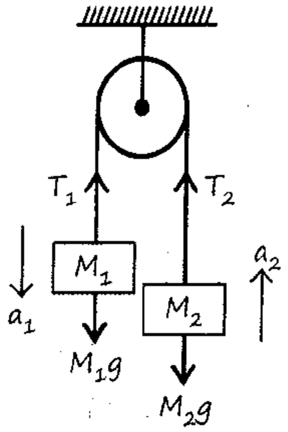
- Accl$^n$ of incline plane so that block over it can free fall:-



$a = g\cot\theta$ $F = Mg\cot\theta$

[illegible][$g\cos\theta$ − [illegible]$\sin\theta$]

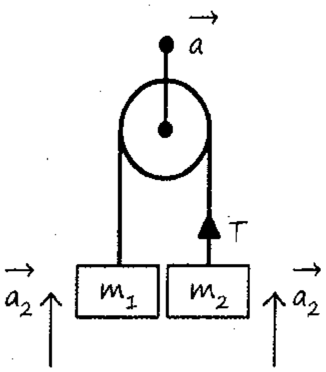
## CONSTRAINT RELATION:-




$T_1a_1 = T_2a_2$

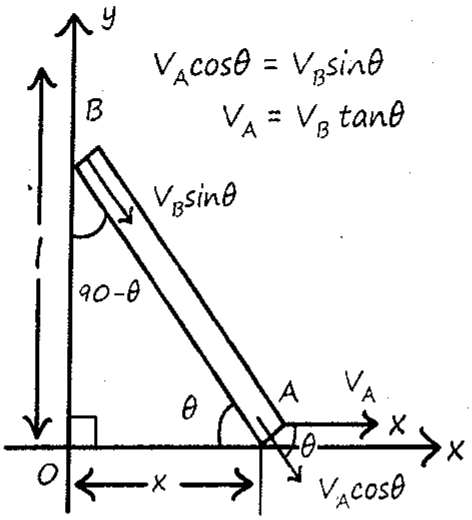
$T_1V_1 = T_2V_2$

$T_1x_1 = T_2x_2$




$$\vec{a} = \frac{\vec{a_1}+\vec{a_2}}{2}$$

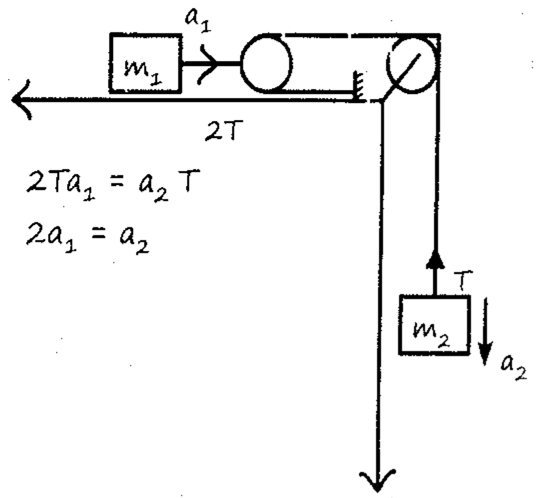
$$T = \frac{2m_1m_2}{m_1+m_2}(g+a)$$




$V_A\cos\theta = V_B\sin\theta$

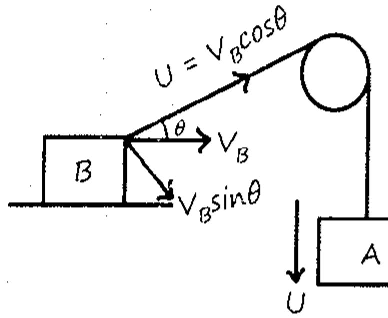
$V_A = V_B\tan\theta$

- Component of velocity along the length of Rod will be same.

- 

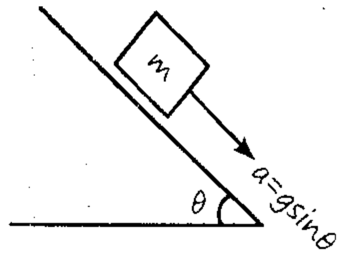

$2Ta_1 = a_2T$

$2a_1 = a_2$

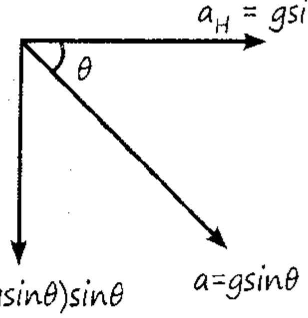
- 


$$V_B = \frac{U}{\cos\theta}$$

Q. Block is sliding on smooth inclined plane, then component of acceleration in vertical direction?




Sol.




$a_H = g\sin\theta.\cos\theta$

$a_V = (g\sin\theta)\sin\theta$

$= g\sin^2\theta$

MR* "तुम किताबों के सामने झुक जाओ। ये तुम्हारे सामने दुनियां झुका देंगी।।"

# 7 Friction

Friction → Component of contact force acts parallel to contact surface.

→ Oppose relative motion or tendency of relative motion.

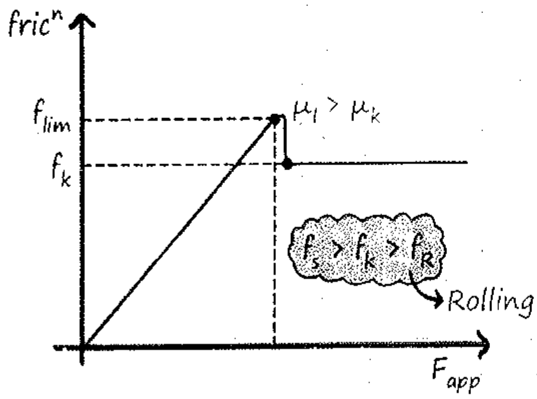
**Friction** → Kinetic $(f_k)$ ○ Object is sliting

Static $(f_s)$ ○ Object is at rest, not sliding

Limiting $(f_L)$ ○ Object about to move

Self adjusting     Friction की औकाद

$f_s = F_{app}$    $f_k = \mu_k N$    $f_l = \mu_l N$




Ramlal is walking in east then friction on Ramlal is static and direction along east.

**MR* For question solving**

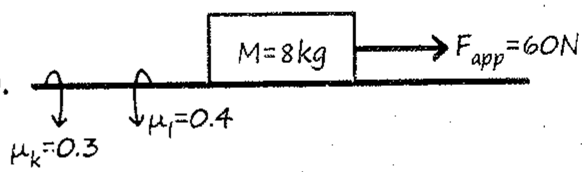
i> Find limiting friction Force.

ii> Compare it with Fapp

$F_{lim} > F_{app}$. [Rest] $[f_{app} = f_{static}]$

$F_{lim} < F_{app}$. [move]

Q.



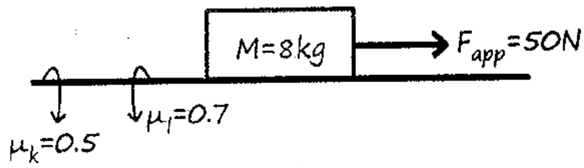

$$F_{limiting} = \mu N = \frac{4}{10} \times 80 = 32N$$

$F_{app} > F_{lim}$ object will move

$$a = \frac{60-\mu_k N}{M} = \left(\frac{60-24}{8}\right) m/s^2$$

$f_{kinetic} = 24N$

Q.




$F_{lim} = \mu_R N = .7 \times 80 = 56N$

Object will not move

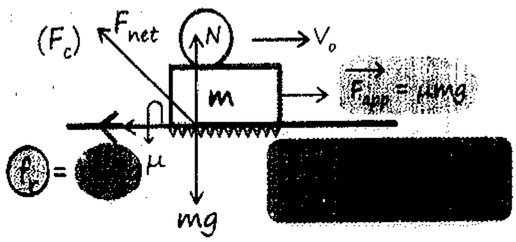
Friction static = 50N

$a = 0$

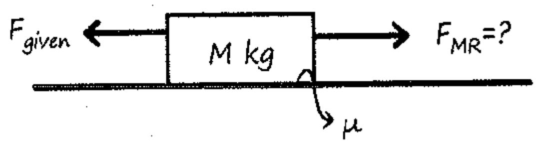
○ Object is thrown with velocity $V_0$ on rough surface of coefficient of friction "$\mu$" then stopping distance and time is:-

$$a = \mu g \qquad s = \frac{V_0^2}{2\mu g} \quad t = \frac{V_0}{\mu g}$$

○ Object is moving with constant velocity as shown in figure then find a contact force between ground and block



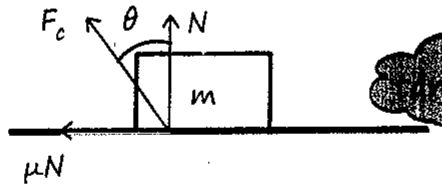

○ Max. & Min value of Friction such that block wont slide:-



$$\left.\begin{aligned}(F_{MR})_{min} &= F_{given} - F_{lim} \\ (F_{MR})_{max} &= F_{given} + F_{lim}\end{aligned}\right\} F_{lim} = \mu N$$

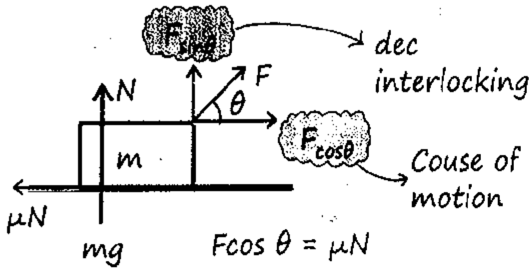
## ANGLE OF FRICTION

Angle b/n N & $F_c$




$\tan\theta = \mu$

- $F_{min}$ req to move an object:-



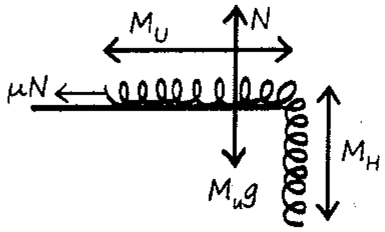

$F\cos\theta = \mu N$

$N = mg - F\sin\theta$

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

[redacted]

## TIME CHAIN QUE:-

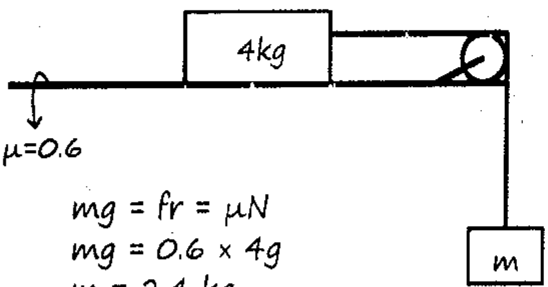



$T = M_Hg = \mu N$

$N = M_ug$

$M_H$ = Hanging Mass. $M_U$ = Upper Mass

Q. Find m so that block just start sliding.

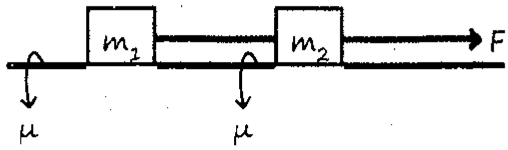



$mg = fr = \mu N$

$mg = 0.6 \times 4g$

$m = 2.4$ kg

Q. Find acceleration and tension in wire?




$F_{limiting} = f_l1 + f_l2$

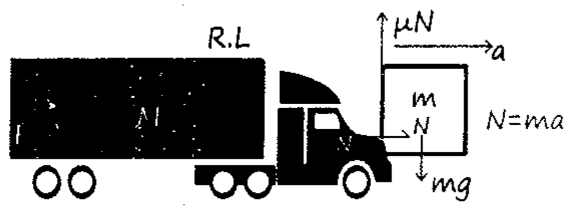
$= \mu m_1 g + \mu m_2 g$

If $F_{limiting} > F$ then rest (a=0)

If $F_{limiting} < F$ then $a = \dfrac{F - f_{lim}}{m_1 + m_2}$

Now find tension by making f.b.d. of $m_1$

## CAR BLOCK QUE:-

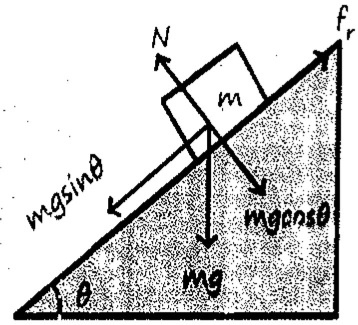



$fr = Mg \qquad \mu N = mg$

$\mu = \dfrac{g}{a} \qquad F = (M + m)\,a = \dfrac{(M + m)_g}{\mu}$

## FRICTION FORCE ON INCLINED PLANE:-

$f_{lim} = \mu mg\cos\theta$




## ANGLE OF REPOSE:-

Max. angle (θ) a rough inclined plane with horizontal such that the block kept on it remains at rest

$\mu = \tan\theta$ Just about to slide

**MR*** When object is placed on rough inclined plane.

- Object chalega ki nai??

  $\mu = \tan\theta$ Just slide

  $\mu < \tan\theta$ Motion

  $f_r = up$ → $a = g\sin\theta - \mu g\cos\theta$ (Down)

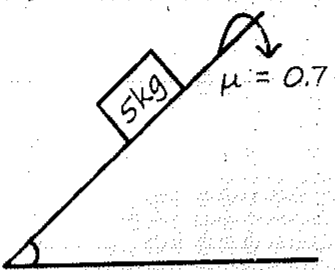
  $f_r = down$ $a = g\sin\theta + \mu g\cos\theta$ (Up)

  $\mu > \tan\theta$ REST

  → $F_c = \sqrt{N^2 + f_r^2}$

  $F_c = Mg$

Q. Find acc, friction and contact force.




$\theta = 30°$

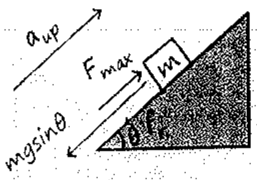
Ans. $a=0$ because $\mu > \tan\theta$

$$fr = mg\sin\theta = 5 \times 10 \times \frac{1}{2} = 25N$$

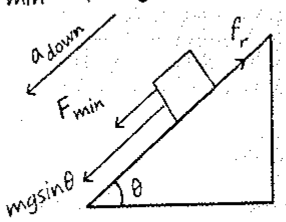
$$F(\text{contact}) = mg = 50N$$

- Max & Min Force req. to move an object on incline:-

$$F_{max} = mg\sin\theta + \mu mg\cos\theta \text{ (up)}$$




$$F_{min} = \mu mg\cos\theta - mg\sin\theta \text{ (down)}$$

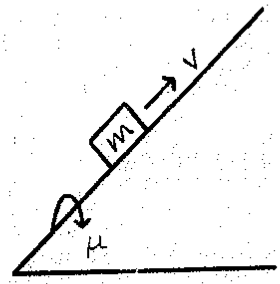



- If ($\mu < \tan\theta$) then force required to keep the object at rest

$$F = mg\sin\theta - \mu mg\cos\theta$$

- Object is moving up to inclined plane

$$a \text{ (downward)} = \mu g\cos\theta + g\sin\theta$$




$$S \text{ (Stoping distance on inclined)} = \frac{V^2(\text{intial})}{2(\mu g\cos\theta + g\sin\theta)}$$

**Note:-**

1> Friction kishika saga nai hai uska koi fix dir^n nai

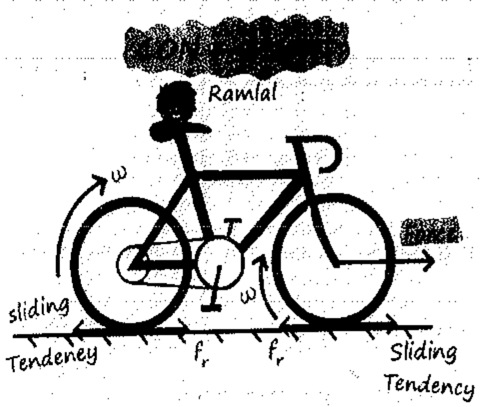
2> Uska ekhi udeesh hai woh relative motion nai hone dega!

\# If object sliding downward then acceleration, $a = g\sin\theta - \mu g\cos\theta$

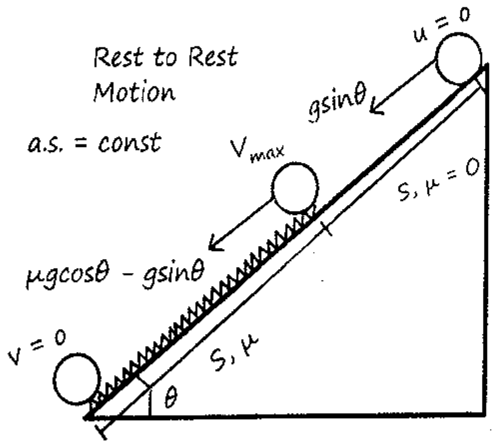
\# If object is sliding upward then acceleration, $a = g\sin\theta + \mu g\cos\theta$

**When Ramlal applies break:-**

**$f_r$ will be backward on both tyres.**




The upper half of incline plane of the inclination is perfectly smooth and the lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom if the coefficient of friction between the block and lower half of the plane is given by:-




**MR ☆**

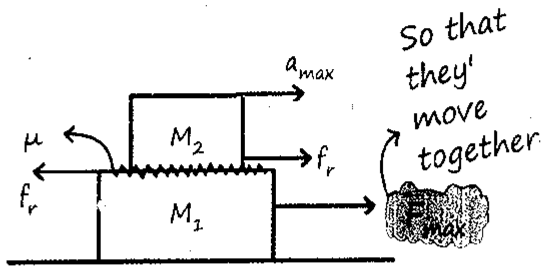
$g\sin\theta . s = s . (\mu g\cos\theta - g\sin\theta)$

$2\sin\theta . = \mu\cos\theta$

[redacted] V.imp

## BLOCK OVER BLOCK SYSTEM:-

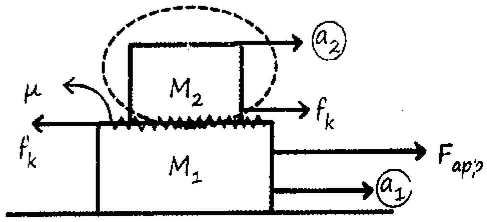
**1> $F = (M_1 + M_2)\mu g$** (साथ - साथ)




$F_{max} = (M_1 + M_2)a_{max}$

$a_{max} = \mu g$ } Friction की max औकात है Iss $Accl^n$ से Chalane Ki

**2> $F_{app} > (m_1 + m_2)\,\mu g$**




**$M_2$**:- Iska maximum $acc^n$ $\mu g$ hi hoga. Isse jayada nahi ho sakta hai.

"$a_1$" Ki value "F" पर depend karegi "$a_1$", ka koi limit nai hai!

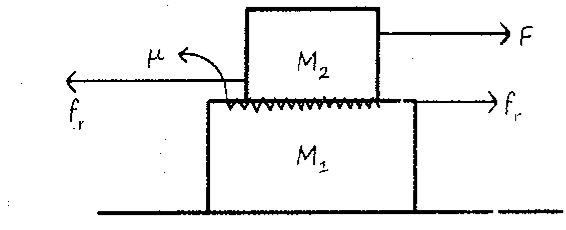
$a_2 = \mu g$ } Upar wala issi $accl^n$ से jayega

Kyuki isko friction hi leke jaare wala hai!

| $a < a_{max}$ | $a = a_{max}$ | $a > a_{max}$ |
|---|---|---|
| Move Together | About to slip! | slip! |
| $f_r$ = static | $f_r$ = limiting | $f_k$ = kinetic diff. $accl^n$ |

$\because a = \dfrac{F_{app}}{M_{net}}$ [redacted]

Note:- If $F_{app}$ on upper block:-

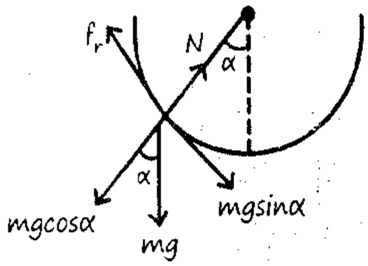



$a = \dfrac{F}{M_1 + M_2}$ } They''' move together

**MR ☆**

$$a_{max} = \mu g\left[\frac{\text{Upar wala mass}}{\text{Niche wala mass}}\right]$$

* An insect crawls up a hemispherical surface very slowly as shown in figure. The coefficient of friction between the insect and the surface is $\frac{1}{3}$. If the line joining the centre of the hemispherical surface to the insect makes an angle $\alpha$ with the vertical, the maximum possible value of $\alpha$ is given by



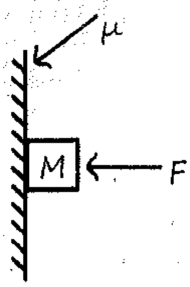

$f_r = mg\sin\alpha$

$\mu mg\cos\alpha = mg\sin\alpha$

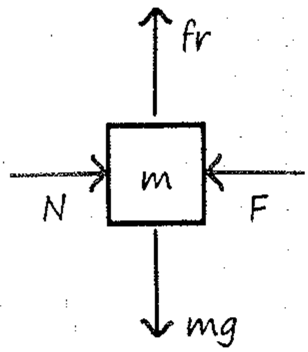
$\mu = \tan\alpha$

[redacted]

Q. If object is at rest then find friction force acting on this object?



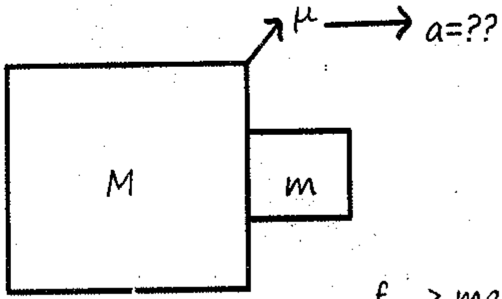

Ans. Object is at rest hence static friction force will act on object

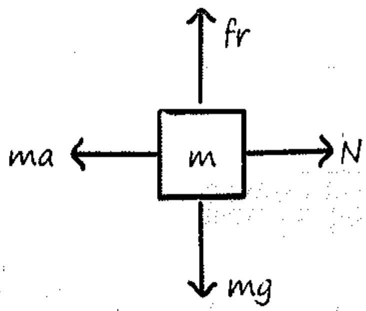



$(Fr)_{static}$ = applied force which create tendency of motion = mg

$f_{limiting} = \mu N = \mu F$ this will act when object about to move

Q. Find $acc^n$ so that block does not slide down?



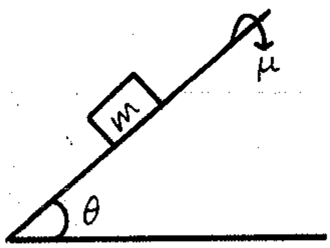

$f_{lim} \geq mg$

$\mu N \geq mg$

$\mu ma \geq mg$

$a \geq \frac{g}{\mu}$.

**MR* Sawal**




If Angle of Inclination θ increase from 0° then contact force remains constant then decreases.

**MR***

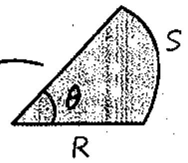
'ख्वाहिशों का कैदी हूँ मैं,
मुझे हक़ीकतें सज़ा देती हैं।
आसान चीजों का शौक नहीं,
मुझे मुश्किलें ही मज़ा देती हैं।।'

# 8 Circular Motion

## 1> GENERAL FORMULA

1. S (distance)

   = Arc length = $R\theta$




   Displacement = $2R\sin\dfrac{\theta}{2}$

   Angular displacement = $\theta$

   ⟶ always in Radian

   ⟶ Anti-clockwise ⊙

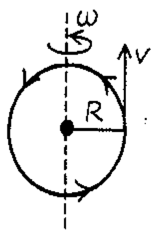
   ⟶ Clockwise ⊗

   Angular disp$^m$ in $\pi$ rotation,

   $\theta = \pi(2\pi) = 2\pi^2$

2. V (speed) = $R\omega$ $\boxed{a_t = R\alpha}$

   $\vec{V} = \vec{\omega} \times \vec{R}$




3. Certripetal Accl$^n$:-

   $a_c = \dfrac{V^2}{R} = R\omega^2 = \omega V$

   ↳ Ye always lagega agar circular motion hai toh.

4. Tangential acceleration:-

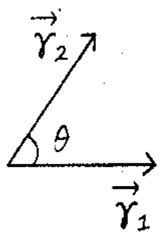
   $a_t = \dfrac{d|v|}{dt} = \alpha R$

   ↳ Ye sirf varying speed Ke samay.

5. $|\Delta\vec{\gamma}| = 2r\sin\dfrac{\theta}{2}$

   **MR****

   $|\Delta\vec{v}| = 2v\sin\dfrac{\theta}{2}$




6. $\vec{\omega}_{inst} = \dfrac{d\theta}{dt}$ $\qquad \vec{\omega}_{Avg} = \dfrac{\Delta\theta}{\Delta t}$

   ⟶ axial vector $\qquad$ ⟶ ω(clock) ⊗

   ⟶ ω(Anti) ⊙

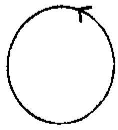
7. T.P. = $\dfrac{2\pi}{\omega}$ $\quad$ Freq = $\dfrac{1}{T.P.}$ $\quad \omega = 2\pi f$

8. Angular acceleration ($\alpha$)

   $\vec{\alpha}_{Avg} = \dfrac{\Delta\omega}{\Delta t}$

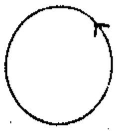
   (Axial Vector)

   $\vec{\alpha}_{Inst} = \dfrac{d\omega}{dt} = \omega\dfrac{d\omega}{d\theta} = \dfrac{d^2\theta}{dt^2}$



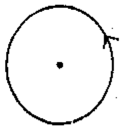
   ω ⊙ Anti = cost$^n$

   $\alpha = 0$



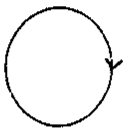
   ω ⊙ increase ↑

   α ⊙



   ω ⊙ Anti ↓ decreases

   α ⊗ Clock



   ω ⊗ ↓

   α ⊙

| | | | |
|---|---|---|---|
| $\vec{a}$ | $\vec{a}_t$ | $\vec{a}_c$ | $\vec{\alpha}$ |

$\boxed{a_t = R\alpha}$ $\quad \vec{a}_c = V^2/R \quad \vec{a} = \vec{a}_t + \vec{a}_c$

$|\vec{a}| = \sqrt{a_t^2 + a_c^2}$

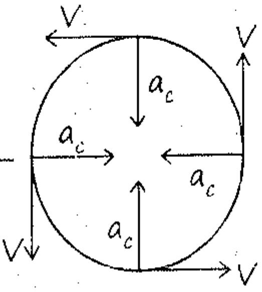
## UNIFORM CIRCULAR MOTION:-

| • Constant | • Zero | • Variable |
|---|---|---|
| ⟶ Speed (V) | $a_t = 0$ | Velocity |
| ⟶ Angular Speed (ω) | $\alpha = 0$ | direction |
| ⟶ K.E. | Work = 0 | acc$^n$ |
| ⟶ $\vec{L}$ (Angular Momentum) | $\tau = 0$ | Force Momentum |

Tangential acc = 0

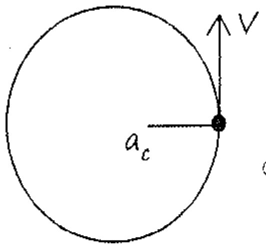
$\vec{a} = \vec{a}_c = \frac{V^2}{R}$ towards direction

$(\vec{a}_c)\ \text{Avg} = \frac{V^2}{R}\ \frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}}$




Dir$^n$ of centripetal acc$^n$ variable and magnitude constant

## CENTRIPETAL FORCE:-



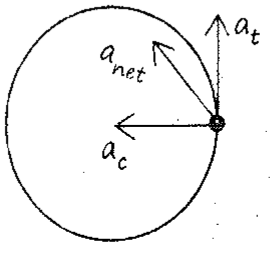

* $F_{CP} = \frac{mV^2}{R} = mR\omega^2$

o WD = 0, Power = 0

Two Car moving with different speed $V_1$ & $V_2$ on circular path of radius $r_1$ & $r_2$ with same time period then ratio of angular speed $\omega_1 : \omega_2 = 1 : 1$

## NON-UCM

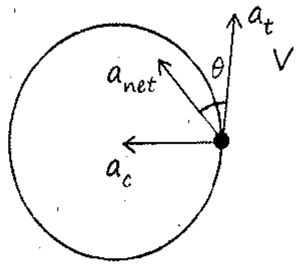
$$\alpha_{avg} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{\partial t} = \frac{\partial \vec{\omega}}{\partial t}$$




$$a_{net} = \sqrt{a_c^2 + a_t^2}$$

$$a_{net} = \vec{\alpha} \times \vec{R} + \vec{\omega} \times \vec{V}$$

$$\vec{a}_{net} = \vec{a}_t + \vec{a}_c$$




$0 < \theta < 90°$

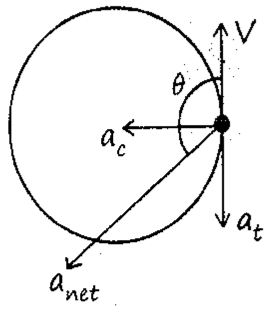
Speed up

$\tan\theta = \frac{a_c}{a_t}$

$a_{net} = \sqrt{a_c^2 + a_t^2}$

$a_t = a_{net}\cos\theta$

$a_c = a_{net}\sin\theta$




$90° < \theta < 180°$

Speed down

If $\theta = 90°$ b/w

$\vec{V}$ & $\vec{a}$ then

Speed = Cost$^n$

**Q. Object starts circular motion from rest and tangential acceleration 4 m/s² find acceleration of object if radius 48m at 3sec.**

Sol. Non uniform circular motion

$at = 4m/s^2 \qquad V = u + at$

$V = 4 \times 3 = 12\ m/s$

$a_c = \frac{V^2}{R} = \frac{12 \times 12}{48} = 3m/s^2$

$a = \sqrt{a_t^2 + a_c^2} = 5m/s^2$

**Q. A particle moves on circle of 5cm with constant time period 0.2πs then find acceleration.**

$a_c = w^2 R = \frac{4\pi^2}{T^2} R = 5m/s^2$

## CIRCULAR MOTION

Kinematical Equation:-

| | |
|---|---|
| $\omega = \omega_0 + \alpha t$ | $V = U + at$ |
| $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ | $S = Ut + \frac{1}{2}at^2$ |

| | |
|---|---|
| $\omega^2 - \omega_o^2 = 2\alpha\theta$ | $V^2 - U^2 = 2aS$ |
| $\theta_n^{th} = \omega_o + \alpha\left[\frac{2n-1}{2}\right]$ | $S_n^{th} = U + a\left[\frac{2n-1}{2}\right]$ |

**MR****

Motion starts from rest with constant Angular acceleration then Angle rotated in 1s, 2s & 3s are in ratio 1 : 4 : 9 & in $1^{st}$ sec : $2^{nd}$ sec : $3^{rd}$ sec = 1 : 3 : 5.

**MR****

Stopping Angle, $\theta = \dfrac{\omega_o^2}{2\alpha}$  ($\alpha$ = Angular retardation)

**MR**** → "$F_{pseudo}$" is used to validate Newton's law in Non-inertial frame.

[redacted]

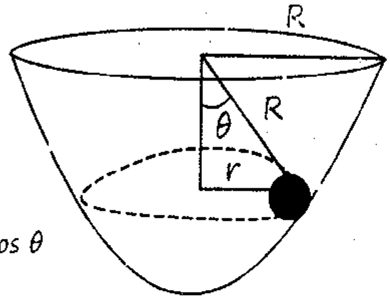
| Non-inertial | Inertial |
|---|---|
| $a \neq 0$ | $a = 0$ |
| Newton L:- ✗ | Newton L:- ✓ |
| $F_{pseudo}$:- ✓ | $F_{Real}$:- ✓ |
| $F_{CPF}$ :- Acts here! | |

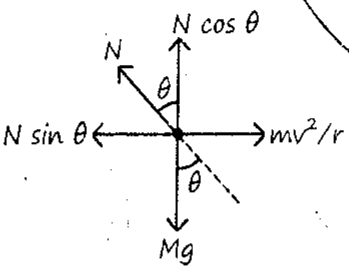
**MR****

1. Semi spherical bowl:-

$$\frac{N\sin\theta}{N\cos\theta} = \frac{mv^2}{r\,mg}$$

$$\left[\tan\theta = \frac{V^2}{rg}\right]$$

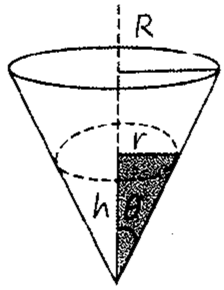



$r = R\sin\theta$

$$\tan\theta = \frac{v^2}{R\sin\theta g}$$

2. Cone:-




$$\tan\theta = \frac{r}{h}$$

$$\cot\theta = \frac{h}{r} = \frac{v^2}{rg}$$

$$h = \frac{v^2}{g}$$

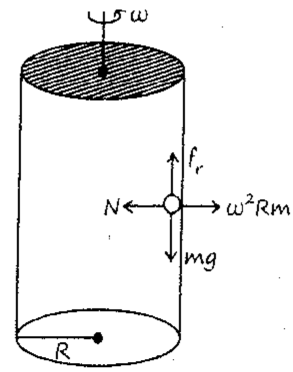
3. Death well:-

$N = m\omega^2 R$  (1)

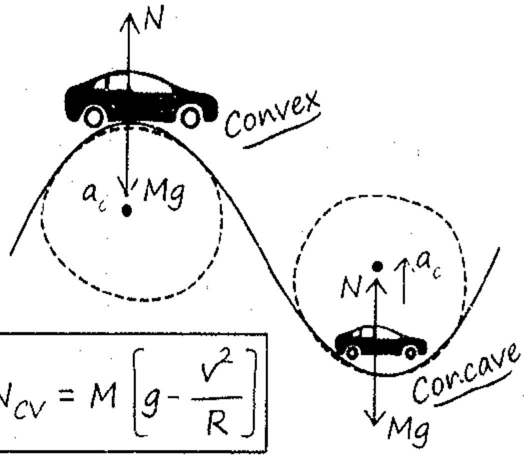
$(f_r)s = mg = \mu N$

$mg = \mu\omega^2 Rm$

$$\boxed{\omega^2 = \frac{g}{\mu R}}$$




4. Car moving on a convex & concave bridge with uniform speed:-

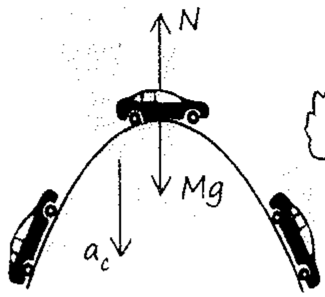



$$\boxed{N_{CV} = M\left[g - \frac{v^2}{R}\right]}$$

$$\boxed{N_{C.C.} = M\left[g + \frac{v^2}{R}\right]}$$

$$N_{Concave} > N_{convex}$$

5. Max. Speed of a vehicle to move on a convex bridge:-




$$\boxed{V_{max} = \sqrt{Rg}}$$

**BANKING OF ROADS:-**

Case-I:- Rough horizontal Road:-

(Sirf Friction)

$$V_{max} = \sqrt{\mu Rg} \qquad \omega_{max} = \sqrt{\frac{\mu g}{R}}$$

Case-II:- Smooth Banked Road:-

(Sirf Banking)

$$V_{max} = \sqrt{Rg \tan\theta}$$

Case-III:- Rough Banked Road:-

(Banking + Friction)

$$V_{max} = \sqrt{Rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}$$

$$V_{min} = \sqrt{Rg\left[\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right]}$$

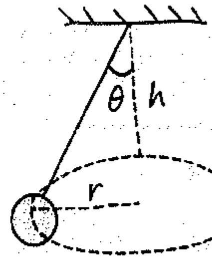
Safe Ride:- $V_{min} < V < V_{max}$.

- Bending of cyclist: $\tan\theta = \dfrac{v^2}{Rg}$

  $\theta$ = Angle bend by cyclist from vertical, V = velocity of cyclist, R = Radius of circular path.

**CONICAL PENDULUM:-**

$$\tan\theta = \frac{v^2}{rg}$$



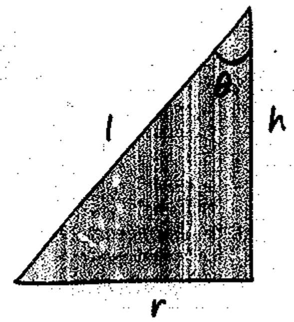

**MR* Special:-**

- "g" - Value of a planet using Conical Pendulum:-

$$V = \sqrt{Rg\tan\theta} = \frac{2\pi r}{T} \qquad \boxed{g = \frac{4\pi^2 r}{T^2 \tan\theta}}$$

- T.P. of Conical Pendulum:-

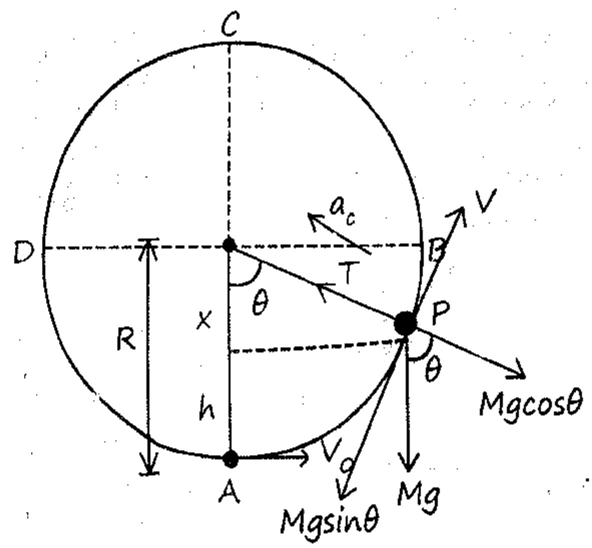
$$T_P = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$




$$T_P = 2\pi\sqrt{\frac{h}{g}}$$

$$T_P = 2\pi\sqrt{\frac{(l^2 - r^2)^{1/2}}{g}}$$

**VERTICAL CIRCULAR MOTION:-**



$$T_P = mg\cos\theta + \frac{MV^2}{R}$$

$$V^2 = V_o^2 + 2gR(\cos\theta - 1)$$

$$T_P = \frac{MV_o^2}{R} - 2mg + 3\,mg\cos\theta$$

$$\boxed{Max}\quad T_A(\theta = 0) = \frac{MV_o^2}{R} + mg$$

$$T_B(\theta = 90) = \frac{MV_o^2}{R} - 2mg$$

$$\boxed{Min}\quad T_C(\theta = 180) = \frac{MV_o^2}{R} - 5mg$$

$$T_D = \frac{MV_o^2}{R} - 2mg$$

$$T_A - T_C = 6mg$$

$$T_A - T_B = 3mg$$

$$T_A - T_D = 3mg$$

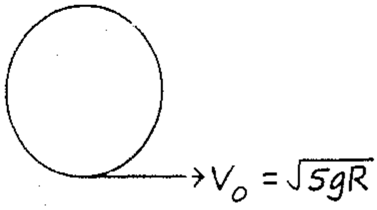
$$T_B - T_D = 0$$

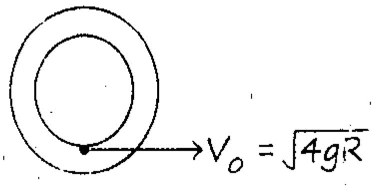
- If reference is at 'A' for potential then total M.E $= \frac{5}{2}MgR$.
- Work done by tension is zero.
- In case of critical condition $V = \sqrt{5gR}$
- If $0 < V \le \sqrt{2gR} \rightarrow$ oscillate

  $\theta_{max} \le 90° \rightarrow$ At extreme $T \ne 0$ ; $V = 0$

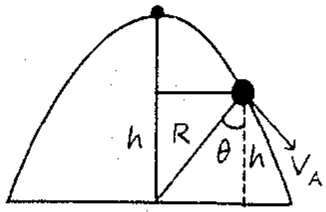
  Ex: $V_o = \sqrt{gR}$ $\quad \theta_{max} = 60°$

  $V_o = \sqrt{2gR}$ $\quad \theta_{max} = 90°$
- If $\sqrt{2gR} < V_o \le \sqrt{5gR}$ Parabolic path at $\theta_{max}$ $V \ne 0$, $T = 0$, $90 < \theta_{max} \le 180°$.
- If $V_o = \sqrt{5gR}$ Critical velocity to complete vertical circle.
- If object (bob) connected with massless rod, then $V_o = \sqrt{4gR}$ to complete vertical circular motion.



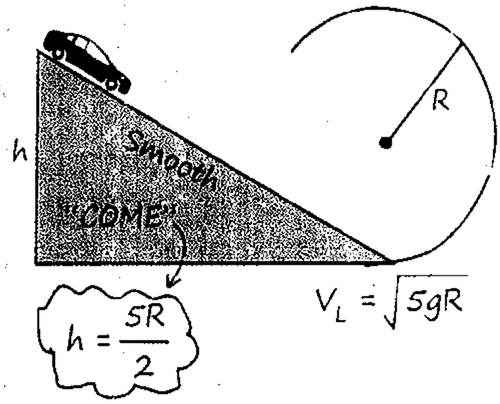



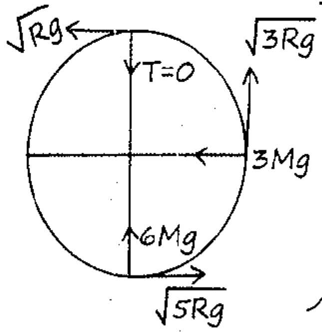

## PARTICLE LEAVING CONTACT WITH SEMISPHERE:-




$\cos\theta = \frac{h}{R}$

$V_P = \sqrt{2gR[1 - \cos\theta]}$

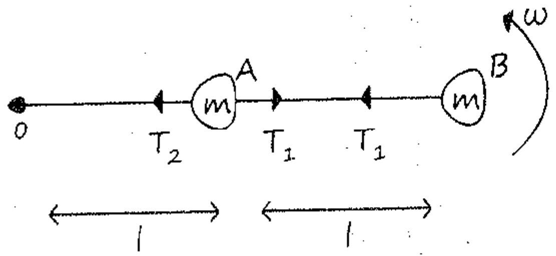



For String Highest Ps:- $T = 0$ $V \ne 0$

For Rod:- Highest Ps:- $T \ne 0$ $V = 0$

**Q. Two blocks of mass m connected with string of length $l$ then. Find $T_1$ & $T_2$.**




FBD of A and B w.r.t. Non inertial frame.

$T_1 = m\omega^2 (2l)$

$T_2 = T_1 + m\omega^2 (l) = m\omega^2 (2l) + m\omega^2 (l)$

$T_2 = 3\, m\omega^2 l$

**Q. Object is given velocity $V = \sqrt{3gR}$ at mean position where it will leave circular path.**

Ans. V (given) > $\sqrt{2gR}$ hence it will leave circular path where tension becomes zero.

$$T = \frac{mV^2}{l} - 2mg + 3mg\cos\theta$$

Put value of V and T find $\theta$

$$\cos = -\frac{1}{3}$$

**Q. If V (given) $V > \sqrt{gR}$ then it will leave circular path where velocity becomes zero**

$V = V_0^2 + 2gl(\cos\theta - 1)$

Put value of $V_g = \sqrt{gR}$ and V=0 then find $\cos\theta = \frac{1}{2}$ $\theta = 60°$

**MR***

"जिस समय जिस काम के लिए प्रतिज्ञा करो,
ठीक उसी समय पर उसे करना ही चाहिये,
नहीं तो लोगों का विश्वास उठ जाता है।"

# 9 Work, Energy, and Power

Force
- How long (time): $F \cdot \Delta t = \Delta P$
- How Far (disp$^m$, S): $\vec{F} \cdot \Delta \vec{S}$ = Work

## 1> WORK DONE :-

- **By Constant Force:-**

$W = \vec{F} \cdot \vec{S} = FS \cos \theta$

where,

F = Force, $\theta$ = Angle b/w Force and the displacement,

S = displacement of point of application of force

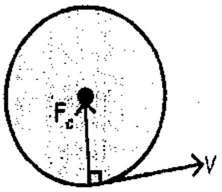
→ Scalar, unit (Joule). $ML^2T^{-2}$

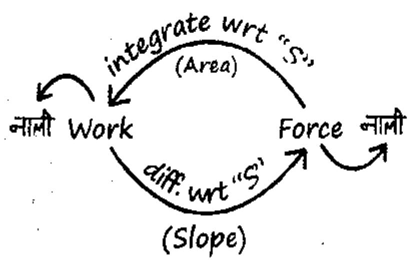
→ depends on Frame

$W_{Total}$ = Scalar sum of all the work by individual force.

- **By Variable Force:-** $\int dW = \int F.dS$

$$W_{Total} = \int_{x_1}^{x_2} F_x d_x + \int_{y_1}^{y_2} F_y d_y + \int_{z_1}^{z_2} F_z d_z$$

| Work = +ve | Work = 0 | Work = –ve |
|---|---|---|
| $0° \le \theta < 90°$<br>Speed ↑<br>KE ↑ | $\theta = 90°$<br>w = 0<br>Speed = const$^n$<br>K.E. = const$^n$<br> | $90° < \theta \le 180°$<br>Speed ↓<br>KE ↓ |




* Man is moving up or down on stairs then work done by normal force is zero.

## 2> WD BY GRAVITY:-

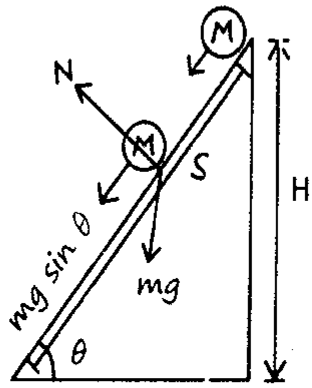
* $WD_{gravity} = F_{vertical} (S_{vertical})$

$= mg\,H$

$= mg\,(S \sin \theta)$

$\boxed{H = S \sin \theta}$

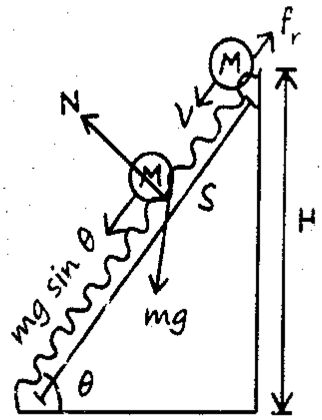
* Path independent.




## 3> WD BY FRICTION:-

$WD = -f_r \cdot S$

* Path dependent

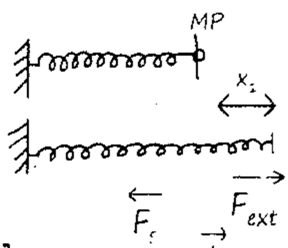



## 4> WD BY SPRING FORCE:-

- $W = -\frac{1}{2} Kx^2$

here, x = compression or elongation

- $W = -\frac{1}{2} K\left[x_2^2 - x_1^2\right]$



Q. Object is moving on a straight line $4y = 3x + 4$ and force acting on object is $F = 3\hat{i} - 4\hat{j}$ then work done by this force.

**Ans.** Work = 0 because disp$^m$ is perpendicular to force. Product of slope of force and displacement is $-1$

Q. Force acting on object $\vec{F} = 2x\hat{i} + 3y^2\hat{j}$ find work when object displace from (0, 2, 3) to (2, 2, 0)

**Ans.** $dW = F_x dx + f_y dy$

$$\int dW = \int 2x\, dx + \int 3y^2 dx$$

$$\text{Work} = 2\left(\frac{x^2}{2}\right)_0^2 + 3\left(\frac{y^2}{3}\right)_2^2 = (2)^2 + 0 = 4J$$

Q. If $\vec{F} = y\hat{i} + x\hat{j}$ then find work when object displace from (1, 2, 3) to (4, 6, 7).

**Ans.** $dW = F_x dx + F_y dy$

Hint: $ydx + xdy = d(xy)$

$$\int dW = \int (ydx + xdy)$$

$$\text{work} = \int d(xy) = (xy)_{1,2}^{4,6}$$

$$= 6 \times 4 - 1 \times 2 = 22J$$

## 5> KE AND MOMENTUM:-

$P = mv,\quad KE = \frac{1}{2}mv^2 = \frac{P^2}{2m}$

P = magnitude of momentum

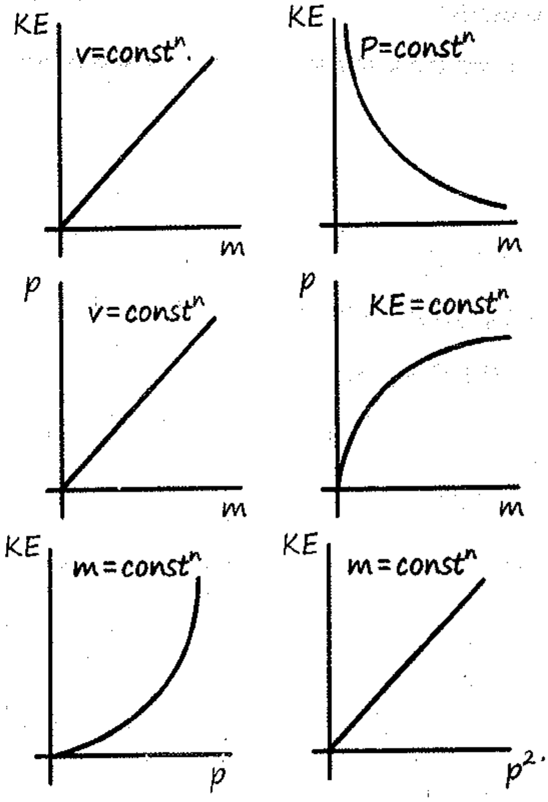
KE = Energy stored due to motion (Scalar)

$P = \sqrt{2m\,(K.E)}$

Statement-1: Two object having same mass and momentum having same K.E. → True

Statement-2: Two object having same mass and K.E. having same momentum → False

o Graphs:-




o % Change Calculation:-

| Small Change:- ≤5% | Large Change:- |
|---|---|
| $\frac{\Delta KE}{KE} \times 100$ | $\% \Delta KE = \frac{K_2 - K_1}{K_1} \times 100$ |
| $= \frac{2\Delta P}{P} \times 100$ | |

## 6> WORK ENERGY THEOREM:-

$W_{all\ Force} = \Delta KE.$

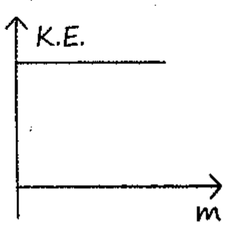
All means all no except$^n$ at all

$W_{C.F.} + W_{N.C.F.} + W_N + W_T + W_{Pseudo} + W_{etc}$
$= \Delta K.E.$

o Special Case:-

$\left.\begin{matrix} F = \text{constant} \\ S = \text{constant} \end{matrix}\right\} \rightarrow$ तो KE independent of Mass.

$KE \propto m^0.$



**Q.** If K.E. of a body is increased by 44%, what is the percentage increase in momentum?

**Ans.** Let initial K.E. is 100% and intial P is 100%

Given in question $K.E_f = 144\%$

$$K.E_f = \frac{144}{100} \qquad P = \sqrt{2m\,(K.E)}$$

Ignore const term

$$P_f = \sqrt{K.E_f}$$

$$P_f = \sqrt{\frac{144}{100}} = \frac{12}{10}$$

for % change

$$P_f = \frac{12}{1\cancel{0}} \times 10\cancel{0} = 120\%$$

Hence increase by 20%

**Q.** If momentum of a body is decreased to 25% then find % change in K.E.

**Ans.** Let initial K.E. and momentum is 100

Now decrease to 25% hence $P_f = 25\%$

$$P_f = \frac{25}{100} = \frac{1}{4}$$

$$K.E_f = P_f^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

for % change $\frac{1}{16} \times 100 = 6.25\%$

$$\begin{bmatrix} \text{decreased to } 6.25\% \\ \text{and decreased by } 93.75\% \end{bmatrix}$$

**Q.** Position of object of mass 2kg $x = \frac{t^2}{3}$ then find work done in first three sec.

**Ans.** $V = \frac{dx}{dt} = \frac{2t}{3}$

$$V_i = 0$$

$$V_f = \frac{2}{3} \times 3 = 2\text{ m/s}$$

$$W = \Delta K.E = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$W = \frac{1}{2}2\,[(2)^2 - 0]$$

$$= 4J$$

**Q.** Ball of mass 5kg is dropped from 2 m height then its velocity at ground is 10 m/s find work done by friction.

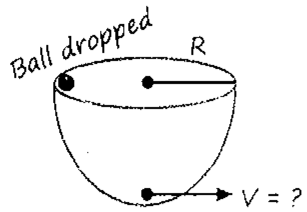
**Ans.** $W_g + W_{air\ frication} = \Delta K.E.$

$$mgH + W_{af} = \frac{1}{2}m\,[v_f^2 - v_i^2]$$

$$5 \times 10 \times 20 + W_{af} = \frac{1}{2}5\,[(10)^2 - 0]$$

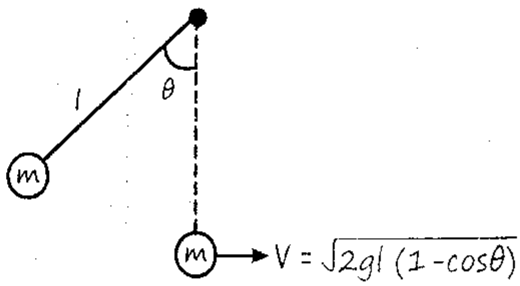
$$W_{af} = 250 - 1000 = -750\text{ J}$$

**Q.** Smooth hemispherical surface find velocity of ball at bottom point.

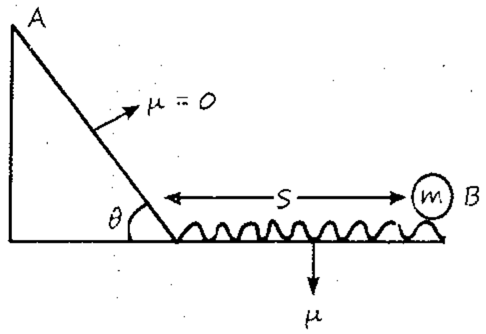



**Ans.** $V = \sqrt{2gR}$

**Q.** Speed of mass m?




**Q.** Ball is droped from A and comes to at rest at B find horizontal distance moved by Ball on rough surface




**Ans.** Work – Energy therom

$$W_g + W_N + W_{fr} = \Delta K.E$$

$$mgH + 0 - \mu mgs = 0$$

$$H = \mu s$$

$$S = \frac{H}{\mu}$$

7>

**Force**

### CONSERVATIVE

- Path independent
- Concept of Potential is defined here!
- W = Reversible
- $W = \oint \vec{F} \cdot \vec{dr} = 0$

### NON-CONSERVATIVE

- Path dependent
- Concept of potential is not defined.
- W = Irreversible.
- $W = \oint \vec{F} \cdot \vec{dr} \neq 0$

- For Conservative Force:-

$$\vec{F}_{CF} = -\left(\frac{dU}{dr}\right)$$

Potential Energy gradient (Vector)
(Potential Energy is scalar)

$\therefore \quad \Delta U = -W_{C.F.} = -\vec{F}_{C.F.} \cdot d\vec{r}$

Change in potential energy does not depends upon reference, and path.

Potential Energy → depends upon reference not have unique/absolute value.

$$\vec{F}_{CF} = \left(-\underset{y \& z = const^n}{\frac{\partial U}{\partial x}\hat{i}}\right) - \left(\underset{x \& z = const^n}{\frac{\partial U}{\partial y}\hat{j}}\right) - \left(\underset{x \& y = const^n}{\frac{\partial U}{\partial z}\hat{k}}\right)$$

$$\Delta U = -\int F_x d_x - \int F_y d_y - \int F_z d_z$$

* Apne -Aap Joh Kaam Hota hai Usmein PE Hamesha Ghatega!

## VERY IMPORTANT!

- WET:- [Heart of Physics]

$\boxed{W_{CF} + W_{NCF} = \Delta KE}$ → Baap

$\boxed{\Delta U = -W_{CF}}$ Mother always valid

$W_{CF} = -\Delta U$

- COME:-

If $W_{NCF} = 0$ → Cond$^n$ Compulsory
If $W_{NCF} = 0$ → NCF may be acting or not acting but $W_{NCF} = 0$

MR* $\boxed{(KE + PE)_i = (KE + PE)_f}$ Beta

* Jaha Hum$^{MR}$ Waha COME$^{NCF}$ nai!

* Jaha COME, waha hum nai

* Hum kisi se kam nai

- → KE = Const$^n$ [Slow - Slow]

  → NCF = Acting against C. Force.

$\boxed{\Delta U = W_{NCF}}$ Beti

Q. If potential energy $U = x^2y + y^2z + z^2x$ then find force acting on it

Ans. $\left(\frac{dU}{dx}\right)_{y \text{ and } z \text{ constant}} = 2xy + z^2$

$\left(\frac{dU}{dy}\right)_{x \text{ and } z \text{ constant}} = x^2 + 2yz$

$\left(\frac{dU}{dz}\right)_{y \text{ and } z \text{ constant}} = y^2 + 2zx$

$\vec{F} = -(2xy + z^2)\hat{i} - (x^2 + 2yz)\hat{j} - (y^2 + 2zx)\hat{k}$

Q. Work done in bringing object from A to B is 40J then find potential energy at B if potential energy at A is −30 J

Ans. $W_{N.C.F} = \Delta U$

$40 \text{ J} = U_B - (-30)$

$40 - 30 = U_B$

$U_B = 10\text{J}$

## 8> POTENTIAL ENERGY:-

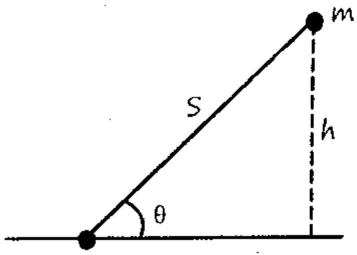
Energy due to shape size and position.

- Point Objects:-

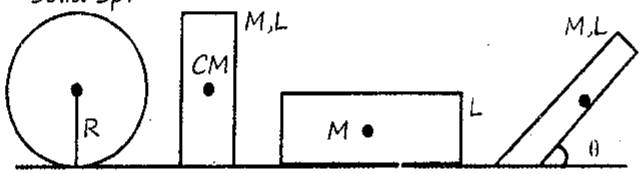
$U = mg\,h$

$\quad = mg\,S \sin\theta$

$\because \; h = s \sin\theta.$



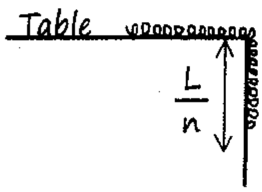

- Extended Objects:-




$U = MgR \qquad U = Mg\left(\frac{L}{2}\right) \qquad U = Mg\,\frac{L}{2} \qquad U = Mg\,\frac{L}{2}\sin\theta$

Chain Problem:-

$$U = \frac{-MgL}{2n^2}$$




- Speed of chain when it becomes vertical!

$$V = \sqrt{gL\left[1 - \frac{x^2}{L^2}\right]}$$

x = Initial hanging length
L = Length of Chain.

Spring Force Energy:-

$$\Delta U = U_{x_2} - U_{x_1}$$

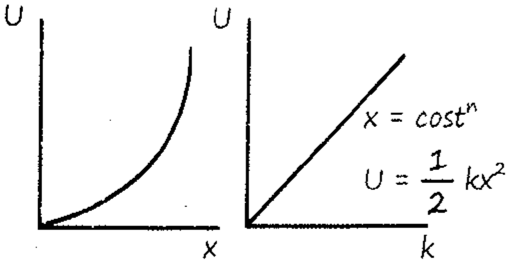
$$= \frac{1}{2}K\left[x_2^2 - x_1^2\right]$$

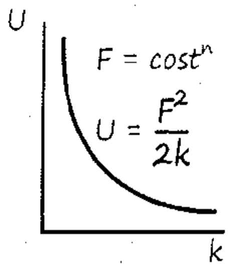
k = Force constⁿ

$$U = \frac{1}{2}Kx^2 \text{ [PE Store]}$$

- For a Spring:-

$U \propto x^2$




$$\boxed{U = \frac{1}{2}\frac{F^2}{K} = \frac{1}{2}Fx}$$

Q. A chain on a frictionless table one fifth of it's length hanging over edge. If chain has length L Mass M then work done to pull back hanging part on table

Ans. refⁿ taken on the table then

$$U_f = 0$$

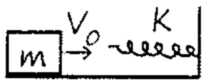
$$U_i = -\frac{m}{5}g\frac{l}{10}$$

$$W = \Delta U = U_f - U_i$$

$$= 0 - \left(-\frac{Mgl}{50}\right)$$

$$= \frac{Mgl}{50}$$

Q. Surface is smooth then maximum compression in spring.




Ans. $\underline{\text{C.O.M.E}}$ $(K.E + U)_i = (K.E + U)_f$

$$\frac{1}{2}mV_0^2 + 0 = \frac{1}{2}Kx^2 + 0$$

$$x = \sqrt{\frac{mV_0^2}{K}}$$

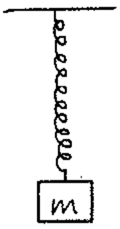
Q. Initially mass m is held such that spring is in relaxed condition then find elongation in spring, if mass m is

(i) Suddenly released

C.O.M.E applicable

loss in gravitational P.E. = Gain in spring P.E

$mgx = \frac{1}{2}kx^2$ and $x = \frac{2mg}{K}$



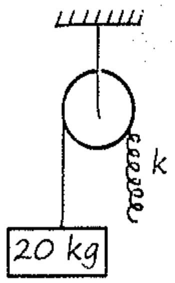

(ii) Slowly released

Non-conservative force (हमारा) is working to released slowly hence COME not applicable

$F = mg = Kx$

$x = \frac{mg}{K}$

Q. Find minimum mass hanged from spring so that it can just pull up 20 kg object?




Ans. $m_{mini} = 10$ kg

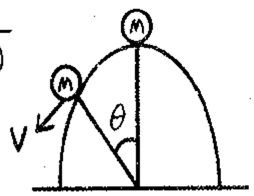
Spring me suddenly elongation

$x = \frac{2mg}{K}$ hoga.

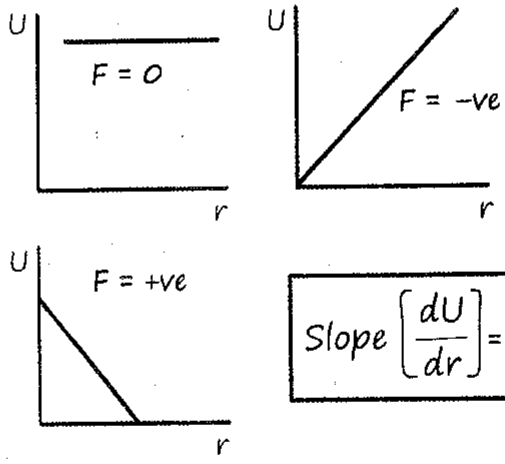
o Ball of mass m, is released with an angle $\theta$ from vertical where it'll loose contact!

* $V = \sqrt{2gR(1-\cos\theta)}$

* $\cos\theta = \frac{2}{3}$

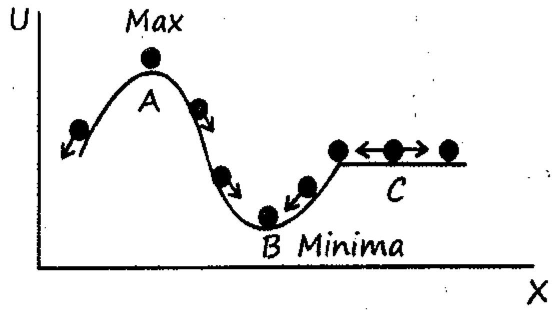



U–x graphs:-



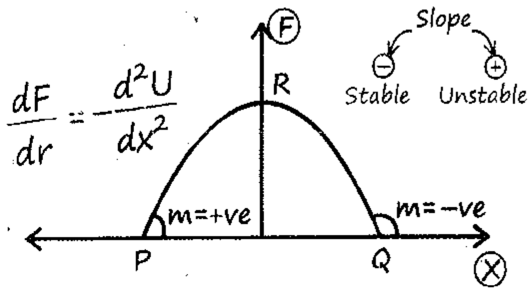

$$\boxed{\text{Slope}\left[\frac{dU}{dr}\right] = -F}$$

**Equilibrium!**




| At A | At B | At C |
| --- | --- | --- |
| F = 0 | F = 0 | F = 0 |
| **Unstable** | **Stable** | **Neutral** |
| $\frac{d^2U}{dx^2}$ = –ve Maxima | $\frac{d^2U}{dx^2}$ = +ve Minima | $\frac{d^2U}{dx^2} = 0$ |

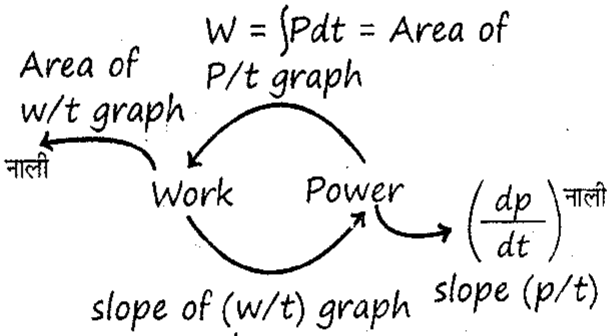
$$\frac{dF}{dr} = \frac{d^2U}{dx^2}$$




o Stable $Eq^m = Q\left[\frac{dF}{dr} = -ve\right]$ Slope

o Unstable $Eq^m = P\left[\frac{dF}{dr} = +ve\right]$ Slope

## 9> POWER




$$P = \frac{dw}{dt}$$

$$P_{avg} = \frac{W_{Total}}{t_{Total}} \qquad P_{inst} = \frac{dW}{dt} = F.v$$

$$P_{avg} = \frac{\int P.dt}{\int dt.} \qquad P_{inst} = ma.v = Fv\cos\theta$$

Unit:- Watt = J/s

**1 Hp = 746 Watt = $\frac{3}{4}$ K Watt.**

- Efficiency $(\eta) = \frac{Output}{Input}$

* For a = cosnt$^n$
F = const$^n$

$\boxed{P \propto t}$ $\boxed{V \propto t}$

NOTE: -

- $F = \rho A V^2$ (Pump)
- Power of pump = $\rho A V^3$
- Rate at which K.E Provided to liquid $= \frac{1}{2}\rho A V^3$
- $P = F.v = \rho A V^3$
- When Power Const$^n$:-

$x \propto v^3 \quad x \propto t^{3/2} \quad v \propto t^{1/2}$

Q. Ball projected with speed u at angle θ then power at maximum height and at time 't' by gravitational force

Ans. $P_{inst} = mg\, u_x \cos 90° = 0$

at maximum height velocity and force is perpendicular

at time 't' $P = mg\,(u \sin\theta - gt)$

Q. An engine pumps 800 kg water through height 10 m in 80 sec. Find the power of engine if its efficiency is 75%

Ans. $75\%\ P = \frac{mgh}{t} \qquad P = \frac{4}{3}$ kW

MR*

"NEET - JEE preparation is not just about being a doctor or engineer. Its always about time management and pressure handling."

# 10 Center of Mass

## Centre of Mass:-

→ A point where whole mass of the system can be assumed there COM lies near to heavier obj.

→ It can be inside or outside the body.

→ It always on the axis of symmety and where two axis of symmetry will cut each other.

→ Position of COM depends upon frame of reference and choice of co-ordinates.

→ Centre of mass does not depend on choice of co-ordinate

○ COM of discrete particle:-

$$\vec{r}_{cm}\text{(Position of C.O.M.)} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\vec{X}_{cm} = \frac{m_1\vec{x}_1 + m_2\vec{x}_2}{m_1 + m_2}$$

$$\vec{Y}_{cm} = \frac{m_1\vec{y}_1 + m_2\vec{y}_2}{m_1 + m_2}$$

$$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

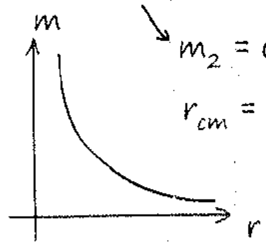
$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$$

Moment of Mass:

MR = constant.

$$m_1 r_1 = m_2 r_2$$

$m_1 = m_2$, $r_{cm} = \frac{r}{2}$

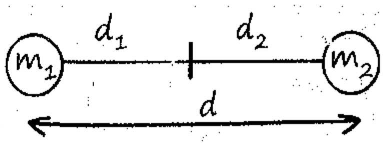



$m_2 = 0$, $r_{cm} = 0$

*COM is Closer to Massive body.

* Internal Force की औकाद नहीं है की वो COM की $\vec{V}_{c.m}$ change करदे !

## COM OF TWO PARTICLE SYSTEM:-




$$d_1 = \frac{m_2 d}{m_1 + m_2} \qquad d_2 = \frac{m_1 d}{m_1 + m_2}$$

# If external force is zero then location of centre of mass will not change → False

# If fext = 0 then state of COM will not change. [$\vec{V}_{cm}$ = $cost^n$]

# Shift in C.O.M.

$$\gamma_{c.m.} = \frac{m_1\overrightarrow{\Delta\gamma_1} + m_2\overrightarrow{\Delta\gamma_2}}{m_1 + m_2}$$

If C.O.M. does not shift its position then

$$m_1\overrightarrow{\Delta\gamma_1} = - m_2\overrightarrow{\Delta\gamma_2}$$

## Com of Continuous System:-

| | | |
|---|---|---|
| $\lambda = \frac{dm}{dL}$ | $\sigma = \frac{dm}{dA}$ | $\rho = \frac{dm}{dV}$ |
| $dm = \lambda dL$ | $dm = \sigma dA$ | $dm = \rho dV$ |
| 1-D ○ | 2-D | 3-D ● |

$$X_{CM} = \frac{\int x \cdot dm}{\int dm}, \; dm = \lambda \cdot dx$$

MR** C.O.M of Rod:-

$X_{CM} = L/2 \quad \lambda = const^n$

$X_{CM} > L/2 \quad \lambda \propto x$

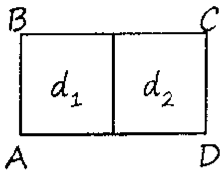
$X_{CM} > 3L/4 \quad \lambda \propto x^2$

MR*

\# If density of rod varies linearly $\lambda = \lambda_0 x^n$ then position $L/2 < x_{cm} < L$

\# If density of rod $\lambda = \alpha + \beta x$ then COM will be at $x_{cm} = L/2$ if $\beta = 0$

Q. Half of the uniform rectangular plate of Length 'L' is made up of material of density '$d_1$' and the other half with density $d_2$. The perpendicular distance of center of mass from AB is




(a) $\dfrac{2d_1+3d_2}{d_1+d_2} \times \dfrac{L}{4}$

(b) $\dfrac{d_1+3d_2}{d_1+d_2} \times \dfrac{L}{4}$

(c) $\dfrac{3d_1}{d_1+d_2} \times \dfrac{L}{4}$

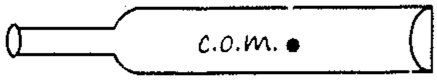
(d) $\dfrac{3d_2}{d_1+d_2} \times \dfrac{L}{4}$

MR* If $d_1 = d_2$

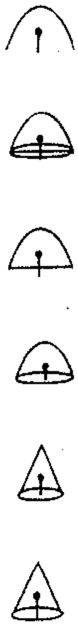
C.O.M. at $\dfrac{L}{2}$ then option (b) correct.

o If we break cricket bat from c.o.m. then bottom part will have large mass, because c.o.m. divide system in two equal moment of mass.

MR = const.



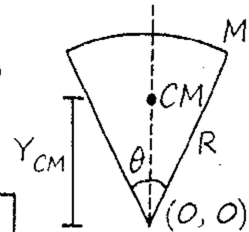

Trick for Com:- Com of continuous mass system



| | | |
|---|---|---|
| Ring S.C. | | $2R/\pi$ |
| Solid H.S. | | $3R/8$ |
| Disc S.C. | | $4R/3\pi$ |
| Hollow H.S. | | $R/2$ |
| Triangle | / Hollow Cone. | $h/3$ |
| Cone Solid. | | $h/4$ |

COM of Circular ARC:-

$$Y_{CM} = \frac{R \sin \theta/2}{\theta/2}$$

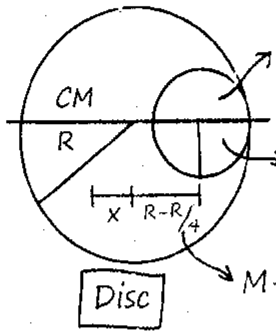



Shift in COM:-

$$\Delta r_{cm} = \frac{m_1 \Delta r_1 + m_2 \Delta r_2}{m_1 + m_2}$$

MR** For COM of Remaining Portion:-

A Disc of radius R/4 is removed from a disc of mass M and radius R.




$\frac{R}{4}$ → $n = 4$

Solid Sphere $\dfrac{M}{n^3}$

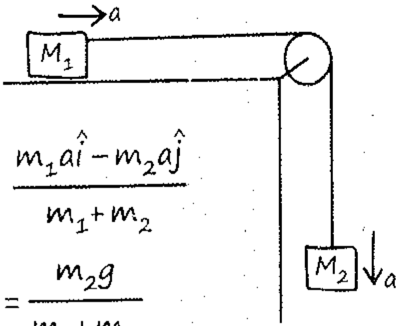
$\boxed{\dfrac{M}{n^2}} = \dfrac{M}{16} = m'$

$M - M' = M - \dfrac{M}{16} = \dfrac{15M}{16}$

$$\frac{15\cancel{M}}{\cancel{16}}x = \frac{\cancel{M}}{\cancel{16}}\left(R-\frac{R}{4}\right)$$

$$x = \frac{\cancel{3}R}{4\times\cancel{15}\,5} = \frac{R}{20}$$

#

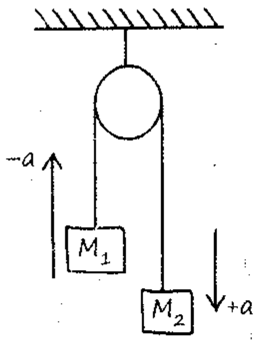



$$\vec{a}_{cm} = \frac{m_1 a\hat{i} - m_2 a\hat{j}}{m_1+m_2}$$

$$\text{Put } a = \frac{m_2 g}{m_1+m_2}$$

MR Ratta*

$$\vec{a}_{cm} = \left[\frac{M_1-M_2}{M_1+M_2}\right]^2 g.$$




Note:-

Agar Fext = 0.

State of COM will not change.

Q. Two ball of mass $m_1$ and $m_2$ projected with $u_1$ and $u_2$ in upward and at 30° from horizontal respectively then acceleration of c.o.m. will be?

Sol. $\vec{a}_{c.m} = g$ (downward) because both have same acc$^n$ g downward.

Aag lage chahe basti mein COM Rahe apne masti mein

Conservation of Linear Momentum of System:

**Condition:-**

Fext = 0. $\quad \vec{P}_{cm}$ = Same.

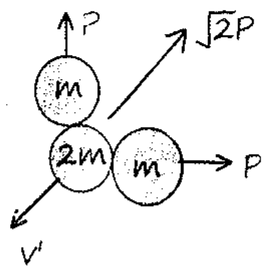
A stationary object explodes in two unequal part then :-

- External force is zero, hence momentum will be conserved.
- Both part will have equal momentum in opposite direction.
- Smallar mass will have greater kinetic energy.
- Work done by internal force = change in K.E. of system = K.E. of both part.
- For same momentum both have unequal velocity in opposite direction.
- Internal force can change kinetic energy of system → True
- Internal force can change momentum of system → false.
- A body falling vertically downward under gravity breaks in two unequal part in that case c.o.m. will continous vertical motion does not shift horizontal.
- A shell following parabolic path explode somewhere in many part but c.o.m. will continue parabolic path.

Q. A body of mass (4m) is lying in x-y plane at rest. It suddenly explodes into three pieces. Two pieces, each of mass (m) move perpendicular to each other with equal speeds (u). The total kinetic energy generated due to explosion is

Sol.:-

MR**



$$2\cancel{m}v' = \sqrt{2}\,\cancel{m}v$$

$$v' = \frac{v}{\sqrt{2}}$$

$$KE_{gen} = \left(\frac{1}{2}mv^2\right) \times 2 + \left(\frac{1}{2}\frac{2mv^2}{2}\right)$$

$$= \frac{3mv^2}{2}$$

Q. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to other end of the plank. If the mass of the plank is M/3, then the distance that the man moves relative to ground is :

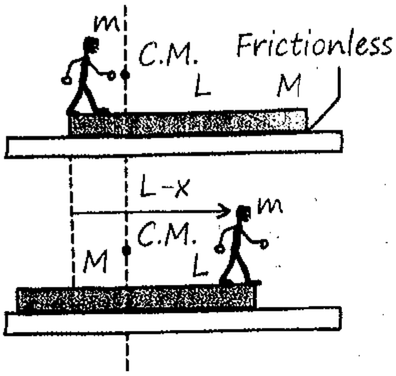
Sol.

m = Mass of walking Man

$$x = \frac{mL}{m+m/3} = \frac{mL}{4m/3} = \underline{\underline{\frac{3L}{4}}}$$

Displacement of man relative to ground =

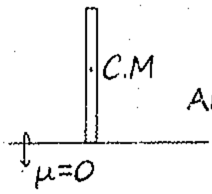
$$L - x = L - \frac{3L}{4} = \frac{L}{4}$$




Q. A bomb explodes into two parts 6 kg and 8 kg if velocity of 6kg is 10m/s then find K.E. of 8kg will be

Sol. $K.E = \frac{P^2}{2m} = \frac{6\times6\times10\times10}{2\times8}$
$= 225\ J$

Q. A vertical rod is placed on smooth ground and released then path of C.O.M will be




Ans straight line in vertical

Q. Object is projected with u at angle θ at maximum height it breaks into two equal part, if one just fall below maximum height then range of other from point of projection.

Sol.

C.O.M ka range तो R हि होगा

$$\vec{r}_{c.m} = \frac{mr_1 + m_2r_2}{m_1 + m_2}$$

$$R = \frac{m\,R/2 + m\,r_2}{2m}$$

$$2R = \frac{R}{2} + r_2$$

$$r_2 = \frac{3R}{2}$$

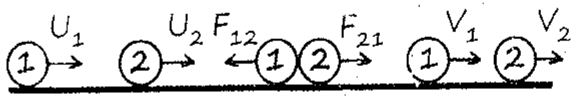
MR*
"Jaldi karo
jaldbazi nahi."

# 11 Collision

In all Colisions,

$P$ = Conserved.

Momentum of individual mass is not conserved but $P_{system}$ = Conserved!




**Coefficient of Restitution:-**

$$e = \frac{V_{sep^r}}{V_{approach}} = \frac{\vec{V}_2 - \vec{V}_1}{\vec{U}_1 - \vec{U}_2}$$

| Elastic Collision | Inelastic | Perfectly Inelastic |
|---|---|---|
| KE = Conserved <br> $e = 1$ | Not Conserved. <br> $0 < e < 1$ | $M_1U_1 + M_2U_2 = (M_1+M_2)V$ <br> $e = 0$ |
| $V_1 = \frac{(m_1 - m_2)\vec{U}_1}{m_1+m_2} + \frac{2m_2\vec{U}_2}{m_1+m_2}$ | $\vec{V}_1 = \frac{(m_1 - em_2)\vec{U}_1}{m_1+m_2} + \frac{(e+1)m_2\vec{U}_2}{m_1+m_2}$ | $V = \frac{m_1u_1+m_2u_2}{m_1+m_2}$ <br> $\to V_{com}$ |
| $V_2 = \frac{(m_2 - m_1)\vec{U}_2}{m_1+m_2} + \frac{2m_1\vec{U}_1}{m_1+m_2}$ | $\vec{V}_2 = \frac{(m_2 - em_1)U_2}{m_1+m_2} + \frac{(e+1)m_1U_1}{m_1+m_2}$ | |
| $e = \frac{\vec{V}_2 - \vec{V}_1}{\vec{U}_1 - \vec{U}_2}$ | $\Delta KE_{loss} = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(U_{rel}^2)(1-e^2)$ | $\begin{pmatrix}\Delta KE \\ max\end{pmatrix}_{loss} = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}U_{rel}^2$ |

Elastic collision b/w moving ($m$) and ($nm$) at rest

Neet

$$\frac{KE_{Trans}}{KE_{initial}} = \frac{4n}{(1+n)^2}$$

($n$) = Kitna bada hai ye matter Krta hai Kon bada hai ye nai.

Fraction of retained K.E. $= \left(\frac{1-n}{1+n}\right)^2$

Note: Elastic collsion of two object in which one is at rest.

## MR** Ek Soch

Q. A ball of mass 2 kg moving with a speed of 5 m/s collides directly with another ball of mass 3 kg moving in the same direction with a speed of 4 m/s. The coefficient of restitution is 2/3. Find the velocities after collision.

→ Formula ek bar he lagana hai dusre object Ki velocity tum direct nikal dena Momentum Conservation after collision से!

$$\vec{V}_1 = \left(\frac{2 - \frac{2}{3}\times 3}{5}\right)5 + \frac{\left(\frac{2}{3}+1\right)3\times 4}{5}$$

$$\vec{V}_1 = 4\ m/s$$

(2) → 4 m/s (3) → V

$$\boxed{P_i = P_f} \quad 10 + 12 = 8 + 3V$$

$$V = 14/3$$

**Elastic Collision B/N Two Object of Same Mass:-**

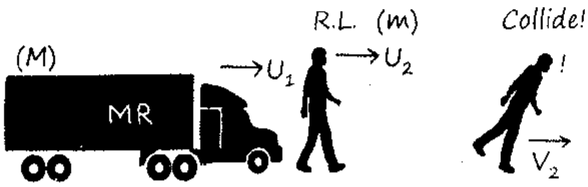
**Woh dono apni velocity interchange Krlenge i.e.**

$$\vec{V}_1 = \vec{U}_2$$

$$\vec{V}_2 = \vec{U}_1$$

Elastic Collision B/N Two Object in Which one Having Mass very Much Greater then other:- (M>>>>m)

Case-I.

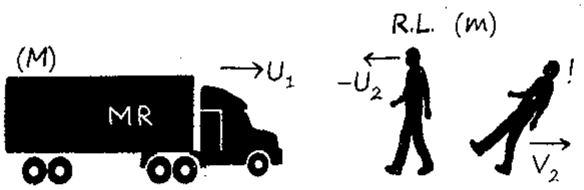



∴ Velocity of Ramlal after Collision

= bade ka double – Khud ka velocity

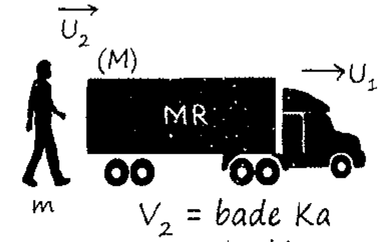
$$\boxed{\vec{V}_2 = 2\vec{U}_1 - \vec{U}_2}$$

Case-II.




$\vec{V}_2$ = bade ka double + Khud ka velocity

$$\boxed{\vec{V}_2 = 2\vec{U}_1 + \vec{U}_2}$$



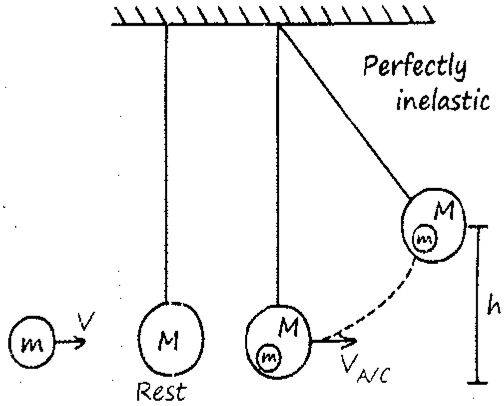

$V_2$ = bade Ka double – Khudka


$$\boxed{V_2 = 2\vec{U}_1 - \vec{U}_2}$$

Tum Heavy object Ko Kahise Collide Karo us Ka Dash Farak Nai Padhta'

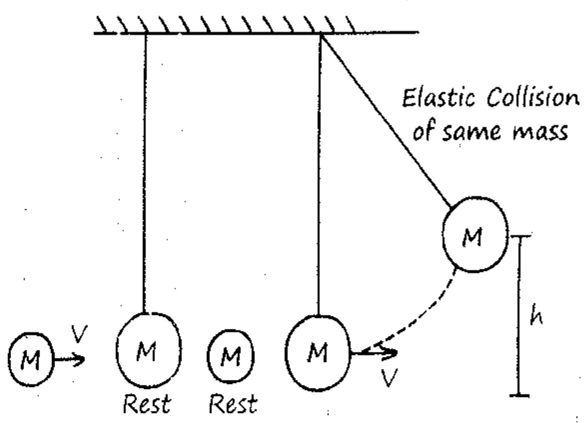
MR-Wala Sawaal:-

1>




$$h = \frac{V_{A/C}^2}{2g} \quad mV + 0 = (M+m)V_{A/C}$$

2>




$$\left.h = \frac{V^2}{2g}\right\} \text{COME.}$$

Q. A neutron makes a head on elastic collision with a stationary deutron. The fraction of energy transferred to the deutron and retained K.E. in neutron.

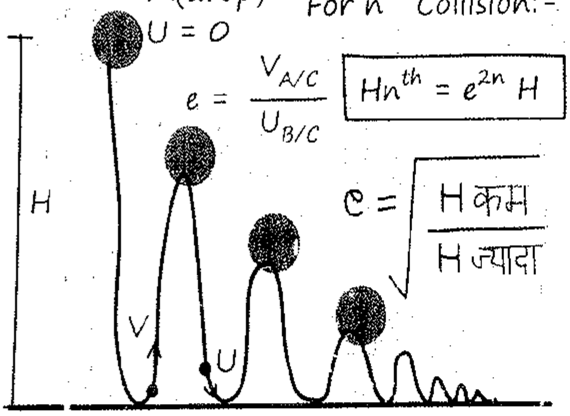
Sol.

Fraction of transferred K.E. =

$$\frac{4n}{(1+n)^2} = \frac{4\times 2}{(1+2)^2} = \frac{8}{9}$$

$$\text{Fraction of retained K.E.} = \left(\frac{1-2}{1+2}\right)^2 = \frac{1}{9}$$

**Ball is Drop From Height H:-**

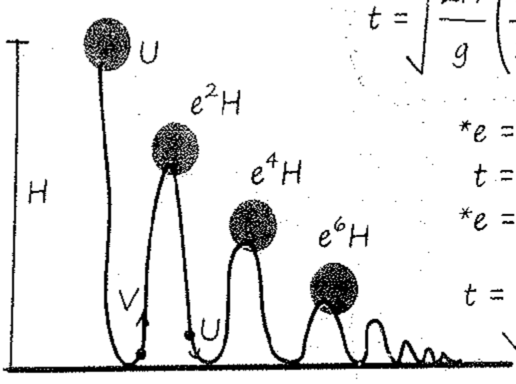



For $n^{th}$ Collision:-

$$e = \frac{V_{A/C}}{U_{B/C}} \qquad \boxed{Hn^{th} = e^{2n}\,H}$$

$$e = \sqrt{\frac{H\,\text{कम}}{H\,\text{ज्यादा}}}$$

**Ball is Dropped from Height H Then Total Time of Flight and total Distance before coming to Rest.**




$$t = \sqrt{\frac{2H}{g}}\left(\frac{1+e}{1-e}\right)$$

$*e = 1$

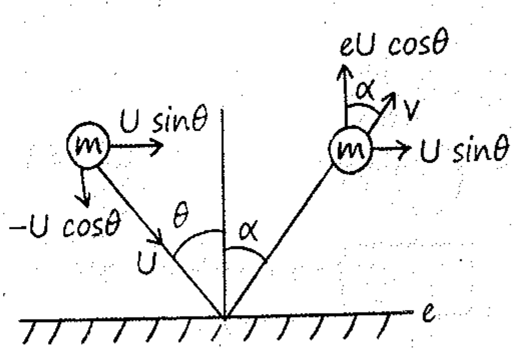
$t = \infty$

$*e = 0$

$$t = \sqrt{\frac{2H}{g}}$$

$$\text{Total distance} = H\left(\frac{1+e^2}{1-e^2}\right)$$

**Oblique Inelastic Collision:-**




$$V = \sqrt{(U\sin\theta)^2 + (Ue\cos\theta)^2}$$

$$\Delta P = mu\cos\theta\,(1 + e)$$

$P_{conserved}$ → x-axis ✓

└→ y-axis ✗

$$\tan\alpha = \frac{U\sin\theta}{eU\cos\theta} \qquad \tan\alpha = \frac{\tan\theta}{e}$$

- Sawaal mein agar "$\theta$" horizontal se liye toh:-

$*\Delta P = mu\sin\theta + emu\sin\theta$

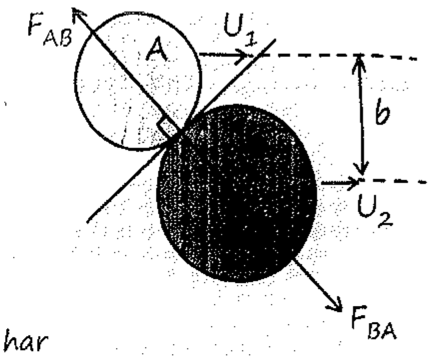
$= mu\sin\theta\,(1 + e)$

1d Head on Collision:-

$(b = 0)$

2d Oblique Collision:-

$(b > 0)$




*System Ki Momentum har dir^n mein conserve hai!

* Force jiss line Ke along lagega uske $\perp^{er}$ body Ka P Conserved hoga!

Q. A sphere P of mass m and velocity $\vec{V}$ undergoes an oblique and perfectly elastic collision with an identical Q initially at rest. The angle $\theta$ between the velocities of the spheres after the collision shall be

**Sol.**

According to the law of conservation of linear momentum,

$$mv_i + m \times 0 = mv_{Pf} + mv_{Qf}$$

$$\Rightarrow v_i = v_{Pf} + v_{Qf}$$

$$\text{Now, } (v_i \cdot v_i) = (v_{Pf} + v_{Qf})^2$$

$$= v_{Pf}^2 + v_{Qf}^2 + 2v_{Pf}\, v_{Qf} \cos\theta \quad ...(i)$$

Using conservation of kinetic energy, we get

$$\frac{1}{2} mv_i^2 = \frac{1}{2} mv_{Pf}^2 + \frac{1}{2} mv_{Qf}^2$$

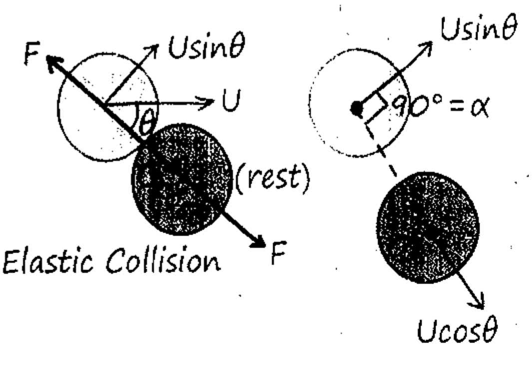
$$\Rightarrow v_i^2 = v_{Pf}^2 + v_{Qf}^2 \quad ...(ii)$$

On comparing (i) and (ii), we get

$$\cos\theta = 0 \Rightarrow \theta = 90°$$

## Wait for MR*

Jis direction me collision hoga velocity interchange ho jayega.

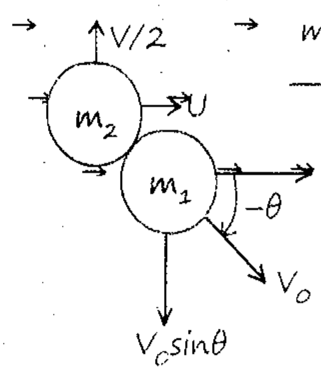



**Note:-**

- 2-D Elastic Collision wale sawaal mein:- KE = Conservation lagao.
- Perfectly Inelastic wale Sawaal mein:- P = Conservation lagao.

Q. Two sphere A and B of masses $m_1$ and $m_2$ respectively collide. A is at rest initially and B is moving with velocity v along x-axis. After collision B has a velocity $\frac{v}{2}$ in a direction perpendicular to the original direction. The mass A moves after collision in the direction. (AIPMT-2012)

Sol.




$$m_2 \frac{u}{2} = m_1 V_o \sin\theta$$

$$m_2 U = m_1 V_o \cos\theta$$

$$\frac{1}{2} = \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\theta = \tan^{-1}\left(-\frac{1}{2}\right) \text{ to the x-axis}$$

Q. Two identical block of mass m moving with speed u perpendicular to each other then find their velocity when they stick after collsion.

Sol.

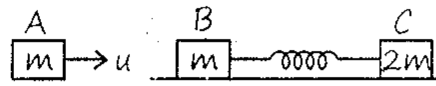
K.E. → Not conserved

$$P_i = P_f$$

$$mu\hat{i} + mu\hat{j} = 2m\vec{V}$$

$$\vec{V} = \frac{u\hat{i} + u\hat{j}}{2}$$

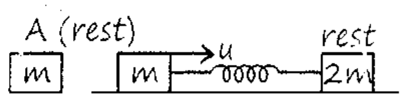
$$|\vec{V}| = \frac{\sqrt{2}u}{2} = \frac{u}{\sqrt{2}}$$

Q. 

if collision b/w A & B is elastic then maximum compression in spring is?

Sol.

Just after collsion (elastic collsion of same mass)



$$(K.E.)_{loss} = \frac{1}{2}Kx^2 \quad \text{(conservation of M.E. in COM frame)}$$

$$\frac{1}{2}\frac{m\,(2m)}{m + 2m}u^2 = \frac{1}{2}Kx^2$$

$$\frac{1}{3}mu^2 = \frac{1}{2}Kx^2$$

$$x = \sqrt{\frac{2mu^2}{3K}}$$

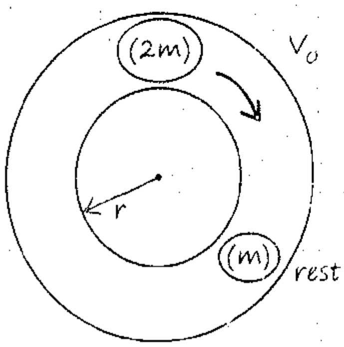
Q. Smooth horizontal circular track, as shown in Fig. then find time taken b/w $1^{st}$ and $2^{nd}$ collision if collsion is elastic, before collision m is at rest and 2m is moving with $V_0$

Sol.

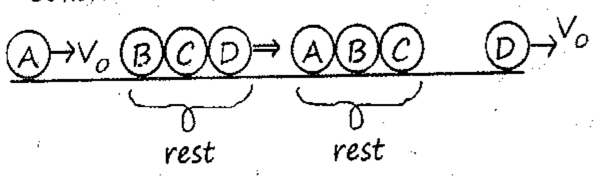
$$t = \frac{2\pi r}{V_0}$$

**MR***

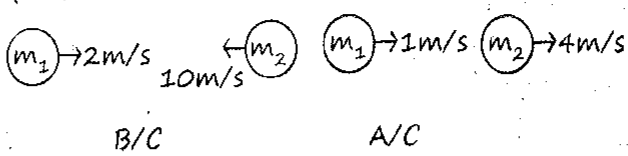
In elastic collision $V_{sep} = V_{app}$




Q. Four identical ball placed on horizontal table then find their velocity after collsion.




Q. Find 'e'




Sol $e = \dfrac{4 - 1}{10 + 2} = \dfrac{3}{12} = \dfrac{1}{4}$

Q. Two Identical object moving with velocity 4m/s and 10m/s towards each other find their velocity after collsion if e = 0.5.

Sol. $\vec{V}_1 = \dfrac{m - 0.5m}{2m}\,4 - \dfrac{(0.5 + 1)m \times 10}{2m}$

$$= \frac{+0.5}{2} \times 4 - \frac{1.5 \times 10}{2}$$

$$= +1 - 7.5 = -6.5 \text{ m/s}$$

Now Conserved momentum of system

$$\vec{P}_i = P_f$$

$$4m - 10m = -6.5m + mv$$

$$-6 = -6.5 + V$$

$$V = 6.5 - 6$$

$$V = 0.5 \text{ m/s}$$

**MR***

"Dusro ke liye kab tak taali bajaoge? Ab aisa karo ki duniya tumhare liye taali bajaye."

# Rotational Motion

## 1> MOMENT OF INERTIA:-

**Ghumane ka कष्ट**

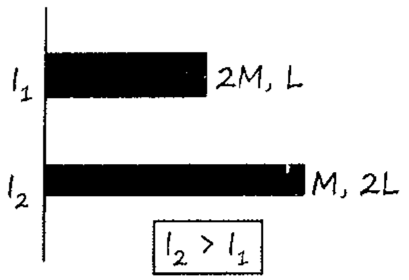
Property of object by which object oppose cause of change in rotational state.

- Unit Kg $m^2$
- Dimension $[ML^2T^0]$

**MR***

$I \propto$ (कितना Mass) × (कितनी Dur)$^2$

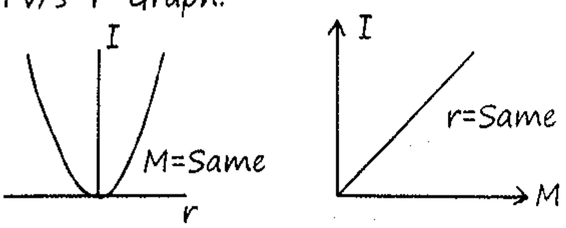
depends more on distance rather than mass




$\boxed{I_2 > I_1}$

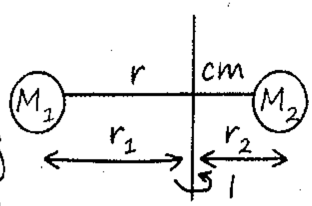
→ Tensor. → Calculated from Axis of Rotation.

- I v/s r Graph:-



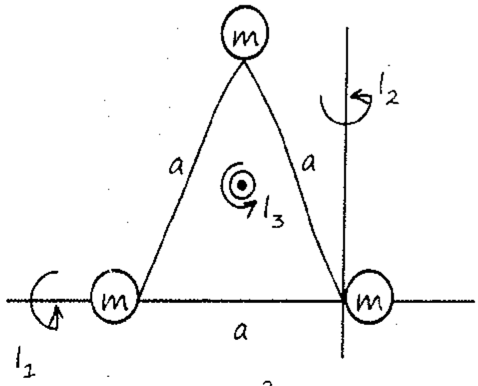

- Mass:-
  1. Point:- $I = Mr^2$ ↗ $\perp^{er}$ dist$^n$ from AOR
  2. "n":- $I = M_1r_1^2 + M_2r_2^2 + ...$
- M.O.I. about C.O.M. & $\perp^{er}$ to line:-

$$I = \frac{M_1M_2}{M_1+M_2}r^2$$




- Object of mass m is placed at (x, y, z) then moment of

  Inertia about x axis $I = m(y^2 + z^2)$
  about y axis $I = m(x^2 + z^2)$
  about z axis $I = m(y^2 + x^2)$
- Three identical mass placed on the corner of equilateral triangle of side (a).

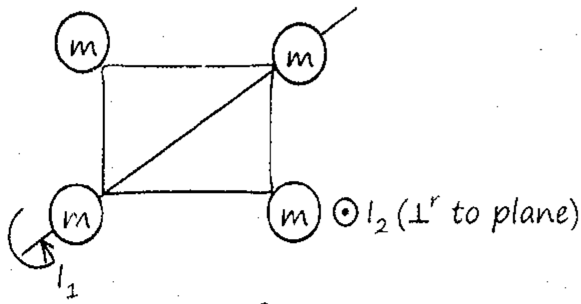



# $I_1 = m\left(\frac{\sqrt{3}\,a}{2}\right)^2 = \frac{3ma^2}{4}$

# $I_2 = ma^2 + \frac{ma^2}{4} = \frac{5ma^2}{4}$

# $I_3 = 3\left[m\left(\frac{a}{\sqrt{3}}\right)^2\right] = ma^2$

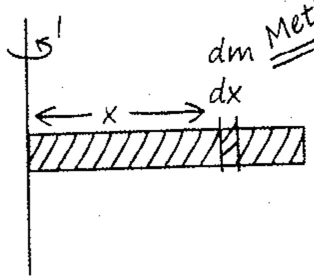
- 4-Point mass placed on the corner of square




$I_1 = 2\left[m\left(\frac{a}{\sqrt{2}}\right)^2\right] = ma^2$

$I_2 = ma^2 + ma^2 + m(\sqrt{2}\,a)^2 = 4ma^2$

- M.O.I. of Continuous body:-




Method

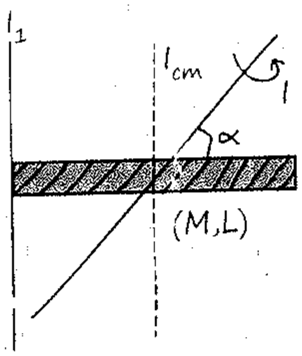
$$\int_0^I dI = \int x^2 \cdot dm$$

$$\lambda = \frac{dm}{dx}$$

Non Uniform body.

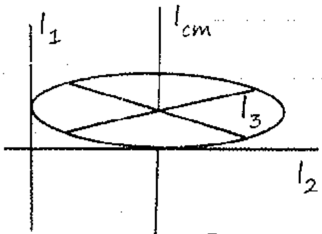
$$I = \int x^2 \cdot \lambda . dx = \int x^2 . dm....$$

## MOI OF ROD:-




- $I_{cm} = \frac{ML^2}{12}$
- $I_1 = \frac{ML^2}{3}$
- $I = \frac{ML^2}{12} \sin^2 \alpha.*$

## MOI OF RING:-




- $I_{cm} = MR^2$
- $I_1 = 2MR^2$
- $I_2 = \frac{3MR^2}{2}$
- $I_3 = \frac{MR^2}{2}$

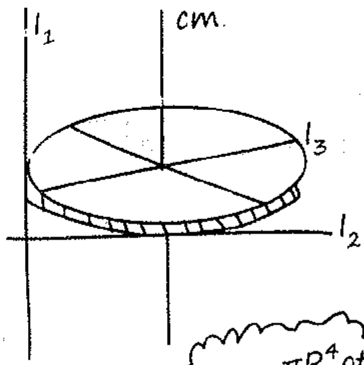
$$I_{cm} = 2\pi A\rho R^3$$

A = cross-section area
(Linear mass density) $\lambda = A\rho$
($\rho$ = volumetric density)

- M.O.I. of half ring about centre perpendicular to plane.

$I = MR^2$

## MOI OF DISC:-




$I_{cm} = \frac{MR^2}{2}$

$I_1 = \frac{3MR^2}{2}$

$I_2 = \frac{5MR^2}{4}$

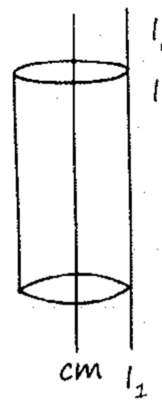
$I_3 = \frac{MR^2}{4}$

$$I_{cm} = \frac{\pi R^4 \rho t}{2}$$

t = thickness

## MOI OF HOLLOW CYLINDER:-

(a)

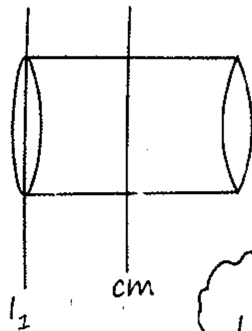



$I_{cm} = MR^2$
$I_1 = 2\,MR^2$

$$I_{cm} = \frac{ML^2}{12} + \frac{MR^2}{2}$$

Rod Ring

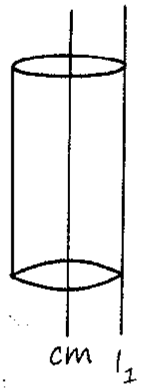
(b)




$$I_1 = \frac{ML^2}{3} + \frac{MR^2}{2}$$

## MOI OF SOLID CYLINDER:-

(a)



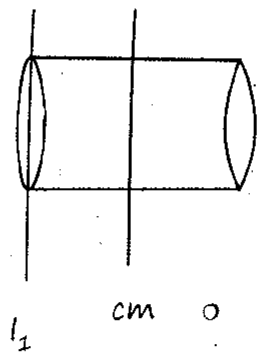

$$I_{cm} = \frac{MR^2}{2}$$

$$I_1 = \frac{3MR^2}{2}$$

o $I_{cm} = \frac{ML^2}{12} + \frac{MR^2}{4}$

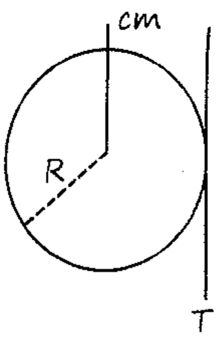
Rod Disc

(b)




o $I_1 = \frac{ML^2}{3} + \frac{MR^2}{4}$

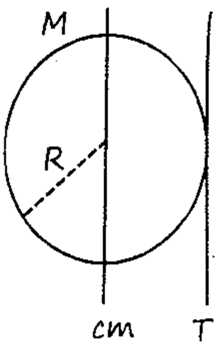
## MOI OF SOLID SPHERE:-




$$I_{cm} = \frac{2}{5}MR^2$$

$$I_T = \frac{7}{5}MR^2$$

## MOI OF HOLLOW SPHERE:-

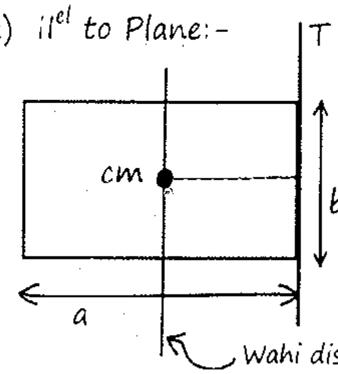



o $I_{cm} = \frac{2}{3}MR^2$

o $I_T = \frac{5MR^2}{3}$

## MOI OF RECTANGULAR PLATE:-

(a) $\parallel^{el}$ to Plane:-

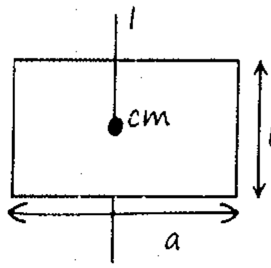



$$I_{cm} = \frac{Ma^2}{12}$$

$$I_T = \frac{Ma^2}{3}$$

Wahi distance Kaam ka hoga joh AOR के $\perp^{er}$ hoga.

(b) $\perp^{er}$ to Plane:-

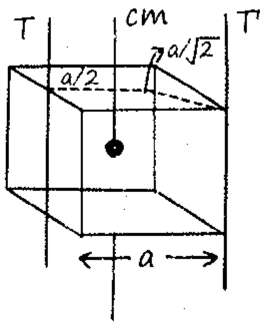



$$I = \frac{M}{12}(a^2 + b^2)$$

*Square Plane Sheet:- $a = b$.

$$I = \frac{Ma^2}{6}$$

## MOI OF CUBE:-

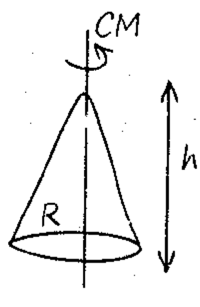



$$I_{cm} = \frac{Ma^2}{6}$$

$$I_{T'} = \frac{2Ma^2}{3}$$

$$I_T = \frac{5Ma^2}{12}$$

## MOI OF CONE:-

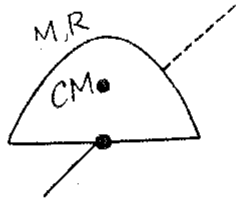



$$I_{cm} = \frac{3}{10}MR^2$$

*Triangular Plate.

$$I_{cm} = \frac{Ma^2}{6}$$

## MOI OF SEMICIRCULAR DISC:-




$$I = \frac{MR^2}{2}$$

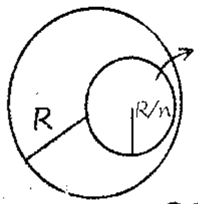
$$r_{com} = \frac{4R}{3\pi}$$

$$^*I_{cm} = MR^2\left[\frac{1}{2} - \frac{16}{9\pi^2}\right]$$

## THEOREMS:-

| Parallel axis → valid for all type of body | Perpendicular axis → valid for planer object |
|---|---|
| $I_O = I_{cm} + Md^2$ | $I_z = I_x + I_y$ |
| d = distance b/w axis passing through C.O.M. and 'O' | $I_x$ & $I_y$ → axis parallel to plane |
| | $I_z$ → $I_r$ to plane |

## CONCEPT OF M.O.I. OF CUTTING SECTION:-




M' → For Area = $M/n^2$

→ For Volume = $M/n^3$ (Solid Sphere)

$$I_{remain} = I_{complete} - I_{removed}$$

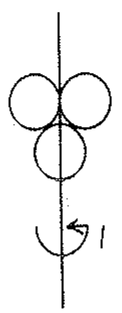
## RADIUS OF GYRATION:-

→ COM : not valid. So we use it.

○ $I = MK^2$ * Yaad rakhna

→ Mass dyan से lena.

○ 3 SPHERICAL SHELL (M,R)




$$I = \frac{2}{3}MR^2 + \left(\frac{5}{3}MR^2\right) \times 2$$

$$I = \frac{12}{3}MR^2 = 4MR^2$$

Q. Two identical disc placed perpendicular to each other then find radius of gyration about axis passing through centre of disc parallel to one disc.

Sol. $I = \frac{MR^2}{2} + \frac{MR^2}{4} = \frac{3MR^2}{4} = 2MK^2$

$$K = \frac{1}{2}\sqrt{\frac{3R}{2}}$$

## TORQUE:-

\# Torque → Cause of change in rotational state of the body

\# Torque oppose rotational motion → false

\# Axial vector

\# Unit → Nm

\# Dim$^n$ → $[ML^2T^{-2}]$

$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta$ → CW ⊗ / ACW ⊙

Thumb ($\vec{\tau}$), Four Fingers ($\vec{r}$), Slap ($\vec{F}$)

$$\vec{\tau}_{net} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \ldots$$

$$\vec{\tau} \perp \vec{r} \perp \vec{F}$$

$$\vec{\tau} = I\vec{\alpha} \qquad \vec{\tau} = \frac{d\vec{L}}{dt}$$

## STATEMENT:-

If $F_{net} = 0$ then $\tau_{net}$ must be zero → False

If $F_{net} \neq 0$ then $\tau_{net}$ must be non-zero → False

If $\tau_{net} = 0$ then net force must be zero → False

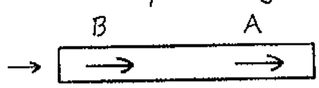
If $\tau_{net} \neq 0$ then net force must be non-zero → False

### MR*

Torque hamesha hinge point के about lagega aur who ek toh body Ko Ghumayega ya toh Ghumte huye object Ka state change Karega.

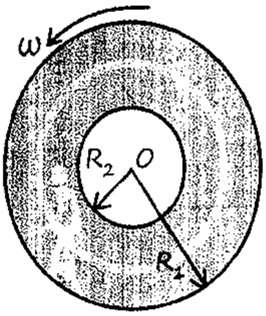
$\vec{\tau}_{net} = 0$ } Rotational Eq$^n$.

○ Concept of Rigid body:-



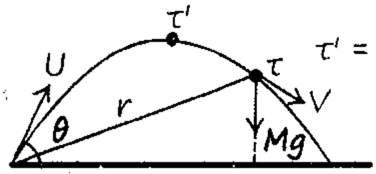

$V_B = V_A$

## M.O.I. OF ANNULAR DISC:-




$$*I_O = \frac{M}{2}(R_2^2 + R_1^2)$$

*Imp Que:-*




U cosθ t

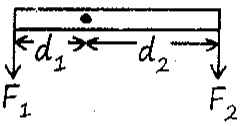
$$\tau = Mg\,(U\cos\theta)t$$

$$\tau' = \frac{Mg\,U^2 \sin 2\theta}{2g}$$

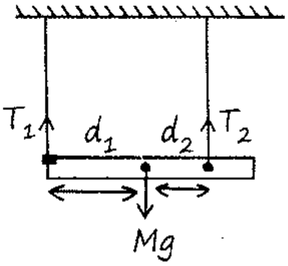
at Hmax

## ROTATIONAL EQUILIBRIUM:-

$\tau_{net} = 0 \quad F_1 d_1 = F_2 d_2$




$$\text{M. Advantage} = \frac{F_1}{F_2} = \frac{d_2}{d_1} \quad (\text{M.A.} > 1)$$




$T_1 + T_2 = mg$ ...(i)

$T_1 d_1 = T_2 d_2$ ...(ii)

## ROTATIONAL KINEMATICS:-

**U.C.M.:-**

$\vec{\omega} = const^n \qquad \alpha = 0 = a_t.$

$\text{Speed} = Const^n \quad \vec{a_c} = \frac{v^2}{r}$

$\theta = \omega t.$

**Non-UCM:-**

(a) $\alpha = const^n$.

Eqn of Motion:

$$\omega_2 - \omega_1 = \alpha t$$

$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$

$$\omega_2^2 - \omega_1^2 = 2\alpha\theta$$

$$\theta = \left(\frac{\omega_2 + \omega_1}{2}\right)t = n2\pi.$$

(b) $\alpha$ = Variable

$\Delta\theta = \int \omega . dt \qquad \Delta\omega = \int \alpha . dt$

$\theta \qquad \omega \qquad \alpha$

$\alpha = \frac{d\omega}{dt}$

$\omega = \frac{d\theta}{dt} \qquad \alpha = \omega\frac{d\omega}{d\theta}$

- Rest to Rest Motion:-

| $\alpha_1(+)$ | $\omega$ | $\alpha_2(-)$ |
|---|---|---|
| $\theta_1 \quad t_1$ | $t$ | $\theta_3 \quad t_2$ |

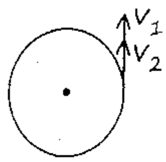
$\alpha_1 t_1 = \alpha_2 t_2$ , $\alpha_1\theta_1 = \alpha_2\theta_3$

$\theta_1 = \frac{1}{2}\alpha_1 t_1^2 \qquad \theta_2 = \omega t$

$\theta_3 = \frac{1}{2}\alpha_2 t_2^2$

- The time at which two particle with different speed, start moving from same position Meet?

$$V_{relative} = \frac{2\pi R}{T}$$




- Pure Motion:-

1> Rotational Motion:-

1. 2. 3.

↑ Fix

$\omega_1 = \omega_2 = \omega_3$

$\alpha_1 = \alpha_2 = \alpha_2$

$v_1 \neq v_2 \neq v_3$

$1 \longrightarrow V_1$

$2 \longrightarrow V_2$

$3 \longrightarrow V_3$

2> Translational Motion:-

$v_1 = v_2 = v_3$

## ANALOGY

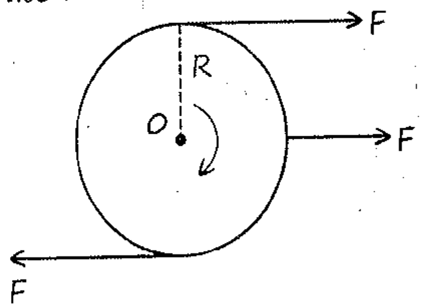
| Translation | Rotational |
|---|---|
| * $S$ | * $\theta$ |
| * $V = \dfrac{dx}{dt}$ | * $\omega = \dfrac{d\theta}{dt}$ |
| * $a = \dfrac{dv}{dt}$ | * $\alpha = \dfrac{d\omega}{dt}$ |
| * $F = \dfrac{dP}{dt} = ma$ | * $\tau = \dfrac{dL}{dt} = I\alpha$ |
| * $P = mV$ | * $L = I\omega$ |
| * $w = F.s.$ | * $W = \tau.\theta$ |
| * $KE = \dfrac{1}{2}mv^2$ | * $KE = \dfrac{1}{2}I\omega^2$ |
| * $P = F.v.$ | * $P = \tau.\omega$ |
| * Impulse = m.V. | Impulse = $I\omega$ |

## ANALOGY

| Translation | Rotational |
|---|---|
| * $S = ut + \dfrac{1}{2}at^2$ | $\theta = \omega t + \dfrac{1}{2}\alpha t^2$ |
| * $V = U + at$ | $\omega = \omega_0 + \alpha t$ |
| * $V^2 - U^2 = 2as$ | $\omega^2 - \omega_0^2 = 2\alpha\theta$ |
| * $S_{n^{th}} = U + \dfrac{a}{2}(2n-1)$ | $\theta_{n^{th}} = \omega_0 + \dfrac{\alpha}{2}(2n-1)$ |
| * $S = \left(\dfrac{U+V}{2}\right)T$ | MR* $\theta = \left(\dfrac{w_i+\omega_f}{2}\right)T$ |
| * $S = \dfrac{1}{2}\left(\dfrac{\alpha\beta}{\alpha+\beta}\right)T^2$ | $S = \dfrac{1}{2}\left(\dfrac{\alpha\beta}{\alpha+\beta}\right)T^2$ |
| * Stoping $dist^n$. $= \dfrac{U^2}{2a}$ | * $\theta = \dfrac{\omega_0^2}{2\alpha}$ |
| $V_{max} = \left(\dfrac{\alpha\beta}{\alpha+\beta}\right)T$ | $\omega_{max} = \left(\dfrac{\alpha\beta}{\alpha+\beta}\right)T$ |

Q. A solid sphere (M, R) hinged about centre and free to rotate then find angular $acc^n$.

Sol.




$$\tau_O = I\alpha$$

$$2FR = \frac{2}{5}MR^2\alpha$$

$$\alpha = \frac{5F}{MR}$$

Q. A solid cylinder of mass 2 kg and radius 4 cm rotating about its axis at the rate of 3 rpm. The torque required to stop after $2\pi$ revolution is

Sol. Using Work Energy Theorem

$$W = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$$

Here $\theta = 2\pi$ revolution

$= 2\pi \times 2\pi = 4\pi^2$ rad

$$W_i = 3\times\frac{2\pi}{60}\text{ rad/s}$$

$$\Rightarrow -\tau\theta = \frac{1}{2}\times\frac{1}{2}mr^2(0^2-\omega_i^2)$$

$$\Rightarrow -\tau = \frac{\frac{1}{2}\times\frac{1}{2}\times 2\times(4\times10^{-2})^2\left(-3\times\frac{2\pi}{60}\right)^2}{4\pi^2}$$

$$\Rightarrow \tau = 2\times10^{-6}\,Nm$$

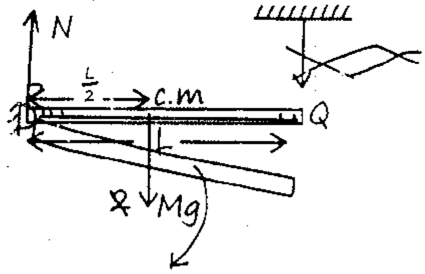
Note:-

$$a_t = R\alpha \qquad a_c = R\omega^2 = \frac{V^2}{R}$$

$$|\vec{a}| = \sqrt{a_t^2 + a_c^2}$$

Q. A rod PQ of mass M and length L is hinged at end P. The rod is kept horizontal by a massless string tied to point Q as shown in figure. When string is cut, the initial angular acceleration of the rod is (NEET 2013, AIP MT – 07/11/ IIT-4 X).

$[\tau_p = I_p \alpha_p]$ hinged point/axis of Rot$^n$




(a) $\frac{2g}{L}$

(b) $\frac{2g}{2L}$

(c) $\frac{3g}{2L}$

(d) $\frac{g}{L}$

$$N \times 0 + \cancel{m}g\frac{\cancel{L}}{2} = \frac{\cancel{M}L^{\cancel{2}\,L}}{3}\alpha$$

$$\frac{g}{2} = \frac{L\alpha}{3}$$

Angular acc$^n$ of Rod (every point of Rod) about 'p'.

$$* \ \alpha = \frac{3g}{2L}$$

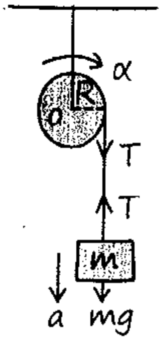
Find liner acc$^n$ of C.O.M.??

$a_t = r(\alpha)$ diff$^n$ for all Points

$$a_{c.m} = \frac{L}{2}\left(\frac{3g}{2L}\right) = \frac{3g}{4}\ m/s^2$$

**(Ask 4X in IIT) (Most Imp. Que.)**

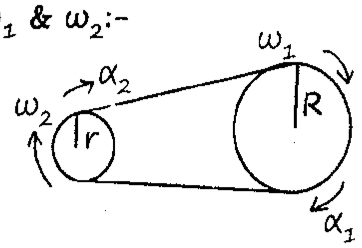
**MR****




$$\text{accl}^n = \frac{F_{net}}{\text{Transl}^n\ \text{Mass} + \text{Rotat}^n\ \text{Mass}}$$

$$a = \frac{mg}{m + I/R^2}$$

o **Relation b/n $\omega_1$ & $\omega_2$:-**



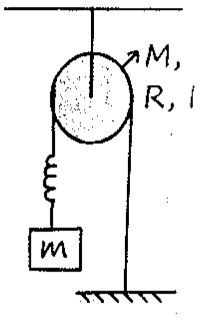

* $V_1 = V_2$

* $\omega_1 R = \omega_2 r$

* $a_{t_1} = a_{t_2}$

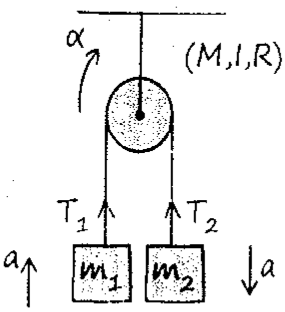
* $\alpha_1 R = \alpha_2 r$

o **T.P. of SHM:-**




$$T = 2\pi\sqrt{\frac{m + I/R^2}{K}}$$

o




Sufficient friction

$$a = \frac{m_2 g - m_1 g}{m_1 + m_2 + I/R^2}$$

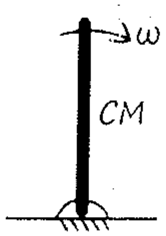
$T_1 \neq T_2$

We can find $T_1$ & $T_2$ by F.B.D.

**APPLICATION OF COM IN PURE ROTATIONAL MOTION:-**

$$KE = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

e.g.:- Rod is released from vertical position Find ω when rod becomes horizontal.



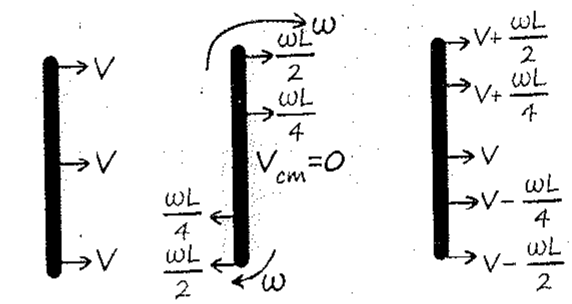

$$\frac{MgL}{2} + 0 = \frac{1}{2}I\omega^2 + 0$$

$$\frac{\cancel{M}g\cancel{L}}{\cancel{2}} = \frac{1}{2}\frac{\cancel{M}L^{\cancel{2}}}{\cancel{3}}\omega^2$$

$$V_{cm} = \frac{L}{2}\omega$$

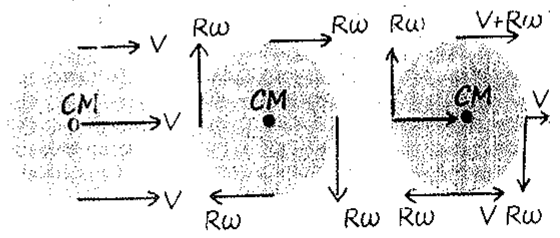
$$\omega = \sqrt{\frac{3g}{L}}$$

# ROLLING MOTION:-

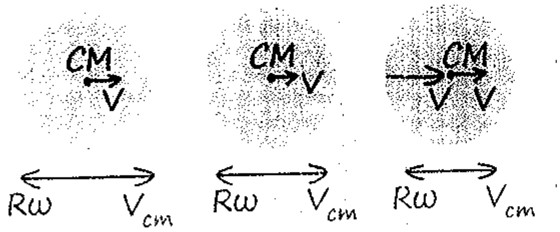



Transl$^n$ Rotat$^n$ Rolling

$$*KE = \frac{1}{2} mV_{cm}^2 + \frac{1}{2} I\omega^2 .*$$



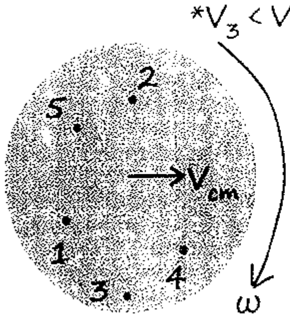

Transl$^n$ Rotat$^n$ Rolling




| Forward Sliping | Backward Sliping | No Sliping |
|---|---|---|
| $V_{cm} > R\omega$ | $V_{cm} < R\omega$ | $V_{cm} = R\omega$ |
| $f_k$ = Back | $f_k$ = Front | Lowest point at rest |

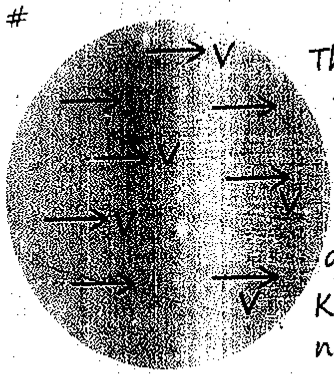
NOTE:-

$*V_3 < V_4 < V_1 < V_5 < V_2*$



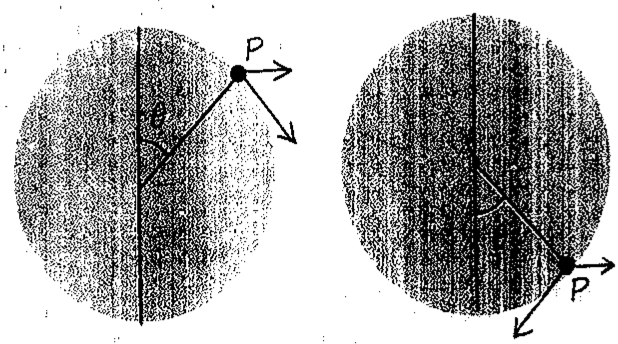

Speed.

#




Thara bhai Translation Motion itni jagah "v" dedega Kishine sochi nai hogi!

o Note:-




$\vec{V}_p = 2V\cos\frac{\theta}{2}$ $\vec{V}_p = 2V\sin\frac{\theta}{2}$

TE:- MR**

$$KE_{Total} = KE_{Trans} + KE_{Rot.}$$

$$KE_{Total} = \frac{1}{2} mV_{cm}^2 + \frac{1}{2} I\omega^2$$

$$\boxed{KE_{Total} = \frac{1}{2} mV_{cm}^2 \left[1 + \frac{K^2}{R^2}\right]}$$

MR*

$$\frac{KE_{Trans}}{KE_{Total}} = \frac{1}{1+\frac{K^2}{R^2}} = \beta$$

## MR * TABLE (THE PRO VERSION)

| Physical quantity | Ring hollow cylinder | Hollow sphere | Disc, solid cylinder | Solid sphere |
|---|---|---|---|---|
| $\beta$ | $0.5 = \frac{1}{2}$ | $0.6 = \frac{3}{5}$ | $0.66 = \frac{2}{3}$ | $0.71 = \frac{5}{7}$ |
| $KE_{Trans}/KE_{Total}$ | $0.5 = 50\%$ | $0.6 = 60\%$ | $0.66 = 66\%$ | $0.71 = 71\%$ |
| $KE_{Rot}/KE_{Total}$ | $0.5 = 50\%$ $(1/2)$ | $0.4 = 40\%$ $(2/5)$ | $0.33 = 33\%$ $(1/3)$ | $0.28 = 28\%$ $(2/7)$ |
| $KE_{Trans}/KE_{Rot}$ | $1:1$ | $3:2$ | $2:1$ | $5:2$ |
| Acclⁿ on inclined → [redacted] | $\frac{g\sin\theta}{2}$ | $\frac{3}{5}g\sin\theta$ | $\frac{2}{3}g\sin\theta$ | $\frac{5}{7}g\sin\theta$ |
| Time req. to come down → | [redacted] → | → | → | |
| Velocity at bottom of inclined [redacted] | $V = \sqrt{gH}$ | $V = \sqrt{\frac{6gH}{5}}$ | $V = \sqrt{\frac{4gH}{3}}$ | $V = \sqrt{\frac{10gH}{7}}$ |
| $H_{max}$ attained by particle | $H = \frac{V_{cm}^2}{2g\beta}$ <br> $H = \frac{V_{cm}^2}{g}$ | $H = \frac{5V_{cm}^2}{6g}$ | $H = \frac{3V_{cm}^2}{4g}$ | $H = \frac{7V_{cm}^2}{10g}$ |
| Friction on inclined | $f_r = Mg\sin\theta\,(1-\beta)$ <br> $f_r = Mg\sin\theta/2$ | $f_r = \frac{2}{5}Mg\sin\theta$ | $f_r = \frac{Mg\sin\theta}{3}$ | $f_r = \frac{2}{7}Mg\sin\theta$ |
| $\mu_{min}$ to start pure rolling | $\mu = (1-\beta)\tan\theta$ <br> $\mu = \tan\theta/2$ | $\mu_s = \frac{2}{5}\tan\theta$ | $\mu = \frac{\tan\theta}{3}$ | $\mu = \frac{2}{7}\tan\theta$ |

* Jahan "g" wahi "β" * Konse bhi sawaal mein Rolling aayega toh β lagado.*

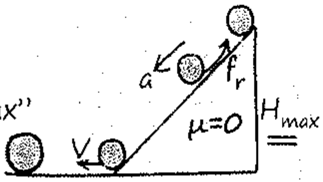
○ Caution:-

Rolling on Smooth inclined plane:-

* $a = g \sin\theta$ ⟶ independent of Mass, Shape, Size.

* $V = \sqrt{2gh}$ ⟶ Velocity at bottom.

* $H = \dfrac{V_{cm}^2}{2g}$ ⟶ "Hmax"




○ Rolling Motion on Rough inclined plane:-

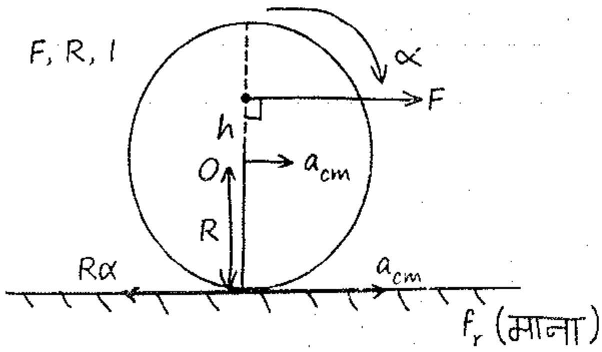
$V = \sqrt{2gh\beta}$ $\quad a = \beta g \sin\theta$

$H = \dfrac{V_{cm}^2}{2g\beta}$

Object Upar jaye ya niche

*$f_r$ always acts upwards!

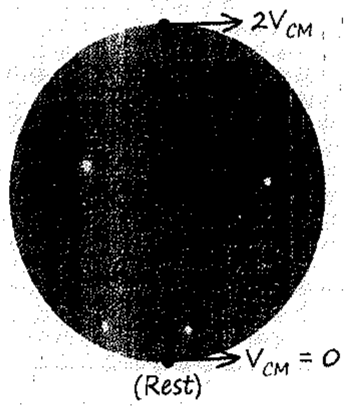
## PURE ROLLING ON A HORIZONTAL PLANE:-




$$\frac{1}{1+\frac{K^2}{R^2}}$$

MR**

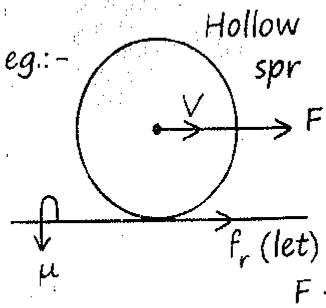
$$a_{cm} = \frac{F\beta}{m} \times \text{(Coefficient of velocity where force acts)}$$




$$a_1 = \frac{F\beta\,(2)}{m} \qquad a_2 = \frac{F\beta}{m}\left(\frac{3}{2}\right)$$

$$a_3 = \frac{F\beta\,(1)}{m} \qquad a_4 = \frac{F\beta}{m}\left(\frac{1}{2}\right)$$

eg.:-




$$a_{cm} = \frac{F}{m}\left(\frac{3}{5}\right)$$

$$F + f_r = ma_{CM}$$

$$f_r = \frac{3F}{5} - F = -\frac{2F}{5} \text{ (backward)}$$

* Pure rolling motion can start on smooth horizontal surface

## ANGULAR MOMENTUM:-

○ Depends on Frame of Ref.

$L = rmv \sin\theta$

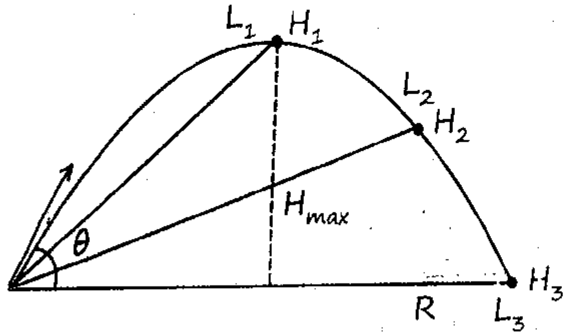
$L = rP \sin\theta$

$\vec{L} = \vec{r} \times \vec{P}$ (Axial Vector)

$\theta$ = Angle b/n $\vec{r}$ & $\vec{P}$

→ When object is moving on straight line with const. velocity then:- L = same.

- *Object is projected with speed "U" at an angle "θ" with horizontal then find angular momentum in projectile:-*




$$L_1 = mu\cos\theta \times H_{max} = mu\cos\theta.\frac{U^2\sin^2\theta}{2g}$$

$$L_3 = 4L_1 = mu\cos\theta.\frac{2U^2\sin^2\theta}{g}$$

L when about to Collide = 4 L at $H_{max}$ *

General point

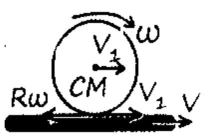
$$L_2 = \frac{mg\,U\cos\theta t^2}{2} \qquad \tau = Mg\,U\cos\theta t$$

$$\int dL = \int MgU\cos\theta t.dt$$

- Pure Rotational Motion:-

$$L = I\omega \qquad KE = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

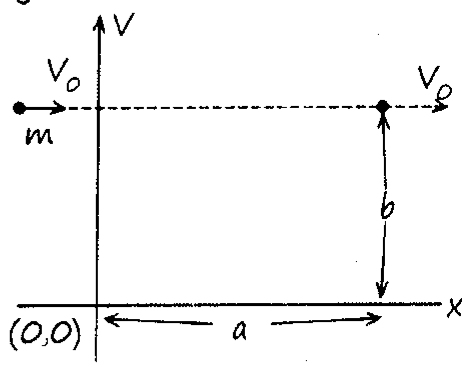
- A rolling body is rolling without slipping on a moving plank.




$v_1 - R\omega = v$

$v_1 - v = R\omega.$

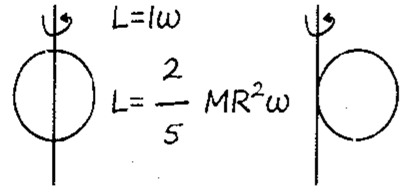
- Angular momentum of object w.r.t. origin:-




$$\boxed{L = mV_0 b}$$

Q. A solid sphere is rotating with angular speed ω then angular momentum about given axis:-

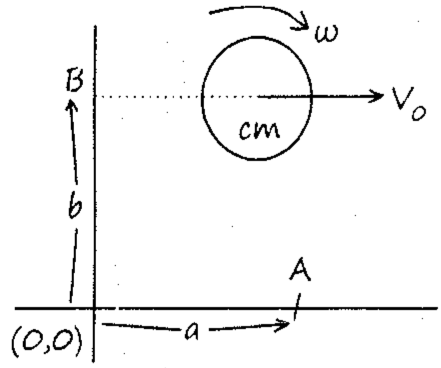
⇒



$L = I\omega$

$L = \frac{2}{5}MR^2\omega$ $\qquad$ $L = \frac{7}{5}MR^2\omega$

Q.




⇒ $L_A = L(0,0) = mV_0 b + I_{cm}\omega$

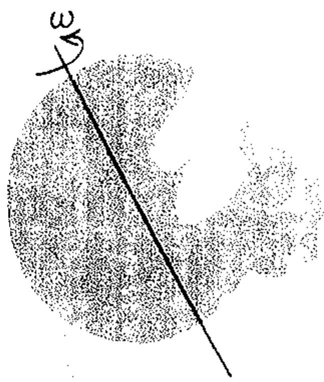
$L_B = I_{cm}\omega$

## CONSERVATION OF ANG. MOMENTUM:-

$\tau = 0 \qquad \vec{L} = \text{const}^n$

$I_1\omega_1 = I_2\omega_2$

1>

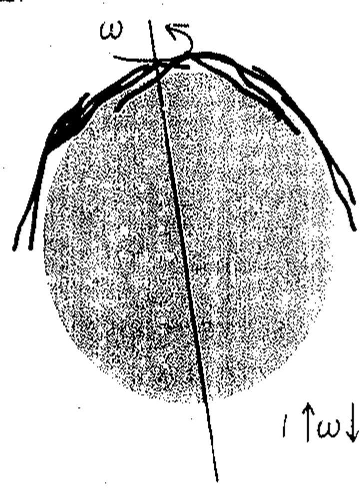



$I\uparrow\omega\downarrow = \text{Const}^n$

$\downarrow\omega = \frac{2\pi}{T\uparrow}$

T.P. will inc.

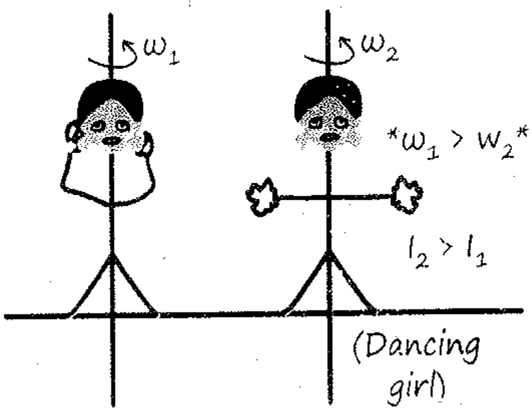
2>




Ice on pole will melt, then.

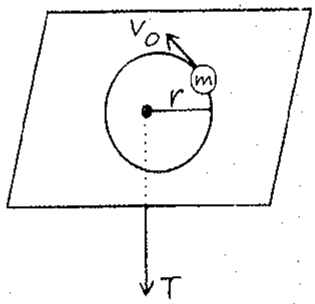
$I\uparrow\omega\downarrow \quad \underline{\underline{T\uparrow}}$

3>




Q. 'm' is moving on circular path of radius r with speed $v_0$, then find its speed when radius becomes $\frac{r}{2}$ by increasing 'T'.

Sol. $L_0 = \cos^n$




$mv_0 r = mv \frac{r}{2}$ $\quad$ $K.E_i = \frac{1}{2}mv_0^2$

$v = 2v_0$ $\quad$ $K.E_f = 2mv_0^2$

Imp:-

Same $dir^n$ = +

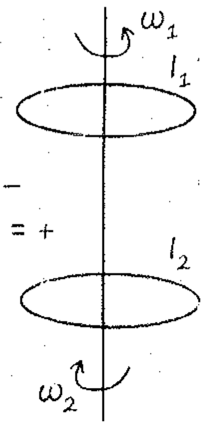
Oppo = −

$$\omega = \frac{I_1\omega_1 \pm I_2\omega_2}{I_1 + I_2}$$

Same = −

Oppo = +

$$\underset{loss}{\Delta KE} = \frac{1}{2}\frac{I_1 I_2}{I_1 + I_2}\omega_{rel}^2$$



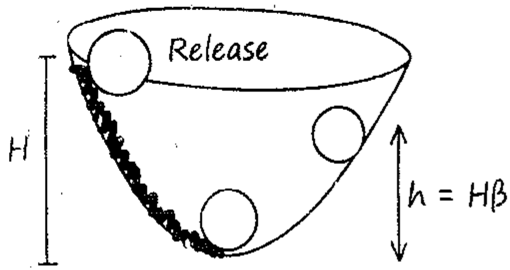

Imp. Model Problems MR*

1> A heavy body is thrown on a horizontal rough surface with initial velocity "U" without rolling. Find "V" when it start pure rolling:-

MR*

$\boxed{V = \beta U}$ $\quad$ $\beta = \frac{1}{1 + \frac{K^2}{R^2}}$

2> IIT - Adv 2014.




$\boxed{h = H\beta}$ $\quad$ $h = \frac{V_0^2}{2g}$, $V_0 = \sqrt{2gH\beta}$.

3> A Rotating body with "$\omega_0$" Placed on rough surface then find Angular velocity when it starts pure rolling motion:-

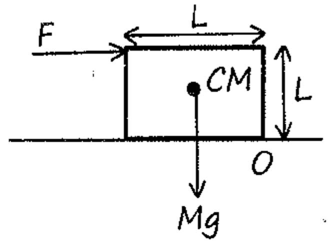
$*\boxed{\omega = (1-\beta)\omega_0}$

4> A body rolls on horizontal floor. Find W (work) to stop it:-

$$\boxed{W = \frac{KE_{Transl}}{\beta} = K.E_{Total}}$$

**TOPPLING:-**

MR*




$\tau_0 = 0$

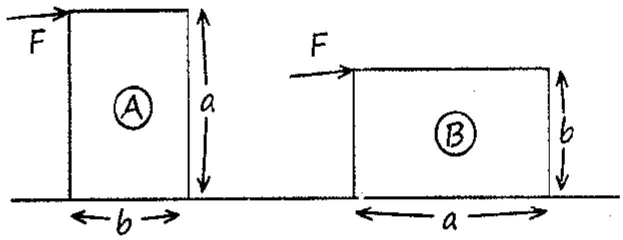
$Mg \frac{\cancel{L}}{2} = F.\cancel{L}$

Don't take $\tau_{CM} = 0$ take

$\tau_0 = 0$

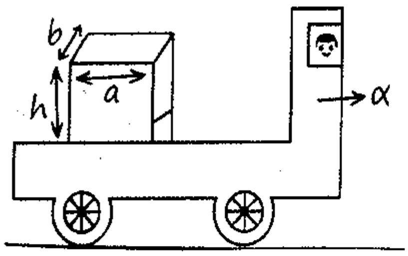
Toppling में Normal Ko Shift KrKe Object apne appko palatne se bachata hai!

Q. In which case probability of toppling is high:-

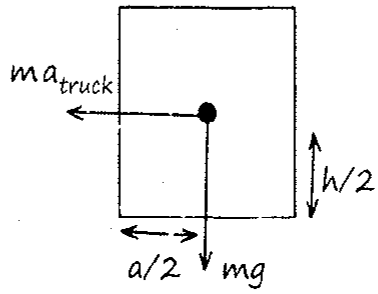



Sol. In A probability of toppling is high

Q. What will be the value of maximum acceleration of the truck in the forward direction so that the block kept on the back does not topple:-




Sol.




Block will not topple if

$$\tau_{ma_{truck}} \leq \tau_{mg}$$

$$\frac{ma_{truck}h}{2} \leq mg\,\frac{a}{2}$$

$$a_{truck} \leq \frac{ag}{h}$$

$$\therefore\ a_{truck} = \frac{ag}{h}$$

Q. Ring, solid sphere and disc of mass M and radius R rotating with same angular speed $\omega$, then work to stop it is :-

Sol. $W_{ring} > W_{Disc} > W_{solid\ sphere}$

MR

"Koi kam Sahi galat nahi hota, bas ush kam ko krne ka samay sahi galat hota hai."

# 13 Gravitation

Gravitational Force-Long range, conservative, follow inverse-square law, central, medium independent, mediated by graviton.

## 1> NEWTON'S LAW OF GRAVITATION:-

Valid for point and spherical object.

$$F = \frac{Gm_1m_2}{r^2} \quad G = 6.67 \times 10^{-11}\ kg^{-1}m^3s^{-2}$$

$$= M^{-1}L^3T^{-2}$$

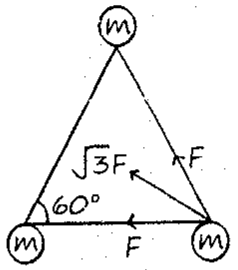
$$\vec{F} = \frac{Gm_1m_2}{|r^3|}\vec{r}$$

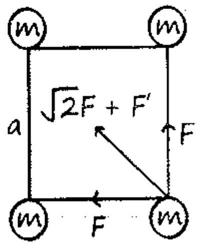
## 2> NEUTRAL POINT:-

$$x = \frac{d}{\sqrt{n}+1}$$ x is from smaller mass

## 3> SUPERPOSITION THEOREM:-

Net force on one object is a vector sum of all other forces acting on it due to other masses.

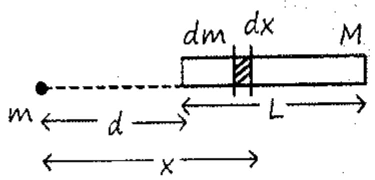



$$F_{net} = \sqrt{3}F = \frac{\sqrt{3}Gm^2}{a^2}$$

$$F_{net} = \frac{Gm^2}{a^2}\left(\frac{2\sqrt{2}+1}{2}\right)$$

4>




$$F_{net} = \frac{GMm}{d(d+L)}$$

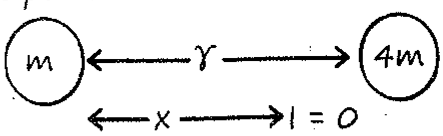
due to rod on particle

## 5> G. FIELD INTENSITY :-

DIRECTION PARALLEL TO FORCE

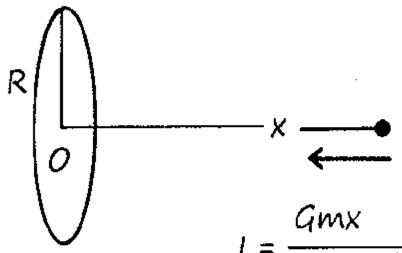
$$\vec{I} = \frac{\vec{F}}{m}\ N/Kg,\ I = \frac{GM}{r^2}$$

Q. Find x so that field at that point will be zero

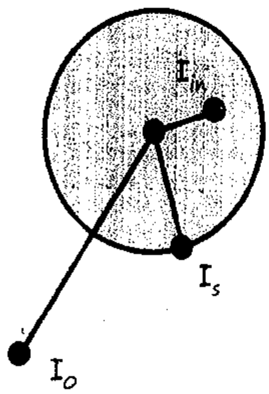



Ans. $x = \frac{r}{\sqrt{4}+1} = \frac{r}{3}$

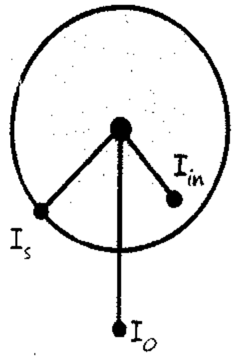
- Gravitational filed intensity due to ring.




$$I = \frac{Gmx}{(R^2 + x^2)^{3/2}}$$




| HOLLOW SPHERE. | SOLID SPHERE. |
|---|---|
| $I_{out} = -\frac{GM}{\gamma^2}\hat{\gamma}$ | $I_{out} = -\frac{GM}{\gamma^2}\hat{\gamma}$ |
| $I_{sur} = -\frac{GM}{R^2}\hat{R}$ | $I_{sur} = -\frac{GM}{R^2}\hat{R}$ |
| $I_{inside} = 0$ | $I_{in} = \frac{GMr}{R^3}$ |




$$F_{in} = \frac{GMmr}{R^3}$$

## 6> ACCELERATION DUE TO GRAVITY:-

a. On Surface:-

$\rho_{earth} = 5.5 \times 10^3\ kg/m^3$

$$g_0 = \frac{GM_e}{R_e^2} = \frac{4}{3}\pi RG\rho$$

$\nearrow$ $M = const^n$ $\quad g_0 \propto \frac{1}{R^2}$

$\searrow$ $\rho = const^n$ $\quad g_0 \propto R$

b. Above Surface:-

$$g_h = \frac{g_0 R^2}{(R+h)^2} \qquad g_h = g_0\left[1 - \frac{2h}{R}\right] \leftarrow h <<< R$$

c. Below Surface:-

$$g_d = g\left[1 - \frac{d}{R}\right]$$

% change in g at height h $= \frac{-2h}{R} \times 100$

% charge in g at depth d $= \frac{-d}{R} \times 100$

$\Big]$ For small change

d. Variation of "g" due to shape of Earth:-

$R_e = R_p + 21$ km, $g_p > g_e$

**Note:-** Mass = Prop of matter, remains same everywhere.

W(weight)$\uparrow$ as we move from equator to pole.

e. Variation of "g" due to rotation of earth about its own axis:-

$g_{eff} = g_0 - R\omega^2\cos^2\theta$ Equator pr accl$^n$ due to gravity 0.34% kam hoti hai pole se!

$\theta$ = measured from equator

Q. The depth 'd' at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the earth's surface is (R radius of earth)

Ans. Acceleration due to gravity at depth d under the surface $g' = g_s\left(1 - \frac{h}{R}\right)$

Given : $g' = \frac{g_s}{n}$

$\therefore \frac{g_s}{n} = g_s\left(1 - \frac{d}{R}\right)$

or $\frac{1}{n} = \left(1 - \frac{d}{R}\right)$

$\Rightarrow d = R\left(\frac{n-1}{n}\right)$

Q. The height from earth's surface at which acceleration due to gravity becomes is $\frac{g}{4}$ (where g is acceleration due to gravity on the surface of earth and R is radius of earth).

Ans. $g_h = g_0 \frac{R^2}{(R+h)^2}$

$\frac{g_0}{4} = g_0 \frac{R^2}{(R+h)^2}$

$\frac{R}{R+h} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

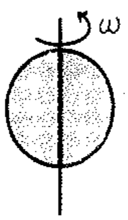
$2R = R + h$

$h = R$

## 7> SPECIAL POINTS:-

a> T = 84.6 min i,e If earth rotates 17 times its present rotational speed so body at equator feels weightless.

b> If $\omega \uparrow$ g = dec. at all place except pole

c> 


W → E

So Rocket also projected in W → E dir?

## 8> GRAVITATIONAL PE:-

$\boxed{W_{CF} = -\Delta U}$ $U_B = -[W_{A\to B}]_{GF}$

## 9> G. POTENTIAL ENERGY PER UNIT MASS:-

$$V = \frac{U}{M} \quad \vec{I} = \frac{\vec{F}}{M} \quad \vec{I} = -\frac{dv}{dr} \quad \vec{F}_{cF} = -\frac{dU}{dr}$$

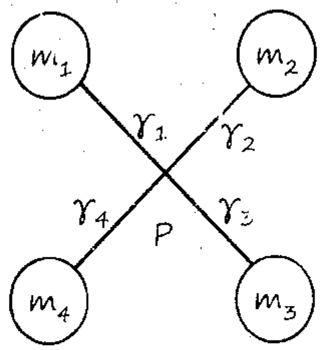
$$\vec{I} = -\left[ \frac{\delta v}{\delta x}\hat{i} + \frac{\delta v}{\delta y}\hat{j} + \frac{\delta v}{\delta z}\hat{k} \right]$$

$$\Delta V = -\int \vec{I}.\, d\vec{r} \qquad \Delta U = \int \vec{F}.\, d\vec{r}$$

## 10> G.P DUE TO POINT MASS AT A DIST$^N$ "$\gamma$":-

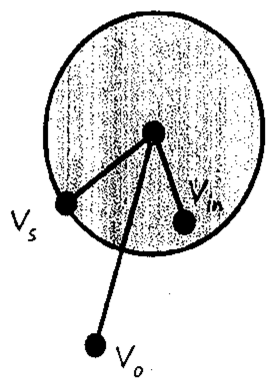
$$U = -\frac{GMm_o}{\gamma} \qquad V = -\frac{GM}{\gamma}$$

## 11> G.P DUE TO COMBINATION OF POINT MASS:-




$$V_p = -\frac{Gm_1}{\gamma_1} - \frac{Gm_2}{\gamma_2} - \frac{Gm_3}{\gamma_3} - \frac{Gm_4}{\gamma_4}$$

## 12> G.P DUE TO :-

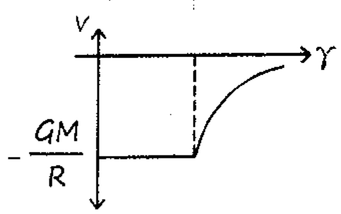



**a> Uniform thin shell**

$$V_{out} = -\frac{GM}{\gamma}$$

$$V_{sur} = -\frac{GM}{R}$$

$$V_{in} = -\frac{GM}{R}$$



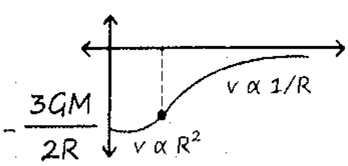

**b> Uniform Solid Sphere**

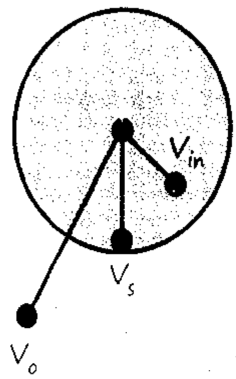
$$V_{out} = -\frac{GM}{r}$$

$$V_{sur} = -\frac{GM}{R}$$

$$V_{in} = -\frac{GM}{2R^3}[3R^2 - \gamma^2]$$

$$V_{centre} = -\frac{3}{2}\frac{GM}{R}$$



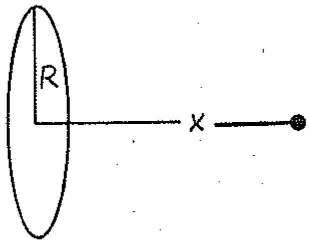



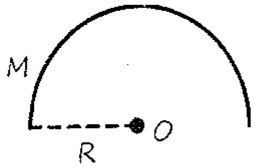
## 13> G.P DUE TO UNIFORM RING :-

$$V_p = -\frac{GM}{\sqrt{x^2+R^2}}$$

At center $(x = 0)$

$$V = -\frac{GM}{R}$$



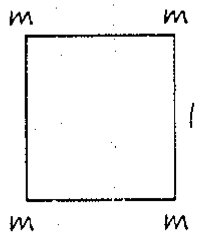



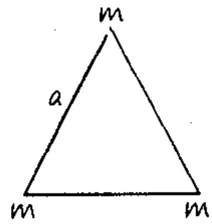

at centre of half ring $V_O = -\dfrac{GM}{R}$

## 14> G.P ENERGY:-

$$V_p = -\frac{GMm}{r}$$

Total no. of terms $= \dfrac{N(N-1)}{2}$



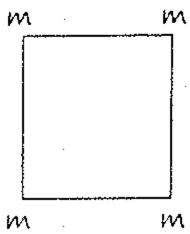



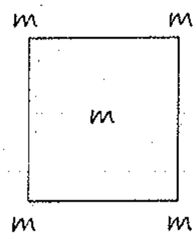

$$U = -\frac{4Gm^2}{l} - \frac{2Gm^2}{\sqrt{2}\,l}$$

$$U = -\frac{3Gm^2}{a}$$

Q. Four point mass m placed at cornor of square of side a, find work done in bringing $5^{th}$ mass m from infinity to centre of square.

Ans.




$U_i = U_o$ (Let)

$$U_f = U_o - \frac{4Gm^2}{a/\sqrt{2}}$$

$$W = \Delta U = U_f - U_i$$

$$= -\frac{4Gm^2\sqrt{2}}{a}$$

## 15> G. P.E. OF EARTH-MASS SYSTEM:-

From surface to height "h":-

$$\Delta U = \frac{mgh}{\left(1+\frac{h}{R}\right)} = \frac{GMmh}{R^2\left[1+\frac{h}{R}\right]}$$

Q. Object of mass m raised to height $h = 2R$ from earth surface change in its potential energy.

Ans. $\Delta U = \dfrac{mgh}{1+\frac{h}{R}} = \dfrac{mg2R}{1+\frac{2R}{R}} = \dfrac{2}{3}mgR$

## 16> ESCAPE VELOCITY:-

$$V_e = \sqrt{\frac{2GM}{R}} \qquad V_e = \sqrt{2gR} \qquad V_e = \sqrt{\frac{8}{3}G\rho\pi R^2}$$

$\rho = const^n \qquad V_e \propto R$

$M = const^n \qquad V_e \propto 1/\sqrt{R}$

Escape velocity does not depend upon angle of projection & mass of object.

## 17> VELOCITY OF OBJECT AT ∞ WHEN PROJECTED WITH $V_g > V_e$ :-

$$V_\infty = \sqrt{Vg^2 - Ve^2}$$

Q. Object is projected with double velocity of escape then its velocity in space

Ans. $V_{space} = \sqrt{Vg^2 - Ve^2} = \sqrt{4V_e^2 - V_e^2}$

$= \sqrt{3}\, V_e$

## 18> $V_g < V_e$ THEN $h_{max}$ = ?

$$h_{max} = \frac{R}{\left[\frac{Ve^2}{Vg^2} - 1\right]}$$

Q. Object is projected with one-fourth of escape velocity then maximum height attained?

Ans. $h = \dfrac{R}{\left(\dfrac{V_e}{V_g}\right)^2 - 1} = \dfrac{R}{16-1} = \dfrac{R}{15}$

## 19> ESCAPE VELOCITY FROM HEIGHT "h" FROM SURFACE:-

$$V = \sqrt{\frac{2GM}{R+h}}$$

For earth to become a black hole $\quad R = 1.48$ mm

If object is projected with speed $V_g$ then,

$V_g > V_e \longrightarrow$ ME = + $V_e$, Hyperbolic

$V_g = V_e \longrightarrow$ parabolic, ME = 0

$V_o < V_g < V_e \longrightarrow$ Elliptical close, ME = $- V_e$

$V = V_o \longrightarrow$ Circular TE = − ve

$V < V_o \longrightarrow$ Elliptical TE = − ve

$V <<<<< V_o \longrightarrow$ Projectile open path

## 20> ORBITAL VELOCITY:-

$$V_o = \sqrt{\frac{GM}{R+h}} \qquad V_e = \sqrt{2}\,V_o$$

## 21> SATELLITE NEAR SURFACE OF EARTH:-

$$V_o = \sqrt{gR} = \sqrt{\frac{4}{3}\pi G \rho R^2}$$

# $V_{satellite}$ inc. by $\sqrt{2}$ times or 41.4 % then it'll escape to $\infty$

Q. The radii of the circular orbits of two satellites A and B of the earth are 4R and R, respectively. If the speed of satellite A is 3v, then the speed of satellite B will be

Ans. $V \propto \frac{1}{\sqrt{R}}$

$$\frac{3V}{V'} = \sqrt{\frac{R}{4R}}$$

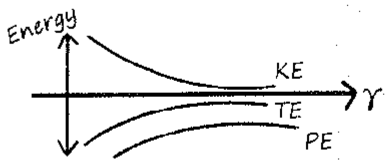
$$V' = 3V \times 2 = 6V$$

## 22> ENERGIES OF SATELLITE:-

$$U = -\frac{GMm}{R+h} \qquad K = \frac{GMm}{2(R+h)} \qquad TE = -\frac{GMm}{2(R+h)}$$

$$BE = \frac{GMm}{2r}$$

$$P : K : T : BE := -1 : \frac{+1}{2} : \frac{-1}{2} : \frac{+1}{2}$$




- B.E. of a particle placed on earth:-

$$BE = \frac{GMm}{R}$$

- $[BE]_{Body} = 2\,[BE]_{Satellite}$

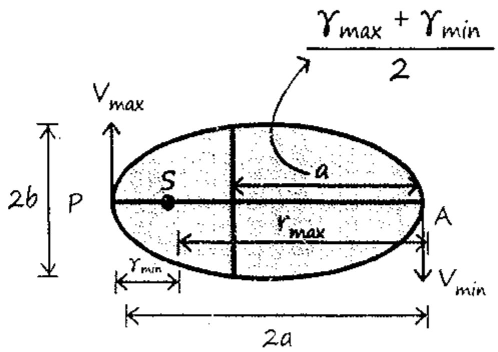
## 23> TIME PERIOD OF SATELLITE :- $F \propto \gamma^n \quad T.P. \propto \gamma^{(1-n)/2}$

$$T = \frac{2\pi\gamma}{V_o} = 2\pi\sqrt{\frac{(R_e+h)^3}{GM_e}}$$ independent of mass of satellite

$$T^2 \propto \gamma^3 \qquad T = \sqrt{k}\,\gamma^{3/2}$$

$$K = \frac{4\pi^2}{GM}$$

## 24> KEPLER'S LAW :-




$$\frac{V_{max}}{V_{min}} = \frac{V_{max}}{V_{min}} = \frac{a(1+e)}{a(1-e)}$$

$$\gamma_{max} + \gamma_{min} = 2a$$

$$\frac{dA}{dt} = const^n \qquad \frac{L}{2m} = const^n$$

$$\gamma V = const^n \qquad \gamma_{mean} = \frac{\gamma_{max} + \gamma_{min}}{2}$$

$$TE = -\frac{GMm}{(\gamma_{max} + \gamma_{min})} \qquad T^2 \propto a^3 \propto \left[\frac{\gamma_{max} + \gamma_{min}}{2}\right]^3$$

Semi major axis

## 25> GEOSTATIONARY SATELLITE:-

Direction of rotation, time period ⟶ same as earth.

$h = 6R$ from surface

$W \longrightarrow E$

$h = 36000$ km

$$V_o = \sqrt{\frac{GM}{7R}}$$

$T = 24hr$

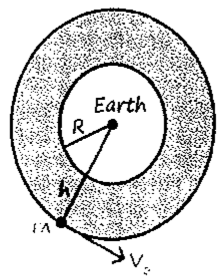
$V = 3.1$ km/hr

Satellite Cover's 1/3rd Area of Earth

## 26> POLAR SATELLITE :-

$T = 100$ min Move about Pole.

## 27> SATELLITE ON CIRCULAR PATH :-




$|\vec{\gamma}| = const^n$

All energies $= const^n$

$L = const^n \qquad \vec{\tau} = 0$

$\vec{P}$ = variable

$\vec{v}$ = variable (due to $dir^n$)

$\omega = 0 \qquad V = const^n$

$$\tau = \frac{dL}{dt} \qquad F = \frac{dP}{dt}$$

28> ESCAPE ENERGY $= + \dfrac{GMm}{R_e}$

29> $V_g = KV_e$

$h_{max}$ from Surface of Earth $= \dfrac{RK^2}{1-K^2}$

30>

If $F \propto \dfrac{1}{r^n}$

$\therefore V_o \propto \dfrac{1}{r^{n-1/2}}$

$\therefore$ T.P. $\propto r^{n+1/2}$

Q. A satellite of mass m moving around planet of mass M in circular path radius $r_1$ then energy required to shift his path to radius $r_2$.

Ans. $E_{given} = E_{total\ final} - E_{total\ initial}$

$$= -\frac{GMm}{2r_2} - \left(-\frac{GMm}{2r_1}\right)$$

$$= \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

**MR***

"Question krte time Galat option ko dhoondo, sahi apne aap mil jayenge."

# 14 Elasticity

1> Rigid Body

Quartz is near approach to perfect elastic body.

$\gamma = \infty \qquad e = 0.$

2> Stress:- $\dfrac{F_{restoring}}{Area}$

# Tensor.

Stress → Normal, Volume, Shear

Normal → Longitudinal ($\Delta L$)

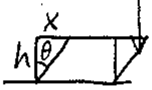
Volume ($\Delta V$)

Shear → Shear stress

$L.S. = \dfrac{F_R}{A} \qquad V.S. = \dfrac{F_R}{A} \qquad S.S. = \dfrac{F_t}{A}$

* Volume Stress = $\Delta P$

3> Strain:- → Longitudinal, Volumetric, Shear




| Longitudinal | Volumetric | Shear |
|---|---|---|
| $\dfrac{\Delta L}{L}$ | $\dfrac{\Delta V}{V}$ | $\theta = \dfrac{x}{h}$ |

Strain is unitless and dimensionless

4> Hooke's Laws:- Stress $\alpha$ Strain

# Within elastic limit.

Material Prop. Modulus of Elasticity (E) = $\dfrac{Stress}{Strain}$

→ Young's modulus, Bulk, Shear/rigidity

Slope of stress strain graph is Young's modulus.

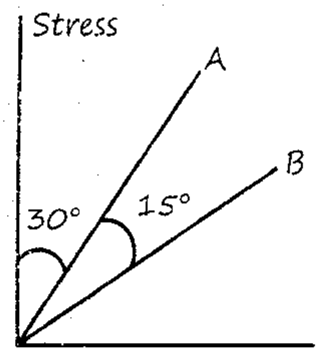
5> Young's Modulus:-

$$\gamma = \frac{\text{Longitudinal Stress}}{\text{Longitudinal Strain}}$$

$$\boxed{\gamma = \frac{FL}{A\Delta L}}$$

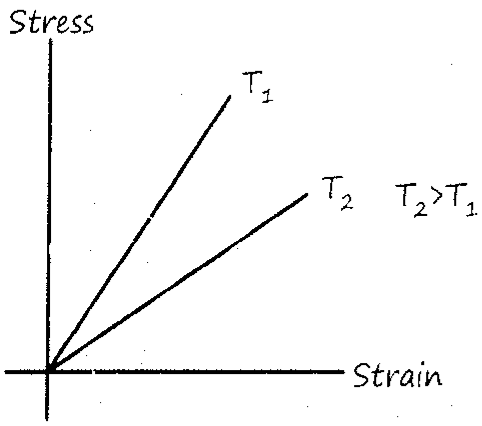
Q. Find ratio of young's modulus

o Slope of stress-strain graph is young's modules.




$$\gamma = \frac{\gamma_A}{\gamma_B} = \frac{(Slope)_A}{(Slope)_B} = \frac{\tan 60^\circ}{\tan 45^\circ} = \sqrt{3} : 1$$

Young Modulus $\alpha \dfrac{1}{\text{Temprature}}$



6> Bulk's Modulus:-

$$\beta = \frac{\text{V. Stress}}{\text{V. Straio}}$$

$$\boxed{\beta = -V\left[\frac{\Delta P}{\Delta V}\right]}$$

Isothermal (T=const) $\beta = P$

Adiabatic (Q=const) $\beta = P\gamma$

Isobaric (P=const) $\beta = 0$

Isochoric (V=const) $\beta = \infty$

$$\gamma = \frac{C_P}{C_V}$$

Compressibility-

$$C = \frac{1}{\beta} = \frac{1}{-V}\frac{\Delta V}{\Delta P}$$

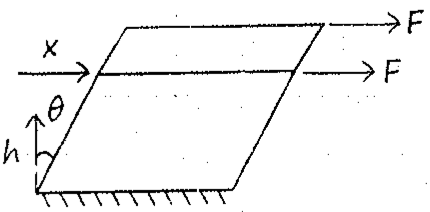
# Density of Compressed Liq. ↑

$$\rho' = \rho\left[1 + \frac{\Delta P}{K}\right] = \rho\,[1 + C\Delta P]$$

7> Shear Modulus:-

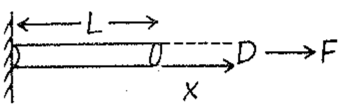
$$\theta = \frac{x}{h}$$

$$\frac{F}{A} = \eta\theta$$




$\eta$ = Coefficient of rigidity

8> Potential Energy Stored in Wire:-




$WD = \frac{1}{2}Fx$

x = elongation

work done in elongation x

w = Fx

o Energy stored $= \frac{1}{2}Fx = U$

$$U = \frac{1}{2}\frac{A\gamma x^2}{L}$$

using Hooks chacha ka law

9> Potential Energy Stored in wire unit volume:-

$$\frac{U}{V} = \frac{1}{2}\gamma\,(\text{Strain})^2 = \frac{1}{2}\frac{(\text{Stress})^2}{\gamma}$$

$$= \frac{1}{2}\text{ Stress} \times \text{Strain.}$$

Breaking Stress (P) :- **Breaking stress does not depends upon area.**

# Material Property.

$$P = \frac{\text{Breaking Force}}{\text{Area}}$$

o For a Particular Material:- **B.Force $\alpha$ A**

Q. If the longitudinal strain of a iron rod is 0.01 and its poisson's ratio of 0.2, then the lateral strain will be

Since,

$$\text{Poisson's ration} = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

⇒ Lateral strain = (Poisson's ratio) × (Longitudinal strain)

⇒ Lateral strain = (0.2) (0.01) = 0.002

Q. A wire elongates by L mm when a load W is hanged from it. If the wire goes over a pulley and two weighs W each is hung at the two ends, the elongation of the wire will be (in mm).

Sol. Due to the arrangement of the pulley, the length of wire is L/2 on each side and so the elongation will be L/2. For both sides, elongation = L

**Pcisson's Ratio**

$$\sigma = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

$$\boxed{\sigma = -\frac{\Delta D/D}{\Delta L/L}}$$

Volume strain = 2 × lateral strain + longitudinal strain

# $\sigma = 0.5$ V = constant.

Q. If the elastic potential energy density store in a material is $3 \times 10^4 J/m^3$ due to the application of longitudinal stress of $1 \times 10^{11} N/m^2$ then, the strain developed in it would be

$$U_e = \times \frac{1}{2} \text{stress} \times \text{strain}$$

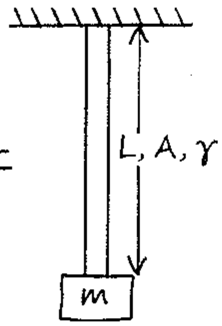
$$\text{strain} = \frac{2U_e}{\text{stress}}$$

$$\text{strain} = \frac{2 \times 3 \times 10^4}{10^{11}} = 6 \times 10^{-7}$$

**Special Case:-**

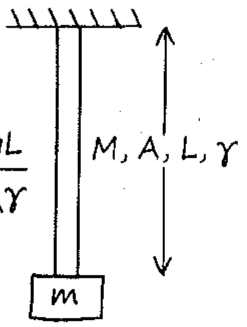
1> Elongation in massless rod due to attatched block of mass m:

$$\Delta l = \frac{mgL}{AY}$$

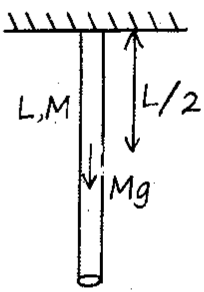



2> Rod have mass M:

$$\text{elongation} = \left(m + \frac{M}{2}\right)\frac{gL}{AY}$$




3> Elongation due to own weight:-




$$\Delta L = \frac{Mg(L/2)}{AY}$$

$$\Delta L = \frac{1}{2}\frac{\rho L^2 g}{Y}$$

4> $l_{max}$ of a wire which can hang under his own weight:-

$$\boxed{l_{max} = \frac{P}{\rho g}}$$

5> L. Strain = $\alpha \Delta T$

L. Stress = $\gamma . \alpha \Delta T$

Q. Two wires are made of the same material and have the same volume. However, wire 1 has a cross-sectional area A and wire 2 has cross-sectional area 3A. If the length of the wire 1 increases by $\Delta x$ on applying force F, how much force is needed to stretch wire 2 by the same amount?

For the same material, Young's modulus is the same and it is given that the volume is the same and the area of the cross-section for the wire $L_1$ is and that of $L_2$ is 3A

$V = V_1 = V_2$

$V = A \times L_1 = 3A \times L_2 \rightarrow L_2 = L_1/3$

$Y = (F/A)/(\Delta L/L)$

$F_1 = YA(\Delta L_1/L_1)$

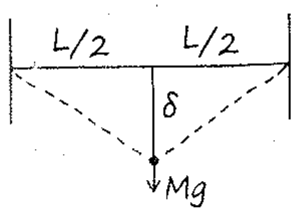
$F_2 = Y3A(\Delta L_2/L_2)$

Given $\Delta L_1 = \Delta L_2 = \Delta x$ (for the same extension)

$F_2 = Y3A(\Delta x/(L_1/3)) = 9.(YA\Delta X/L_1)$
$= 9F_1 = 9F$

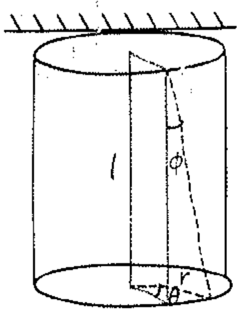
6. Fractional Change in Radius of Sphere:-

$\beta$ = Bulk Modulus. $\frac{dR}{R} = \frac{F}{3A\beta} = \frac{Mg}{3A\beta}$

7> 


$$\delta = \frac{L}{2}\left[\frac{Mg}{AY}\right]^{1/3}$$

8> Relation between shear angle and angle of twist.




$\theta$ = Twist Angle

$\phi$ = Shear Angle

$$PE = \frac{1}{2}C\theta^2 = \frac{1}{2}Ke^2$$

$$r\theta = l\phi$$

9> Relation among Y, K, $\eta$ & $\sigma$ :-

$$Y = \frac{9\eta K}{3K + \eta} \qquad Y = 2\eta\,[1 + \sigma]$$

$$Y = 3K\,[1 - 2\sigma]$$

$$Y = \frac{3K - 2\eta}{6K + 2\eta}$$

10> If length of wire $l_1$ at tension $T_1$ and $l_2$ at tension $T_2$, then find original length

$T_1 \rightarrow l_1 \Rightarrow T_2 \rightarrow l_2$

$$L = \frac{T_1 l_2 - T_2 l_1}{T_1 - T_2} = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$$

Q. When a block of mass M is suspended by a long wire of length L, the length of the wire becomes (L+l). The elastic potential energy stored in the extended wire is:

Using Formula

$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$$

$$\text{stress} = \frac{\text{Force}}{\text{Area}} = \frac{Mg}{A}$$

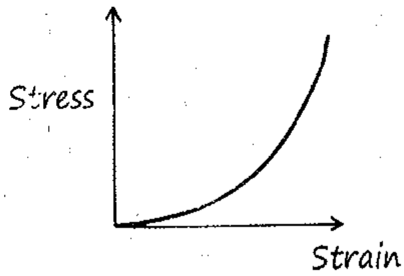
strain = $\Delta L/L = l/L$

volume = area × length = $A \times L$

Hence, $U = \dfrac{1}{2} \times \dfrac{Mg}{A} \times \dfrac{l}{L} \times A \times L$

$$U = \frac{1}{2}Mgl$$

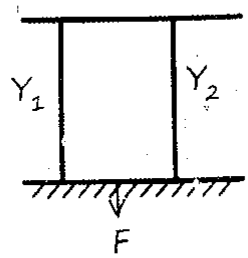
Stress-Strain graph for Elastomer:- e.g., Tissue of Aorta,




- Parallel & Series Combination of Young's Modulus:-

1> Parallel Combination:-

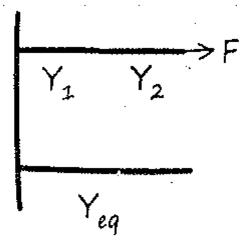
In both wire have same elongation but different stress




$$Y_{eq} = \frac{Y_1 + Y_2}{2}$$

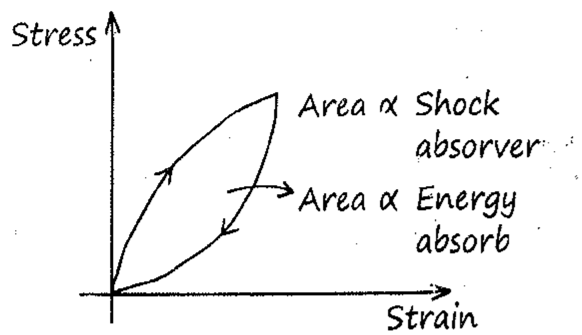
2> Series Combination:-

In both wire have same stress but different elongation.

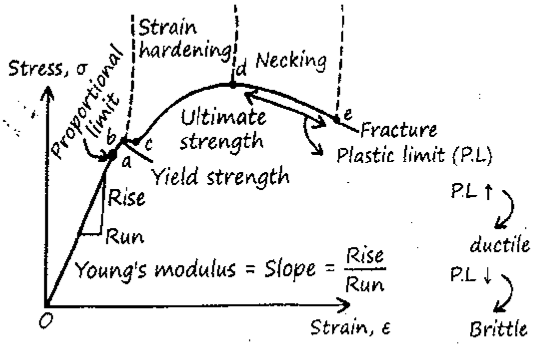



$$Y_{eq} = \frac{2Y_1Y_2}{Y_1 + Y_2}$$

- NOTE: On removing deforming Force Solid regain its shape by this graph.



NOTE:-




- O–a:- Hooke's law Valid. (Stress ∝ Strain)

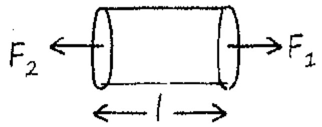
  $F_{elastic}$ Conservative.

  Completely regain its shape.
- O–b:- Conservative limit.

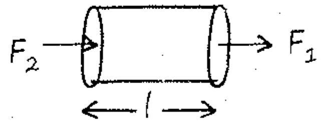
  Stress ≠ Strain

  Almost Regain its shape.

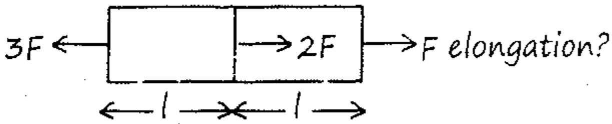
  $F_{elastic}$ Conservative.
- b–c:- $F_{elastic}$ is not Conservative
- Special Type Questions:-

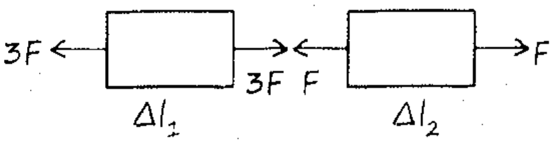



$$\Delta l = \frac{F_1+F_2}{2}\,\frac{l}{A\gamma}$$

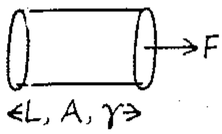



$$\Delta l = \frac{F_1-F_2}{2}\,\frac{l}{A\gamma}$$




$$\Delta l_1 = \Delta l_1 + \Delta l_2 = \frac{3Fl}{A\gamma} + \frac{Fl}{A\gamma} = \frac{4Fl}{A\gamma}$$




$$\Delta l\ (\text{elongation}) = \frac{Fl}{2A\gamma}$$

**MR***

"Push Yourself, because No One Else is going to do it for you."

**FLUID:**-That can flow. ex-liquid and gas.

Static Fluid:-

- Relative density:-

$$R.D. = \frac{\rho_{obj.}}{\rho_{water}} = \text{Unitless.}$$

$\rho_w = 1\ gm/cm^3$

$\rho_{oil} = 0.8\ gm/cm^3$

$\rho_{Hg} = 13.6\ gm/cm^3$

$\rho_{milk} = 1.04\ gm/cm^3$

- Density of Mixture:-

$$\rho_{mix} = \frac{M_{mix}}{V_{Total}}$$

(a) M = same  ρ = diff.

$$\rho_{mix} = \frac{2m}{\frac{m}{\rho_1} + \frac{m}{\rho_2}} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$$

(b) V = same  ρ = diff.

$$\rho_{mix} = \frac{\rho_1 V + \rho_2 V}{2V} = \frac{\rho_1 + \rho_2}{2}$$

- Pressure (P):-

$$P = \frac{F}{A}\ \text{Scalar, } N/m^2, [ML^{-1}T^{-2}]$$

$1\ atm = 1.05 \times 10^5\ N/m^2$

*Variation of "P" with depth:-

$\Delta P = \rho g h$

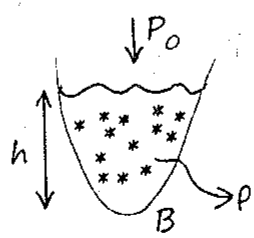
ρ = density of liquid.

h = depth.

ΔP does not depend upon amount of liquid, shape of container.

- Pascal's Law:-

Static liq, "P" on horizontal level must be same.




$^*P_B = P_o + \rho g h.^*$

- Relation between Absolute & Gauge Pressure:-

$P_{absoluble} = P_{net}$

$P_{gauge} = P_{\text{due to liq. only.}}$

$$P_{absolute} = P_o + P_{gauge}$$

*Moving Container "P" Calculation:-

(Be "$g_{eff}$" lelena hai!)

1. Lift up:- (with "a")

   $P_{net} = \rho(g + a)H$

2. Lift down:- (with "a")

   $P_{net} = \rho(g - a)H$

3. Free Fall:- (a = g)

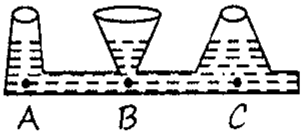
   $P_{net} = 0$

*Note:-

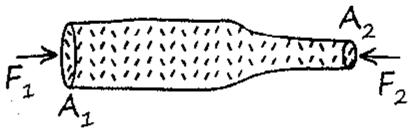
1>


Pascal चाचा :-
Static fluid me pressure balance kerte hai

$$\frac{F_1}{\pi r_1^2} = \frac{F_2}{\pi r_2^2}$$




$$P_A = P_B = P_C$$




$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

When air bubble move from bottom to surface of lake then $p_1V_1 = p_2V_2$.

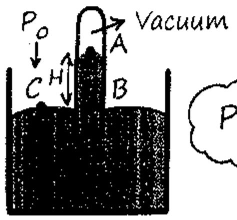
2> MR**

Agar Koi Cheez linearly vary Krti hai toh uska Avg:-

$$X_{avg} = \frac{X_i + X_f}{2}$$

## Measurement of ATM. Pressure:-

1. Barometer:- "Torricilli"




$$P_B = P_o = \rho_{Hg}\, gH$$

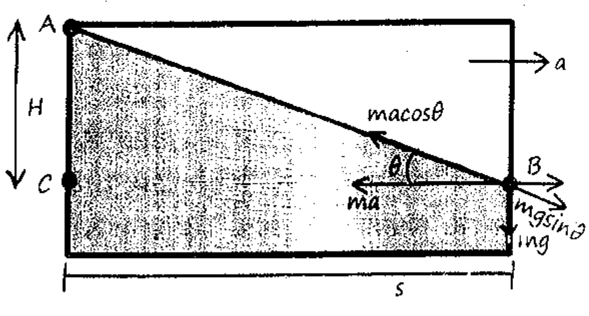
H = 76 cm of Hg = 1 atm

The Garib Ramlal Exp:-

Height of water column in Barometer:-

$$H = 10.1\ m$$

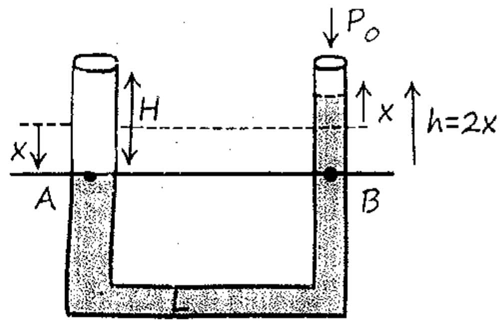
## Horizontal Accelerating Lift:-




$$\tan\theta = \frac{a}{g} = \frac{H}{l} \qquad \sin\theta = \frac{a}{\sqrt{a^2+g^2}}$$

## U-Tube:-

Initialy one liquid is on same horizontal level, other liquid is put in left arm of container then $1^{st}$ liquid moves x down from initial level in left arm of U-tube.




$$P_A = P_B \qquad \rho_w gH = \rho_L g(2x)$$

ek aaise line select Karo jiske about "P" same hona chahiye.

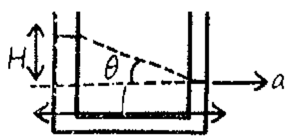
## Important Case:-

1> When U-tube is given horizontal accl$^n$

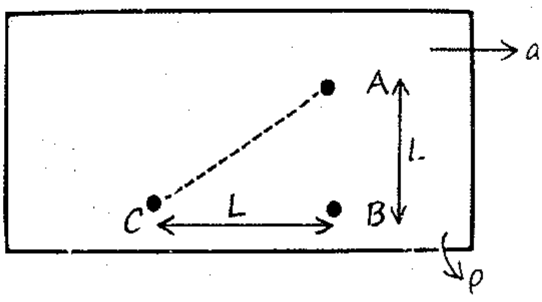
(a) rise is liq. column:-

$$\frac{a}{g} = \frac{H}{l} = \tan\theta$$

$$H = al/g$$



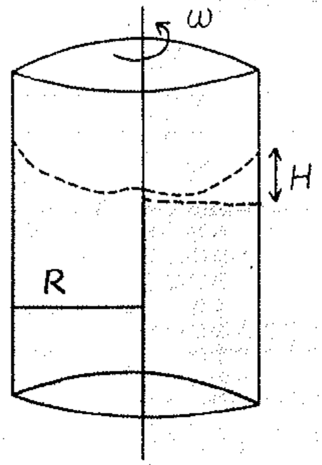

2>



$$P_C - P_A = \rho L(a+g)$$

$$P_B - P_A = \rho g L \qquad P_C - P_B = \rho a L$$

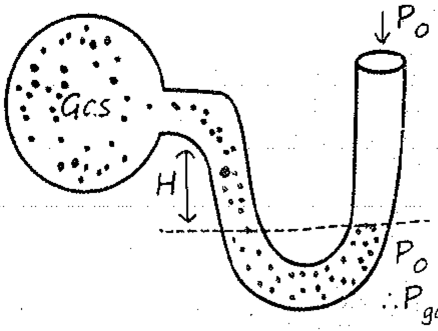
3> A vessel is rotated about vertical axis. Find rise in water "H":-




$$H = \frac{R^2 w^2}{2g}$$

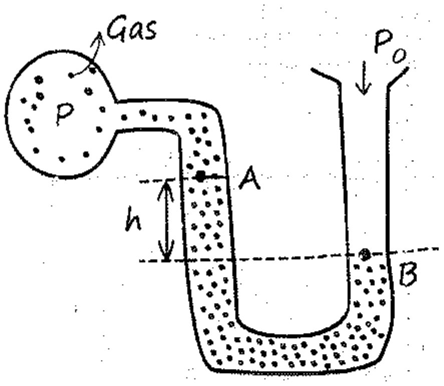
## Open-Tube Manometer

I>




$P_0 = \rho g H + P_{gas}$
$\therefore P_{gas} = P_0 - \rho g H$

II>




$$P_{gas} + \rho g h = P_B$$

## Archimedes Principle:-

$F_B$ (Buoyant Force) = Weight of displaced liquid.

→ depends on density of liquid, volume of solid submerged in liquid.

* does not depends on density, location of object inside liquid.

Volume of displaced liq. = Volume of Solid in liq.

$$F_B = mg = \underset{\text{liquid}}{\rho}\, V_{in}\, g$$ } Reason is "P" difference

* Apparent Weight:-

$\sigma$ = object $\qquad \rho$ = Liquid

| $\sigma > \rho$ | $\sigma = \rho$ | $\sigma < \rho$ |
|---|---|---|
| Sink! | Submerge & Float. | Float on Surface |
| $N = mg_{eff}\left[1 - \frac{\rho}{\sigma}\right]$ | $N = 0$ | $N = 0$ |
| $N = mg_{eff}\left[1 - \frac{\text{Chota}}{\text{bada}}\right]$ | Object Remains where it's Placed! | $\frac{V_{in}}{V_T} = \frac{\sigma}{\rho}$ |
| $N = mg_{eff}\left[1 - \frac{1}{RD}\right]$ | | $\frac{V_{in}}{V_T} = \frac{\text{Chota}}{\text{bada}}$ |

*Object of density "σ" is released then find accl$^n$ of object inside liquid:-

$(\sigma > \rho)$

$$a = g\left[1 - \frac{\rho}{\sigma}\right]$$

*A ball of density "D" is immersed in liquid of density "d" to a depth "h" below the surface of liquid & then released. Upto what height will be ball jump out of the liquid:-

$$\boxed{H = h\left[\frac{d}{D} - 1\right]}$$

*A container is at rest then inside volume is $V_{in}$ and when accl$^n$ up with $a_o$ the inside volume is $V_{in}'$ of an object then:-

$$\boxed{V_{in} = V_{in}'}$$

Container Ke accl$^n$ se up/down volume emerged in liquid depend hi nai Krta.

## Buoyant Force with Cavity:-

$V_{matter} = V_T - V_C$

*$N = \sigma[V_T - V_C]g - \rho V_T g$

To Find $V_{cavity}$ :-

$$\left.\begin{aligned} 1>\ & N = mg - F_B \\ & F_B = \rho_L V_T g \\ & V_{matter} = \frac{m}{\sigma} \\ & \boxed{V_C = V_T - V_M} \end{aligned}\right\} \text{Problem Solving Strategy}$$

*** Rise / Fall of liq:-**

When ICE placed on liquid will Melt.

$\rho_L > \rho_O$ = Rise

$\rho_L < \rho_O$ = Fall

$\rho_L = \rho_W$ = Same.

$\rho_O$ = density of water.

$\rho_L$ = Surr. Liquid.

## Fluid Dynamics-

Ideal Fluid:-

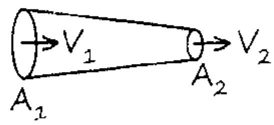
- Non-viscous
- Incompressible
- $\rho = const^n$
- Streamline & Irrational Flow.

Equation of Continuity:-

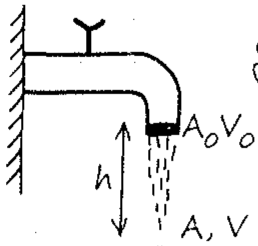
(Conserv$^n$ of Mass)

*Area × Velocity = Const$^n$

Rate of Volume Flow remains Const$^n$.




$$\boxed{A_1V_1 = A_2V_2}$$

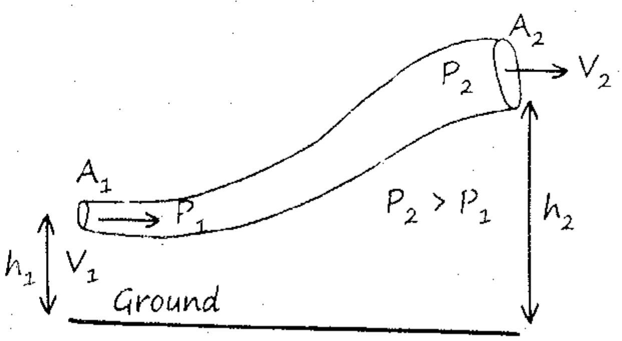



$$\boxed{V^2 = 2gh\left[\frac{A_o^2}{A_o^2 - A^2}\right]}$$

$$\boxed{\text{Rate of Volume Flow} = AA_o\sqrt{\frac{2gh}{A_o^2 - A^2}}}$$

Bernouli's Eqn:-

(Conserv$^n$ of Energy)




PE/V   KE/V

$$\boxed{P + \rho gh + \frac{1}{2}\rho V^2 = Const^n}$$

Divide by $\rho g$.

$$\boxed{\underbrace{\frac{P}{\rho g}}_{\text{Pressure head.}} + \underbrace{h}_{\text{Gravit}^n\text{ head.}} + \underbrace{\frac{V^2}{2g}}_{\text{Velocity head.}} = Const^n}$$

## Ramlal House:-

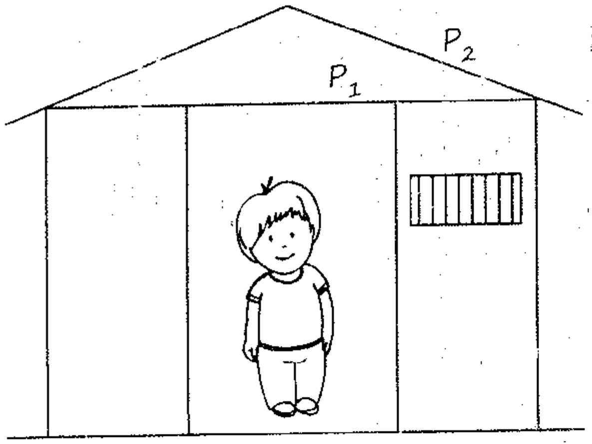
Wind is flowing outside then pressure inside $P_1$ and outside house $P_2$

$P_2 < P_1$

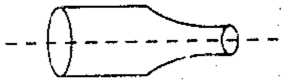
Hence upthrust force will act on roof

$F = (P_1 - P_2) \times \text{Area}$

$$\boxed{F = \frac{1}{2}\rho V^2 A}$$



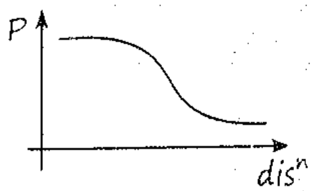

#



$P + \frac{1}{2}\rho V^2 = \text{cost}^n$

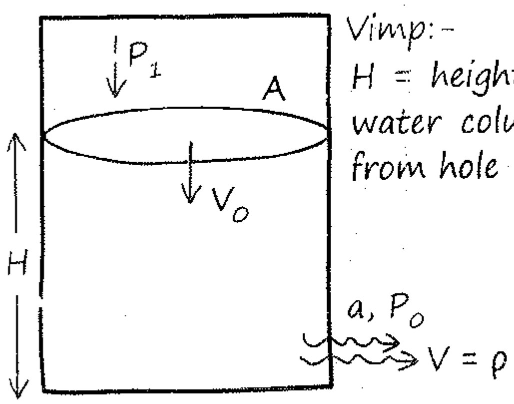
$AV = \text{const}^n$

Potential energy same on horizontal level




## Velocity of Efflux:-

Closed!




Vimp:-
H = height of water column from hole

$$V = \sqrt{\frac{2(P_1 - P_0)}{\rho} + 2gH}$$

$P_1 = P_{Top}$ $\qquad$ ρ = liquid

$P_0$ = atm P. $\qquad$ $AV_0 = av$

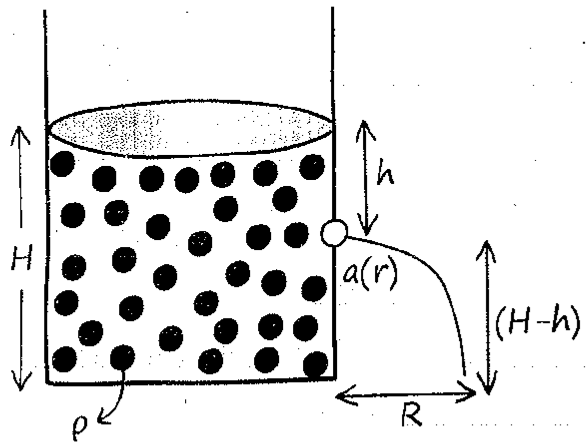
Open Container (Torricilli चाचा )

$P_1 = P_0$

$V = \sqrt{2gh}$ $\qquad$ h:- height of liq. level from hole.

→ Velocity of efflux does not depends on density of liquid.

## Range, Heights, Time of Flight:-




$$\boxed{\text{Rate of Volume Flow} = a\sqrt{2gh}}$$

$$T_f = \sqrt{\frac{2(H-h)}{g}}$$ Take $H_{vertical}$

$$R = U_x T_f = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}$$

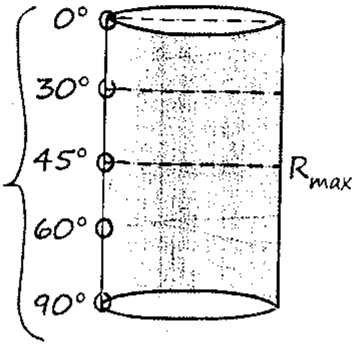
$$R = \sqrt{4h(H-h)}$$

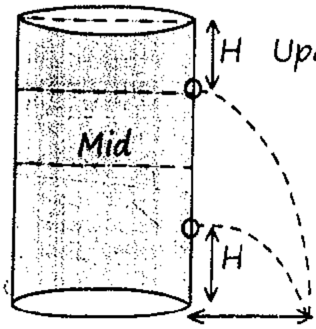
* Height at which Range is maximum & its value:-

$h = \frac{H}{2}$ :- R = max?

$R_{max} = H$

Just like Motion in 1-D




Uparse = Nichese
H H
R = Same.

## Force on Container:-

$F = \rho a V^2$

* $F = 2\rho agH$ *

$\rho$ = density of liq.

H = Height of liquid column from Hole.

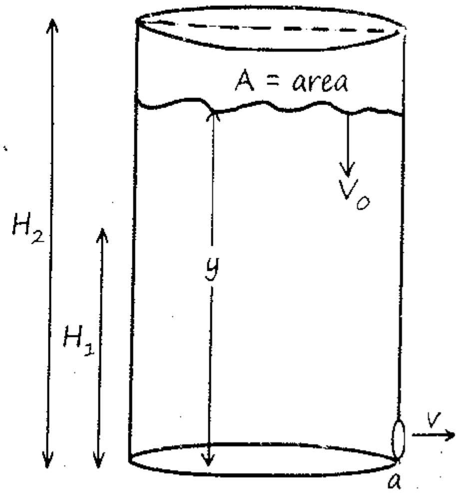
a = Area of orifice.

For massless container, "$\mu_{min}$" to keep container at rest:-

$\mu mg = 2\rho agh$ a = hole
A = Container Area.

$$\mu_{min} = \frac{2a}{A}$$

## Time Taken to Move Liquid from Height $H_2$ to $H_1$:-



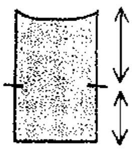

$$V_o = \frac{a}{A}\sqrt{2gy}$$

$$t = \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{H_2} - \sqrt{H_1}\right)$$

## Time to Fall down :-

- To bottom :- $t = \frac{A}{a}\sqrt{\frac{2H}{g}}$
- Ratio from $h \rightarrow \frac{h}{2}$ & $\frac{h}{2} \rightarrow$ bottom

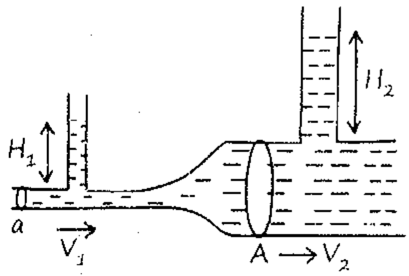
$$\frac{t_1}{t_2} = \frac{\sqrt{2}-1}{1}$$



$T - T/\sqrt{2}$
$T/\sqrt{2}$
1-D का उल्टा !

## Venturimenter:-

- Measure rate of Volume Flow
- Based on Bernoulli's Principle.

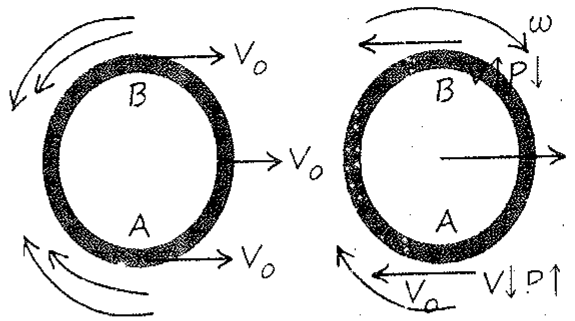



$$V_2^2 = \frac{2g(H_2 - H_1)a^2}{A^2 - a^2}$$

* $aV_1 = AV_2 = V$ = Rate of Volume Flow:

$$P_1 + \frac{1}{2}\rho V_1^2 + 0 = P_2 + \frac{1}{2}\rho V_2^2 + 0$$

## Dynamic Lift & Magnus Effect:-




Football without Spining

$V_A = V_B = V_0 + V$

Football Spining about vertical axis

$V_A = V + V_0 - R\omega$

$V_B = V + V_0 + R\omega$

## Viscocity:-

[Cohesive Force between Liq. molecules]

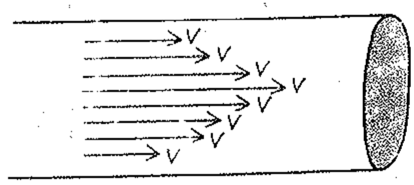
Force between two liq:-

$$f = \eta A \frac{\Delta V}{l}$$

$\eta$ = coeff. of viscocity.

$$\boxed{\eta = \frac{\text{Shear Stress}}{\text{Velocity gradient}} = \frac{\text{Shear Stress}}{\text{Strain Rate}}}$$

"1 Poissulle = 10 Poise."




## Stoke's Law:-

Only For Sphere.

Jab Force between Solid-liq:-

$$F = 6\pi\eta r V$$

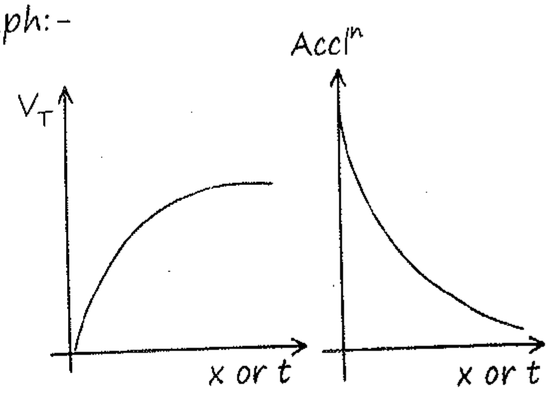
- Terminal Velocity:-

  $mg = F_B + f_r$

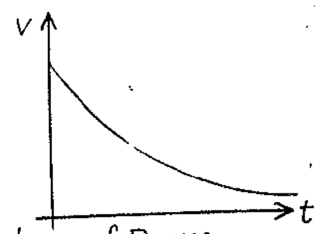
  $$V_T = \frac{2}{9}\frac{r^2}{\eta}(\sigma - \rho)g.$$

  r = object, σ = object, ρ = liquid.

Graph:-




- $V_T \propto r^2$ :- Bigger rain drops velocity is greater!
- Ball is thrown down with velocity greater than terminal velocity in viscous liq. then variation of "v" v/s "t"? (NEET-2022)




- Coalesce of Drops:-

  *$V_T' = n^{2/3} V_T$ * $R = n^{1/3} r$
- Temperature dependence of η:-

  $$\eta_{liq} \propto \frac{1}{T} \qquad \eta_{gas} \propto T$$

Poiseuille Equn:-

↳ For Viscous liq.

↳ Bernoulli is not valid

Woh sirf ideal liq keliye.

Fluid Current:-

$$Q = \frac{\Delta P}{\frac{8\eta l}{\pi r^4}} = \frac{\Delta P}{R_{Fluid}} \qquad R_{fluid} \text{ of pipe} = \frac{8\eta l}{\pi r^4}$$

ΔP = Press difference

r = radius of pipe.

l = length of pipe.

η = coefficient of viscocity

**Series**

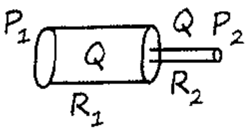
$Q$ = same

$\Delta P$ = Diff.

$$\boxed{Q = \frac{\Delta P}{R_{eq}}}$$

$R_{eq} = R_1 + R_2$

$\Delta P = P_1 - P_2$



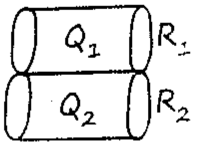

**Parallel**

$Q$ = Diff.

$\Delta P$ = same

$$Q = \frac{\Delta P}{R_{eq}}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$




## Reynolds Number:-

$$R_e = \frac{\text{Inertial Force}}{\text{Viscous Force}} = \frac{\rho V d}{\eta}$$

$d$ = diameter of pipe

$\rho$, $v$ = liquid.

Laminor :- $R_e < 1000$

Turbulent :- $R_e > 2000$

Unsteady :- $1000 < R_e < 2000$

## Surface Tension

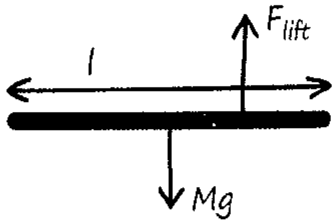
1> Rod :- $S = \dfrac{F}{2l}$

2> Disc :- $S = \dfrac{F}{2\pi r}$

3> Ring :- $S = \dfrac{F}{2\pi r + 2\pi R}$

4> Annular Disc :- ↑

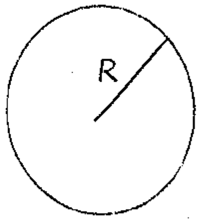
**Rod :-**




$$F_{lift} = F_{ST} + Mg$$
$$= 2Sl + Mg$$

## Surface Energy :-

* E = S.A.



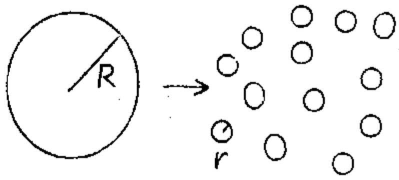

**Sphere.**

S.A = $4\pi R^2$

Vol. = $\frac{4}{3}\pi R^3$

Circum = $2\pi R$

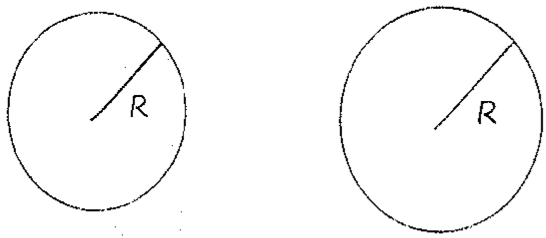
## Splitting of Drops into Droplets :-




$R = n^{1/3} r$

$\Delta A = 4\pi[nr^2 - R^2]$; $\Delta A = 4\pi R^2[n^{1/3} - 1]$

$$\Delta A = 4\pi R^2\left[\frac{R}{r} - 1\right]$$




$A = 4\pi R^2 \times 2$ $A = 4\pi R^2$

Film:- Take Area double.

## Energy Released when Droplets Combine to Form a Drop :-

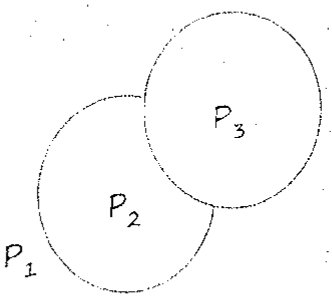
$$\Delta E = E\left(n - n^{2/3}\right) = S4\pi r^2\left(n - n^{2/3}\right)$$

$$\frac{E_f}{E_i} = \frac{1}{n^{1/3}}, \ \% E_{loss} = \left[\frac{1}{n^{1/3}} - 1\right] \times 100$$

$$\Delta E = n4\pi r^2 - 4\pi R^2$$

$$\Delta E = 3VT\left[\frac{1}{r} - \frac{1}{R}\right]$$ $V$ = Vol of larger drop

Pressure is always high in concave side




$P_3 > P_2 > P_1$

### Values of Excess Pressure :-

Drop

$$\Delta P = P_{in} - P_{out} = \frac{2S}{R}$$

$$P_{in} = P_{out} + \frac{2S}{R}$$

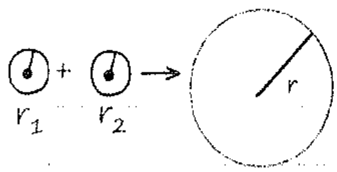
Bubble

$$\Delta P = P_{in} - P_{out} = \frac{4S}{R}$$

$$P_{in} = P_{out} + \frac{4S}{R}$$

### Radius of Coalesce :-

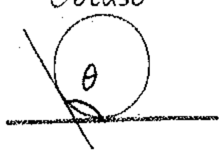
Two drops of radius $r_1$ & $r_2$ coalesce under isothermal condition.




$$\boxed{r = \sqrt{r_1^2 + r_2^2}}$$

### Angle of Contact :-

Obtuse




$\theta > 90°$

Cohesion > Adhes.

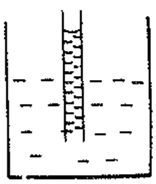
Acute


$\theta < 90°$

Cohe. < Adhes.

### Capillary Tube :-

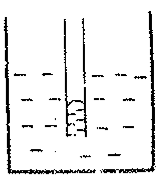
Rise



$\theta < 90°$

Concave Meniscus

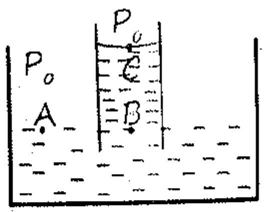
Fall



$\theta > 90°$

Convex Meniscus

#




"$P_A = P_B = P_O$ ...(1)

Due to excess pressure

$$P_O - \frac{2S}{R} = P_C \quad ...(2)$$

Due to pressure variation in liguid

$P_C + \rho g h = P_B = P_O$

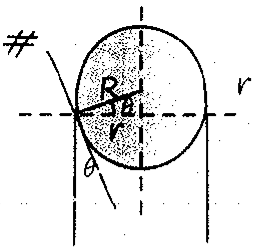
Putting value of $P_C$ from (2)

$$P_O - \frac{2S}{R} + \rho g h = P_O$$

$$\frac{2S}{R} = \rho g h \Rightarrow h = \frac{2S}{\rho g R}$$

$$h = \frac{2S\cos\theta}{\rho g r} \qquad r = \text{radius of tub.}$$

#




$r = R\cos\theta$

**Height of liq. rising in C.T.**

$$\boxed{h = \frac{2S\cos\theta}{\rho r g}}$$

"$g_{eff}$"

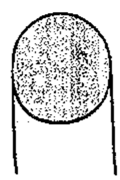
$$\boxed{M = \rho A h = \frac{A2S\cos\theta}{rg}}$$

"Mass of liq. in tube"

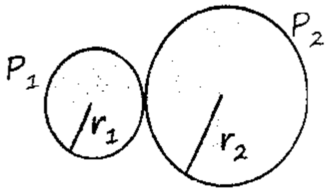
- Rise of liq. in tube of insufficient length :-

$$\boxed{r \propto \frac{1}{h}}$$

don't overflow



Radius of Interface :-




$$r = \frac{r_1 r_2}{r_1 - r_2}$$

- Height/Depth of liquid $\propto 1/r$
- Mass of liquid $\propto r$
- Potential energy of liquid $\propto r^0$
- If container with capillary in a freely falling lift, liquid rise upto complete length and does not overflow.

$$h \propto \cos\theta \propto \frac{1}{\theta}$$

- For two different liguid if h, S and r same then find relation b/w density and contact angle.

$$h = \frac{2S\cos\theta}{\rho g r}$$

$$\cos\theta \propto \rho$$

$$\theta \propto \frac{1}{\rho}$$

**MR***

"धीमे कदमों पर यूं मायूस ना हो,
पहाड़ कभी भाग कर नहीं चढ़े जाते!"

# 16 Thermal Properties of Matter

**Temperature:-** Measure of hotness and coldness.

- Two body A at $T_1$ and B at $T_2$ put in contact.

$$\text{(A)} \text{———} \text{(B)}$$

If $T_1 > T_2$ then
S-1 Some temp$^r$ will flows from A to B $\Rightarrow$ False
S-2 Some heat will flows from A to B $\Rightarrow$ True
S-3 Heat will increase in B $\Rightarrow$ False
S-4 Tempr of A will decrease $\Rightarrow$ True

- Heat can not be stored it can flows from Body A to B.

## Measurement of temperature :-

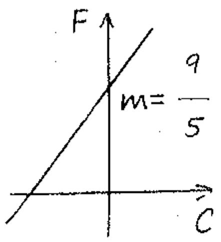
$$\frac{\text{Temp}^r - \text{L.F.P}}{\text{U.F.P} - \text{L.F.P}} = \text{Constant}$$

$$\frac{°C - 0}{100 - 0} = \frac{F - 32}{212 - 32} = \frac{K - 273}{373 - 273}$$

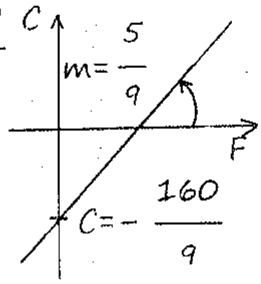
$$= \frac{MR - \text{L.F.P}}{\text{U.F.P} - \text{L.F.P}}$$

- Relation in Fahrenheit & Celcius :-

$$F = \frac{9C}{5} + 32$$



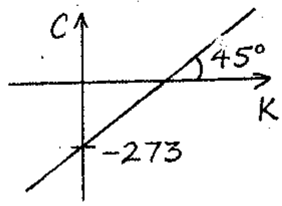

$$C = \frac{5F}{9} - \frac{160}{9}$$



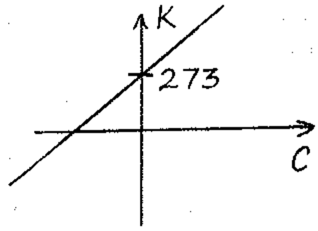

$$\boxed{\frac{\Delta C}{5} = \frac{\Delta F}{9}}$$

- Relation between Kelvin & Celcius :-

$$C = K - 273$$




$$K = C + 273$$




- Relation between °F & Kelvin :-

$$F - 32 = \frac{9}{5}(K - 273)$$

- Change in Temperature

$$\boxed{\Delta C = \Delta K = \frac{5}{9}\Delta F}$$

Q. The freezing point on MR* scale is 20° and boiling point 150°. A temperature of 60° thermometer will be read as.

Ans. Let x is resting at 60°C

$$\frac{x - 20}{150 - 20} = \frac{60 - 0}{100 - 0}$$

$$x = 90°C$$

## Construction of thermometer :-

$$\boxed{\frac{\text{Change in physical quantity}}{\text{per unit raise in temperature}} = \text{constant}}$$

1> Resistance Thermometer :-

$$t = \frac{R_t - R_o}{R_{100} - R_o} \times 100$$

2> Pressure thermometer :-

$$t = \frac{P_t - P_o}{P_{100} - P_o} \times 100$$

3> Volume Thermometer :-

$$t = \frac{V_t - V_0}{V_{100} - V_0} \times 100$$

4> Length Thermometer :-

$$t = \frac{L_t - L_0}{L_{100} - L_0}$$

Q. Length of rod at 20°C is 10m and at 80°C is 40 m then find temperature when its length is 30 m.

Ans. Change in length per unit raise temperature = const

$$\frac{40\text{ m} - 10\text{ m}}{80°C - 20°C} = \frac{\ell - 10\text{ m}}{t - 20}$$

$$\frac{1}{2} = \frac{30 - 10}{t - 20}$$

$t = 60°C$

## Thermal Expansion

1> Linear Exp. :-

$\Delta L = L_0 \alpha \Delta \theta$ ; $L_2 = L_0 (1 + \alpha \Delta \theta)$

2> Areal Exp. :-

$\Delta A = A_0 \beta \Delta \theta$ ; $A_2 = A_0 (1 + \beta \Delta \theta)$

3> Volume Exp. :-

$\Delta V = V_0 \gamma \Delta \theta$ ; $V_2 = V_0 (1 + \gamma \Delta \theta)$

Imp relations :-

Bulk modulus and thermal coefficient of volume expansion.

$$\Delta P = \beta \frac{\Delta V}{V} = \beta \gamma \Delta \theta$$

$\beta$ = Bulk Modulus of Elasticity

$$\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$$

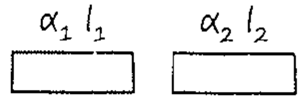
o For anisotropic crystal

$\gamma = \alpha_x + \alpha_y + \alpha_z$

For Isotropic $\alpha_x = \alpha_y = \alpha_z$

$\boxed{\gamma = 3\alpha}$

o For Two Rod




* If difference in length of these two rod independent upon each other then $l_1\alpha_1 = l_2\alpha_2$

Ramlal Ne socha winter me gold buy karunga, summer me sold karunga tab length Increase ho Jayga ☺

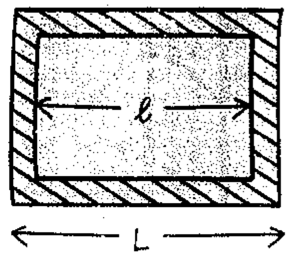
→ Ramalal ko koi benifit nahi hoga

## Cavity Problems :-

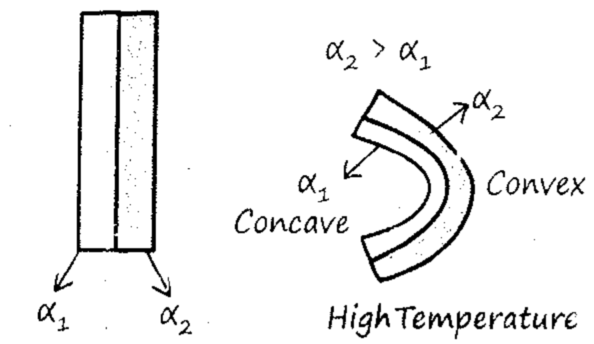
Photographic Enlargement.

T↑ → L↑ l↑

T↓ → L↓ l↓




## Bimetallic Strip :-




## Thermal Stress :-

$$T.S. = \frac{F}{A} = \alpha Y \Delta \theta$$

Y :- Young's Modulus

A rod of length $\ell$ and area A placed on smooth surface then due to to increase in temprature thermal stress is zero.

## Pendulum Clock :-

Note :-

Temperture↑ = Clock Slow = Time loss

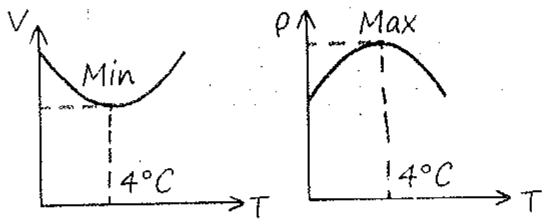
Temperture↓ = Clock Fast = Time Gain

Loss or gain in time, $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} = \frac{1}{2}\alpha\Delta\theta$

## Variation of density with temperature :-

$\rho_2 = \rho_0(1 - \gamma\Delta\theta)$

## Anomalous Behaviour of Water :-




## Apparent Coefficient of Volume Expansion of liquid :-

$\gamma_{app} = \gamma_{real} - \gamma_{container}$

$\gamma_l' = \gamma_l - \gamma_s$

\# If $\gamma_l = \gamma_s$ $\gamma_l' = 0$
Level Unchanged

\# If $\gamma_l > \gamma_s$ $\gamma_l' = +ve$
Liq. Overflows.

\# If $\gamma_l < \gamma_s$ $\gamma_l' = -ve$
Liq. go down

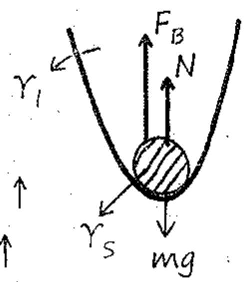
## Effect of Expansion on Apparent Weight in Liquid :-

$N = mg - \rho_l V_s g$

$\gamma_l = \gamma_s$ N = same

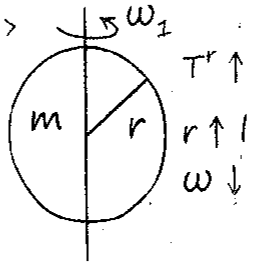
$\gamma_l < \gamma_s$ N = dec. $V_s \uparrow$

$\gamma_l > \gamma_s$ N = inc. $V_s \uparrow$




Imp Note :-

1>




$\omega_2 = \omega_1[1 - 2\alpha\Delta\theta]$

## Meter Scale

Only one type will be asked :-

True length of Rod = Reading taken + $\Delta L$

$\Delta L = L\alpha\Delta\theta$

# CALORIMETRY :-

## Heat Capacities :-

1> Specific Heat Capacity :- Heat required to raise the temperature by unit degree °C of unit mass

$Q = mS\Delta\theta$

raise $T^r$ of "m" Kg by $\Delta\theta$

Unit of S = J/kg kelvin

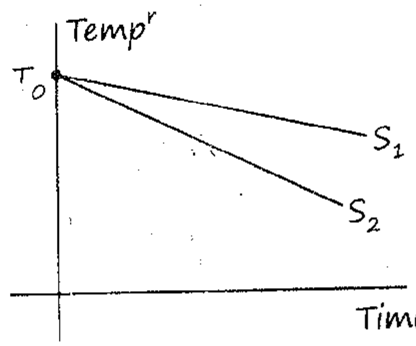
2> Heat Capacity (C) :- Heat required to raise temperature by unit of m mass.

$C = mS$ $(\Delta\theta = 1°C)$

raise $T^r$ of object by 1°C

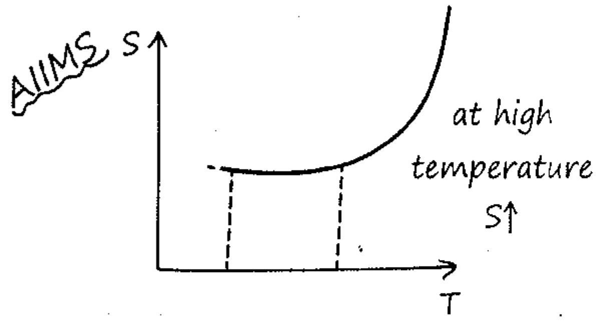
MR*

→ Joh Jaldi Garam hoga woh jaldi Thanda hoga. Uska specific heat capacity kam hai.




$S_2 < S_1$

AIIMS




3> Molar heat Capacity [C]

$Q = nC\Delta\theta$

raise $T^r$ of 1 mole $Sub^s$ by 1°C

**Note :-**

$S_w$ = 1 cal/gm°C = 4200 J/KgK

$S_{ice}$ = 0.5 cal/gm°C = 2100 J/KgK

$S_{steam}$ = 0.5 cal/gm°C = 2100 J/KgK

1 cal = 4.2 J

4> Latent Heat :-

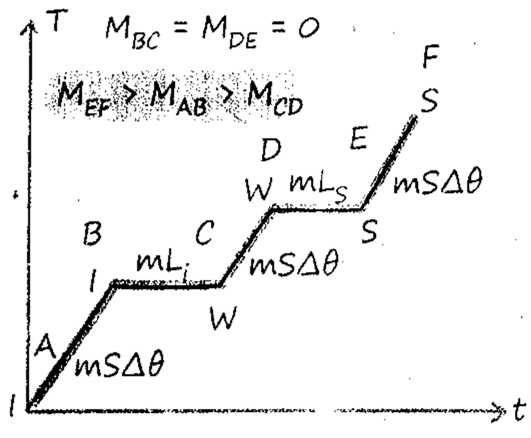
$Q = mL$

heat req. to change state.

$L_{ice}$ = 80 cal/gm

$L_{steam}$ = 540 cal/gm

## Temp.-time graph :-




MR* feel

- Water Can't exist below 0°C. Ice can't exist above 0°C. at 0°C both can exist. Mixture of (Ice + water) only Possible at 0°C. Zero se upar gya matlb sab pani ho gya, zero se Niche sab Ice.
- Steam + water mixture only exist at 100°C.

Q. 10 gm ice at 0°C mixed with 10 gm water at 40°C then mass of water in mixture?

Ans $Q_{ice} = mL = 800$

$Q_{water} = wt = 10 \times 40 = 400$

only 5 grm ice will melt

m (water) = 15 gm.

## Principle of Calorimetry :-

Heat Loss = Heat Gain

### Mixture :-

- Final Temperature of Mix. :-

$$\begin{pmatrix} m_1 \\ S_1 \\ T_1 \end{pmatrix} + \begin{pmatrix} m_2 \\ S_2 \\ T_2 \end{pmatrix} \longrightarrow \begin{pmatrix} M_1 + M_2 \\ T_{mix} \end{pmatrix}$$

Final Temperature of Mixture when two liquid of mass $m_1$ & $m_2$ of specific heat capacity $S_1$ and $S_2$ at temperature $T_1$ and $T_2$ mixed.

$$T_{mix} = \frac{m_1S_1T_1 + m_2S_2T_2}{m_1S_1 + m_2S_2}$$

Mixture of ice & water :-

MR*

- m gram Ice at 0°C mixed with W gram water at T°C
- Bring both ice & water at common $Temp^r$ and same phase.

Required :- Q = mL (m gram Ice melt into m gram water at 0°C)

Supply :- Q = WT = mSΔT (When W gram water cool down from T to 0°C)

1> $mL = WT$

$T_{mix} = 0°C$ $(amt)_{ice} = 0$

$(amt)_w = W + m$

2> $mL > WT$

$T_{mix} = 0°C$ $\quad (amt)_{ice} = m - m'$

$(amt)_W = W + m'$

$$\boxed{m' = \frac{WT}{L}}$$

Amt. of ice converted to water.

3> $mL < WT$

$$T = \frac{WT - mL}{m + W}$$ *Paani hi Rahega.

Mixture of ice & steam-

Required :- $Q = mL = 80\ m$

Supply :- $Q = WL + WS\Delta\theta = 640\ W$

1> $Q_{Supply} = Q_{Req.}$

$M_{ice} = 8W$ $\quad \boxed{T_{mix} = 0°C.}$

$$\boxed{\frac{M_{ice}}{W_{steam}} = \frac{8}{1}}$$

2> Ice $0°C \xrightarrow{80m}$ water $0°C \xrightarrow{100m}$ $\boxed{\text{Water } 100°C}$

Steam $100°C \xrightarrow{540W}$ $\boxed{\text{Water } 100°C}$

$180m = 540W$ $\quad \boxed{\frac{M_{ice}}{W} = \frac{3}{1}}$

The MR*

| | $M_{ice}$ | : | $W_{steam}$ |
|---|---|---|---|
| $T_{mix} \to 100°C$ | 1 | : | 1 |
| | 2 | : | 1 |
| | 3 | : | 1 |
| $T_{mix} \to 0°$ to $100°C$ | 4 | : | 1 |
| | 5 | : | 1 |
| | 6 | : | 1 |
| | 7 | : | 1 |
| $T_{mix} \to 0°C$ | 8 | : | 1 |
| | 9 | : | 1 |

Q. 200 grm ice at −20°C is mixed with 500 grm water it 20°C, then find temperature of mixture and amount of water, ice in mixture

Ans. MR* Dono ko kisi ek phase me same temperature par le ke aao.

$Q_1 = ms\Delta T + mL = 2000$ cal + 16000 cal heat given to ice to melt Δ°C.

$Q_2 = ms\Delta t = 10000$ cal.

Heat given by water when it fall from 20° to 0°C.

$Q_1 > Q_2$ hence complete ice will not melt, out of 10000 cal heat given by water 2000 cal used to increase temperature and 8000 cal use to melt ice

$m'L = 8000$

$m' = 100$ ice will melt

$T_{mix} = 0°$ because (ice + water) mixture

$M_{water} = 600$ grm

$M_{ice} = 100$ grm

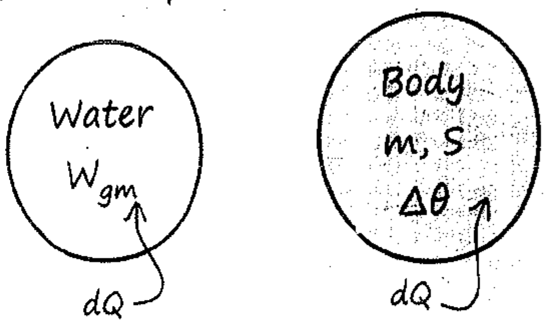
Q. 5 gm water at 30°C and 5 gm ice at −20°C mixed then temperature of mixture.

Ans. $T_{mix} = 0°$ requirement < supply

Water Equivalent :-

Woh liquid utnahi Heat lega ΔT Temp. Rise Keliye jitna $W_{gm}$ water le raha hh!

Toh Aapko uss Liquid Ko Na Assume Karke Water Ko uss liquid Ki tarah Treat Karna hai!




$\therefore\ mS\cancel{\Delta\theta} = W \times 1 \times \cancel{\Delta\theta}$

$$\boxed{W = mS}$$

eg :- Water equivalent = 55g at 40°C.

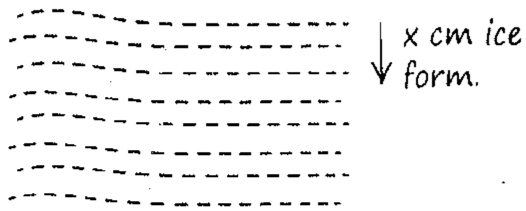
$WT = 55 \times 40 = 2200$ cal.

## V. Imp Question :-

*A bullet of mass "m" moving with "u" hits an ice block of "M" gm Kept on a frictionless floor & gets stuck in it. How much ice will melt if x% of the lost KE goes to ice ? (initial temp$^r$ of block & bullet = 0°C).

$$\boxed{\left[\frac{1}{2}\ \frac{mMu^2}{M+m}\right] x\% = mL}$$

- Ice Formation :-




Formation of ice :-

0 to x : x to 2x : 2x to 3x

t : 3t : 5t

0 to x : 0 to 2x : 0 to 3x

t : 4t : 9t

$$\boxed{t \propto x_2^2 - x_1^2}$$

"It's okay to feel up and down Its normal and natural, don't overthink, move forward and work hard"

# 17. Heat Transfer

## Conduction

Heat flows from hot end to cold end, medium required but particles of medium simply oscillate but do not leave their position.

- Slow process
- Takes places in solid
- Path may be zig-zag
- Temperature of medium increases.

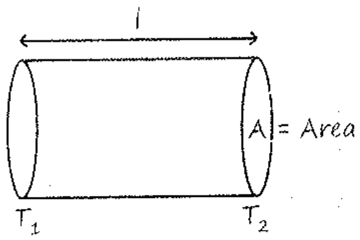
## Convection

- Medium required, each particle of medium absorbs heat and moves from hot end to cold end.
- Slow process
- Occurs in fluid not in solid
- Temp$^r$ of medium increases.

## Radiation

- Heat flows in the form of electromagnetic waves.
- Medium is not required
- Path straight line
- No change in temp$^r$ of medium.

## Law of Thermal Conductivity:-




$$H = \frac{Q}{t} = \frac{KA\Delta T}{L}$$

K = Coefficient of Thermal Conductivity (Material Property)

Q = Heat.

## Heat Current:-

$$H = \frac{\Delta T}{R_T} \quad R_T = \frac{l}{KA}$$

$R_T$ :- Thermal Resistance

## Combination of Rod :-

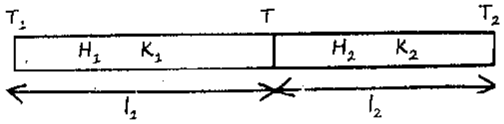
- Series Combination :-

"R" add होगा

$R_{eq} = R_1 + R_2 + R_3.....$

$$\boxed{\frac{l_{eq}}{K_{eq}} = \frac{l_1}{K_1} + \frac{l_2}{K_2} + \frac{l_3}{K_3}}$$

Junction Temp :- Rate of heat flow same in series combination.




$$H_1 = H_2$$

$$\frac{K_1(T_1 - T)}{l_1} = \frac{K_2(T - T_2)}{l_2}$$

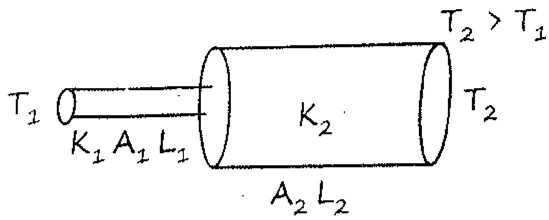
- Parallel Combination...

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

$$\boxed{\frac{K_{eq}A_{eq}}{l} = \frac{K_1A_1}{l} + \frac{K_2A_2}{l} + \frac{K_3A_3}{l}}$$

- Combination of Conductor :-

1> Series :-



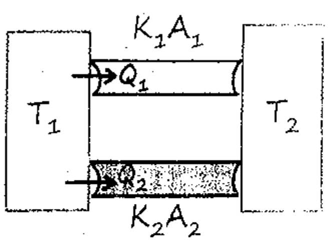

$$R_{eq} = R_1 + R_2 = \frac{L_1}{K_1A_1} + \frac{L_2}{K_2A_2}$$

$$\frac{H}{dt} = \frac{\Delta T}{R_{eq}} = \frac{T_2 - T_1}{R_{eq}}$$

$$K_{eq} = \frac{L_1 + L_2}{\frac{L_1}{K_1} + \frac{L_2}{K_2}}$$

$$\boxed{T_{mid} = \frac{\frac{K_1A_1T_1}{L_1} + \frac{K_2A_2T_2}{L_2}}{\frac{K_1A_1}{L_1} + \frac{K_2A_2}{L_2}}}$$

2> Parallel :-




$T_1 > T_2$

Length Should be same

$L_1 = L_2 = L$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

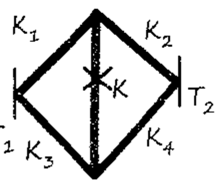
$$\frac{K_{eq}[A_1 + A_2]}{\cancel{L}} = \frac{K_1A_1}{\cancel{L}} + \frac{K_2A_2}{\cancel{L}}$$

$$K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

$$H_{eq} = Q_1 + Q_2$$

$$= \frac{K_1(T_1 - T_2)A_1}{L} + \frac{K_2(T_1 - T_2)A_2}{L}$$

- Wheat stone Bridge :-

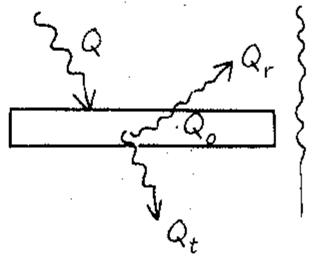



$K_1 . K_4 = K_3 . K_2$

$$\frac{K_1}{K_2} = \frac{K_3}{K_4}$$

**Radiation :-**

Black Body :-




$Q = Q_a + Q_r + Q_t$

$r + a + t = 1$

POWER

Absorptive $a = \frac{Q_a}{Q}$

Reflective $r = \frac{Q_r}{Q}$

Transmitive $t = \frac{Q_t}{Q}$

- Emissive Power [Intensity] :-

$$E = \frac{Q}{At} = \frac{J}{m^2 s} = Watt/m^2$$

Stefan's Law :-

$E = \sigma T^4$ $\quad$ T = Kelvin.

$$\sigma = 5.67 \times 10^{-8} \frac{Watt}{m^2K^4}$$

Emissivity :-

$$e = \frac{\text{Emissive Power of Normal body } (\varepsilon)}{\text{Emissive Power of Black body } (E)}$$

For Black Body (e) = 1.

**Emissive Power of Normal Body ($\varepsilon$) :-**

$$\varepsilon = eE = e\sigma T^4$$

$$P = \frac{Q}{t} = Ae\sigma T^4$$

Stefan-Boltzmann's Law :-

$$P_{loss} = P_{emit} - P_{absorb}$$

$$P_{loss} = \sigma eA [T^4 - T_0^4]$$

$T_0$ = Surr. Temp$^r$, T = body Temp$^r$

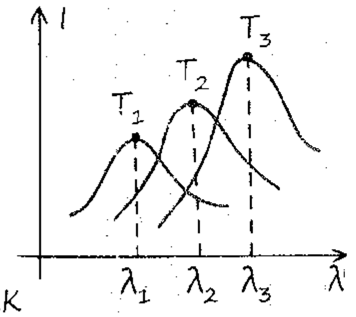
## Wien's Law :-

$\lambda_m T = b$

$\lambda_3 > \lambda_2 > \lambda_1$

$T_1 > T_2 > T_3$

$b = 2.9 \times 10^{-3}$ mK




λ = wavelength corresponding to which Energy is maximum.

## Newton's Law of Cooling :-

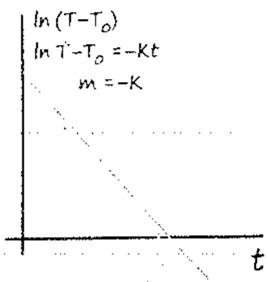
- Time 't' taken to fall temperature $T_1$ to $T_2$ where $T_0$ is the temperature of surrounding.

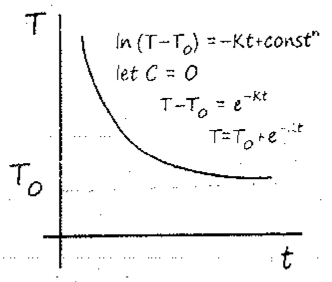
  $$\frac{T_1 - T_2}{t} = \left(\frac{T_1 + T_2}{2} - T_0\right)$$

- Rate of cooling $\propto$ Temperature difference
- $90°C \xrightarrow[t_1]{} 80°C \xrightarrow[t_2]{} 70°C \xrightarrow[t_3]{} 60°C$

  Time $t_1 < t_2 < t_3$

  (Time taken to fall temp$^r$ for every 10°C)




- Kirchoff's Law :-

  A good absorber is a good emitter!

## Solar Constant :-

Total Thermal Energy falling per unit area per sec.

$$\boxed{S = e\sigma T^4 \left[\frac{R^2}{r^2}\right]}$$

$$T_{sun} \propto \left[\frac{Sr^2}{e\sigma}\right]^{1/4}$$

e = emissivity of sun.

σ = Stephan's const$^n$

T = Temp. of Sun.

R = Radius of Sun.

r = Dist$^n$ of sun & earth.

## Weisman-Fraz Law :-

Ratio of thermal conductivity and electrical conductivity at a temp$^r$ is same for all body.

$$\boxed{\frac{K}{\sigma T} = \text{Const}^n}$$

$$\boxed{K \propto \sigma}$$

K = Thermal Conductivity

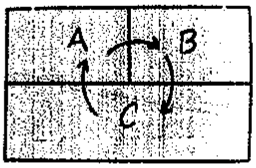
σ = Electrical Conductivity

**MR***

"Jeet ke khatir junoon chahiye, ho ubal ayesa khoon chahiye, Aasman bhi ayega zameen par bas irado main jeet ki goonj chahiye."

# 18 KTG & Thermodynamics

## Zeroth Law of TD :-




$T_A = T_B = T_C$ (They are in thermal contact)

**Work :-** $W = P.\Delta V = \int P.dV$

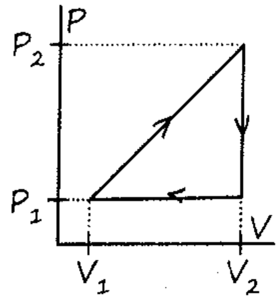
| By gas | On gas |
|---|---|
| Vol inc, expansion | Vol dec, compression |
| $W = +ve$ | $W = -ve$ |

## P–V Graph → Indicator Diagram :-

Work = Area of Close loop in PV Graph

| C. Wise | A.C. Wise |
|---|---|
| $W = +ve$ | $W = -ve$ |

- Expansion → $\Delta V = +ve$ $W \uparrow$
- Compression → $\Delta V = -ve$ $W \downarrow$




$$\text{Work} = +\frac{1}{2}(V_2 - V_1)(P_2 - P_1)$$

## Internal Energy (U) :-

⟶ "Path independent"

$U = KE + PE.$

⟶ "Temp$^r$ dependent."

$U \propto T$

$$U = \frac{1}{2}K_BT = \frac{f}{2}K_BT \left.\right\} \text{Due to 1 molecule}$$

$$U = \frac{f}{2}\frac{R}{N_A}T \times N = \frac{nfRT}{2} \left.\right\} N \text{ molecule}$$

$f$ = degree of freedom.

$N$ = No. of molecules.

$n$ = No. of moles

$N_A = 6.023 \times 10^{23}\ mol^{-1}$

$K_B = \dfrac{R}{N_A}$ = Boltzmann Constant

$K_B = 1.38 \times 10^{-23}\ m^2kg\,s^{-2}k^{-1}$ → $JK^{-1}$

## Degree of Freedom :-

1> Monoatomic Gas :- (Ne, He, Ar)
   $f = 3T + 0R$

2> Diatomic gas :- ($N_2$, $O_2$, $H_2$)
   $f = 3T + 2R$

3> Triatomic Linear gas :- ($CO_2$)
   $f = 3T + 2R$

4> Polyatomic gas/Triatomic Non-linear gas:- ($SO_2$, $CH_4$, $NH_3$)
   $f = 3T + 3R$

When Considering Vibrational Motion:-

- No effect for Monoatomic gas
- Diatomic = $f = 3T + 2R + 2V = 7$
- Triatomic
  - → Linear $f = 3T + 2R + 2V = 7$
  - → Non-Linear $f = 3T + 3R + 2V = 8$

## Heat Capacities :-

1> Specific heat Capacity (S) :-

$$S = \frac{dQ}{m\,dt}$$

2> Heat Capacity (C) :-

$$C = \frac{dQ}{dT}$$

3> Molar heat Capacity :-

$$C_m = \frac{dQ}{n\,dt}$$

Relation between Specific & Molar Sp. H.C. :-

* $\boxed{C_m = M_{wt} S}$ $\quad C_m = \frac{W}{n} S$

› $C_p$ & $C_V$ denote the Specific Heat per unit mass of an ideal gas of Mwt "M" then :-

$C_p - C_V = R \rightarrow$ Molar Sp. heat

$S_p - S_V = \frac{R}{M}$ (Specific heat per unit mass)

## Molar Heat Capacity :-

Const$^n$ P

$$C_p = \frac{dQ}{dt\,n}$$

Const$^n$ V

$$dQ = dU$$

$$C_V = \frac{fR}{2}$$

Note :-

$$\frac{C_P}{C_V} = \gamma$$

$$C_p - C_V = R$$

$$C_V = \frac{R}{\gamma - 1}$$

$$\boxed{\gamma = 1 + \frac{2}{f}}$$

## MR* Table :-

| Gas | DOF (f) | $C_V (Rf/2)$ | $C_p (C_V + R)$ | $\gamma (C_p / C_V)$ |
|---|---|---|---|---|
| Monoatomic | 3 | $\frac{3R}{2}$ | $\frac{5R}{2}$ | $\frac{5}{3} = 1.66$ |
| Diatomic | 5 | $\frac{5R}{2}$ | $\frac{7R}{2}$ | $\frac{7}{5} = 1.44$ |
| Triatomic Linear | 5 | $\frac{5R}{2}$ | $\frac{7R}{2}$ | $\frac{7}{5} = 1.4$ |
| Triatomic Non-linear | 6 | 3R | 4R | $\frac{4}{3} = 1.33$ |
| Diatomic at high Temperature | 7 | $\frac{7R}{2}$ | $\frac{9R}{2}$ | $\frac{9}{7}$ |

## Gas Mixture :-

$$(C_V)_{mix} = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2}$$

$$f_{mix} = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$$

$$(C_P)_{mix} = \frac{n_1 C_{P1} + n_2 C_{P2}}{n_1 + n_2}$$

$$\gamma_{mix} = \frac{(C_P)_{mix}}{(C_V)_{mix}}$$

$$T_{mix} = \frac{n_1 f_1 T_1 + n_2 f_2 T_2}{n_1 f_1 + n_2 f_2}$$

# 1st Law of Thermodynamics :-

$dQ = dU + dW$

Based on Energy Conservation

| $\boxed{dQ}$ | $\boxed{dU}$ | $\boxed{dW}$ |
|---|---|---|
| Given = + To gas | T ↑ = + of gas | by gas = + |
| Taken = − from gas | T ↓ = − of gas | on gas = − |

(a) Gay-Lusaac Law :-

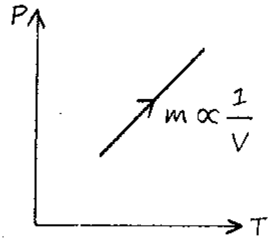
- $V = Const^n$ $(P \propto T)$

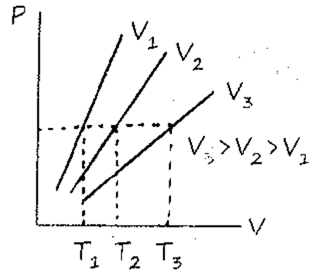
Isochoric Process !

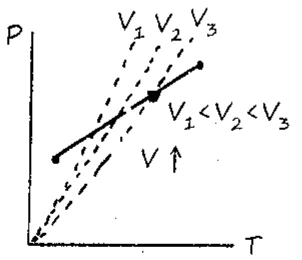
$WD = 0 \qquad Q = \Delta U = nC_V \Delta T$

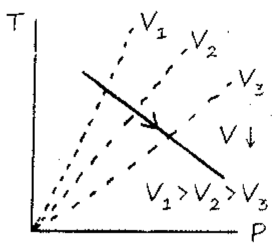
$$Q = \frac{nfR\Delta T}{2}$$

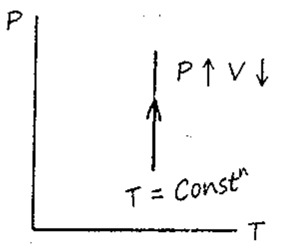
- Graph :-

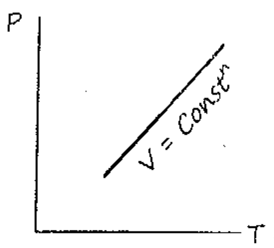


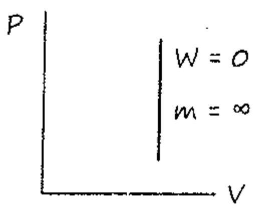



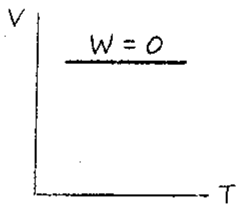




(b) Charles Law :-

$P = Const^n \qquad (V \propto T)$

Isobaric Process !

$W = P\Delta V = P(V_2 - V_1)$

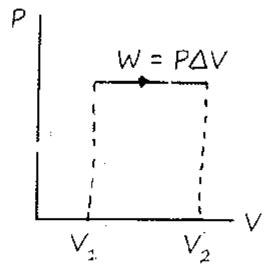
- Fraction of Heat goes in Work :-

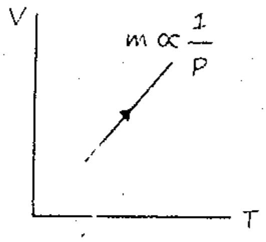
$$\frac{W}{Q} = 1 - \frac{1}{\gamma}$$

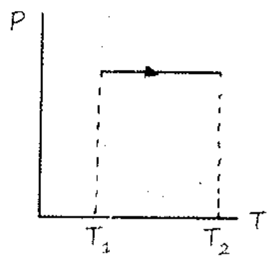
- Fraction of Heat goes in $\Delta U$ :-

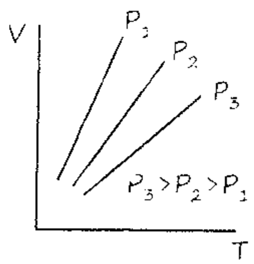
$$\frac{dU}{dQ} = \frac{1}{\gamma}$$

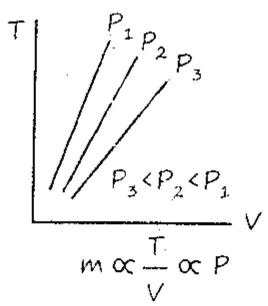
- Graph :-

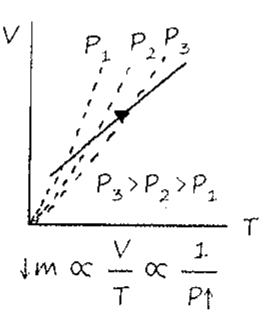



(c) Boyle's Law :-

- $T = Const^n \qquad PV = Const^n$
- Isothermal Process !
- Reversible i.e., V slow.

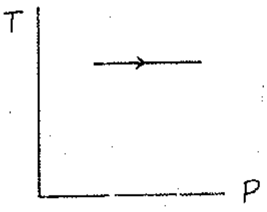
- $\Delta U = 0 \rightarrow$ Hum jitna Kam Karenge sab heat mein jayega!
- $C_m = \dfrac{dQ}{n\Delta T} = \infty$

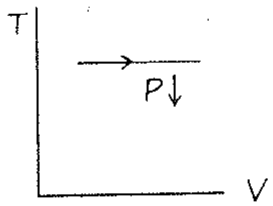
* $Work = nRT\left[\ln\left(\frac{V_2}{V_1}\right)\right]$ *

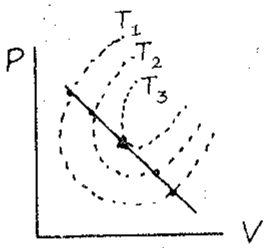
$$W = 2.303\, nRT\left[\log_{10}\left(\frac{V_2}{V_1}\right)\right] = dQ$$

$$W = 2.303\, nRT\left[\log_{10}\left(\frac{P_1}{P_2}\right)\right] = dQ$$

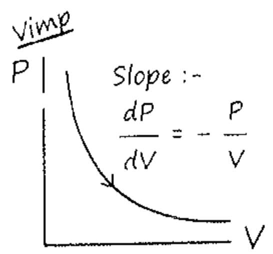
- Graph :-

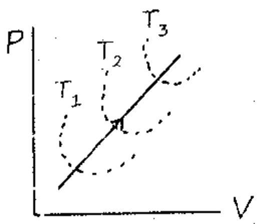



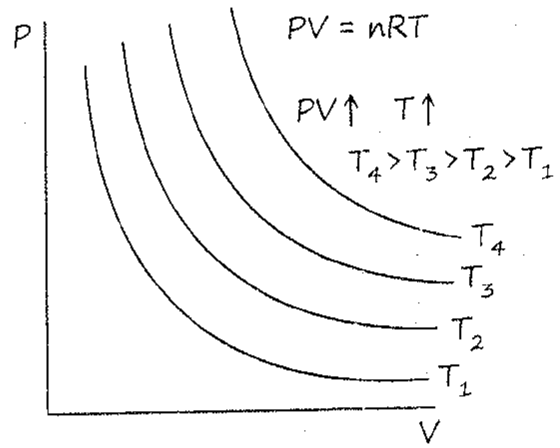




$T = 1^{st}$ inc then dec.




$T_3 > T_2 > T_1$




(d) **Adiabatic Process :-**

- $\Delta Q = 0 \qquad PV^{\gamma} = Const^n$
- $\gamma$ = Adiabatic Coefficient $= \dfrac{C_P}{C_V}$
- Sudden Process.

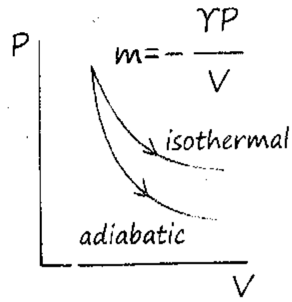
  $dW = -\,dU$

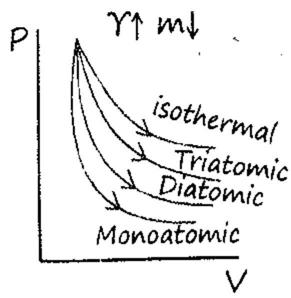
Tyre burst

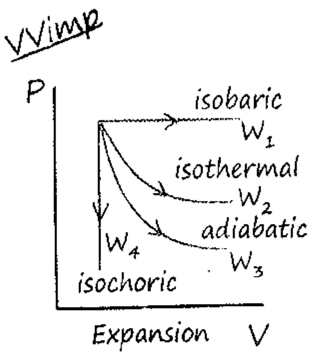
Expansion :- $V\uparrow$ $W = +ve$ $\qquad U = -ve$ $T\downarrow$

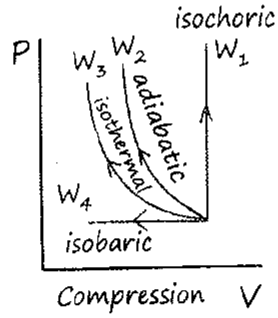
Compression :- $V\downarrow$ $W = -ve$ $\qquad U = +ve$ $T\uparrow$

- $C_m = \dfrac{dQ}{n\Delta T} = 0 \qquad S = 0$
- Note :-
- $dW = -nC_V\Delta T = \dfrac{nR(T_2 - T_1)}{1-\gamma}$
- $dW = \dfrac{P_2V_2 - P_1V_1}{1-\gamma}$
- $PV^{\gamma} = TV^{\gamma-1} = P^{1-\gamma}T^{\gamma} = $ Constant
- Graph :-




$W_1 > W_2 > W_3$

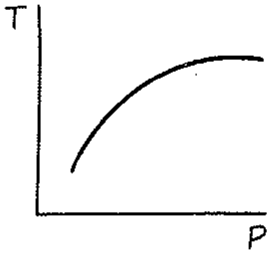
$W_4 = 0$

$W_2 > W_3 > W_4$

$W_1 = 0$

- Note :-

Adiabatic Elasticity = Bulk Modulus = $\gamma P$

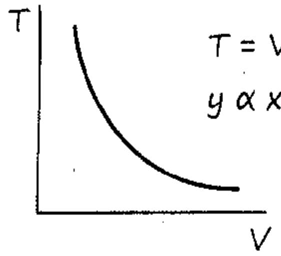


$P^{1-\gamma}T^{\gamma} = Const^n$

$T = P^{\frac{\gamma-1}{\gamma}}$

$\gamma > 1$

Power of $P < 1$



$T = V^{1-\gamma}$

$y \propto x^{-1}$ Rectangular hyperbola

## Polytropic Process :-

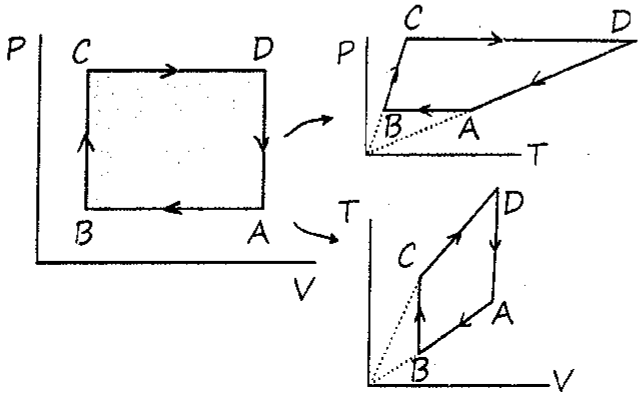
$PV^x = TV^{x-1} = P^{1-x}T^x = const^n$

$$\boxed{W = \frac{nR(T_2 - T_1)}{1-x}} \quad C_m = C_V + \frac{R}{1-x}$$

$$\boxed{C_m = \frac{R}{\gamma-1} + \frac{R}{1-x}}$$

Graph Conversions :-

1>



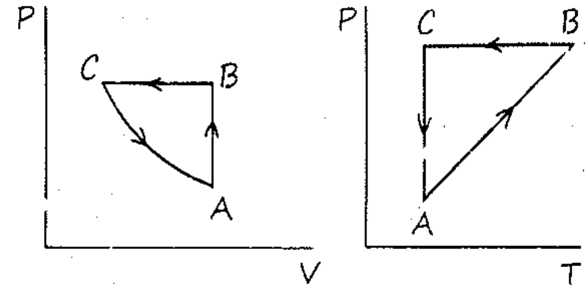
$A \to B = P = Const^n$ V↓ T↓

$B \to C = V = Const^n$ P↑ T↑

$C \to D = P = Const^n$ V↑ T↑

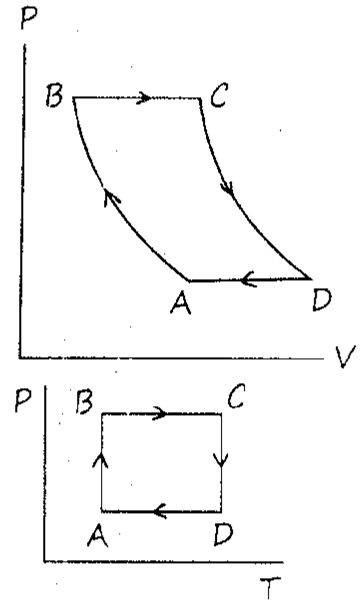
$D \to A = V = Const^n$ P↓ T↓

2>



| | | |
|---|---|---|
| $A \to B$ | P↑ T↑ | $V = Const^n$ |
| $B \to C$ | V↓ T↓ | $P = Const^n$ |
| $C \to A$ | P↓ V↑ | $T = Const^n$ |

3>

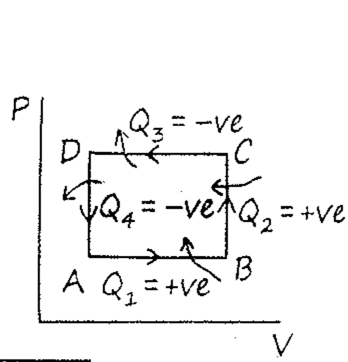


| | |
|---|---|
| $A \to B$ | $T = Const^n$ P↑ V↓ |
| $B \to C$ | $P = Const^n$ T↑ V↑ |
| $C \to D$ | $T = Const^n$ P↓ V↑ |
| $D \to A$ | $P = Const^n$ T↓ V↓ |

## Efficiency ($\eta$) :-

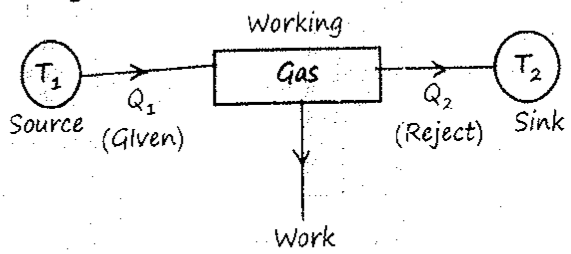
$$\eta = \frac{WD}{Q_{given}}$$

$$\eta = \frac{A_{req}}{Q_1 + Q_2}$$



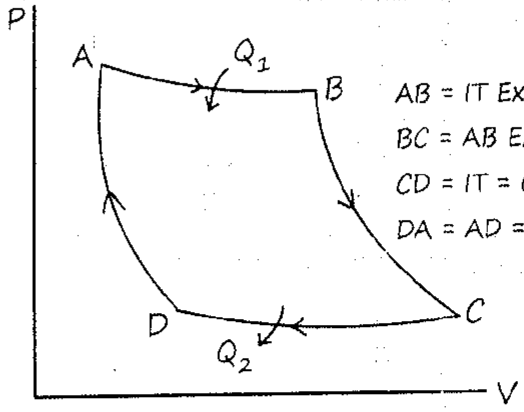
$$\eta = \frac{A_{req}}{\underbrace{nC_P\Delta T}_{AB} + \underbrace{nC_V\Delta T}_{BC}}$$

## Heat Engine :-




$Q_1 = W + Q_2$ $\quad$ $\eta = \dfrac{Q_1 - Q_2}{Q_1}$

1> Carnot Engine :- Output :- Work
Input :- Heat



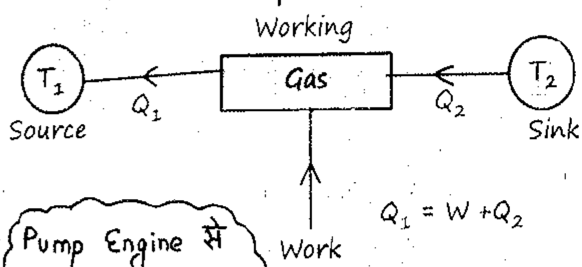

AB = IT Exp.
BC = AB Exp.
CD = IT = Comp
DA = AD = Comp

Carnot Theorem :- $\dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2}$

$\eta = \dfrac{\text{Work}}{Q_1}$ $\qquad$ $\eta = 1 - \dfrac{Q_2}{Q_1}$

$\therefore$ $\eta = 1 - \dfrac{T_{Kam}}{T_{Jyada}}$

2> Heat Pump :- Output : Heat
Input : Work



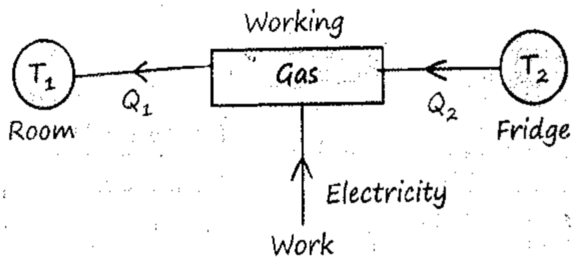

$Q_1 = W + Q_2$

Pump Engine मे उलटा Kaam Krta hai !

$\eta = \dfrac{Q_1}{W} = \dfrac{1}{\eta_{engine}}$

3> Refrigerator :- Output :- Heat
Input :- Work

- Same as Pump.




$\beta = \dfrac{Q_2}{W} = \dfrac{Q_2}{Q_1 - Q_2} = \dfrac{T_2}{T_1 - T_2} = \dfrac{\text{Kam}}{\text{diff.}}$

$\beta = \dfrac{1-\eta}{\eta} = \dfrac{1}{\eta} - 1$ Vimp

Q. The efficiency of a Carnot heat engine is $\frac{1}{3}$ Find the coefficient of performance of Carnot refrigerator when both heat engine and refrigerator are working between similar source and sink.

Sol. Efficiency of heat engine is, $\eta = \dfrac{1}{3}$

The relation between $\beta$ & $\eta$ when same Carnot engine is used,

$$\Rightarrow \beta = \frac{1-\eta}{\eta}$$

$$\therefore \beta = \frac{1 - \frac{1}{3}}{\frac{1}{3}} = 2$$

Q. A carnot engine works as a refrigerator in between 250 K and 300 K. If it acquires 750 calories from heat source at low temperature, then what is the heat generated at higher temperature. (in calories)?

Sol.

$$\eta = \frac{T_1-T_2}{T} = \frac{Q_1-Q_2}{Q_1}$$

$$= \frac{300-250}{300} = \frac{Q-750}{d}$$

$$\Rightarrow \frac{50}{300} = 1 - \frac{750}{Q}$$

$$\Rightarrow \frac{750}{Q} = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\therefore Q = \frac{750 \times 6}{5}$$

$$= 900$$

Q. A system is taken from state A to state B along two different paths 1 and 2. If the heat absorbed and work done by the system along these two paths are $Q_1$, $Q_2$ and $W_1$, $W_2$ respectively, then

(a) $Q_1 = Q_2$

(b) $W_1 = W_2$

(c) $Q_1 - W_1 = Q_2 - W_2$

(d) $Q_1 + W_1 = Q_2 + W_2$

Sol. Internal energy does not depends on path. Heat & work depends (c) $Q_1-W_1 = Q_2-W_2$ is correct.

Q. In a given process, $dW = 0$, $dQ < 0$, then for the gas:

(a) Temperature increases

(b) Volume decreases

(c) Pressure decreases

(d) Pressure increases

Sol. $dW = 0 \quad dQ < 0$

$dU + dW = dQ$

$dU = dQ$

$dU < 0$

Temper decrese

Volume constant

$P \propto T$

Pressure decreases

Q. If 32 gm of $O_2$ at 27°C is mixed with 64 gm of $O_2$ at 327°C in an adiabatic vessel, then the final temperature of the mixture will be:

Sol. For adiabatic process, gain of heat loss of heat =

or, $m_1 S(T-T_1) = m_2 S(T_2-T)$

or, $m_1(T-T_1) = m_2(T_2-T)$

or, $32 \times (T-27) = 64 \times (327-T)$

or, $T-27=2(327-T)$

or, $T-27\ 654-2T$

or, $3T=681$

or, $T=227°C$

Q. If $W_1$ is the work done in compressing an ideal gas from a given initial state through a certain volume isothermally and $W_2$ is the work done in compressing the same gas from the same initial state through the same volume adiabatically, then:

(a) $W_1 = W_2$ (b) $W_1 < W_2$

(c) $W_1 > W_2$ (d) $W_1 = 2W_2$

Sol. (b)

Q. During an experiment and ideal gas is found to obey an additional law $VP^2$ = constant. The gas is initially at a temperature T and volume V. When it expand to a volume 2V, the temperature becomes.

Sol. Ideal gas: $V P^2$ = constant

Again $PV = nRT$ from equation of state,

Hence, $VP \times P$ = constant i.e. $nRT \times P$ = constant

Again, $P = \frac{nRT}{V}$

$\therefore \frac{(nRT)^2}{V}$ = constant

$\frac{T^2}{V}$ = constant

Thus volume V when expanded to 2V, temperature $T_2$

$$T_2 = \sqrt{\frac{2V}{V}} = \sqrt{2}\,T_1$$

Q. A carnot engine having an efficiency of $\frac{1}{10}$ th of heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is:

Sol. Coefficient of performance of a refrigerator

$$\beta = \frac{1-\eta}{\eta}$$

$$\beta = \frac{1-\frac{1}{10}}{1/10} = 9$$

Also $\beta = \frac{Q_L}{W}$ (where W is the work done)

or

$Q_L = \beta \times W = 9 \times 10 = 90J$

Q. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its temperature. The ratio of $C_P/C_V$ for the gas is equal to:

$P \propto T^3$,

$PV = nRT$

$P \propto T^3$

$P \propto (PV)^3$

$P^2V^3$ = constant

$PV^{\frac{3}{2}}$ = constant

$$\gamma = \frac{3}{2}$$

Q. A monoatomic gas at pressure $p_1$ and $V_1$ is compressed adiabatically to $\frac{1}{8}$ th its original volume. What is the final pressure of the gas?

Sol. Correct option is (a)

It is given that $\frac{C_P}{C_V} = \gamma = \frac{5}{3}$

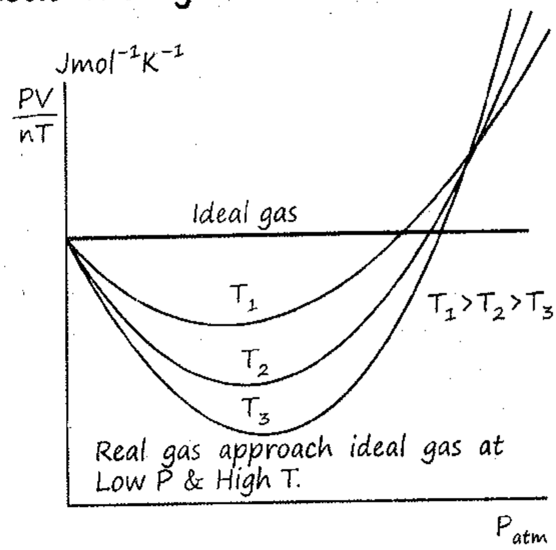
For an adiabatic process,

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

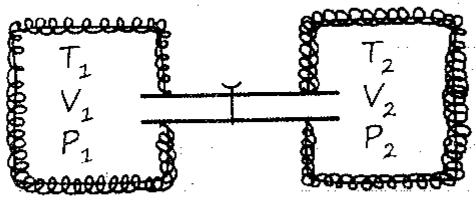
$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{5/3}$$

$$= \left(\frac{8}{1}\right)^{5/3} = 32$$

## Kinetic Theory of Ideal Gas :-




o Note :-




The valve joining the two vessels is opened :- $T_{mix} = ?$

$$T_{mix} = \frac{n_1T_1 + n_2T_2}{n_1 + n_2} = \frac{\frac{P_1V_1}{R} + \frac{P_2V_2}{R}}{\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}}$$

$$\boxed{T_{mix} = \frac{(P_1V_1 + P_2V_2)\,T_1T_2}{P_1V_1T_2 + P_2V_2T_1}}$$

Pressure of ideal gas :-

$P = \frac{1}{3} nmV_{rms}^2$ n = no. density $= \frac{N}{V}$

$V_{rms} = \sqrt{\frac{3P}{nm}}$ m = mass of each molecule

## Kinetic interpretation of temperature :-

$$KE = \frac{3RT}{2}$$

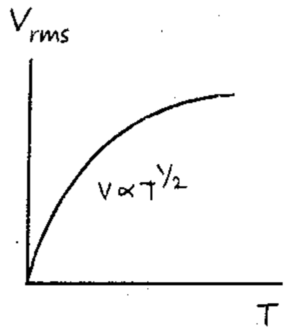
- Energy density

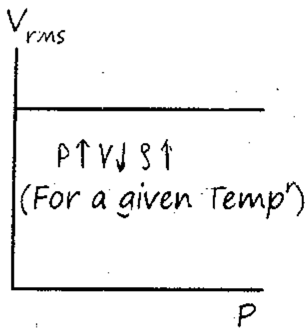
$$\frac{KE}{V} = \frac{3P}{2}$$

- Rotational KE :-

$$KE = \frac{fRT}{2N_A} \qquad KE = \frac{fKT}{2}$$

$$V_{rms} = \sqrt{\frac{3KT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}} \qquad \boxed{V_{rms}}$$



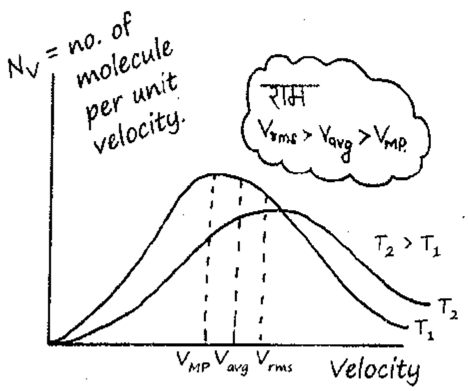




$$V_{avg} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8}{3\pi}}\,V_{rms} \qquad \boxed{V_{avg}}$$

$$V_{avg} = 0.82\,V_{rms}$$

$$V_{MP} = \sqrt{\frac{2RT}{M}} \qquad \boxed{V_{MP}}$$

## Maxwell Distribution Curve :-




- Area $= \int N_V\,dv$ = no. of molecules.
- → Temp$^r$ independent.

## Mean free path :-

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} \qquad n = \text{number density} = \frac{N}{V}$$

$$\lambda = \frac{RT}{\sqrt{2}\pi N_A P} \qquad d = \text{diameter} = 2r$$

## Density of gas :-

$$d = \frac{PM}{RT} = \frac{Pm}{K_B T}$$

# A–R based questions:

A → A liquid is filled in container which is moving with high speed does not have higher temperature.

R → Temp$^r$ of liquid related to internal energy, not to K.E. of liquid.

**Ans.:-** Both are true and correct explanation.

A → A mixture of petrol and air when ignited is not in equlibrium state.

R → Its temperature and pressure not uniform.

**Ans.:-** Both are correct and correct explanation.

A → $\Delta Q$ is extensive.

R → It is propotional to total mass of system.

**Ans.:-** Both are true and correct explanation.

# 19 Simple Harmonic Motion

**Periodic Motion:-**

(Repeat itself after a regular interval of time)

(repeat about mean)
(To & Fro)
Oscillation

- large Amplitude → Not S.H.M
- Small amplitude, Energy conserved, $F \propto -x^n$ → Simple harmonic Motion

(repeat not about mean)
Not Oscillation

\# Vibration → Oscillation with high frequency.

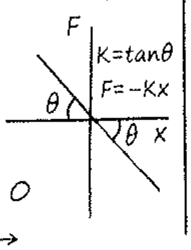
\# Periodic but not oscillation
→ all uniform circular motion
→ earth around sun

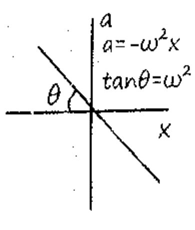
- All SHM is oscillatory and periodic
- All oscillatory is periodic but need not S.H.M
- All periodic need not to be oscillatory & S.H.M.

SHM :- (Amplitude is small)

* $\vec{F} \propto -\vec{x}$
* $\vec{a} = -\omega^2 x$
* $\dfrac{d^2\vec{x}}{dt^2} + \omega^2\vec{x} = 0$

$\vec{a} \propto -\vec{x}$




- Equation of SHM :-

$x = A\sin(\omega t + \phi)$

$v = A\omega\cos(\omega t + \phi)$

$a = -A\omega^2\sin(\omega t + \phi)$

A = Amplitude

$\omega$ = Angular Frequency $= \dfrac{2\pi}{T} = 2\pi f$

$\phi$ = initial phase

$\phi = 0$ for mean से start

$\phi = \pi/2$ for extreme से start

$\phi = \pi/6$ for half of extreme से start

$\phi = \pi/4$ for $x = \dfrac{A}{\sqrt{2}}$ से start

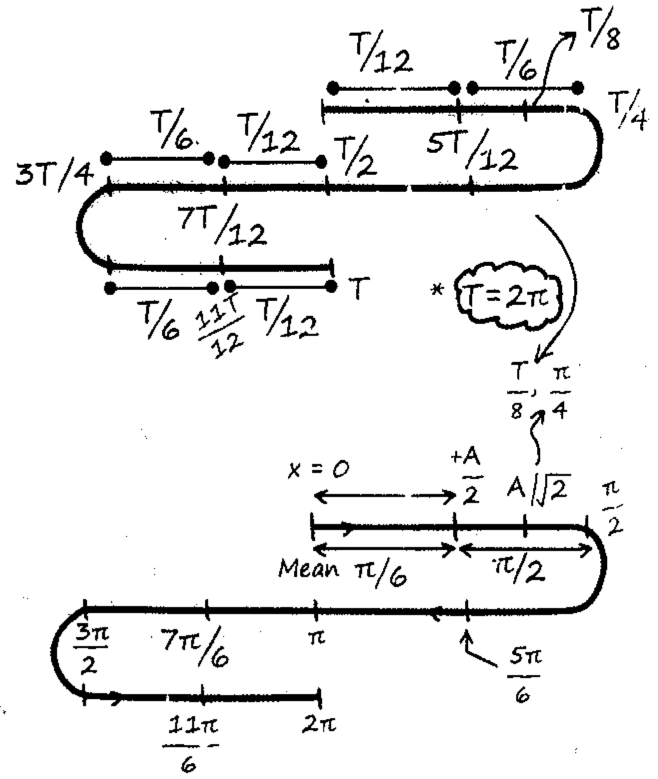
Note:-

$a \propto -x^n$  n = even = Translat$^n$.

n = odd but not 1.
→ "Oscillation but not SHM"

$\boxed{a \propto x^1}$ ⇒ SHM!

## Fire Concept MR*




## Motion From Mean :-

$x = A\sin\omega t \qquad v = A\omega\cos(\omega t)$

$a = -A\omega^2\sin\omega t \qquad a = -\omega^2 x$

$v = A\omega\sqrt{1-\sin^2\omega t}$

$v = \omega\sqrt{A^2-x^2} \qquad \omega = \sqrt{\frac{k}{m}}$

| Mean | Extreme |
|---|---|
| $x = 0$ | $x = A$ |
| $v_{max} = A\omega$ | $v = 0$ |
| $a = 0$ | $a_{max} = -\omega^2 A$ |

MR*

Mai tere piche, wo mere piche hai re kismat mai tum se na mil pau, wo mere se na mil pay

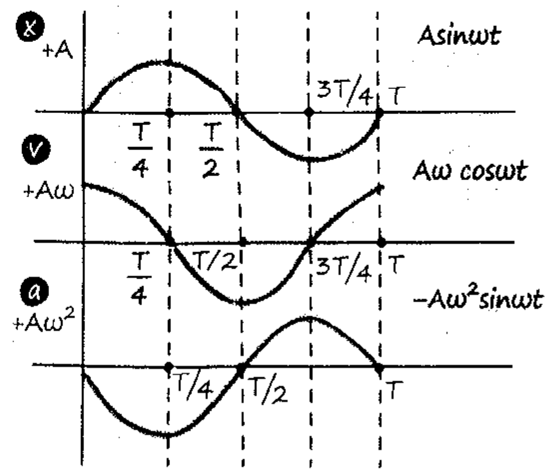
Mai → Velocity tum → $acc^n$

Wo → Position ☺

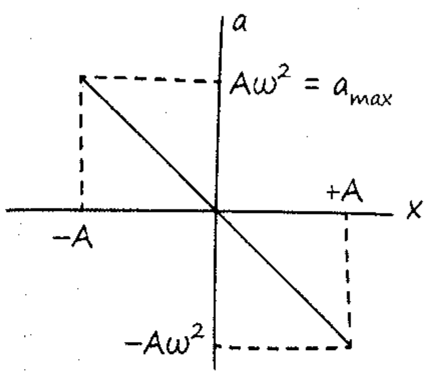
## Graphs :-

$\Delta\phi = +\pi/2$: ⓧ $\xrightarrow{+\pi/2}$ ⓥ $\xrightarrow{+\pi/2}$ ⓐ

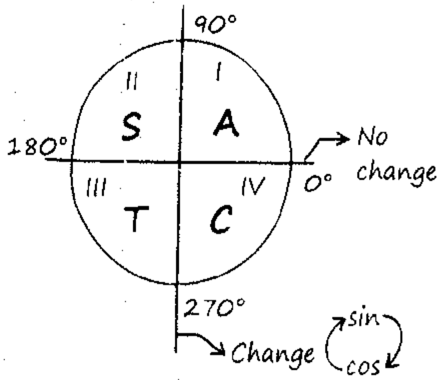
1> x – t :-



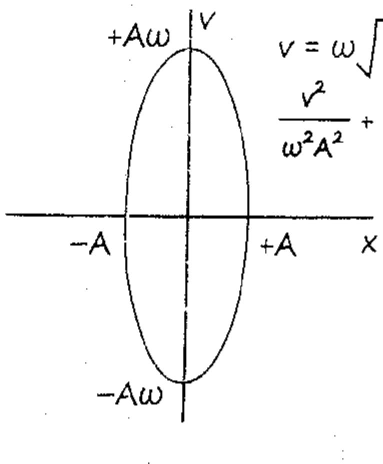

2> $a = -\omega^2 x$




Note :-



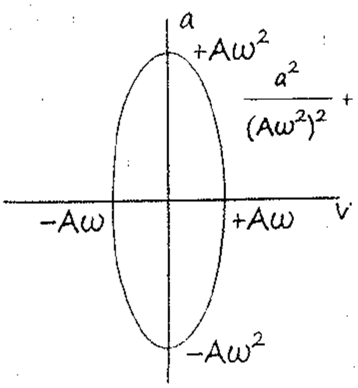

3> v – x :-




$v = \omega\sqrt{A^2-x^2}$

$\frac{v^2}{\omega^2A^2} + \frac{x^2}{A^2} = 1$

4> a - v :-




$$\frac{a^2}{(A\omega^2)^2} + \frac{v^2}{(A\omega)^2} = 1$$

Imp :-

TP of a particle executing SHM along straight line its velocity at position $x_1$ and $x_2$ from mean are $V_1$ and $V_2$ & Then TP :-

$$T = 2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_1^2 - v_2^2}}$$

Energies :-

$$K.E = \frac{1}{2}mv^2$$

we know $m = \frac{K}{\omega^2}$

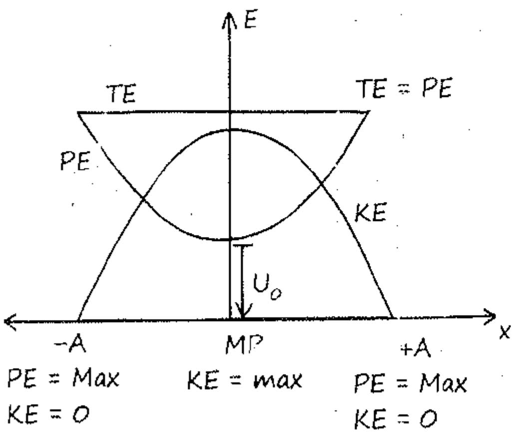
KE $= \frac{1}{2}K(A^2 - x^2) = \frac{1}{2}KA^2\cos^2(\omega t + \phi)$

U $= \frac{Kx^2}{2} + U_0 = \frac{1}{2}KA^2\sin^2\omega t + U_0$

TE $= \frac{1}{2}KA^2 + U_0 = \text{constant}$

$K$ = Force Const$^n$ = $m\omega^2$

Energy graphs :-




Kitne time lagega Repeat Krne mein

Kitne bar given time में अपने Pattern को Repeat करेगा।

MR*

| Physical Quantity | Time Period | Frequency |
|---|---|---|
| Position | T | f |
| Velocity | T | f |
| Speed | T/2 | 2f |
| Acceleration | T | f |
| KE | T/2 | 2f |
| PE | T/2 | 2f |
| KE-PE | T/2 | 2f |
| \|KE-PE\| | T/4 | 4f |
| ME = KE + PF | ∞ | O |

TP of SHM :-

**Force Method**

$F = -Kx$

$a = -\frac{Kx}{m} = -\omega^2 x$

$\omega = \frac{2\pi}{T}$

**Energy Method**

$F = -\frac{dU}{dr}$

$a = \frac{F}{m} = -\omega^2 x$

$\omega = \frac{2\pi}{T}$

TP of Spring Mass System :-

$$T = 2\pi\sqrt{\frac{M}{K}}$$

$(T_{Horizontal} = T_{vertical})$

independent of "g" & shape of object

F = const$^n$ की औकाद नहीं है की SHM करवाए!

MR*

- Equilibrium से जब तूमने खिंचा Mass को तो वो जो F extra आया है वो Oscillate करवाएगा !
- $Eq^n$ likhkar Equilibrium का $Kx_0 = mg$ Put किया तो कटा था कटा है और कटेगा !

## Combination of Spring :-

1> Series Combination :-

F = same, elongation different

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} \qquad \boxed{T^2 = T_1^2 + T_2^2}$$

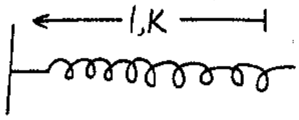
2> Parallel Combination :-

F = different, elongation = same

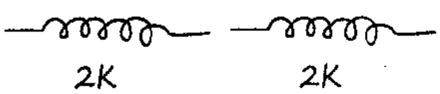
$$\boxed{\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}} \qquad K_{eq} = K_1 + K_2$$

Cutting of Spring :-

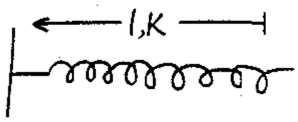
Spring constant $K \propto \dfrac{1}{\text{Length of spring}}$




cut in two equal part then




Q. Cut into three part of ratio 1:2:3 then ratio of spring constant.

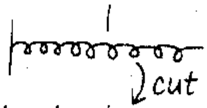



$l_1 : l_2 : l_3 = 1 : 2 : 3 = x : 2x : 3x$

$K_1 : K_2 : K_3 = \frac{1}{x} : \frac{1}{2x} : \frac{1}{3x}$

$K_1 : K_2 : K_3 = 6 : 3 : 2$

$K_1 = 6K \quad K_2 = 3K \quad K_3 = 2K$




eg:-

$$\boxed{K \propto \frac{1}{l}}$$

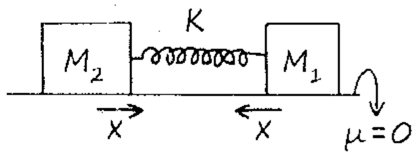
$l_1 : l_2 : l_3 = 1 : 3 : 5$

join $\parallel^{el}$. $= x : 3x : 5x \quad l = 9x \quad x = \dfrac{l}{9}$

Parallel :- $K_{eq} = \dfrac{9K}{1} + \dfrac{9K}{3} + \dfrac{9K}{5}$

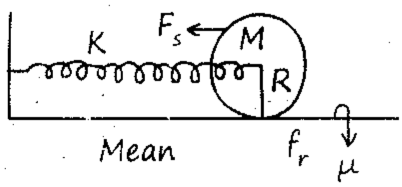
$= 9K + 3K + \dfrac{9K}{5} = \dfrac{69K}{5}$

- Reduced mass Concept :-




$$T = 2\pi\sqrt{\frac{M_1M_2}{(M_1+M_2)K}}$$

- Rotation + Translation wale Que. :-

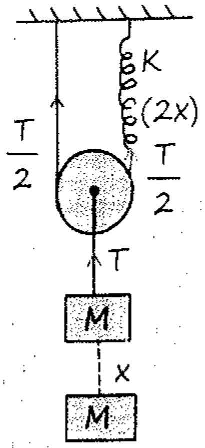



$$T = 2\pi\sqrt{\frac{M + I/R^2}{K}}$$ I = Moment of inertia

$$T = 2\pi\sqrt{\frac{M}{K}} \text{ if } \mu = 0$$

## ○ Constrain Motion :-

1>




$$K(2x) = \frac{T}{2}$$

$$T = 4Kx$$

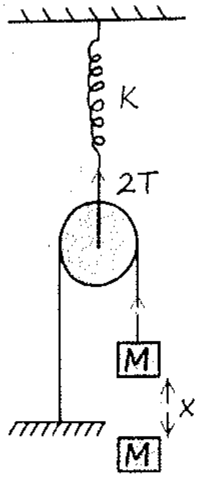
$$a = \frac{4K}{M}x$$

$$\omega = \sqrt{\frac{4K}{M}}$$

MR*

$$T = 2\pi\sqrt{\frac{M}{4K}}$$

2>

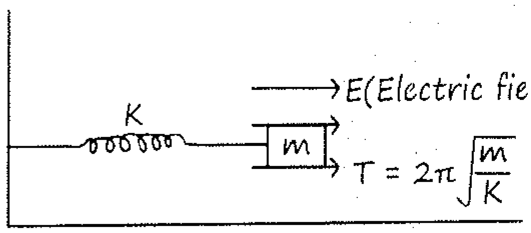



$$2T = \frac{Kx}{2}$$

$$T = \frac{Kx}{4}$$

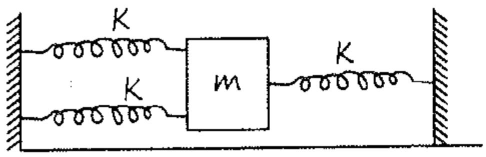
$$T = 2\pi\sqrt{\frac{4m}{k}}$$

3>



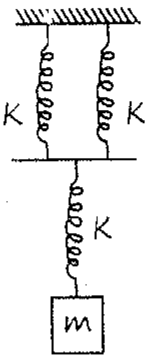

$$T = 2\pi\sqrt{\frac{m}{K}}$$

4>




$$T = 2\pi\sqrt{\frac{m}{3K}}$$

5>




$$T = 2\pi\sqrt{\frac{3m}{2K}}$$

## Time Period of Simple Pendulum :-

$$T = 2\pi\sqrt{\frac{l}{g_{eff.}}}$$

○ **Special Cases :**

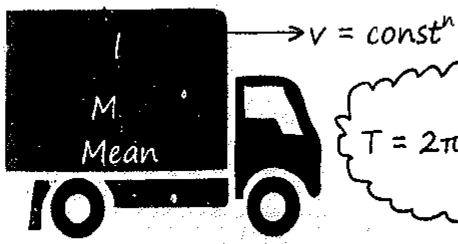
1> Lift :-

○ Up :- $T = 2\pi\sqrt{\frac{l}{g+a}}$

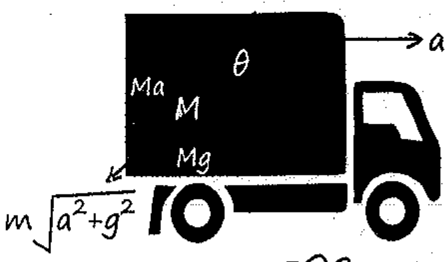
○ Down :- $T = 2\pi\sqrt{\frac{l}{g-a}}$

○ Free Fall :- $T = \infty$

2> Car :-

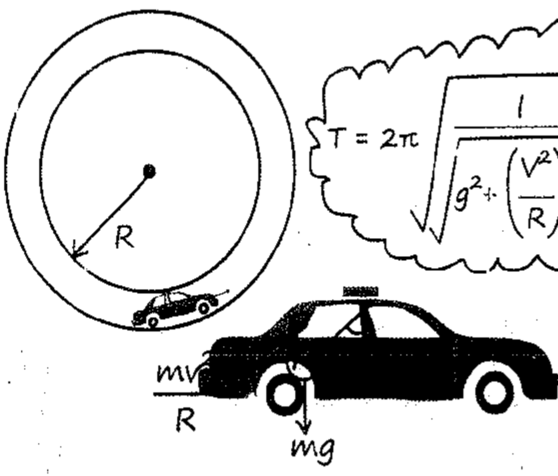



$$T = 2\pi\sqrt{\frac{l}{g}}$$




$$T=2\pi\sqrt{\frac{l}{\sqrt{a^2+g^2}}}$$

3> Car at Circular Track :-




$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{V^2}{R}\right)^2}}}$$

4> Pendulum in liquid :-


$$T = 2\pi\sqrt{\frac{l}{g - \frac{\rho V g}{M}}}$$

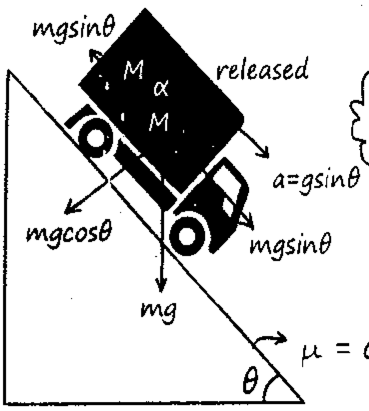
V = Vol. of bob.
ρ = density of liquid.
M = Mass of bob.

$$T = 2\pi\sqrt{\frac{l}{g\left(1 - \frac{\rho}{\sigma}\right)}}$$

σ = density of bob.

Pendulum को किसी Fluid मे डाला तो उसका T.P. बढेगा !

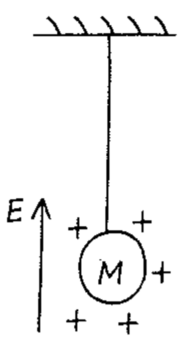
5> Angle made by pendulum with ceiling is 90°.



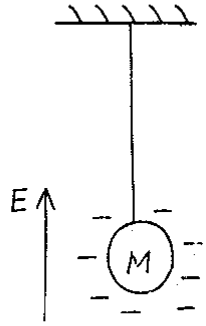

$$T = 2\pi\sqrt{\frac{l}{g\cos\theta}}$$

$\alpha = 0°$

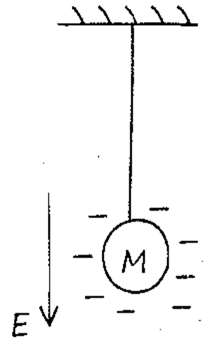
6> T.P. in E.F. :-



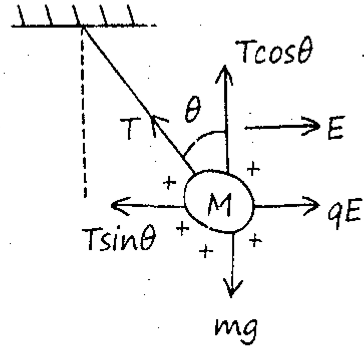

$$T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{M}}}$$

- 


$$T = 2\pi\sqrt{\frac{l}{g + \frac{qE}{M}}}$$

- 


$$T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{M}}}$$

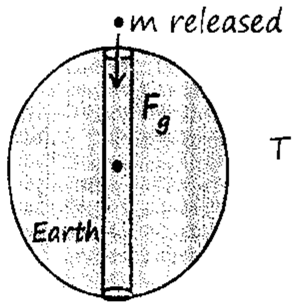
- 


$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{qE}{M}\right)^2}}}$$

7> T.P. when length of Simple Pendulum is very large :-

| | |
|---|---|
| $*T = 2\pi\sqrt{\dfrac{1}{g\left(\frac{1}{l} + \frac{1}{R}\right)}}$ | ○ $l >>> R$<br>$T = 2\pi\sqrt{\dfrac{R}{g}}$ |
| $*l = R$<br>$T = 2\pi\sqrt{\dfrac{R}{2g}}$ | ○ $l <<< R$<br>$T = 2\pi\sqrt{\dfrac{l}{g}}$<br>T = 84.6 min. |

8>




$$T = 2\pi\sqrt{\frac{R}{g}}$$

9> Second Pendulum :
length = 1m    Time period = 2 sec
f = 0.5 Hz

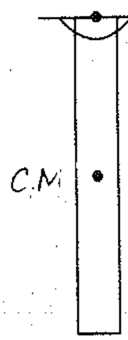
## TP of Physical Pendulum :-

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

I = M.O.I of object wrt point of suspension.

d = distⁿ of O & CM !


## Vertical Rod hinged about one end



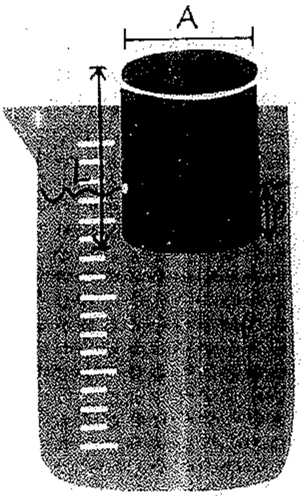

$$I = \frac{ML^2}{3}$$

$$T = 2\pi\sqrt{\frac{ML^2}{3mg\frac{L}{2}}}$$

$$T = 2\pi\sqrt{\frac{2L}{3g}}$$

Special Case :-

1> T.P. of Solid Cylinder performing SHM if it is slightly displaced downward & released :-




$$T = 2\pi\sqrt{\frac{m}{\rho A g}}$$

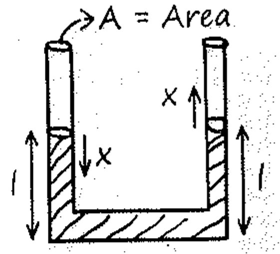
$$T = 2\pi\sqrt{\frac{\rho_0 L}{\rho g}}$$

$\rho l = \rho_0 L$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

l = length of cylinder inside liquid.

2> Oscillation of liquid column (I) :-




$$T = 2\pi\sqrt{\frac{m}{2\rho A g}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$*2l = L \quad l = \frac{L}{2}$

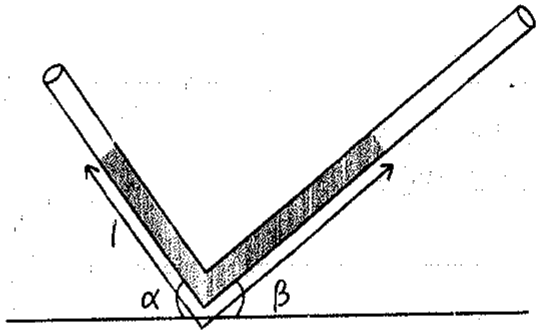
m = mass of liquid

ρ = density of liquid

A = Area of liquid column.

L = Total length of liquid column.

Oscillation of liquid column (II) :-




L = total length of water column

$$T = 2\pi\sqrt{\frac{L}{g(\sin\alpha+\sin\beta)}}$$

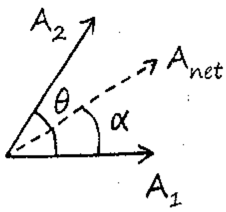
## Superposition of SHM of two object Oscillating in Same Direction :-

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$$

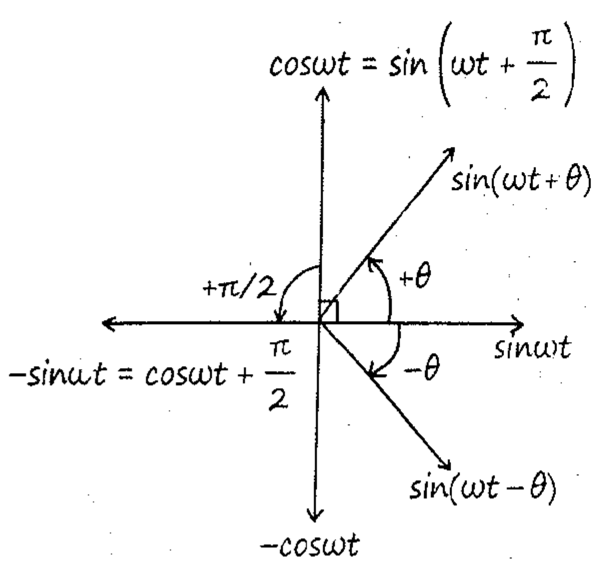
θ = initial phase difference.

$$\tan\alpha = \frac{A_2\sin\theta}{A_1 + A_2\cos\theta}$$

$\alpha$ = Angle b/n $A_{net}$ & 1st SHM



- Phasor diagram :-




| **Equation of S.H.M of 1st** | **Equation of S.H.M of 2nd** | **Equation of Superimposed S.H.M** |
|---|---|---|
| $x_1 = A\sin(\omega t)$ | $x_2 = A\cos(\omega t)$ | $x = \sqrt{2}\,A \sin(\omega t + \pi/4)$ |
| $x_1 = A\sin(\omega t)$ | $x_2 = A\cos(\omega t + \pi/6)$ | $x = A \sin(\omega t + \pi/3)$ |
| $x_1 = A\sin(\omega t - \pi/6)$ | $x_2 = A\cos(\omega t - \pi/3)$ | $x = \sqrt{3}\,A\sin(\omega t)$ |
| $x_1 = A\sin(\omega t)$ | $y_1 = A\cos(\omega t)$ | **Circle** $x_1^2 + y_1^2 = A^2$ |
| $x_1 = A\sin(\omega t)$ | $y_2 = A\sin(\omega t)$ | **Straight line** $x_1 = y_2$ |
| $x_1 = A\sin(\omega t + \pi/3)$ | $x_2 = A\cos(\omega t + \pi/6)$ | $x = \sqrt{3}\,A\cos(\omega t)$ |

Damped SHM :-

* $\frac{mdx^2}{dt^2} + K\vec{x} + \frac{bd\vec{x}}{dt} = 0$ $\quad \vec{F} = -b\vec{v}$ (air friction)
* $\frac{dx^2}{dt^2} + \omega^2 x + \frac{b}{m}\frac{d\vec{x}}{dt}$ $\quad A_0$ = initial Amplitude
* $Y = A_0 e^{-bt/2m}\sin(\omega t)$
* $A_t = A_0 e^{-bt/2m}$ $\quad A_t$ = Amplitude at "t"

$A_n = A_0\, e^{-kn}$ ← no. of oscillation

→ Amplitude after n-oscillation

Forced Oscillation (Resonance) :-

$F(t) = F_0\cos\omega_1 t$
$x(t) = A\sin\omega_2 t$
} "$\omega_1$" must be equal to "$\omega_2$"

$$\frac{mdx^2}{dt^2} + \frac{bdx}{dt} + Kx = F_0\cos\omega t$$

Joh aapka Force का Freq. होगा वो आपके Body के Freq. के बराबर होना चाहिए resonance के Liye.

Q. Amplitude becomes half in 4-oscillation then find amplitude after 16 - oscillation.

Sol. $A = A_0\, e^{-n}$ $\qquad A = A_0\, e^{-16}$

$\frac{A_0}{2} = A_0\, e^{-4}$ $\qquad = A_0\, e^{-4\times 4}$

$e^{-4} = \frac{1}{2}$ $\qquad = A_0\left(\frac{1}{2}\right)^4$

$e^{-4} = \frac{1}{2}$ $\qquad A = \frac{A_0}{16}$

MR
"Failure is the first step of success."

# 20 Wave Motion

Tum wave si ho main tumhara medium, Tum aati ho meri zindagi mein Mera use krti ho, main apne mean position ke about aage piche oscillate krta rehjata hu tumhare liye, aur tum Mera use krke aage nikal jaati ho...

WAVE

| Mechanical Wave | Non-Mech. Wave |
|---|---|
| Medium ✓ | Medium × |
| # Sound, Wave on string, wave on water. | # EM Wave i.e., Light Wave. |

### Mechanical Waves :-

1> Transverse Wave :-

Particle upar niche wave aage.

Ex. :- String Wave

2> Longitudinal Wave :-

Particle age-piche aur wave aage.

Ex. :- Sound.

### Equation of Propagating Harmonic Wave :-

$$Y = A \sin(\omega t \pm Kx \pm \phi)$$

SHM ek particle Ka oscill$^n$ hai wave hazaro lakho particle Ka oscillation hai !

A = Amplitude of particle.

$\omega$ = Ang. freq.

Angular wave no., $K = \dfrac{2\pi}{\lambda}$

wave no., $\bar{v} = \dfrac{1}{\lambda}$

$\phi$ = initial phase $[t = 0 \quad x = 0]$

- Longitudinal Wave :- ↕ ↕

Wave in "y" $y = A \sin[Ky + \omega t + \phi]$ Particle in "y"

Dono Same

- Transverse Wave :- ↔ ↔

Wave is moving in "y" $x = A \sin(Ky + \omega t)$ Particle oscillates in "x"

Dono different

$$y = A \sin 2\pi \left[\frac{x}{\lambda} + \frac{t}{T}\right]$$

# Velocity of Wave :-

$$V_{wave} = \frac{\omega}{K} = \frac{\lambda}{T} = \lambda f$$

1> $y = A \sin(Kx - \omega t)$ ek ⊕ dusra ⊖ !

Wave is moving in +x-axis.

2> $y = A \sin(Kx + \omega t)$ dono ⊕

Wave is moving in $-\underline{x}$-axis.

Note :- $V_p = A\omega \cos(Kx + \omega t)$

$(V_P)_{max} = A\omega$

### Relation Between wave Velocity & max Particle Velocity :-

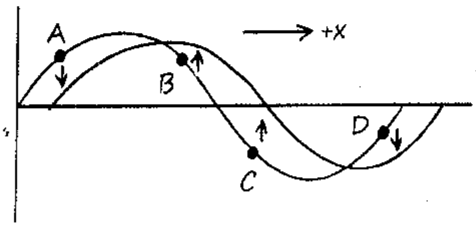
$*(V_P)_{max} = AKV_{wave}$.

### Estimation of Particle Whether IT'LL Go UP/DOWN :-

$$\boxed{V_{particle} = -V_{wave} \times \text{Slope of wave}}$$

MR*

*Na Tumhe Formula likhna na slope dhekhna direct answer bs wave jis direction mein travel Kr rahi hai uss direction mein thoda shift Krdo !*




Condition of Wave Eq. :-

$$\frac{d^2y}{dt^2} = \frac{\omega^2}{K^2}\cdot\frac{d^2y}{dx^2} \quad \Bigg| \quad \frac{d^2y}{dx^2} = \frac{d^2y}{dt^2}\cdot\frac{1}{V^2}$$

"y" → Finite hona chahiye at all position of "x".

$$y = f(ax + bt) = \frac{f}{(ax + bt)}$$

## IMP Relation :-

$$\boxed{\frac{2\pi}{\phi} = \frac{\lambda}{\Delta x} = \frac{T}{\Delta t}}$$

Q. The maximum particle velocity is 3 times the wave velocity of a progressive wave. If A is the amplitude of oscillating particle, find phase difference between two particles of separation x.

Sol. The maximum particle velocity = 3 time wave velocity

$$Aw = 3v$$

$$\therefore w = \frac{3v}{A}$$

As we know

$$v = \lambda f$$

$$\therefore \lambda = \frac{v}{f}$$

$$\text{or } \lambda = \frac{V}{\frac{W}{2\pi}} = \frac{2\pi A}{W} = \frac{2\pi v}{\frac{3v}{A}}$$

$$\therefore \lambda = \frac{2\pi A}{3}$$

$$\phi = \text{phase difference} = \frac{2\pi}{\lambda} \times x$$

$$= \left(\frac{2\pi}{\frac{2\pi A}{3}}\right)x$$

$$= \frac{3x}{A}$$

Q. The velocity of waves in a string fixed at both ends is 2 m/s. The string forms standing waves with nodes 5.0 cm apart. The frequency of vibration of the string in Hz is

Sol. Here the distance between the two nodes is half of the wavelength

$$\frac{\lambda}{2} = 5.0\text{cm} \Rightarrow \lambda = 10\text{cm}$$

$$\text{Hence } n = \frac{v}{\lambda} = \frac{200}{10} = 20\text{Hz}$$

## Velocity of T. Wave in A String :-

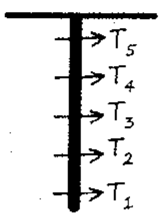
$$\text{Strain} = \frac{\Delta l}{l} = \alpha\Delta T$$

$\alpha$ = coefficient of liner expansion

$$V = \sqrt{\frac{\gamma\alpha\Delta T}{\rho}}$$

$\mu = M/L$   Y = Young Modulus

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{\text{Stress}}{\rho}} = \sqrt{\frac{\text{Y. Strain}}{\rho}}$$



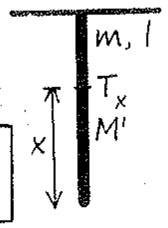

$V \propto \sqrt{T}$   $T_5 > T_4 > T_3 > T_2 > T_1$

Jaise-Jaise Upar jaoge

$T\uparrow \;\therefore\; V_{wave}\uparrow$

MR*

$m' = \mu x \quad T_x = m'g = \mu g x$

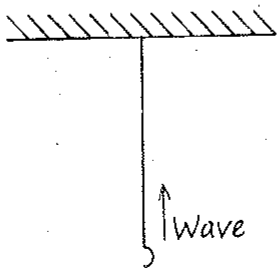



$$\boxed{V = \sqrt{\frac{T}{\mu}} = \sqrt{gx}} \leftarrow \boxed{\text{General Point}}$$

$\therefore$ V & a of wave at L/2 :-

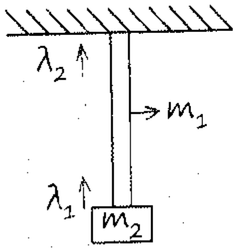
$$V = \sqrt{\frac{gL}{2}} \qquad \boxed{a = \frac{g}{2} = \text{Const}^n}$$

Time taken to reach top point




$$T = 2\sqrt{\frac{L}{g}} = \sqrt{\frac{4L}{g}}$$

Ratio of transvers wave at bottom to top




$V = \lambda f \rightarrow$ frequency same

$\lambda \propto V \propto \sqrt{\text{tension}}$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1 + m_2}}$$

## Sound Wave :-

- Speed of :-
  - 1> Sound :- $V_s > V_{liq} > V_{gas}$
  - 2> Light :- $V_{vacuum} > V_{gas} > V_{liq} > V_s$
- Range :-
  - Infrasonic Sound :- $f < 20$ Hz
  - Audible Sound :- $20 \text{ Hz} \le f \le 20$ KHz
  - Ultrasonic Sound :- $f > 20$ KHz

## Speed of Sound :-

$$V_s = \sqrt{\frac{Y}{\rho}} \qquad V_G \text{ or } V_L = \sqrt{\frac{\beta}{\rho}}$$

$Y$ = Young modules $\quad \beta$ = Bulk modules

- Newton's Formula :- (isothermal P.)

$$V_{G\,\&\,L} = \sqrt{\frac{P}{\rho}} \qquad \beta = P \quad V \doteq 280 \text{ m/s}$$

- Laplace's Correction :- (Adiabatic P.)

$$V_{G\,\&\,L} = \sqrt{\frac{\gamma P}{\rho}} \qquad \gamma = 1 + \frac{2}{f}, \beta = \gamma P$$

$$V = \sqrt{\frac{\gamma RT}{M}} \qquad V \propto \sqrt{T} \propto \frac{1}{\sqrt{M}}$$

Note :-

$V_{moist\ air} > V_{dry\ air}$

$\rho_{moist\ air} < \rho_{dry\ air}$

$V_{rms} > V_{sound\ in\ gas'}$

**Speed of sound in gas varies with temperature. Let speed of sound is $V_o$ at 0°C. Speed of sound is $V_t$ at T°C. Then find the relation between them**

$$V_t = V_o\left[1 + \frac{t}{546}\right] \Big\} \text{ Sab °C mein Chalega}$$

$$\Delta V = \frac{V_o t}{546} = 0.61t$$

Change in Velocity of Sound is $\boxed{0.61 \text{ m/s}}$ per unit raise in temperature i.e. $\boxed{0.18\%}$

**Sound Wave Travel Due to Pressure & Density Variation. :-**

| | Compression | Rarefraction |
|---|---|---|
| $P \rightarrow$ | Max | Min |
| $\rho \rightarrow$ | Max | Min |

$S \rightarrow$ Min  Max

$\phi$ between disp$^m$ (s) & Pressure wave in sound wave is $\pi/2$.

## Relation Between Pressure Amplitude & Displacement Amplitude :-

$$\Delta P = \beta A K$$

$\Delta P \rightarrow$ P. amplitude

$\beta \rightarrow$ Bulk Modulus

$A \rightarrow$ Disp$^m$ amplitude

$K \rightarrow$ Angular wave no.

## Intensity of Wave :-

$$I = \frac{E}{At} = \frac{1}{2}\rho V A^2 \omega^2$$

$$I = \frac{J}{m^2 sec} = \frac{watt}{m^2}$$

## Energy Density of Wave :-

$$U = \frac{Energy}{Volume} = \frac{I(Intensity)}{V(Velocity)} = \frac{1}{2}\rho A^2 \omega^2$$

Where, $I = \frac{1}{2}\rho V A^2 \omega^2$

MR* $\rightarrow$ direction se feel karo

## Intensity Variation Due to Diff. Sources :-

| Point | Linear | Planar |
|---|---|---|
| $I \propto \frac{A^2}{r^2}$ | $I \propto \frac{A^2}{r}$ | $I \propto r^0$<br>$A \propto r^0$ |

## Loudness of Sound Wave :-

$$I_0 = 10^{-12}\ \frac{Watt}{m^2}$$

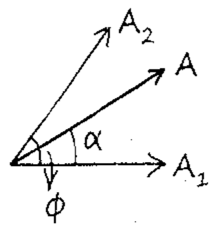
$$L = 10 \log_{10}\left(\frac{I}{I_0}\right) dB \quad 1 Bel = 10\ dB$$

MR*

$$\Delta L = 10 \log_{10}\left(\frac{I_f}{I_i}\right) = 10 \log_{10}\left(\frac{I_{max}}{I_{min}}\right)$$

## Principle of Superposition of Waves :-

$$\tan \alpha = \frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi}$$




$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \phi$$

$$I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} \cos \phi$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2} = \frac{\left(\frac{A_1}{A_2} + 1\right)^2}{\left(\frac{A_1}{A_2} - 1\right)^2}$$

## Interference :-

Aaise doh wave Ka superposition jiska $\lambda$ & f same hai !

| Constructive | Destructive |
|---|---|
| $A_{max} = A_1 + A_2$ | $A_{min} = A_1 - A_2$ |
| $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ | $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ |
| $\cos \phi = 1$ | $\cos \phi = -1$ |
| $\phi = 0, 2\pi, 4\pi, 6\pi,$ | $\phi = \pi, 3\pi, 5\pi, 7\pi,$ |
| $\phi = 2n\pi$ | $\phi = (2n+1)\pi$ |
| $\Delta x = 0, \lambda, 2\lambda, 3\lambda,$ | $x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$ |
| $\Delta x = n\lambda$ | $\Delta x = (2n+1)\frac{\lambda}{2}$ |

Two wave same amplitude $A_0$ :-

| Constructive | Destructive |
|---|---|
| $A_{max} = 2A_0$ | $A_{min} = 0$ |
| $I_{max} = \left(2\sqrt{I}\right)^2 = 4I$ | $I_{min} = 0$ |

Q. The intensity ratio of the two interfering beams of light is $\beta$. What is the value of $[(I_{max} - I_{min}) / (I_{max} + I_{min})]$?

(a) $\sqrt{\beta}$ (b) $2\sqrt{\beta} / (1 + \beta)$

(c) $2 / (1 + \beta)$ (d) $(1 + \beta) / 2\sqrt{\beta}$

Sol. $\Rightarrow \dfrac{2\sqrt{\beta}}{1+\sqrt{\beta}}$

Apply MR*

If. $\beta = 1$

then $\dfrac{I_{max}-I_{min}}{I_{max}+I_{min}} = 1$

## Reflection of T. Wave in String :-

(String और Sound का Same hai)

(a) Rarer $\longrightarrow$ Denser :-

| | Reflected | Transmited |
|---|---|---|
| f | f | f |
| Speed | Same | U↓ |
| Wavelength | Same | λ↓ |
| Phase diff. | $\pi$ | 0 |

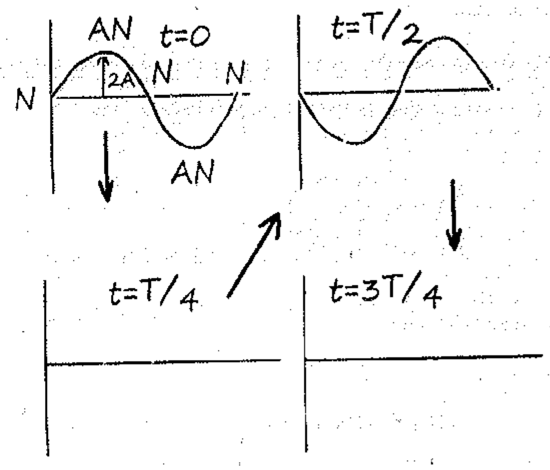
(b) Denser $\longrightarrow$ Rarer :-

| | Reflected | Transmited |
|---|---|---|
| f | f | f |
| Speed | Same | U↑ |
| $\lambda$ | Same | λ↑ |
| $\phi$ | 0 | 0 |

## Stationary Wave :-

Aaise doh wave ka superposition jiska sab Kuch same hoga bs direction opposite

1> $Y = 2A \sin(Kx) \cos(\omega t)$




| Position of :- | |
|---|---|
| Node | Antinode |
| $\phi = Kx = n\pi$ | $\phi = Kx = (2n+1)\dfrac{\pi}{2}$ |
| $x = \dfrac{n\lambda}{2}$ | $x = (2n+1)\dfrac{\lambda}{4}$ |
| integral | odd |

Different equation of Stationary Wave :-

$Y = 2A \sin(Kx) . \cos(\omega t)$

$Y = 2A \cos(Kx) . \sin(\omega t)$

$Y = 2A \sin(Kx) . \sin(\omega t)$

$Y = 2A \cos(Kx) . \cos(\omega t)$

Q. If node is formed at origin then amplitude which is at between node and antinode.

Sol. $x = 2A \sin(kx)$, $x = \dfrac{\lambda}{8}$

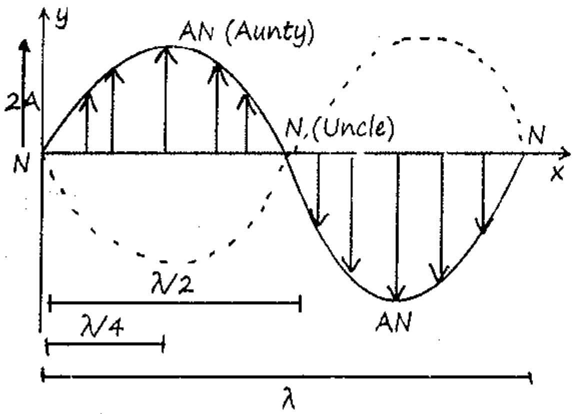
$= 2A \sin\left(\dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{8}\right)$

$= 2A \sin\dfrac{\pi}{4} = \sqrt{2}A$

## Difference Between Stationary & Progressive Wave :-

| Stationary wave | Progressive wave |
|---|---|
| 1> Particle at Node → rest | 1> No Particle at rest. |
| 2> No Transfer of E & P. | 2> Transfer of E & P occurs. |
| 3> Sabka "A" diff. | 3> "A" same. |
| 4> All in same phase between N. | 4> All are in diff. phase |
| 5> All particles cross MP with diff. speed at same time. | 5> All particles cross MP with same speed at diff. time. |

Stationary Wave :-




$$y = \underbrace{2A \sin (Kx)}_{\text{Amplitude}} \underbrace{\cos (\omega t)}_{\text{SHM.}}$$

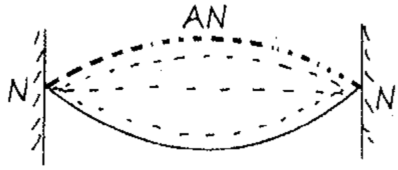
## Formation of Stationary Wave in String Sonometre Wire :-

$$f' = \frac{V}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{T}{\rho A}}$$

$$f' = \frac{1}{2L}\sqrt{\frac{\text{Stress}}{\rho}}$$

(a) Fundamental or $1^{st}$ harmonic :-

$$f' = \frac{V}{\lambda} = \frac{V}{2L}$$

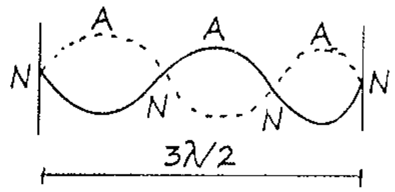



(b) $2^{nd}$ harmonic or $1^{st}$ overtone :-

$$f = \frac{V}{\lambda} = \frac{V}{L}$$
$$= 2f'$$


(c) $3^{rd}$ harmonic or $2^{st}$ overtone :-

$$f = \frac{V}{\lambda} = \frac{3V}{2L}$$
$$f = 3f'$$




Vimp

$n$ - harmonics or $(n-1)$ overtone :-

$$f = n\left(\frac{V}{2L}\right)$$

No. of AN = $n$

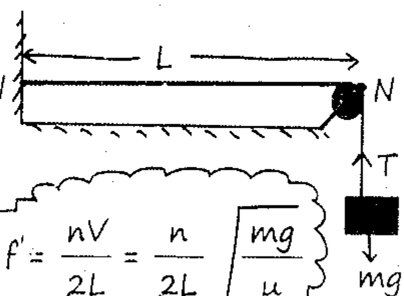
No. of N = $n + 1$

Difference between any two consecutive harmonics ;-

$$\Delta f = \frac{V}{2L} = f$$ NEET

- Sonometer Wire :-

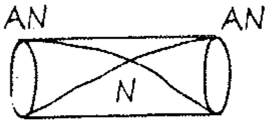
$\mu$ = linear mass density of wire.




$$f' = \frac{nV}{2L} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}$$

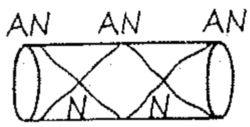
## Open Organ Pipe :-

(a) Fundamental Freq. or $1^{st}$ harmonic :-



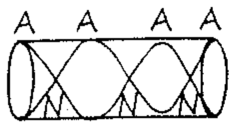

$$f' = \frac{V}{2L}$$

(b) $2^{nd}$ harmonic or $1^{st}$ overtone :-




$$f = 2f'$$

(c) $3^{rd}$ harmonic or $2^{nd}$ overtone :-




$$f = 3f'$$

$$*\ f = \frac{nV}{2L} = \frac{n}{2L}\sqrt{\frac{\gamma P}{\rho}} = \frac{n}{2L}\sqrt{\frac{\gamma RT}{M}}$$

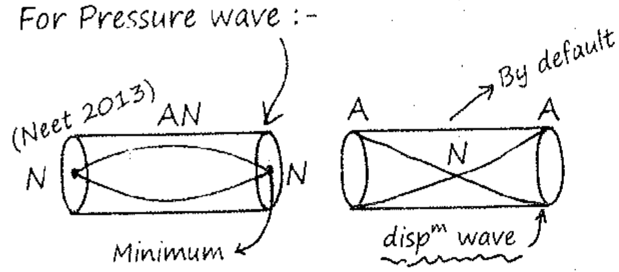
- $n$ = harmonic  $(n-1)$ = overtone

  No. of N = $n$  No. of AN = $n + 1$

Vimp integral

$$f_1 : f_2 : f_3 = 1 : 2 : 3 : \ldots$$

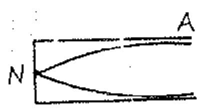
For Pressure wave :-




∴ Disp$^m$ wave & Pressure wave have $\phi = \pi/2$.

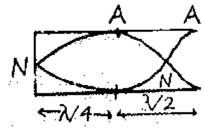
## Closed Organ Pipe :-

(a) Fundamental or $1^{st}$ harmonic :-



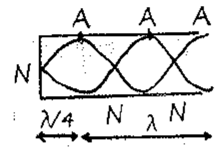

$$f' = \frac{V}{\lambda} = \frac{V}{4L}$$

(b) $3^{rd}$ harmonic or $1^{st}$ overtone :-




$$f_2 = \frac{3V}{4L} = 3f'$$

(c) $5^{th}$ harmonic or $2^{nd}$ overtone :-




$$f_3 = \frac{5V}{4L} = 5f'$$

- $(2n + 1)$ = Hormonic  $n$ = Overtone

$$*\ f = (2n+1)\frac{V}{4L} *$$  $f_1 : f_2 : f_3 = 1:3:5$

Difference between any two consecutive harmonics :-

$$\Delta f = 2f' = \frac{2V}{4L}$$ **NEET**

The MR*

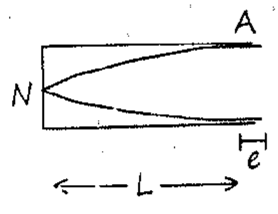
Frequency Ka ratio likhdo aur uss ratio mein dhekho Konse wave form Ki baat chalrahi jiss wave form Ki baat chal rahi hogi wahi tumhara N & AN hoga.

$$f_1 : f_3 : f_5 : f_7 = 1 : 3 : 5 : 7$$

| 5 | 7 |
|---|---|
| $2^{nd}$ overtone | $3^{rd}$ overtone |
| 3 wave form | 4 wave form |
| 3N & 3AN | 4N & 4AN |

## End Correction :-

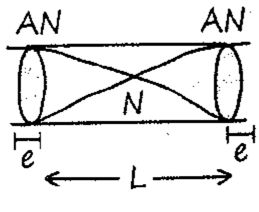
Closed OP :-




$$f = \frac{(2n+1)V}{4(L+e)}$$

$e = 0.6\ r.$

Open OP :-




$$f = \frac{nV}{2(L+2e)}$$

$e = 0.6\ r.$

Q. The length of an open organ pipe is twice the length of another closed organ pipe. The fundamental frequency of the open pipe is 100 Hz. The frequency of the third harmonic of the closed pipe is.

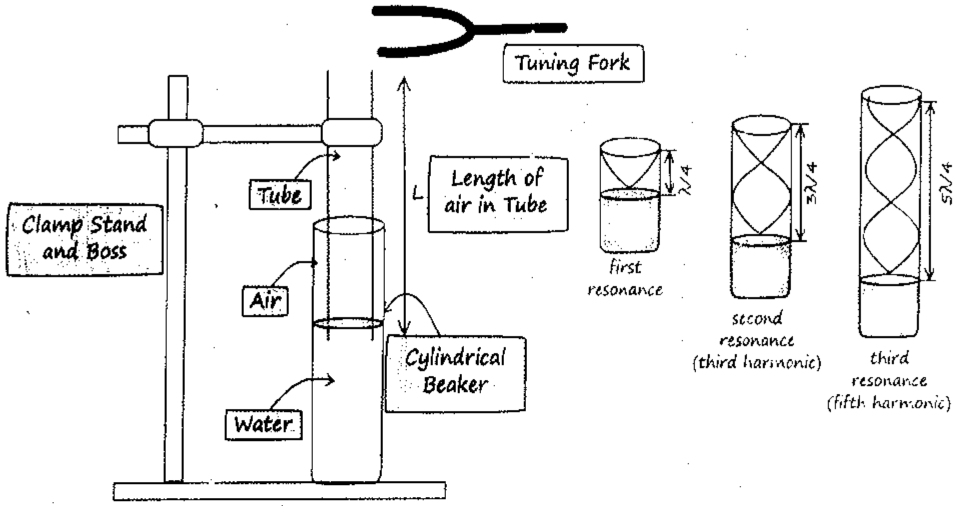
Sol. The fundamental frequency of open pipe

$= f_o = \frac{V}{2l} = 100Hz$

Hence the third harmonic of closed organ pipe is $\frac{3V}{4(l/2)}$

$= 300Hz$

## Resonance Tube :-




$$f_1 = \frac{V}{4L_1} = f_T \qquad f_2 = \frac{3V}{4L_2} = f_T$$

"Resonance"

NEET $\frac{V}{4L_1} = \frac{3V}{4L_2}$ $\quad$ $L_2 = 3L_1$

o End Correction using resonance Tube :-

$$e = \frac{l_2 - 3l_1}{2}$$

## Doppler Effect :-

There is apparent change in frequency due to relative motion of source and observer.

Valid in sound and electromagnetic wave also.

Doppler effect is observed when

(i) The source is moving observer is stationary.

(ii) The observer is moving source at rest.

(iii) Both are moving.

Condition when doppler effect will not occur:

(i) When both are at rest.

(ii) When both are moving with same velocity.

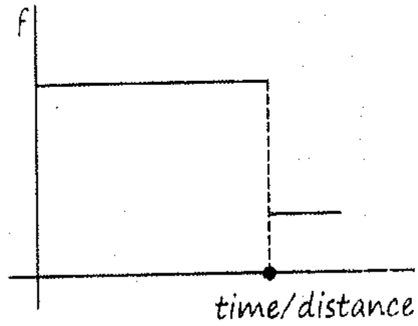
(iii) Both are moving exactly perpendicular to each other.

(iv) When they are moving greater than speed of wave.

(v) When once is moving on circular-path and other is exactly at centre.

(vi) Does not depends on distance b/w them.

Q. Man is standing and source is crossing him then graph of frequency observed with distance or time.




Sol. Frequency only depends on relative velocity does not depends on distance and time.

The MR*

(The pro-version of Mr*)

1>


$$f' = f_0\left(\frac{V - V_0}{V - V_S}\right)$$

2>


$$f' = f_0\left(\frac{V - V_0}{V + V_S}\right)$$

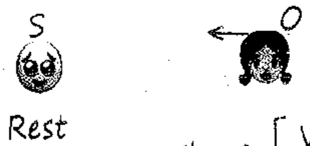
3>


$$f' = f_0\left(\frac{V + V_0}{V + V_S}\right)$$

4>


$$f' = f_0\left(\frac{V + V_0}{V - V_S}\right)$$

5>




$$f' = f_0\left[\frac{V + V_0}{V}\right]$$

6>

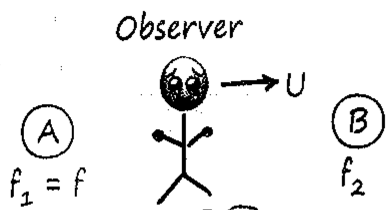

$$f' = f_0\left[\frac{V}{V - V_S}\right]$$

7>


$$f' = f_0\left[\frac{V}{V + V_S}\right]$$

8> IMP Case :-

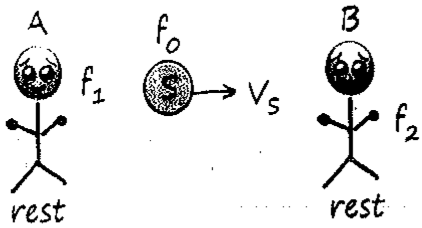
(a)




$f_1 = f$ $\qquad$ $f_2$

$$\Delta f = \frac{2f_0U}{V}$$

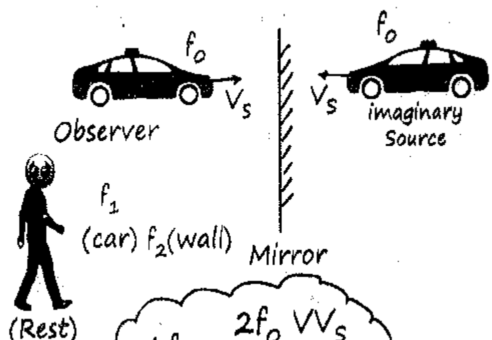
(b)




$$\Delta f = \frac{2f_0 VV_S}{(V^2 - V_S^2)}$$

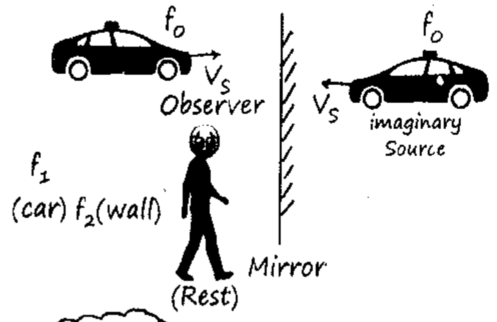
9> Car is Moving Towards Stationary Wall :-

(a) Man is standing behind CAR




$$\Delta f = \frac{2f_0 VV_S}{(V^2 - V_S^2)}$$

III (b) Man is standing b/w CAR and wall at rest.




$$\boxed{\Delta f = 0}$$

The MR*

(The pro-version of MR*)

10> When medium is moving opposite to the direction of sound :-

$V_{sound} = V - V_M$

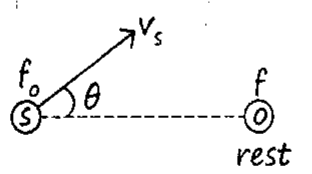
$$f = f_o\left(\frac{V - V_M}{V - V_M}\ \frac{V_o}{V_s}\right)$$ Baki Sign dheklena

11> When medium is moving in direction of sound :-

$V_{sound} = V + V_M$

$$f = f_o\left(\frac{V + V_M}{V + V_M}\ \frac{V_o}{V_s}\right)$$ Baki Sign dheklena

12>




$$f = f_o\left(\frac{v}{v - v_s\cos\theta}\right)$$

Beats!

Aaisi doh wave jinka "A" same hai lekin "f" slightly different. $\Delta f < 10$.

$$Y = \underbrace{2A}_{\text{Amplitude}}\left[\underbrace{\sin\left(\frac{\omega_1 + \omega_2}{2}t\right)}_{\text{oscill}^n} \cdot \underbrace{\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)}_{\text{oscill}^n}\right]$$

Ye Amplitude Ko decide Karega !

Angular freq of A $= \dfrac{\omega_1 - \omega_2}{2}$

Angular freq of I $= \omega_1 - \omega_2$

Angular beat freq $= \omega_1 - \omega_2$

Vimp $\boxed{f_B = f_1 - f_2}$

o Tuning Fork :-

Sharp :- f↑

Waxing :- f↓

Waxing :- $\dfrac{I_{max}}{I_{min}} = \dfrac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$

Waning :-

Q. Two tuning forks when sounded together produced 4 beats/sec. The frequency of one fork is 256. The number of beats heard increases when the fork of frequency 256 is loaded with wax. The frequency of the other fork is

Sol. $f_A - f_B = 4Hz$

$f_A = 256$ Hz → 260Hz

$f_B = ??$ → 252Hz (Possible)

In waxing of A $f_A$ will decrease but frequency difference is increasing hence 260 Hz will be answer.

Q. A tuning fork A produces 2 beats per second when sounded with a tuning fork of frequency 200Hz. When A is loaded with wax the beats stop. What is the frequency of fork A?

Sol. There are 2 beats per second

$\therefore f_1 - f_2 = 2$

$\Rightarrow 200 - f_2 = \pm 2$

$f_2 = 200 \pm 2$

$f_2 = 202$ or $f_2 = 198$

When loaded with wax the beats stop since the frequency decreases on loading.

$\therefore f_2 = 202$ Hz

# 21 Electrostatics

| Charge | E.F. | MF | EM Wave |
|---|---|---|---|
| Rest | ✓ | ✗ | ✗ |
| $V = Const^n$ | ✓ | ✓ | ✗ |
| Accelerated | ✓ | ✓ | ✓ |

1> Properties of Charge :-

- Scalar, Conserved, Quantized

  $Q = ne$, $n$ = integer

- Invariant does not depends upon speed
- Two type; Positive charge → deficiency of electron, Negative charge → excessness of electron
- Charge can't exist without mass but mass can exist without charge.
- Same charge → repel (may attract)
- Opposite charge → must attract
- Sure check of charge body is repulsion
- SI unit 1 C = $3\times10^9$ esu. 1C = $\frac{1}{10}$ emu.
- Smallest unit frankline = 1 esu
- Largest unit faraday 1 faraday = 96500 C
- One charge may attract other Neutral

Quarks :- Does not exist in free state

| | |
|---|---|
| up | $+\frac{2e}{3}$ |
| down | $+\frac{1}{3}e$ |

Q. Which of the following charge is possible:

(a) $\frac{1C}{100}$ (b) $\frac{1e}{50}$

(c) $4.8\times10^{-21}C$ (d) $1.56\times10^{-18}C$

Ans. Use $Q = ne$ (a) is possible

2> Charge को Sharp Point पसंद है!

3> Charging of Body by conduction :-

For conductor only

$Q \propto R$.

$$\text{New Charge} = \frac{\text{Uska } R}{\text{Total } R}[Q_{Total}]$$

$$Q_1' = \frac{R_1}{R_1 + R_2}[Q_1 + Q_2]$$

4> Charging of body by friction :-

For insulator only equal and opposite charge on two rubbing object

5> Charging by induction :-

For conductor and dielectric → equal or lesser charge of positive nature induced.

6> Charge density :-

| Linear object | Areal object | Volumetric object |
|---|---|---|
| $dq = \lambda dl$ | $dq = \sigma dA$ | $dq = \rho dV$ |
| $\lambda = \frac{dq}{dl}$ c/m | $\sigma = \frac{dq}{dA}$ c/m$^2$ | $\rho = \frac{dq}{dv}$ c/m$^3$ |
| Linear charge density | Areal charge density | Charge density |

7> Gold leaf experiment :-

- Device used to detect charge, not use to measure charge.
- Method involves conduction or induction.
- Diverge angle $\propto$ charge of leaf.

8> Coulomb's Law :-

- Valid for point or spherical charge symmetry
- Conservative, long range, follow inverse square law $\left(F \propto \frac{1}{r^2}\right)$, central, mediated by photon.

$$F = \frac{Kq_1q_2}{r^2} \qquad \varepsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{Nm^2}$$

$$K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{Nm^2}{C^2}$$ Mag:- Formula से

Dir$^n$:- Buddhi से.

* Another Medium between Charges :-

$$K = \varepsilon_r = \frac{\varepsilon_m}{\varepsilon_0} \quad \left(F' = \frac{F_0}{K}\right) \text{ dec. by K.}$$

- Electrostatic force on $q_2$ due to $q_1$ does not depends upon medium or presence of other charge → True.
- Net electrostatics force on $q_2$ due to $q_1$ depends upon medium or presence of other charge → True.

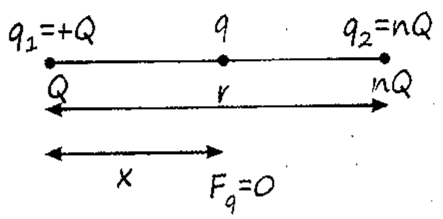
Q. Two identical charge 'q' repel each other with a force of 100 N, one of the charge is increased by 10% ond decreased by 10% then new force of repulsion at the same distance?

Ans. $q_1' = 110\%q,\ q_2' = 90\%q$

$$F = \frac{Kq_1q_2}{r^2},\ F' = \frac{Kq_1'q_2'}{r^2} \Rightarrow F' = 99N$$

9> Neutral Point :-

Find position of 3$^{rd}$ charge where force on that charge will be zero




Like Charge

$$x = \frac{d}{\sqrt{n}+1}$$

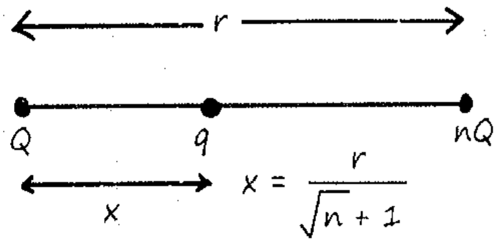
Unlike Charge

$$x = \frac{d}{\sqrt{n}-1}$$

$$n = \frac{\text{Bada charge}}{\text{Chota charge}}$$

10> If we Divide charge Equally, they repel each other with $F_{max}$.

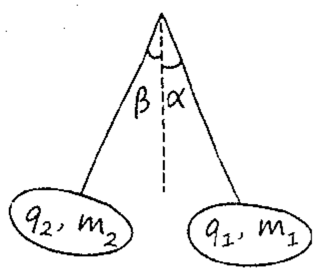
11> Find Position & Value of "q" so that System will be in Equilibrium :-




$$x = \frac{r}{\sqrt{n}+1}$$

$$q = \frac{-nQ}{(\sqrt{n}+1)^2}$$

12> Pendulum Problem :-



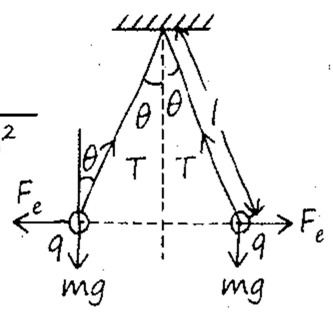

- Value of α and β depends on charges and masses but ratio of α and β does not depends on charges and depends only on masses

$$\frac{\tan\alpha}{\tan\beta} = \frac{m_2}{m_1}$$

- If $m_1 = m_2$ then $\alpha = \beta$
- If $m_1 > m_2$ then $\alpha < \beta$

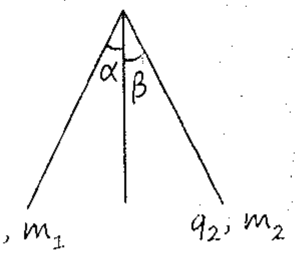
$$\tan\theta = \frac{F_e}{mg}$$

$$T = \sqrt{F_e^2 + (mg)^2}$$



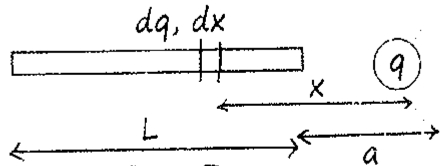
Q. If $q_1 > q_2$ but $m_1 = m_2$ then which of the following is correct?

(a) $\alpha = \beta$ (b) $\alpha > \beta$ (c) $\alpha < \beta$




Ans. (a)

13> Force on Rod Due to Point Charge :-




$$F = \frac{Kq(\lambda L)}{a(a+L)}$$

14> Coulomb's Law in Vector Form :-

$$\vec{F}_{21} = \frac{Kq_1q_2}{|\vec{r}_{21}|^2}\hat{r}_{21} = \frac{Kq_1q_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

15> Electric Field Intensity :-

Electrostatic force per unit positive charge.

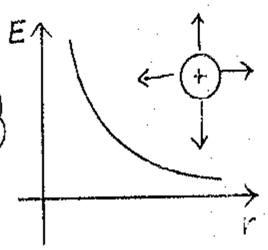
$$\vec{E} = \frac{\vec{F}}{q} = N/C. \quad \text{Vector !}$$

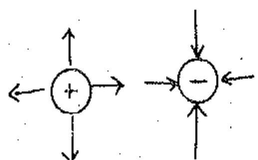
E → +ve से dur
→ −ve के Pass

F → on + = along $\vec{E}$
→ on − = opposite $\vec{E}$

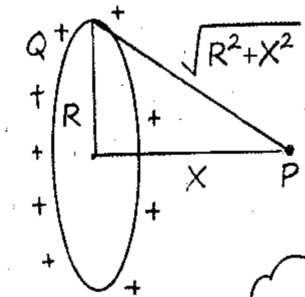
(A) Point charge :-

$$E = \frac{KQ}{r^2}$$



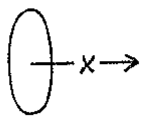
(B) Ring :-

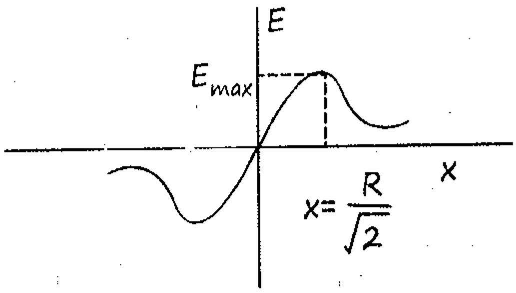



$$E_{max} = \frac{\text{Radius}}{\sqrt{2}}$$

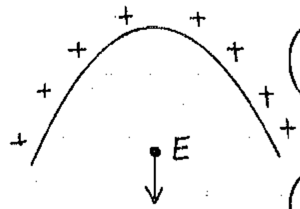
$$E_P = \frac{KQx}{(R^2+X^2)^{3/2}}$$

- Graph B/w electric field and distance



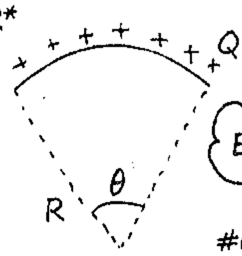

(C) Half ring :-




$$E = \frac{2K\lambda}{R}$$

$$E = \frac{2KQ}{\pi R^2}$$

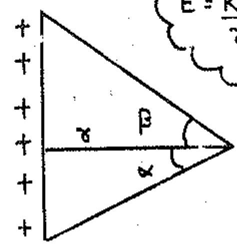
MR*




$$E = \frac{2K\lambda}{R}\sin(\theta/2)$$

#Garda Visualisat$^n$.

16> Electric Field Due to Line Charge :-




$$E = \frac{K\lambda}{r}\left[(\sin\alpha + \sin\beta)\hat{i} + (\cos\alpha - \cos\beta)\hat{j}\right]$$

(a) $\infty$ – line :- $E = \dfrac{2K\lambda}{r}$

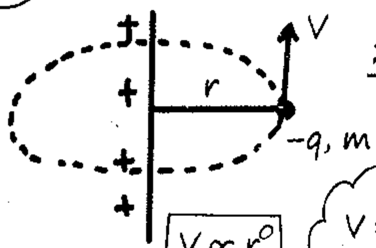
(b) Semi- $\infty$ line :- $E = \dfrac{K\lambda}{r}\left(\hat{i}-\hat{j}\right)$

(c) Finite line :- $E = \dfrac{K\lambda}{r}[(\sin\alpha + \sin\beta)\,\hat{i} + (\cos\alpha + \cos\beta)\,\hat{j}]$

(d) Finite line at 'r' when $(\alpha=\beta)$ :-

$$E = \frac{2K\lambda}{r}\sin\alpha\hat{i}$$

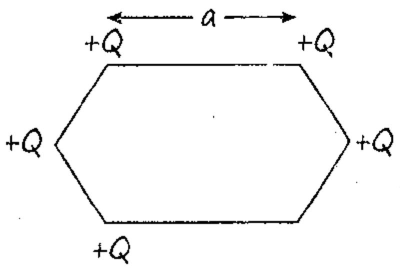
IMP Case  Charge $(-q,\ m)$ moving around infinite line charge with speed V.




$$\frac{2K\lambda}{r}q = \frac{mv^2}{r}$$

$\boxed{V \propto r^0}$  $V = \sqrt{\dfrac{2K\lambda q}{m}}$

Q. Find electric field at centre ?




Ans. $E = \dfrac{KQ}{a^2}$

MR* Put +Q and –Q charge on the corner, where no charge is present.

17> Motion of Charged Particle in Uniform E.F. :-

1> Charge is drop/released :-

$$a = \frac{qE}{m} \quad V = \frac{qEt}{m} \quad P = qEt$$

$$S = \frac{1}{2}\frac{qEt^2}{m} \quad KE = \frac{1}{2}\frac{q^2E^2t^2}{m}$$

$P^+ = e, m$. Deuteron = e, 2m.

$\alpha$-particle = 2e, 4m.

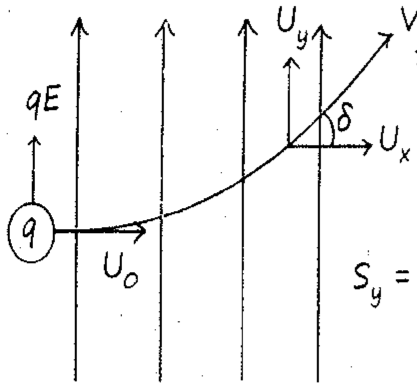
2> Charge is projected with "$U_o$" in dir$^n$ of E.F. :-

$$a = \frac{qE}{m} = \text{Const}^n \quad V = U_o + \frac{qE}{m}t$$

$$\Delta P = qEt$$

$$S = U_o t + \frac{1}{2}\frac{qEt^2}{m} \quad P_t = m\left[U_o + \frac{qEt}{m}\right]$$

3> Charge is projected $\perp^{er}$ to E.F. :-




$$\tan\delta = \frac{qEd}{mU_o^2}$$

$$S_y = \left[\frac{qE}{2mU_o^2}\right]d^2$$

Q. Proton, Deutron and $\alpha$-particle projected in electric field perpendicular to it, then find ratio of deviation if:

(i) They projected with same speed

$\delta_P : \delta_D : \delta_\alpha = 2 : 1 : 1$ $\boxed{\delta \propto q/m}$

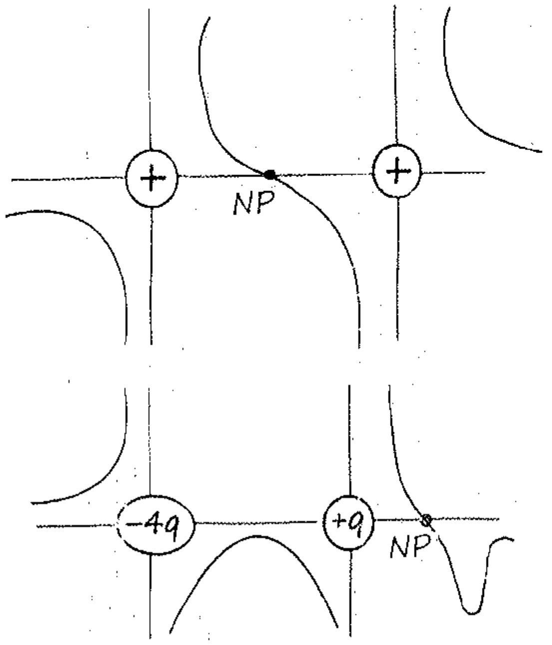
(ii) They are projected with same K.E.

$\delta_P : \delta_D : \delta_\alpha = 1 : 1 : 2$ $\boxed{\delta \propto q}$

(iii) Projected with same momentum

$\delta_P : \delta_D : \delta_\alpha = 1 : 2 : 8$ $\boxed{\delta \propto qm}$

18> Graph of EF with R due to Combination of Point Charge :-




19> Electric Dipole :-

Two equal and opposite charge placed at small distance.

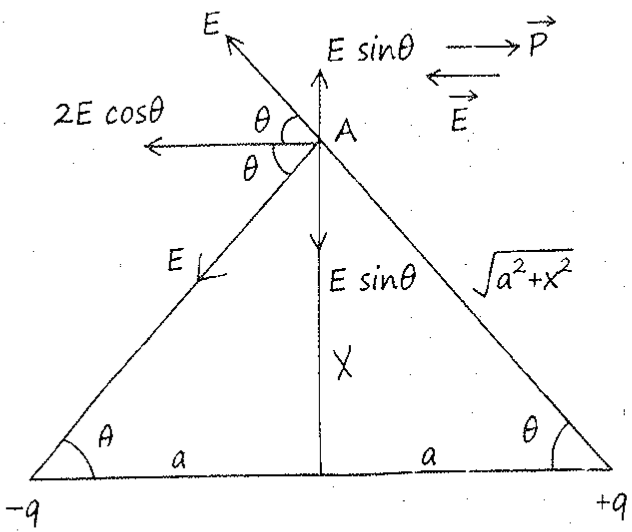
Unit = C.m, Net charge = 0, Ideal dipole = small dipole

$$\vec{P} = \begin{bmatrix}\text{Mag. of} \\ \text{Q.}\end{bmatrix} \times \begin{bmatrix}\text{Dist}^n\text{ between} \\ \text{two charge}\end{bmatrix}$$

Direction of dipole moment -ve to +ve charge.

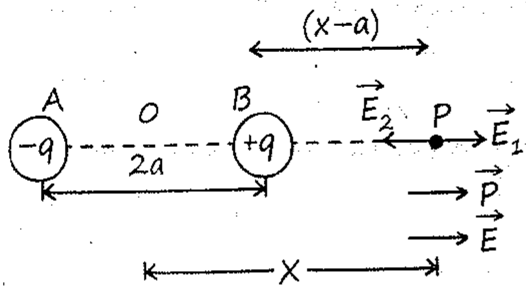
(a) Equitorial line :-

$$\vec{E} = \frac{-K\vec{P}}{(a^2+x^2)^{3/2}} \qquad \vec{E} = \frac{-K\vec{P}}{x^3}$$




(b) Axial line :-

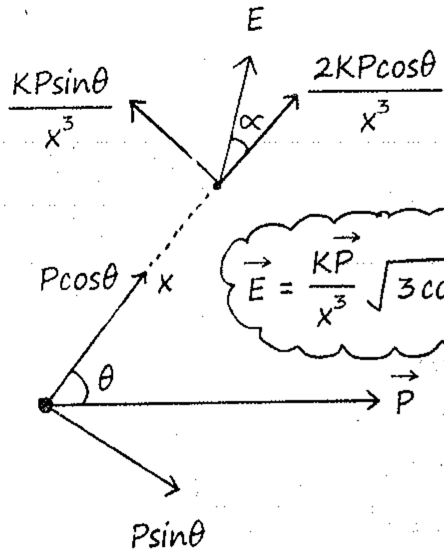
$$\vec{E} = \frac{2K\vec{P}x}{(a^2-x^2)^2} \qquad \vec{E} = +\frac{2K\vec{P}}{x^3}$$




$$\therefore \boxed{\frac{E_{equi}}{E_{axial}} = \frac{1}{2}}$$

Axial पर E.F. Same line mein hogi aur E.F. equitorial line pr $\perp^{er}$ hogi aur dipole Ke opposite hogi.

(c) θ from ideal dipole




$$\vec{E} = \frac{K\vec{P}}{x^3}\sqrt{3\cos^2\theta + 1}$$

$\therefore$ Angle between $\vec{E}$ & $\vec{P}$ = $\theta + \alpha$

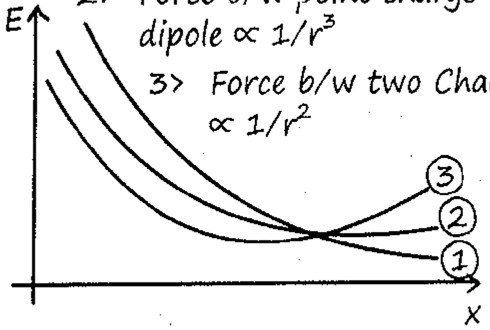
$$\tan\alpha = \frac{KP\sin\theta/x^3}{2KP\cos\theta/x^3} = \frac{\tan\theta}{2}$$

$\theta = \tan^{-1}(\sqrt{2})$ The Angle from dipole at which $\vec{E} \perp \vec{P}$

1> Force b/w two dipole $\propto 1/r^4$

2> Force b/w point charge & dipole $\propto 1/r^3$

3> Force b/w two Charge $\propto 1/r^2$




*Jiska Power ↑ woh sabse niche?

20> Electric oscillate in electric field :-

$\vec{\tau} = \vec{P} \times \vec{E} = PE \sin\theta$ Vector !

$\theta$ = Angle between E & P.

Torque hamesha dipole को E.F. की $dir^n$ में Align करना चाएगा!

$$T = 2\pi\sqrt{\frac{I}{PE}}$$

$I$ = Moment of Inertia

$T$ = Time period of oscillation

$P$ = Electric dipole moment

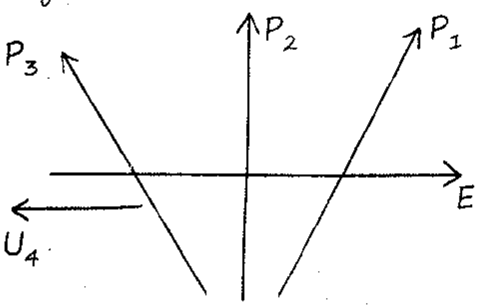
$E$ = Electric field.

- Electric field on the axis of electric dipole is always parallel to electric dipole moment → False
- Electric field on the equitorial line of dipole is anti-parallel to dipole → True

21> Electrostatic P.E. Stored in Dipole in Uniform E.F. :-

**Scalar.**

$U_\theta = -\vec{P}\cdot\vec{E} = -PE\cos\theta.$




$U_4 > U_3 > U_2 > U_1$

| $\theta = 0^\circ$ | $\theta = 90^\circ$ | $\theta = 180^\circ$ |
|---|---|---|
| $\vec{P} \parallel^{el} \vec{E}$ | $\vec{P} \perp \vec{E}$ | $\vec{P}_{anti} \parallel^{el} \vec{E}$ |
| $F = 0$ | $F = 0$ | $F = 0$ |
| $\tau = 0$ | $\tau = PE$ ⊗ | $\tau = 0$ |
| $U_{min} = -PE$ | $U = 0$ | $U_{max} = PE$ |

- Work Done to Rotate dipole $W = \Delta U = U_f - U_i$
- Work done by E.F. to rotate dipole $= w = -\Delta U$

*Special Case :-

- **Work Done to Rotate from Stable to Unstable :-** $U = 2PE$.

अगर जहासे Rotate किया वही पर वापस लाया तो W.D. = 0!

22> Electric Flux :-

$\phi = \text{flux} = \vec{E}\cdot\vec{A} = EA\cos\theta$

$\theta$ = Angle between $\vec{E}$ & $\vec{A}$

→ Gives the idea of electrostatic energy passing through given area.

→ Counting of field lines passing through given crosss-section area

→ Scalar

→ Unit (Volt - meter) & ($Nm^{+2} C^{-1}$)

→ $d\phi = \oint \vec{E}\cdot d\vec{A}$ variable electric field.

Flux :- Aaisi line joh area Ko aarpar ched banakr jarahi hai !

3-D body :- $\phi_{in}$ = −ve $\phi_{out}$ = +ve $\phi_{Total} = \phi_{in} + \phi_{out}$

Uniform E.F. :- $(\phi_{Total})_{close\ surface} = 0$.

Q. A charge Q placed on the cornor of square plate of side 'L' then find flux throug that square plate → zero.

23 Gauss Law :-

$$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

$\phi$ = सिर्फ inside charge से आएगा

E = inside और outside दोनों से !

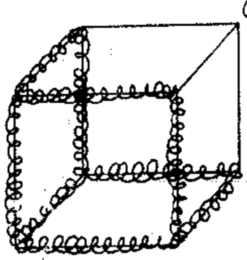
→ Flux from close surface does not depends shape, size of surface and location of charge inside surface and charge outside the surface.

→ Always valid

→ Only applicable to calculate electric field for symmetric charge distribution

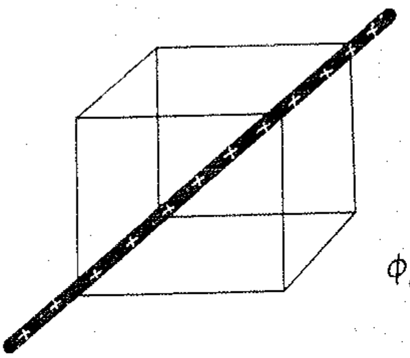
o Special Case :-

1> Flux through faces not touching the corner charges :-




$$\phi = \frac{1}{3}\left[\frac{Q}{8\varepsilon_0}\right]$$

2>



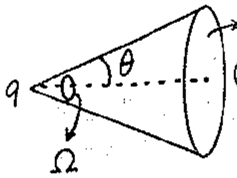
$$\phi_{max} = \frac{\sqrt{3}\, l\lambda}{\varepsilon_0}$$

$$\phi_{cube} = \frac{\lambda l}{\varepsilon_0}$$

Aaise Question mein pahile length nikalo, charge aistribution dekho

$\phi = \dfrac{\lambda l}{\varepsilon_0}$ lagado !

3> Relation between plane angle & solid angle :-



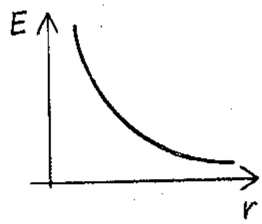

$$d\Omega = 2\pi(1-\cos\theta)$$

$$\boxed{\phi = \frac{q}{2\varepsilon_0}(1-\cos\theta)}$$

Application of gauss law :-

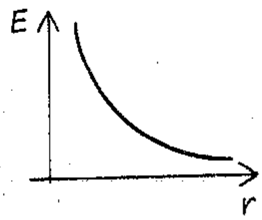
1> E.F. Due to point charge :-

$E = \dfrac{KQ}{r^2}$




2> Infinite line charge :-

$E = \dfrac{2k\lambda}{r}$

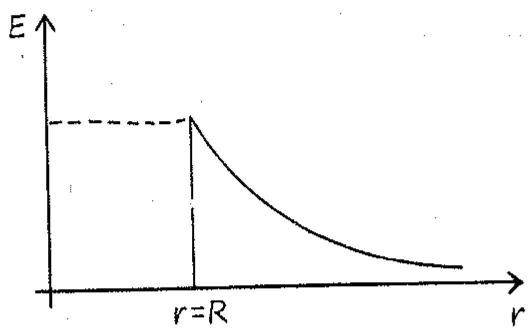



3> Infinite hollow/solid conducting or hollow non-conducting cylinder :-

$$E = \frac{2K\lambda}{x} = \frac{\sigma R}{\varepsilon_0 r}$$

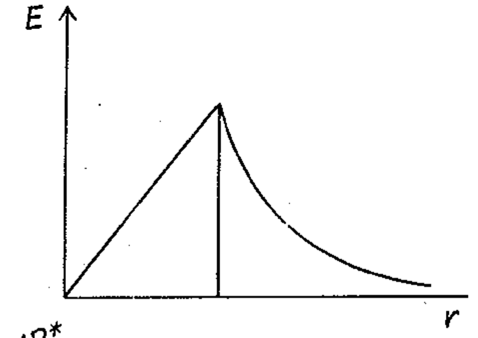
radius

outside point



4> **Solid non-conducting cylinder :-**

$$E_{out} = \frac{\rho R^2}{2\varepsilon_0 r} \qquad E_{in} = \frac{\rho R}{2\varepsilon_0} \quad r = R$$




MR*

$$q_{in} = \lambda l = \sigma[2\pi R]l = \rho[\pi R^2]l$$

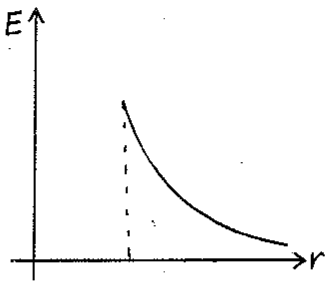
$$E = \frac{2K\lambda}{r}$$

5> **E.F. Due to hollow conducting/N.C. Solid conducting sphere :-**

$$E_{in} = 0$$

$$E_{out} = \frac{KQ}{r^2}$$

$$E_{sur} = \frac{KQ}{R^2}$$

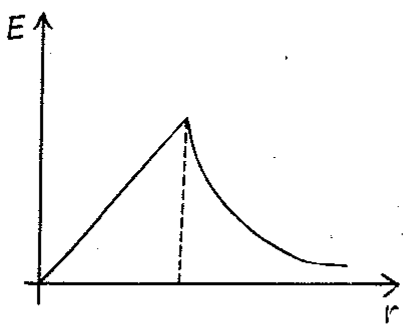



6> **E.F due to solid non-conducting sphere :-**

$$E_{in} = \frac{KQr}{R^3} \qquad E_{out} = \frac{KQ}{r^2}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$E_{sur} = \frac{KQ}{R^2} \qquad E_{in} = \frac{\rho r}{3\varepsilon_0}$$




7> Spherical Shell

(like Hollow Sphere).

$$E_{out} = \frac{KQ}{r^2} \quad E_{sur} = \frac{KQ}{R^2} \quad E_{in} = 0$$

8> Conducting Shell :-

E.F. inside isolated Conductor is zero.

9> E.F. Inside cavity of non-conducting solid sphere :-

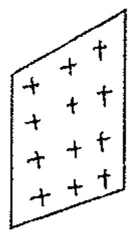
distn b/w centre of sphere to centre of cavity

$$\vec{E}_P = \frac{\rho \vec{r}}{3\varepsilon_0} \text{ (uniform) } \quad \vec{r} = \vec{r}_0 - \vec{r}_1$$

$\vec{r}_0$: from Sphere, $\vec{r}_1$: from Cavity

10> E.F. Due to infinite large $\boxed{\text{non}}$ conducting plate/sheet :-

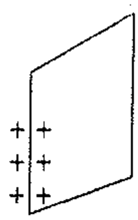
$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$



Kuch na bola jaye toh by default Non-conducting consider Karo!

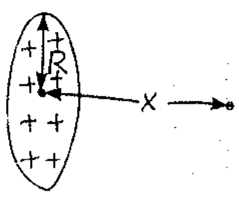
11> E.F. Due to conducting plate :-

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$



Charge Ke Terms मे दोनो sheet Ka Formula same है!

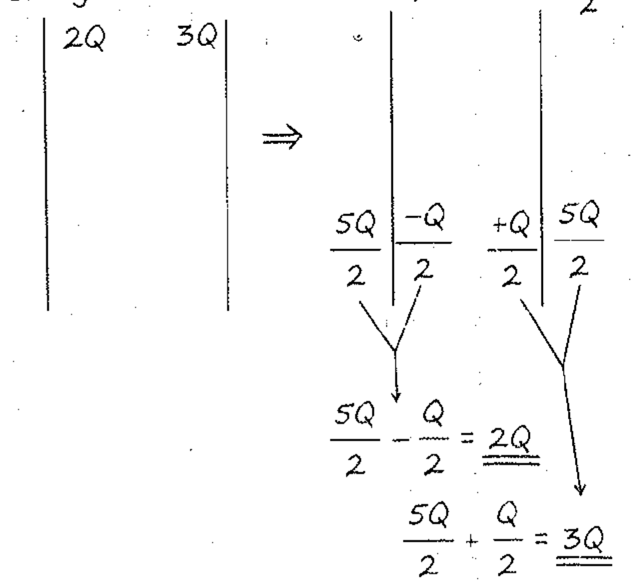
12> Electric field due to charge disc




$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

## 24. Charge Distribution :-

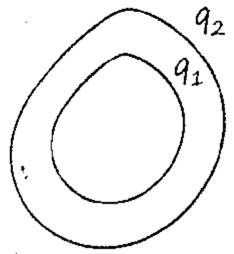
Charges on outer surface of plate $= \dfrac{\text{Total Charge}}{2}$




- Charge distribution on concentric sphere :-

$$q\begin{pmatrix}\text{final charge on outer}\\ \text{surface of outer sphere}\end{pmatrix} = q_1 + q_2$$

- Charge on inner surface of outer sphere $= q_2 - q$




**MR***

"मुश्किल नहीं है कुछ दुनिया में,
तू जरा हिम्मत तो कर।
ख्वाब बदलेंगे हकीकत में,
तू जरा कोशिश तो कर।।"

# Electric Potential

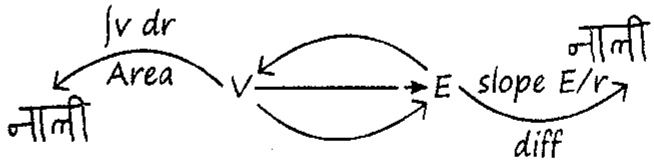
## Electric Potential :-

Work done in bringing unit positive charge from infinity to the point without change in K.E. is called electric potential at that point. (or) Negative work done by electric force in bringing unit positive charge from infinity to that point.

- depends upon refrence
- unit: volt, J/C, weber/sec, N-m/C

## Potential :-

$$\Delta V = -\int \vec{E} \cdot \vec{dr} = \text{Area of } -E/r \text{ graph}$$




$$E = -\frac{dV}{dr} \text{ (diff}^n\text{)}$$

$$E = -(\text{slope of } V/r \text{ graph})$$

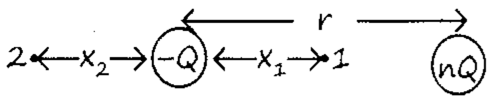
$$\Delta V = -\int_{r_A}^{r_B} \vec{E} \cdot \vec{dr} = -\int_{r_A}^{r_B} \frac{F_{CF}}{q} \cdot dr = -\frac{W_{CF}}{q}$$

- Scalar
- Ref. at $\infty$, $V = 0$.

$$\# \; E = -\frac{dV}{dr} = -[\text{Slope of } V-r \text{ graph}]$$

$$E = -\left[\frac{\partial v}{\partial x}\hat{i} + \frac{\partial v}{\partial y}\hat{j} + \frac{\partial v}{\partial z}\hat{k}\right]$$

- Potential decrease in the direction of Electric Field.
- Positive potential due to positive charge → False
- Due to positive charge potential may be +ve, −ve, zero depends on refrence → True




\# At two point potential will be zero one, b/w the charges and $2^{nd}$ left of smaller charge (V = 0 karne ke liye dono charge opposite nature ke hona Chahiye)

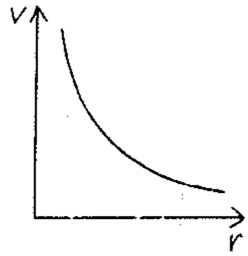
- $x_1 = \frac{r}{n+1}$ • $x_2 = \frac{r}{n-1}$

## Potential due to :-

1> Point charge :-

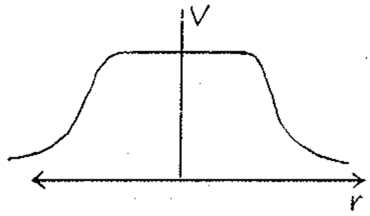
$$\oplus = V_p = \frac{KQ}{r}$$

$$\ominus \; V_p = -\frac{KQ}{r}$$

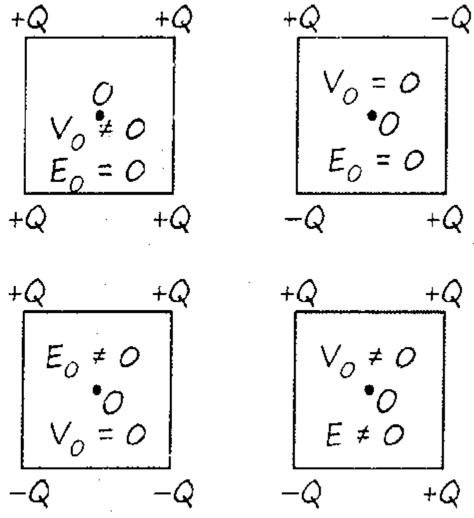



2> Ring :-

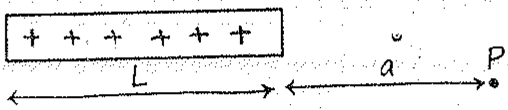
$$V_{axis} = \frac{KQ}{\sqrt{R^2 + X^2}}, \; V_C = \frac{KQ}{R}, \; V_{(X>>>R)} = \frac{KQ}{X}$$




3> Charges at corner of square :-



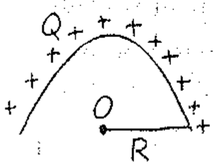
4> Axis of line charge rod :-




$$V_P = K\lambda \log\left[\frac{a+L}{a}\right]$$

5> Half ring :-

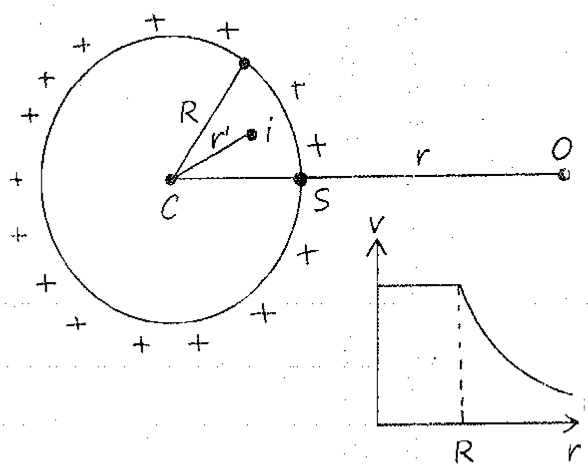
$$V_O = \frac{KQ}{R}$$



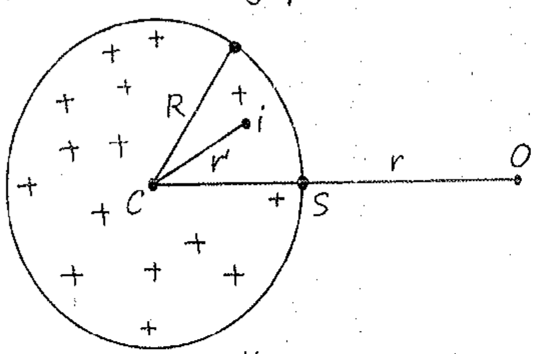

6> Hollow conducting / non-conducting or solid conducting sphere :-

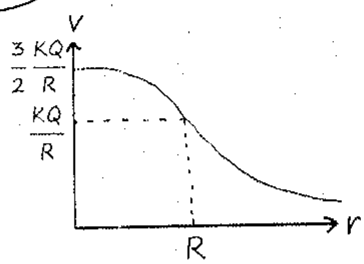
$$V_O = \frac{KQ}{r} \qquad V_S = \frac{KQ}{R}$$

$$V_i = \frac{KQ}{R} \qquad V_C = \frac{KQ}{R}$$




7> Solid non-conducting sphere :-



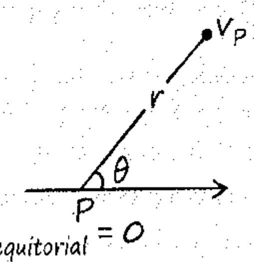




$$V_O = \frac{KQ}{r} \qquad V_S = \frac{KQ}{R}$$

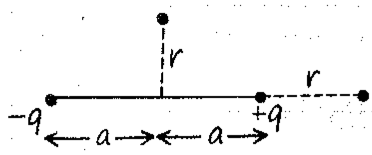
$$V_{in} = \frac{KQ}{2R^3}[3R^2 - r^2] \qquad V_C = \frac{3KQ}{2R}$$

8> Dipole :-

$$V_P = \frac{KP\cos\theta}{r^2}$$



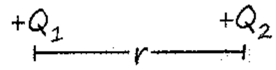

$$V_{axial} = \frac{KP}{(r^2 - a^2)} \qquad V_{equitorial} = 0$$




- If potential zero then electric field must be zero – Flase
- If field is zero then potential must be zero – False
- If potential constant then field is zero – True
- If electric field constant then potential is zero – False
- If we move perpendicular to field potential remains constant. – True
- Potential increases if we move opposite to the direction of electric field. – True

**Potential difference does not depends upon reference point its always same between two points!**

Potential Energy of System of Charged Particles :-

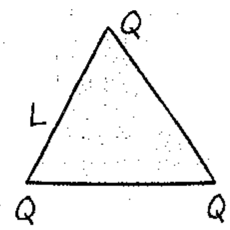


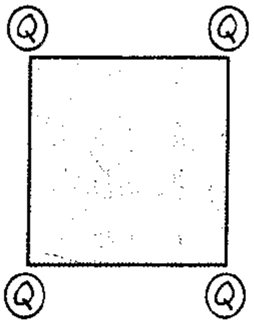

$$U = \frac{KQ_1Q_2}{r}$$

$$U = qV$$

$$U = q[-\int \vec{E}\cdot d\vec{r}]$$

$$U_{net} = \frac{3KQ^2}{L}$$






$$U = \frac{4KQ^2}{L} + \frac{\sqrt{2}KQ^2}{L}$$

Vimp.

$$\boxed{\text{Total no. of terms of PE :- } \frac{n(n-1)}{2}}$$

n = no. of Charges

$W_{ext} = + \Delta U = \Delta KE = q\Delta V$

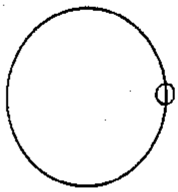
जो काम खुद हो रहा है जैसा होना चाहिए वैसा हो रहा है तो :-

$U\downarrow \qquad \oplus \rightarrow \leftarrow \ominus \qquad r\downarrow$

अगर जबरदस्ती किए :-

$U\uparrow \qquad \oplus \rightarrow \leftarrow \oplus \qquad r\downarrow$

$$\boxed{\text{Pressure :- } \frac{\sigma^2}{2\varepsilon_0}}$$



$E = \dfrac{\sigma}{\varepsilon_0}$

Self Energy :-

1> **hollow conducting / non-conducting or solid conducting sphere :-**

$$U = \frac{KQ^2}{2R}$$

2> **Solid non-conducting sphere :-**

$$U_{Total} = \frac{0.6KQ^2}{R} = \underbrace{\frac{0.5KQ^2}{R}}_{\text{Surface to } \infty} + \underbrace{\frac{0.1KQ^2}{R}}_{\text{Centre to Surface}}$$

## Equipotential Surface :- $\Delta V = 0 \quad W = 0$

1> point charge :- spherical.

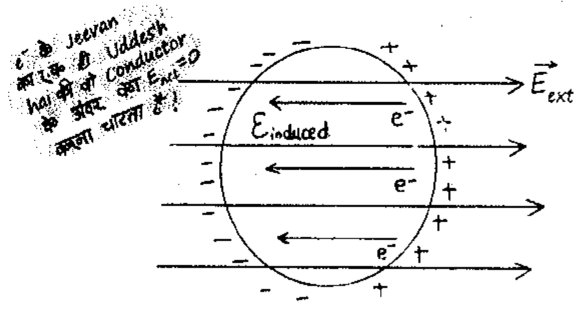
2> infinite line charge - cylindrical.

3> infinite charged plate :- plate.

4> at large distance from ring or any charge distribution :- spherical.

## Electrostatics of Conductor :-

Electrostatic shielding




$\therefore \varepsilon_{net} = \varepsilon_{ext} + \varepsilon_{ind}$

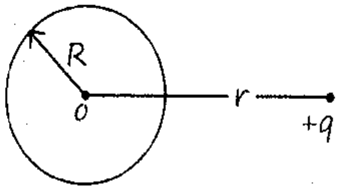
$\varepsilon_{net} = 0.$

$\varepsilon_{ext} = \varepsilon_0$

$\varepsilon_{induce} = \varepsilon_0$

\# Jitna External E. Field hoga Utna opposite में $\varepsilon_{induce}$ hoga.

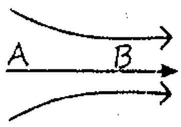
Q. Neutral conducting sphere, find electric field of center due to + q charge.




Ans. $\vec{E}_q = \dfrac{Kq}{r^2} \quad E_{net} = 0$

$\vec{E}_{induced} = -\vec{E}_q$

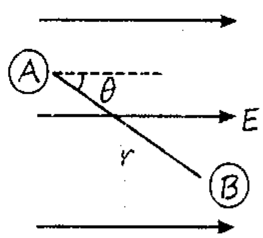
○



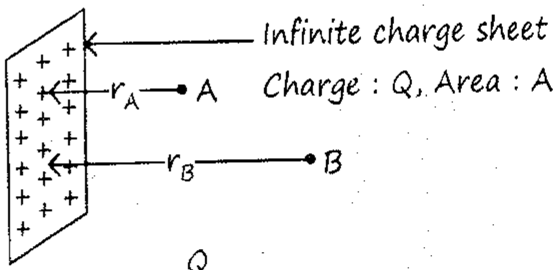

$E_A < E_B$

$V_A > V_B$

○

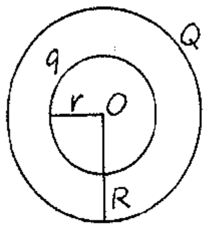



$V_A - V_B = Er\cos\theta$




$$V_A - V_B = \frac{Q}{2A\varepsilon_0}(r_B - r_A)$$

- Find potential difference b/w two concentric sphere



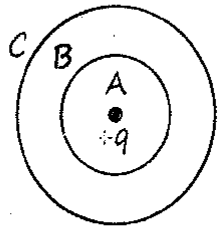

$$\Delta V = Kq\left[\frac{1}{r} - \frac{1}{R}\right]$$

- Potential difference does not depends on charge on outer sphere only on inner charge.
- When two concentric spherical charge connected with wire the potential difference becomes zero, all the charge of inner sphere will flow to outer sphere.
- $W_{ext} = \Delta U = q\Delta V$
- $W_{electric\ field} = -\Delta U = -q\Delta V$
- If external force is absent, we can apply (C.O.M.E.) conservation of mechanical energy or $\Delta U_{loss} = K.E._{gain}$

$$\boxed{K.E. = q\Delta V}$$

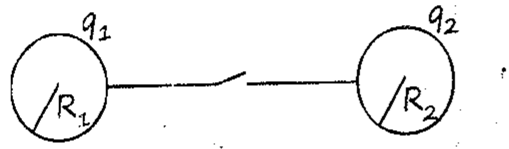
- Electric field inside conductor is zero – False
- Electric field inside isolated conductor where matter is present is always zero – True

- Thick conductor :-




$E_A \neq 0$
$E_B = 0$
$E_C \neq 0$

- When two conducting sphere is connected with wire (Not concentric)



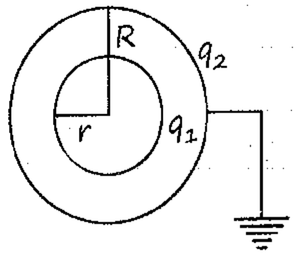

Final charge $q \propto$ Radius (R)

Potential V = Same

Surface charge density $\sigma \propto \dfrac{1}{\text{Radius (R)}}$

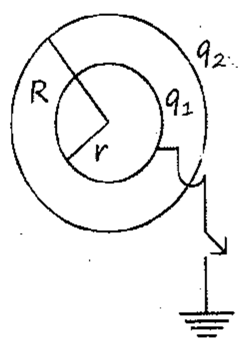
Electric field on surface $E \propto \dfrac{1}{\text{Radius (R)}}$

- After earthing potential of conductor must be zero, charge may or may not be zero.
- Final charge on each sphere when outer sphere is grounded.




Jisko ground nahi kiya hai uska charge same rahega. Jisko ground kiya hai, uska potential zero hoga and final charge on outer sphere

$$q'_2 = -q_1$$

- 


$q_2$ = Same
$q'_1 = -q_2\, r/R$

- From large distance then distance of closest approach


$$r = \frac{2KQq}{mV_0^2}$$

Q. What amount of work is done in moving a charge of 4 coulombs from a point 220 volts to a point at 230 volts?

Sol. Potential difference between the two points

$\Delta V = 230 - 220$ volts

$\therefore \Delta V = 10$ volts

Amount of charge moving $q = 4$ coulombs

Thus work done $W = q\Delta V$

$\therefore W = 4 \times 10 = 40$ joules

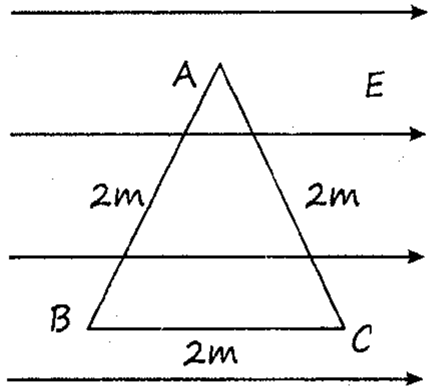
Q. Three point charge -q, +q and -q are placed along straight line at equal distance (say r meter). Electric potential energy of this system of charges will be if +q charge is in the middle.

Sol. $-q \leftarrow r \rightarrow q \leftarrow r \rightarrow -q$

$$U = \frac{-Kq^2}{r} - \frac{Kq^2}{r} + \frac{Kq^2}{2r}$$

$$U = \frac{-Kq^2 3}{2r}$$

Q. In an uniform electric field E = 10N/C as shown in figure, find $V_A - V_B$:




Sol. We know that Electric field, distance and voltage are related by formula,

$E.d = V$

Subsituting values,

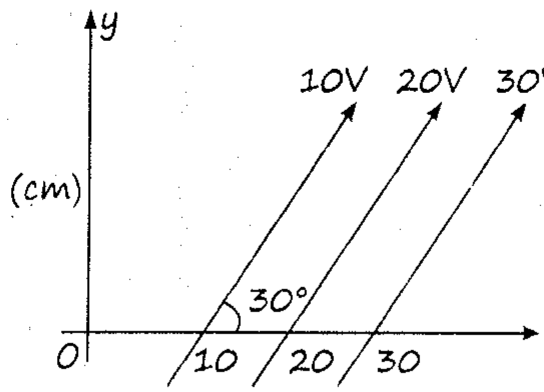
$V = 10 \times 2 \times \cos 150°$

$V = -10V$

Q. The work done to move a charge on an equipotential surface is?

Sol. Workdone in equipotential surface is zero as $W = q(V_A - V_B)$ & $V_A = V_B$

Q. Equipotential surfaces are shown in fig. then the electric field strength will be




Sol. $V = -E.\Delta r\cos\theta$

$$E = \frac{-\Delta V}{\Delta r\cos\theta}$$

$$E = \frac{-(20-10)}{10 \times 10^{-2} \cos 120°}$$

$$= \frac{-10}{10 \times 10^{-2}(-\sin 30°)}$$

$$= \frac{-10^2}{-1/2} = 200 \text{ V/m}$$

Q. If potential at centre of Non conducting sphere is zero then find potential at surface of sphere?

Sol. $\Delta V$ = Same, does not depends on reference

$$V_O - V_S = \frac{3}{2}\frac{KQ}{R} - \frac{KQ}{R}$$

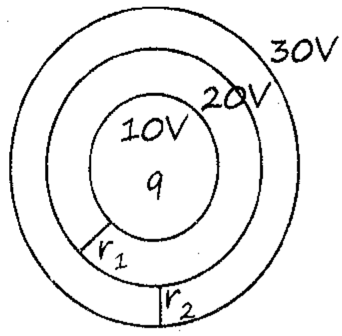
$$0 - V_S = \frac{KQ}{2R}$$

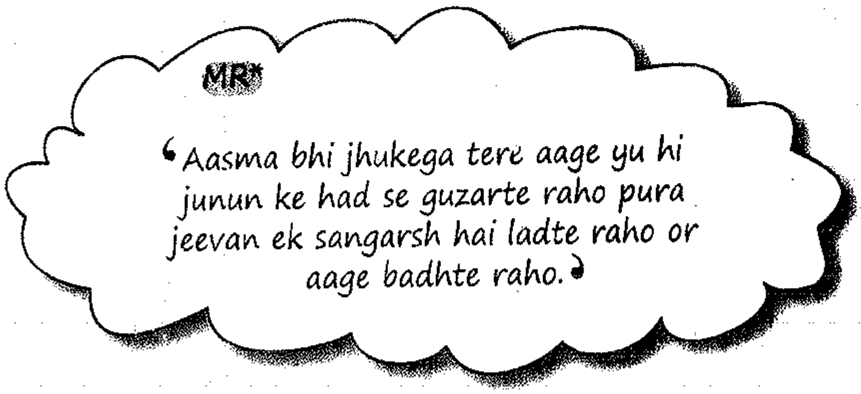
$$V_S = -\frac{KQ}{2R}$$

Q. Equipotential surface due to point charge then compair $r_1$ and $r_2$.

Sol. Hint $E = -\frac{dv}{dR}$

$r_1 < r_2$



# 23 Capacitor

### 1> Capacitor :-

- An electrical device which store electrostatic energy by storing charge.
- Generally it is combination of two conductor, having equal and opposite charge.

$Q = CV \qquad \frac{C}{Volt} = \text{Farad}$

* Depends on size, shape & Medium between them.

$C = \frac{Q}{\Delta V} = \frac{Q}{V_1 - V_2}$


→ Slope of Q/v graph

→ Scalar

→ unit c/v = farad

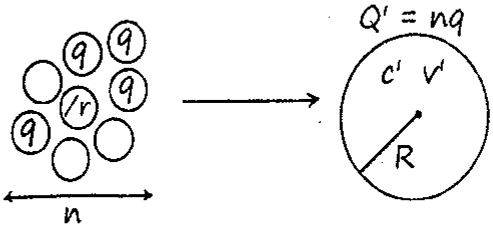
→ Dim → $[M^{-1}L^{-2}T^{4}A^{2}]$

### 2> Spherical Capacitor :-

$C = \frac{Q.R}{KQ} = 4\pi\varepsilon_0 R$

### Special Case :-

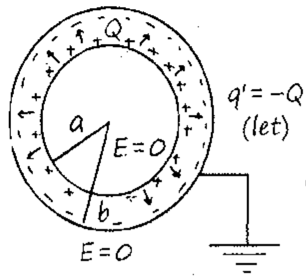
- n small charged sphere combine to form a bigger sphere.




$C' = n^{1/3}C \qquad V' = n^{2/3}V$

$U' = n^{5/3}U_0 \qquad R = n^{1/3}r$

### 3> Combination of Spherical Capacitor :-



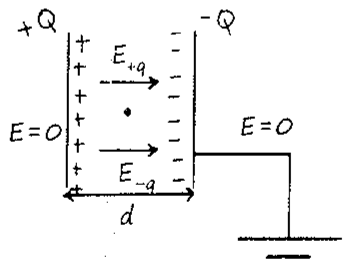

$$C = 4\pi\varepsilon_0\left[\frac{ab}{a-b}\right]$$

MR*

$b = \infty$

$C = 4\pi\varepsilon_0 a$

### 4> Parallel Plate Capacitor :-




A = Effective Area

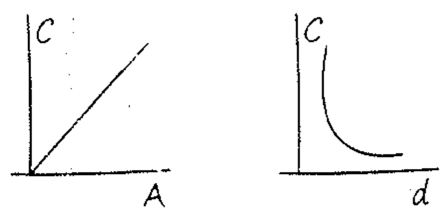
$E \text{ (due to one plate)} = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$

$E_{net} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

$\Delta V = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{A\varepsilon_0}$

$(\text{air})\ C_0 = \frac{Q}{\Delta V} = \frac{A\varepsilon_0}{d}, \qquad (\text{medium})\ C' = \frac{A\varepsilon_m}{d}$

$C' = k\frac{A\varepsilon_0}{d}$




*Dielectric introduce between plates :-

$C' :: KC_0$

Force between the Plates :-

$$F = \frac{Q^2}{2A\varepsilon_0} = \frac{\sigma^2 A}{2\varepsilon_0} \quad \because F = qE.$$

Pressure on the plate

$$P = \frac{F}{A} = \frac{Q^2}{2A^2\varepsilon_0} = \frac{\sigma^2}{2\varepsilon_0}$$

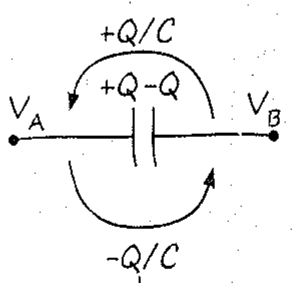
Energy density

$$U = \frac{\sigma^2}{2\varepsilon_0} = \frac{\text{Energy}}{\text{Volume}} = \frac{Q^2}{2CAd}$$

- Between the plate net electric field only due to charge on Inner surface of plate

$$E_{net} = \frac{Q}{A\varepsilon_0}, \; Q = \text{charge of Inner surface.}$$

-




A to B

$V_A - Q/C = V_B$

$V_A - V_B = Q/C$

B to A

$V_B + Q/C = V_A$

$V_A - V_B = Q/C$

5> Energy on Capacitor

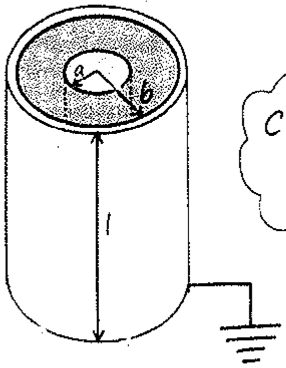
$$E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

- Self Energy :-

For Spherical Capacitor :-

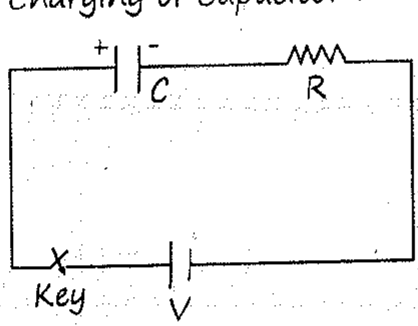
$$U = \frac{Q^2}{8\pi\varepsilon_0 R}$$

6> Cylindrical Capacitor




$$C = \frac{2\pi\varepsilon_0 l}{\log_e\left(\frac{b}{a}\right)}$$

7> Charging of Capacitor :-




- WD by battery in Charging Capacitor $= CV^2$
- WD in Charging Capacitor $= \frac{1}{2}CV^2$
- $U_{loss} = \frac{1}{2}CV^2$ Heat

MR* for Calculating $E_{loss}$

$$\boxed{U_i + W_{battery} = \underline{U_f} + U_{loss}}$$

$\frac{1}{2}CV^2_{initial}$    $Q\,\Delta V$ = charge transfered (emf of battery)    $\frac{1}{2}CV^2_{final}$

Connection of Two Capacitor :-

- Same Terminal Connection :-

$$V_C = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{q_1 + q_2}{C_1 + C_2}$$

Loss in Energy :-

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

Final Charge :- $q'_1 = C_1V_C$

$q'_2 = C_2V_C$ Transfer :- $[q - q'_1]$

- Reverse Polarity Connection :-

$$V_C = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}[V_1 + V_2]^2$$

- An air filled capacitor charged with potential V connected with identical uncharged capacitor filled with k-dielectric then common potential and final charge on each.

$$V_C = \frac{CV+KC\times O}{C+KC} = \frac{CV}{C(1+K)} = \frac{V}{1+K}$$

$$Q(air) = \frac{CV}{1+K} \quad Q(dielectric) = \frac{KCV}{1+K}$$

8> Combination of Capacitor :-

(A) Series combination of capacitors

$Q$ = Same

$V = V_1 + V_2 + \dots$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

Law of Potential drop :-

$Q = CV$ = same. MR* Special $C \propto \frac{1}{V}$

Jiska Capacitance Jyada hoga Usmein Voltage drop Kam hoga.

(B) Parallel combination of capacitors

$V$ = Same

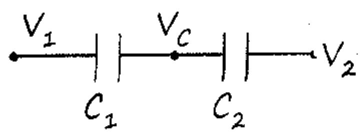
$Q = q_1 + q_2 + \dots$

$C_{eq} = C_1 + C_2 + C_3 + \dots$

Law of Charge Distribution :-

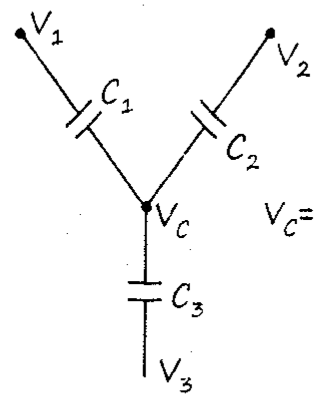
$q = CV$ MR* Special $q \propto C$

Jiska Capacitance jyada hoga woh jyada charge rakh lega!

MR* 


If $V_1 > V_2$

$$V_C = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \qquad Q_1 = C_1(V_1 - V_C) \qquad Q_2 = C_2(V_C - V_2)$$

o 


$$V_C = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}$$

If n-indentical Connected in Series & Parallel :-

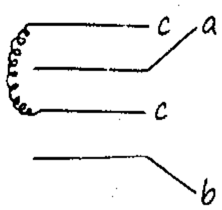
| * Series :- | * Parallel :- |
|---|---|
| $C_{eq} = \frac{C}{n}$ | $C_{eq} = nC$ |
| $C_{eq}$ in smaller than Smallest Capacitor | $C_{eq}$ is larger than largest Capacitor. |

Breakdown Voltage :-

$V_{max}$ that can be applied across a Capacitor.

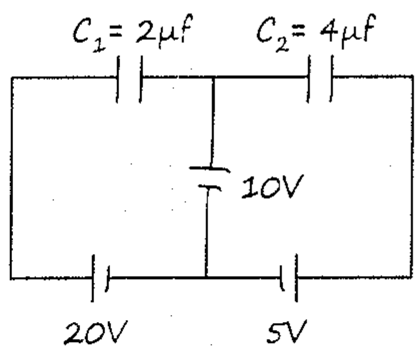
* Series :- $\boxed{V_B = V_1 + V_2 + V_3 + \dots}$
* Parallel :- $\boxed{V_B = V}$

*Special Case :-

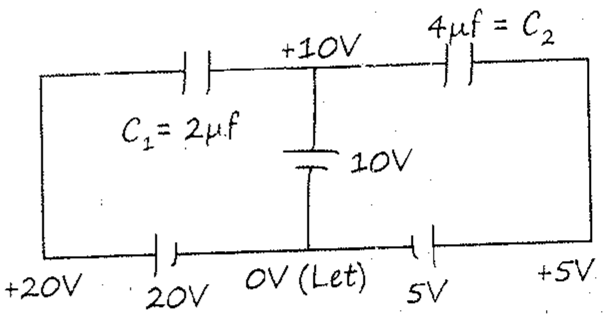



No. of gap = No. of Cap.

Q. Find change and potential difference across each capacitor.



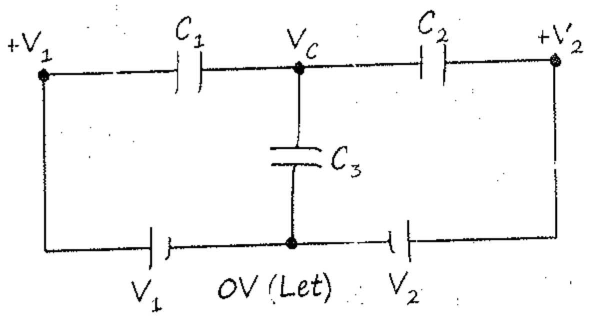

MR* → you can assume potential ka refrence zero potential at any one point of circuit.




(Change on $C_1$) $Q_1 = C_1\,(\Delta V) = 2\mu f \times (10V) = 20\mu c$

(Change on $C_2$) $Q_2 = C_2\,(\Delta V) = 4\mu f \times (10-5) = 20\mu c$

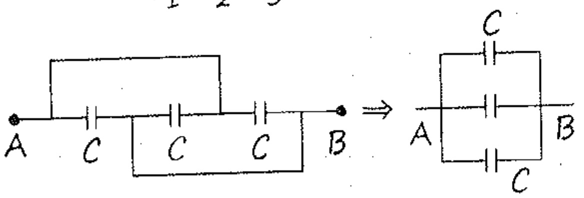
○




MR* Bade aaram है

$$V_C = \frac{C_1V_1 + C_2V_2 + C_3\times 0}{C_1 + C_2 + C_3}$$

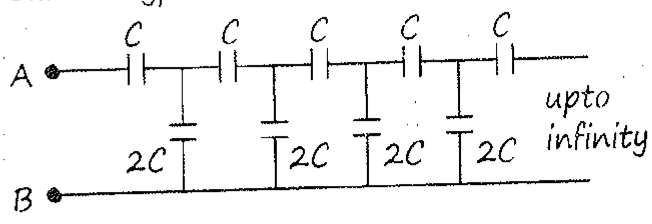
○



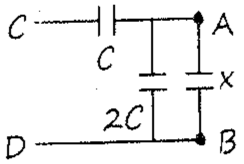

$C_{eq} = 3C$

Ladder Network :-

Ladder Type - 1




Sol.



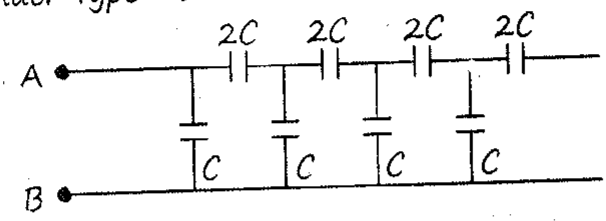

infinity me 1 unit subtract karne se fark nahi padta

X-2C connect ← X&2C parallel

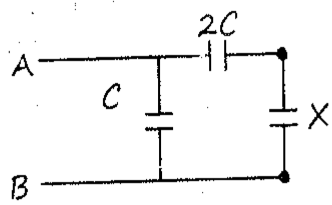
$$C_{eq} = \frac{(X+2C)C}{(X+2C)+C}$$ ← then it with c in series to get $C_{eq}$

MR* In series $C_{eq}$ must be less than smallest hence $\boxed{C_{eq} < C}$

Ladder Type - 2




add 2C & X in series then in parallel with C.



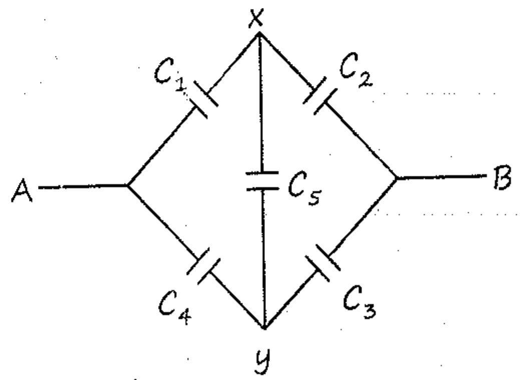

$$\frac{(2C)X}{2C+X} + C = X$$

$\boxed{C < X < 3C}$

MR*

○ Series main $C_{eq}$ less than smallest and in parallel $C_{eq}$ larger than largest

Wheat stone Bridge :-



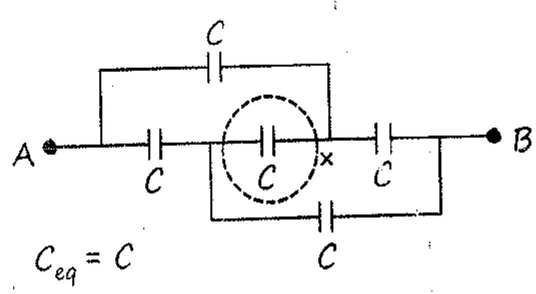

\# $V_x = V_y$

\# Charge on $C_5 = 0$

\# $\dfrac{C_1}{C_4} = \dfrac{C_2}{C_3}$

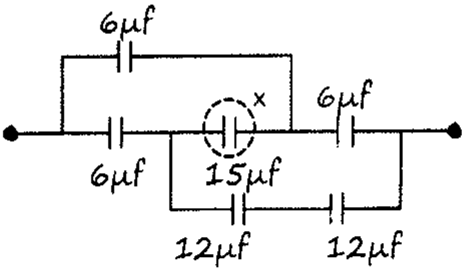
Some example of Wheat Stone Bridge :-

(i)




$C_{eq} = C$

(ii)



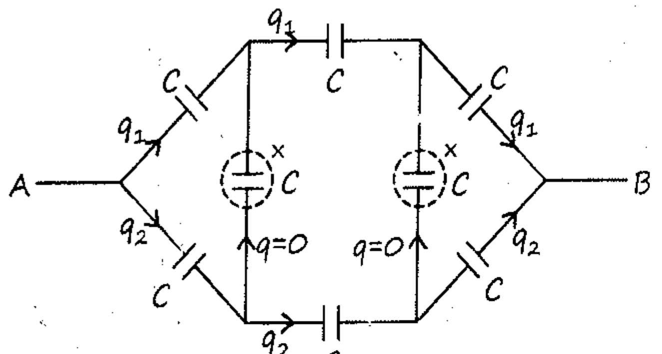

$C_{eq} = 6\mu f$

(iii)


$C_{eq} = \frac{\varepsilon_0 A}{d}$

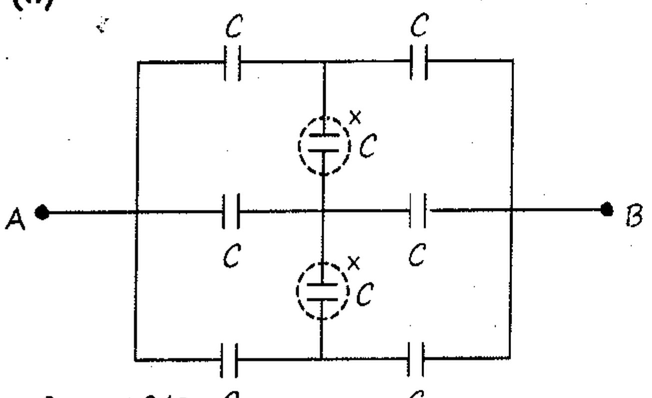
Some symmetric circuit :-

(i)



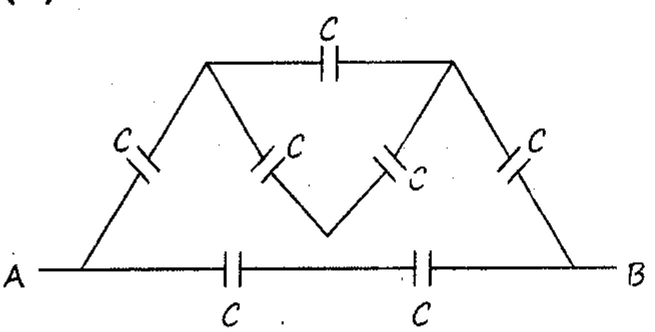

$C_{eq} = 2C/3$

(ii)



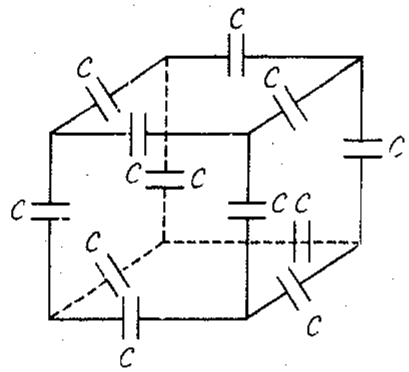

$C_{AB} = 3C/2$

(iii)




$C_{AB} = 7C/8$

o 12-Identical capacitors or resistors in cubical form :-



| MR* | Along side | Along Face diag. | Along Body diag. |
|---|---|---|---|
| | $C_{eq} = \frac{12}{7}C$ | $C_{eq} = \frac{4}{3}C$ | $C_{eq} = \frac{6}{5}C$ |
| | $R_{eq} = \frac{7R}{12}\Omega$ | $R_{eq} = \frac{3R}{4}\Omega$ | $R_{eq} = \frac{5R}{6}\Omega$ |

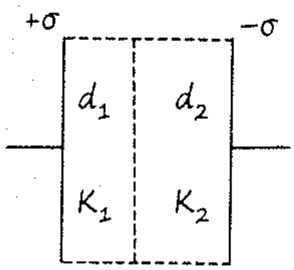
9> Dielectric Medium in an External E. Field :-

$E_{net} = \frac{E_o}{K}$ Jaise Coulomb F. dec. "K" time Similarly E. Field dec. "K" times.

(A) Series Combination of Dielectrics :- Ek Ke baad Ek daalte rehna !

$$C = \frac{A\varepsilon_0}{\left[\frac{d_1}{K_1} + \frac{d_2}{K_2}\right]}$$

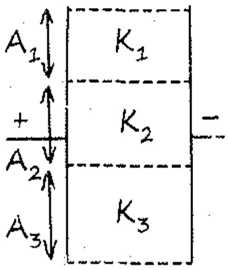
$$K_{eq} = \frac{d}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$



(B) Parallel Combination of Dielectrics :-

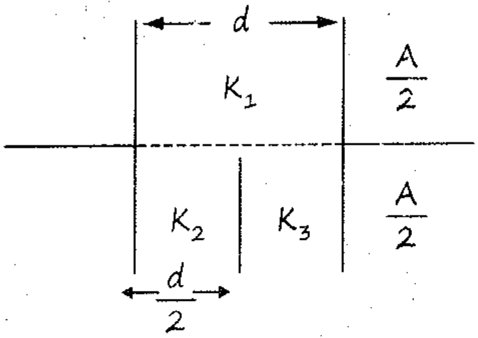
$$\begin{bmatrix}\text{Saare dielectric} \\ \text{ka ek terminal} \\ \text{+ve और Dusra} \\ \text{–ve terminal}\end{bmatrix}$$

$$C_{eq} = \frac{E_0}{d}\,[K_1A_1 + K_2A_2 + \ldots]$$




$$K_{eq} = \frac{K_1A_1 + K_2A_2 + \ldots}{A_1 + A_2 + \ldots}$$

(C)

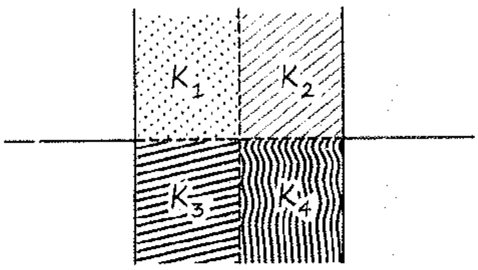



$$C = \frac{\varepsilon_0 A}{d\left(\frac{K_2+K_3}{K_2K_3}\right)} + \frac{\varepsilon_0 AK_1}{2d}$$

$$C = \frac{\varepsilon_0 A}{d}\left[\frac{K_2K_3}{K_2+K_3} + \frac{K_1}{2}\right]$$

MR* If $K_1 = K_2 = K_3$ then $C = \frac{\varepsilon_0 AK}{d}$

(D)




$$C_{eq} = \frac{\varepsilon_0 A}{d}\left(\frac{K_3K_4}{K_3+K_4} + \frac{K_1K_2}{K_1+K_2}\right)$$

MR* If $K_1 = K_2 = K_3 = K_4$ then $C_{eq} = \frac{\varepsilon_0 AK}{d}$

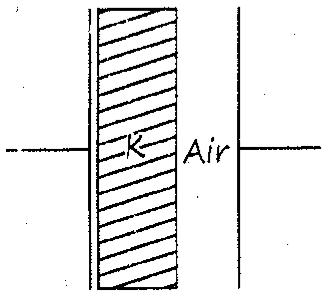
Q. A parallel plate air capacitor has capacitance C. Half of space between the plate is filled with dielectric K as shown to Fig. Then new Capacitance C' is

(a) $C' = C\left[\frac{K}{K+1}\right]$

(b) $C' = C\left[\frac{2K}{K+1}\right]$

(c) $C' = \frac{2C}{K+1}$

(d) $C' = C\left[1 + \frac{K}{2}\right]$




MR* If $K=1$, $C' = C$ | If $K=\infty$ Conductor $C' = 2C$

Introduction of dielectric between plates of capacitor.

o Battery is Connected :-

| | Initially | Finally |
|---|---|---|
| Capacitance | $C_0$ | $KC_0$ |
| Potential | $V_0$ | $*V_0$ |
| Charge | $Q_0 = C_0V_0$ | $KQ_0$ |
| E. Field | $E_0 = V_0/d$ | $E_0$ |
| $U_0$ (Energy) | $U_0 = Q_0^2/2C_0$ | $KU_0$ |
| $F_0$ (Force on Plate) | $F_0 = qE$ | $KF_0$ |

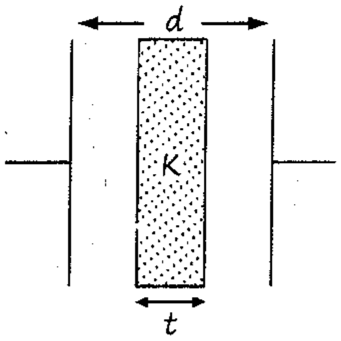
Battery Ziddi hai potential same rakhegi !

- Battery is removed then dielectric is placed :-

| | Initially | Finally |
|---|---|---|
| Capacitance | $C_o$ | $KC_o$ |
| Potential | $V_o = Q_oC_o$ | $^*V_o/K$ |
| Charge | $Q_o = C_oV_c$ | $^*Q_o$ conserved |
| E. Field | $E_o = V_o/d$ | $E_o/K$ |
| $U_o$ (Energy) | $U_o = Q_o^2/2C_o$ | $U_o/K$ |
| $F_o$ (Force on Plate) | $F_o = qE$ | $F_o/K$ |

Plate isolated hai to charge conserved rahega.

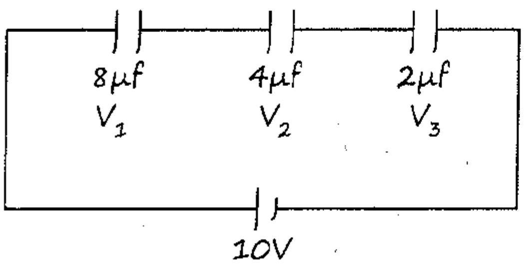
Dielectric of width t placed between capacitor :-




$$C = \frac{\varepsilon_0 A}{(d-t)+\frac{t}{K}}$$

MR* if t=0 $C = \frac{\varepsilon_0 A}{d}$ | d=t $C = \frac{\varepsilon_0 KA}{d}$

Q. Find charge and potential drop across each capacitor.




Sol. MR* $V = \frac{q}{C}$

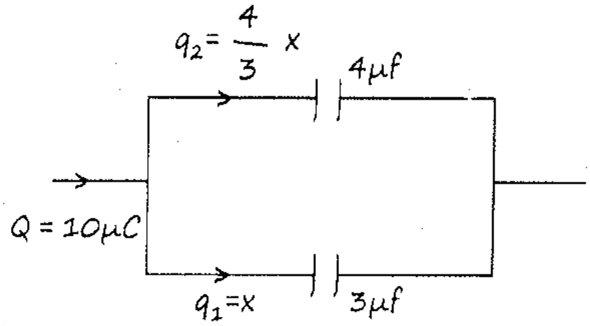
$V_1 = x$(let) $V_2 = 2x$ $V_3 = 4x$

$x+2x+4x = 10V$

$$x = \frac{10}{7} \text{ volt}$$

$$q_1 = C_1V_1 = 8 \times \frac{10}{7} = \frac{80}{7}\ \mu f$$

Q.

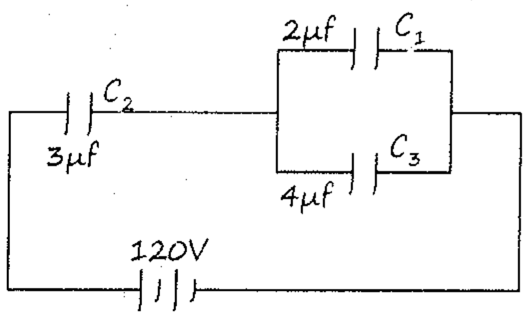



Sol. $x + \frac{4}{3}x = 10$ MR* $Q = CV$

$Q \alpha C$

$$x = \frac{30}{7}\ \mu C$$

Q. The charge on capacitors shown in the figure and the potential difference across each will be respectively.




Sol. The total capacitance of the circuit is,

$C = 2\mu f$

$Q = CV$

So, $Q = 240\mu C$

Voltage across $3\mu f$ capacitor will be,

$$V_1 = \frac{240}{3}\ V$$

$= 80V$

Voltage across $2\mu f$ and $4\mu f$ capacitor will be,

$$V_2 = \left(120 - \frac{240}{3}\right) V$$

$= 40V$

Charge across the $2\mu f$ capacitor will be,

$Q_1 = (2 \times 40)\ \mu C$

$= 80\mu C$

Charge across the $4\mu f$ capacitor will be,

$Q_2 = (4 \times 40)\ \mu C$

$= 160\mu C$

MR

"Duniya me koi kam asambhav nahi, bas hosla aur mehnat ki jarurat hoti hai."

# Current Electricity

1> **Electric Current :-** The rate of directional flow of electric charge is called electric current.

Current

In time 't' — at time 't'

$$I_{Avg.} = \frac{\Delta Q}{\Delta t} = \frac{\int I dt}{\int dt}$$

$$I_{Inst.} = \frac{dq}{dt}$$ = slope of charge -time graph

* Charge on circular path

$$I_{Avg.} = \frac{Q}{T} = \frac{ne}{T} = nef = \frac{nev}{2\pi r}$$

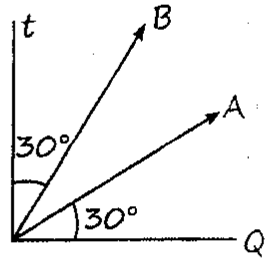
$\int dq = \int I dt$
$\Delta Q$ = Area of current time graph

- scalar,
- unit (Ampere),
- dimension $[A^1]$

Q. Through a given cross section $n_1$ electron per-sec are passing from left to right and $n_2$ proton are passing from right to left simultaneously then the electric current through this cross section.

Ans. $I = (n_1 + n_2)e$

Q. Find ratio of current.




Ans. $\frac{I_A}{I_B} = \frac{\tan 60°}{\tan 30°} = \frac{3}{1}$

2> **Isolated Conductor :-**

$ne^- = 10^{28} e^-/m^3 \quad V = 10^4$ m/s

$V_{avg} = 0 \quad E_{in} = 0 \quad$ (speed $V \propto \sqrt{T}$)

$\frac{1}{2} mv^2 = \frac{3}{2} K_B T \quad K_B = 1.38 \times 10^{-23} \frac{Js}{K}$

3> **Battery Connected to Conductor :-**

$F_{e^-} = qE \quad a = \frac{qE}{m_e} \quad E = \frac{V}{l}$

$\hookrightarrow$ Force on electron

$V_D = \cancel{u}^{\,0} + AT \qquad I = NEAV_D$

$V_D = AT = \frac{eE\tau}{m_e}$

E = electric field
$V_d$ = drift velocity
$l$ = length of conductor
n = no. of electron per unit volume
$\tau$ = relaxation time
$m_e$ = mass of electron
V = emf of battery
e = charge of electron

MR* feel

$$\sigma = \frac{ne^2\tau}{m} \xrightarrow{\rho = \frac{1}{\sigma}} \rho = \frac{m}{ne^2\tau}$$

$$R = \frac{\rho l}{A}$$

$$R = \frac{ml}{ne^2\tau A} \xrightarrow{V = iR} V = \frac{mil}{ne^2\tau A}$$

Micro scopic form of Ohm's law!

5> **Mobility :-** Property of charge carrier.

- Does not depends on drift velocity and electric field.

$$\mu = \frac{V_d}{E} = \frac{e\tau}{m} \qquad \sigma = ne\mu$$

$$\mu_e > \mu_p > \mu_{Deut.} = \mu_\alpha$$

MR*

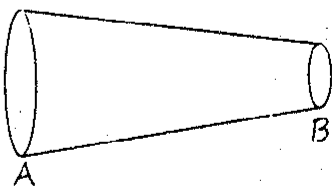
Q. If drift velocity is doubled then what about mobility ?

Ans. Remains same

6> Current Density :- (Vector)

$i = \vec{J}.\vec{A} = JA\cos\theta$

$\vec{J} = ne\vec{V}_d$ $\quad \vec{J} = \sigma\vec{E}$ Vector form of Ohm's law !




Current : $I_A = I_B$

Current density : $J_A < J_B$

Drfit velocity : $V_A < V_B$

7> Variation of Resistance :- Material prop.

$R = \frac{\rho l}{A}$

| | |
|---|---|
| (a) $L \to$ Change<br>$A \to$ Const$^n$<br>$R \propto l$ | (b) $L \to$ Change<br>$V \to$ Const$^n$<br>$R \propto l^2$ |
| (c) $A \to$ Change<br>$l \to$ Const$^n$<br>$R \propto 1/A$ | (d) $A \to$ Change<br>$V \to$ Const$^n$<br>$R \propto 1/A^2$ |
| (e) $R = \frac{\rho l^2}{M} \times$ density<br>$R \propto \frac{l^2}{M}$ | (f) $R = \frac{\rho M}{\text{density } A^2}$<br>$R \propto \frac{M}{A^2}$ |

If a wire of Resistance R stretched to double of its length the new resistance becomes 4R.

Q. The masses of the wire in the ratio of 1 : 3 : 5 and their length are in ratio of 5 : 3 : 1. The ratio of their resistance.

Ans. $R \propto \frac{l^2}{m}$ $\quad R_1 : R_2 : R_3 = 125 : 15 : 1$

8> Ohm;s Law :-

$\vec{J} = \sigma\vec{E}$ $\quad V = \frac{mil}{ne^2\tau A}$ $\quad V = iR$

*Note :-

Temp$^r$↑ → Conductor $\quad \alpha = +ve \quad \sigma\downarrow \ \rho\uparrow$

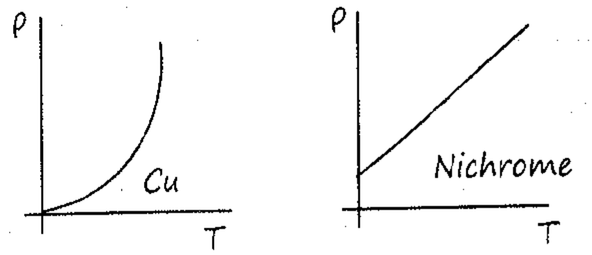
Temp$^r$↑ → Insulator $\quad \alpha = -ve \quad \sigma\uparrow \ \rho\downarrow$

Temp$^r$↑ → Semi-Conductor $\quad \alpha = -ve \quad \sigma\uparrow \ \rho\downarrow$

9> Temperature Dependence of Resistance & Resistivity :-

$\rho_T = \rho_0[1 + \alpha\,\Delta T]$, $\quad \alpha = \frac{\Delta\rho}{\rho_0\,\Delta T}$

unit $K^{-1}$




\# Two resistance $R_1$ and $R_2$ connected in series and their $R_{eq}$ does not depends upon temperature then $R_1\alpha_1 = -R_2\alpha_2$

\# $R_{t°C} = R_0\,(1 + \alpha T)$ always valid

$R_{t_2} = R_{t_1}\,[1 + \alpha\,(T_2 - T_1)]$

Valid for small change in temperature

$$\boxed{\frac{R_{t_2}}{R_{t_1}} = \frac{1 + \alpha t_2}{1 + \alpha t_1}}$$

always valid

$R_0$ = resistance at 0°C

$R_{t_1}$ = Resistance at $t_1$

$R_{t_2}$ = Resistance at $t_2$

10> Relation between Coefficient of :-

(a) linear expansion ($\alpha$)

(b) Resistance ($\alpha_R$)

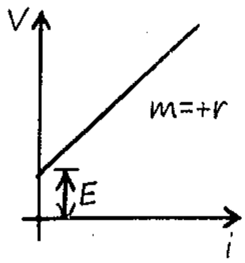
(c) Resistivity ($\alpha_\rho$)

$$R = \frac{\rho l}{A} \qquad \alpha_R + \alpha = \alpha_\rho$$

11> Battery :-

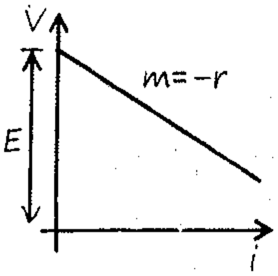
(a) Charging

$\Delta V = E + ir$




(b) Discharging

$\Delta V = E - ir$

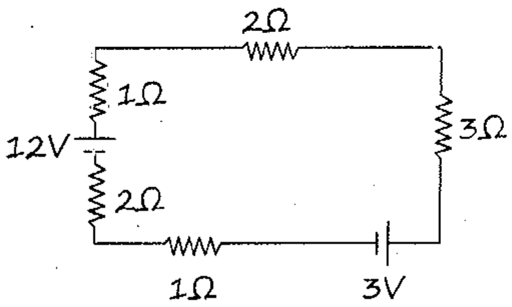



12> Kirchoff's Law :-

Law (i) $\Sigma i = 0$ $\begin{bmatrix}\text{Charge} \\ \text{Conserv}^n\end{bmatrix}$

Law (ii) Energy Conserv$^n$.

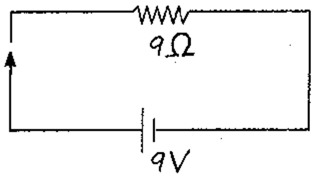
*Household circuit maltab parallel circuit




MR*

Sare resistance ko series me ek sath add kar ke battery ko ek sath polarity ke sath add kare

$$I = \frac{9}{9} = 1A$$

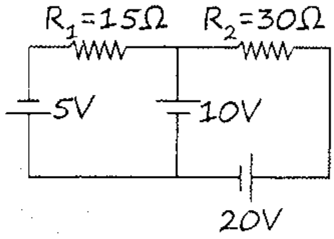



MR*

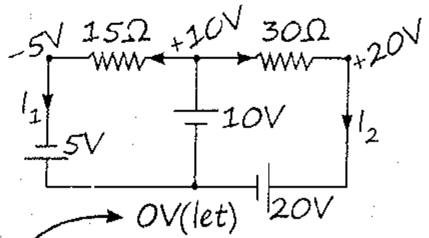
Point potential → current depends on potential difference not on potential and potential difference does not depends on reference, hence you can assume zero potential at any one point of circuit. (Sirf ek point pe hi zero man sakte hai)

Kishi bhi point ka Potential Apan zero Mann Sakte hai!

Q. Find current in 15Ω and 30Ω.




MR*

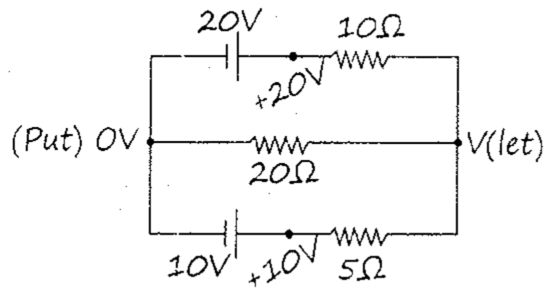



yaha ish liye kyki yha se sare point potential nikalna easy hai.

$$I_1 = \frac{10-(-5)V}{15} = \frac{15}{15} = 1A$$

$$I_2 = \frac{(20-10)}{30} = \frac{10}{30} = \frac{1}{3}A$$

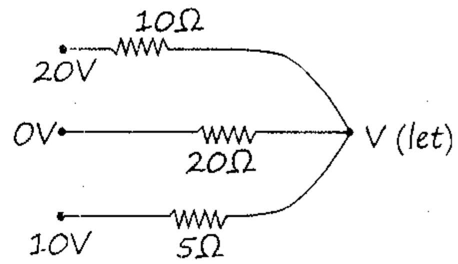
Q.




current through each resistance ?

Ans.

MR*




$$V = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$V = \frac{\frac{20}{10} + \frac{0}{20} + \frac{10}{5}}{\frac{1}{10} + \frac{1}{20} + \frac{1}{5}} = \frac{\frac{2+1+4}{20}}{} \;\; \frac{4}{20}\ldots = \frac{80}{7}$$

Now, we can calculate each current because we have $V = \frac{80}{7}$ hence we have potential difference.

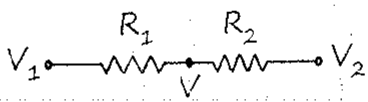
13> Combination of Resistance :-

| Series $i$ = Same | Parallel [v = same] |
|---|---|
| potential different | current different |
| $V_{total} = V_1+V_2+V_3+...$ | $I_{total} = I_1+I_2+I_3+...$ |
| $R_{eq} = R_1+R_2+R_3+...$ | $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$ |
| If n equal resistance, | If n equal resistance, |
| $R_{eq} = nR$ | $R_{eq} = R/n$ |
| $R_{eq}$ will be larger than the largest Resistance | $R_{eq}$ will be smaller than smallest Resistance. |
| $V \propto R$ | $i \propto 1/R$ |
| Jitna Jyada "R" Utna Jyada $V_{drop}$. | Jitna Kam "R" Utna jyada "i" |

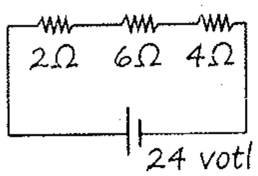
*Household circuit maltab parallel circuit.

## To Calculate Potential at Midpoint :-

$$V_{mid} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$




Q. Find current and potential drop across each resistance




Ans. $V = IR$

$V \alpha R$

$V_1 = x(\text{let}) \qquad V_2 = 3x \qquad V_3 = 2x$

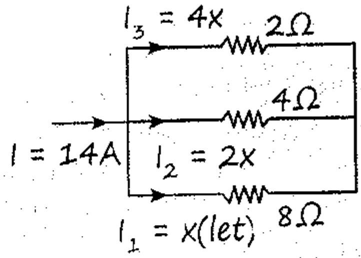
Hence $x + 3x + 2x = 24V$

$V_1 = x = 4$ volt

$V_2 = 12$ volt $V_3 = 8$ volt

$$I = \frac{V_{net}}{R_2} = \frac{24}{12} = 2Amp$$

Q. Find current through each resistance




Ans. $V = IR$

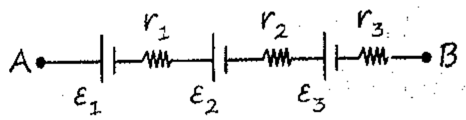
$I \alpha \frac{1}{R}$

$x + 2x + 4x = 17$

$X = 2$ Amp

14> Combination of Battery :-

Series :-




$\varepsilon_{net} = \varepsilon_1 + \varepsilon_1 + \varepsilon_3$

$r_{net} = r_1 + r_2 + r_3$

# If there are "n" identical battery is connected in series :

$\varepsilon_{net} = n\varepsilon$

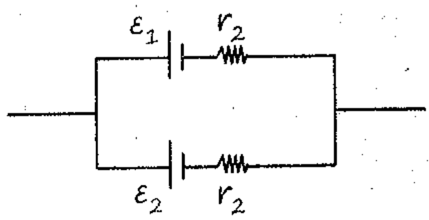
$r_{net} = nr$

# If "n" identical battery is connected in series out of which "m" reversed :

$\varepsilon_{net} = [n-2m]E$

$r_{net} = nr$ [Maximum]

Parallel :-




$$V = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}, \quad \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

# If there are n-identical cell in parallel then,

- $\Sigma_{net} = \mathcal{E}(emf)$
- $r_{eq} = r/n$ [Minimum]

## Mixed Grouping

- n-Identical battery (E,r) connected in series then this series combination connected m-times parallel with external resistance R.

  ∴ – series ⟹ nE, nr

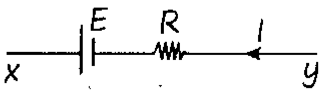
  then m-times parallel $(nE)\left(\frac{nr}{m}\right)$

  $$I = \frac{nE}{R + \frac{nr}{m}}$$

  This I will be maximum when

  $R = \left(\frac{nr}{m}\right)$ (Internal resistance)

- Circuit Mai chalna Important Hai :

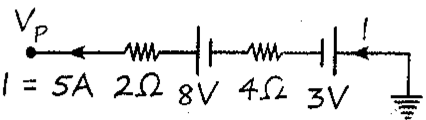



  → $V_y - IR + E = V_x$

  → $V_x - E + IR = V_y$

- Current ki direction me resistance ko cross karne par potential drop hoga (–IR).
- Current ke opposite potential increase hoga (+IR).
- Battery ko lower to higher cross karne pe potential increase hoga (+E).
- Higher to lower cross karne pe potential decrease (–E).
- Current ki direction se fark nahi pedega.

Q. Find $V_P$?




Ans. Move from 'P' to ground

$V_P + 5 \times 2 - 8 + 4 \times 5 + 3 = 0$

$V_P = -33 + 8 = -25$ volt

15> Power :-

$P = iV \qquad P = i^2R \qquad P = \frac{V^2}{R}$

Series :- $P \propto R$

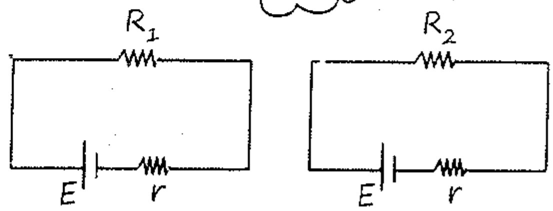
Parallel :- $P \propto 1/R$

Joule's Law of heating :

$H = i^2Rt = ivt = ms\Delta\theta$

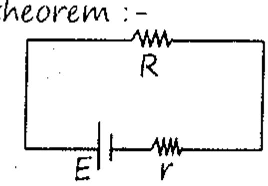
i = Variable :- $H = R\int i^2 . dt$.

Special Case $r = \sqrt{R_1R_2}$



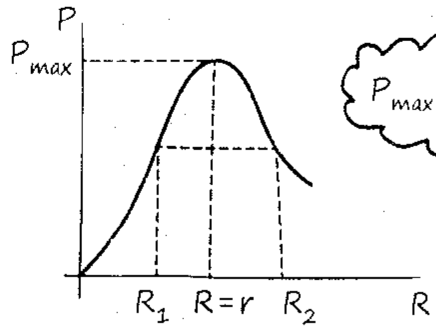

agar ye internal resistance between two circuit iss pattern mein raha toh $P_{drop}$ Same hoga woh doh circuit mein!

Power drop in ext. Circuit with maximum power theorem :-




Power drop across R, $P = I^2R = \left[\frac{E}{R+r}\right]^2 R$

Power drop will be maximum when r=R




$P_{max} = \frac{E^2}{4R}$

16> Bulb :- (Pure Resistance है)

- Rated power and rated voltage given to calculate resistance of bulb.

$R_{bulb} = \frac{V_R^2}{P_R}$

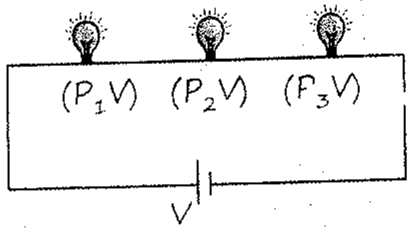
$R_{Bulb} \propto \frac{1}{P_{Rated}}$

$P_{Cons.} = \left[\frac{V_S}{V_R}\right]^2 P_{Rated}$

Q. If two bulb of power (60W, 110V) and (100W, 110V) are connected in series with supply of 220V then?

Ans. Potential drop across 60W bulb will be greater than 110V hence it will fuse.

- Series Combination :-
  Bakwas Combination




$$\frac{1}{P_{cons.}} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}$$

If all are identical bulb then,

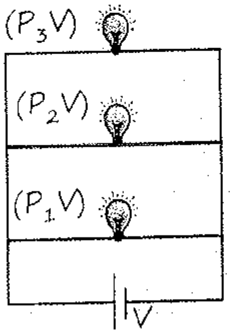
$P_{cons.} = \frac{P}{n}$ $\quad P_{cons.}$ is smaller than the smallest $P_{rated}$ bulb.

$$P_{cons.} \propto R_{bulb} \propto \frac{1}{P_{rated}}$$

$$P_{consumed} = i^2 R_{bulb}$$

Joh Kam $P_{rated}$ Ka hoga woh Jyada Chamkega!

- Parallel Combination :-




$$P_{cons.} = P_1 + P_2 + P_3$$

If all are identical bulb then,

$P_{cons.} = nP \qquad P_{cons.} \propto \frac{1}{R_{bulb}} \propto P_{rated}$

$$P_{consumed} = \frac{V^2}{R_{bulb}}$$

Joh Jyada "$P_{rated}$" Ka hoga woh Jyada Chamkega!

$[1 \text{ kWh} = 36 \times 10^5 \text{J}]$

17> Time Taken by Heater Coils :-

- Series :-

$$t = t_1 + t_2$$

- Parallel :-

$$t = \frac{t_1 t_2}{t_1 + t_2}$$

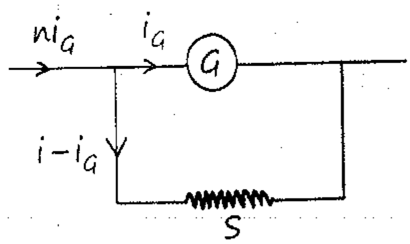
18> Electrical Instruments :-

(a) Galvanometer :-

- An instrument use to detect or measure small current.
- Very sensitive, produce large error.
- $I_G$ = Maximum current that can flow.
- $G$ = resistance of galvanometer

- As Ammeter :- Connected in series in circuit.
  - Small resistance shunt connected in parallel with galvanometer.
  - Ideal R = 0; Behave as simple wire.

$$\% \text{ Error} = \frac{i_T - i_M}{i_T} \times 100$$




$$R_{ammeter} = \frac{GS}{G+S}$$

$$\boxed{S = \frac{G}{n-1}} \quad \begin{matrix} R \downarrow \\ \% \text{ Error} \downarrow \end{matrix}$$

$$n = \frac{i \text{ we want to Measure}}{i \text{ jitna Galvano-meter se jayega.}}$$

- As Voltmeter :- Connected parallel in circuit
  - large resistance connected in series with galvanometer
  - Ideal R = ∞ infinite (Behave as open wire)

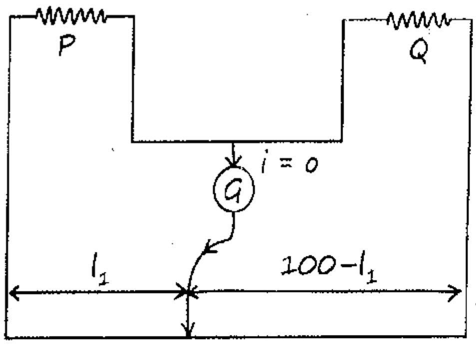

$$\boxed{S = G(n-1)}$$

$$\% \text{ Error} = \frac{V_T - V_M}{V_T} \times 100$$

(b) Meter Bridge :- Use to find value of Resistance.

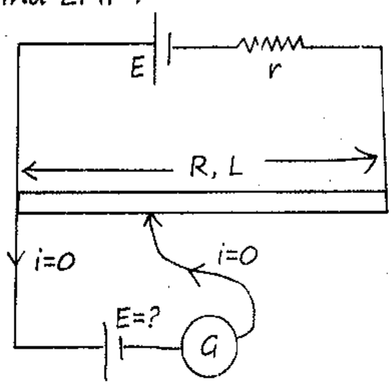
- Working based on wheat-stone bridge.

$$\frac{P}{l_1} = \frac{Q}{100 - l_1}$$




(c) Potentiometer wire :- Working based on potential gradient.

- To Find EMF :-




*Step-i :- $i = \frac{E}{R+r}$

*Step-ii :- $v = iR$

*Step-iii :- $K = \frac{V}{l}$ → Potential gradient

Potential drop per unit length in wire.

$E = kl$

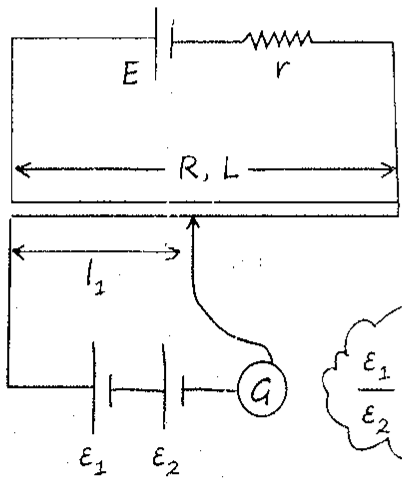
$l$ = Balancing length where current through galvanometer is zero.

Caution !

*EMF of the battery must be less or equal to the Potential drop in wire.

*Upar Ke battery Ki polarity aur niche ke battery Ki polarity supportive honi chahiye nai toh balance point asambhav hai... !

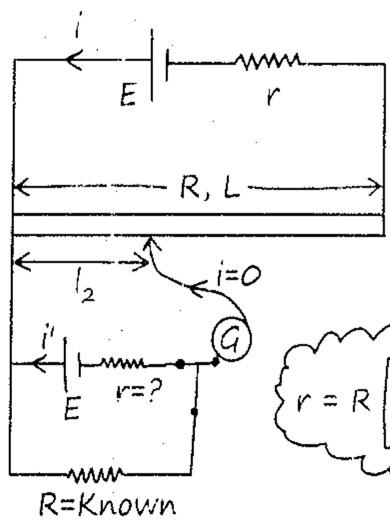
- To Compare EMF :-




$l_1$ & $l_2$ are balance points $E_1$ & $E_2$ are support each other and when oppose each other respectively.

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1 + l_2}{l_1 - l_2}$$

- To Find internal Resistance :-




$$r = R\left[\frac{l_1}{l_2} - 1\right]$$

r = Unknown Resistance

R = Known joined resistance in $11^e$!

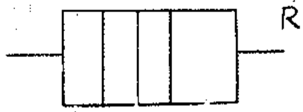
$l_1$ = initial length before connecting known Resistance .

$l_2$ = Final length.

19> Colour Coding :-

| | | | | |
|---|---|---|---|---|
| B | Black | 0 | $10^0$ | |
| B | Brown | 1 | $10^1$ | |
| R | Red | 2 | $10^2$ | |
| O | Orange | 3 | $10^3$ | |
| Y | Yellow | 4 | $10^4$ | |
| G | Green | 5 | $10^5$ | |
| B | Blue | 6 | $10^6$ | |
| V | Violet | 7 | $10^7$ | |
| G | Grey | 8 | $10^8$ | |
| W | White | 9 | $10^9$ | |
| G | Gold | | $10^{21}$ | 5% |
| S | Silver | | $10^{22}$ | 10% |
| No Colour | | | | 20% |

To Calculate Tolerance :- $\dfrac{\Delta R}{R} \times 100$



$R = 470 \pm 5\%$

Yellow Violet Brown Gold

MR*

"खुद की समझदारी भी अहमियत रखती है,
वरना दुर्योधन और अर्जुन दोनों
के गुरु एक ही थे।"

# 25 Magnetic Effect of Electric Current

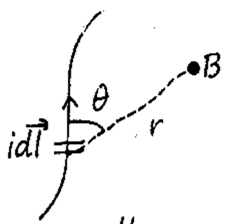
### Oersted Exp :-

- Electric field outside current carying wire is zero
- Electric field inside current carrying wire may or may not be zero.
- But moving charge near to current carrying wire experience force hence there must be a field that is magnetic field.

$F_{rest\ Q} = 0 \rightarrow F_{moving\ Q} \neq 0$

*Current carring wire produces MF around wire.

### Biot-Savart's Law :-




$$\boxed{dB = \frac{\mu_0 i}{4\pi} \frac{dl \sin\theta}{r^2}}$$

(Scalar form)

$$\frac{\mu_0}{4\pi} = 10^{-7}\ Tm/Amp.$$

$$*1T = 10^4\ G = 1\ \frac{wb}{m^2}$$

Vector Form :-

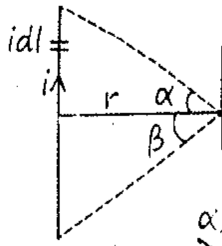
$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{id\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{(id\vec{l} \times \vec{r})}{r^3}$$

MR* To find direction of magnetic field

$$\underset{\text{(Result)}}{\vec{dB}} = \underset{\text{(1}^{st}\text{ vector)}}{id\vec{l}} \times \underset{\text{(2}^{nd}\text{ vector)}}{\vec{r}}$$

Place your four-finger (palm) of right hand along $1^{st}$ vector slap 2nd vector, thumb will represent $\vec{B}$.

### Mag. Field due to Straight Wire :-



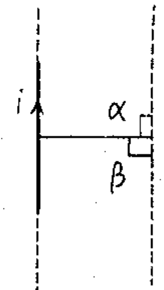

$$\boxed{B = \frac{\mu_0 i}{4\pi r} [\sin\alpha + \sin\beta]}$$

α, β :- Hamesha point से lena hai !

MR* Dimensional Format

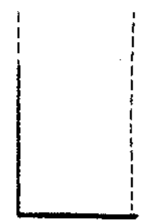
$$B = \frac{\mu_0 i}{dist^n}$$

1> ∞




$$B = \frac{\mu_0 i}{4\pi r} \times 2$$

2> Semi-∞

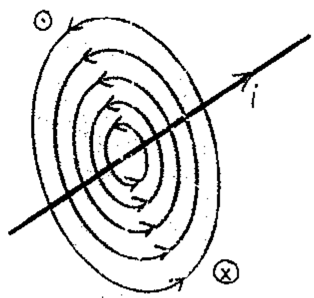


$$B = \frac{\mu_0 i}{4\pi R}$$

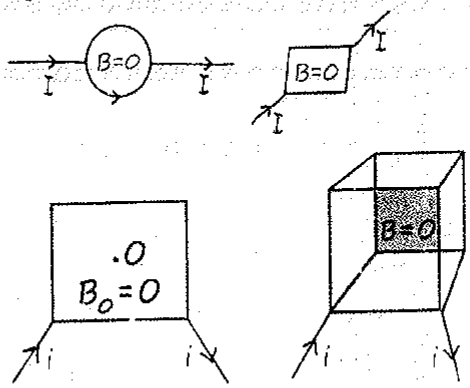
- Dir$^n$ of Magnetic Field :-

Right hand Rule:-

- Place thumb in the direction of current then curling finger will represent direction of magnetic field

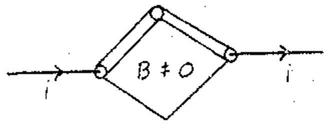


- Symmetrical Object :-

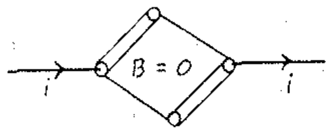



- Combination of two thick and two thin wire

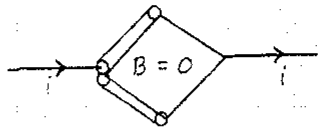
(i)

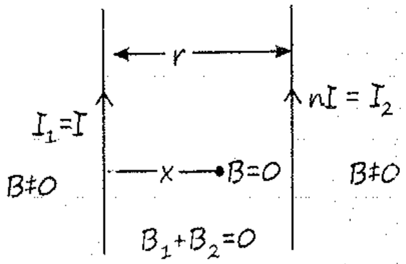



(ii)




(iii)




- 


$$B_1+B_2=0$$

$$\frac{\mu_0 I}{2\pi x} = \frac{\mu_0 I n}{2\pi(r-x)}$$

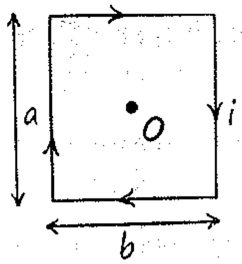
$$r-x = nx$$

$$\boxed{x = \frac{r}{n+1}}$$

- Similarities and difference between Biot Savart law and coloumbs law :-

| | |
|---|---|
| (i) Electric field is produced by scalar source "charge" | (i) Magnetic field produced by vector source "current element $i\vec{dl}$" |
| (ii) Electric field along the position vector from source | (ii) Magnetic field perpendicular to the position vector from source |
| (iii) follow inverse square law | (iii) also follow same |
| (iv) linear inside source | (iv) linear inside source |

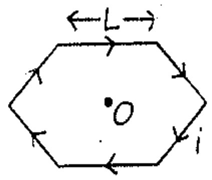
MR wala sawaal :-




$$B = \frac{2\mu_0 i}{\pi ab}\sqrt{a^2+b^2}$$

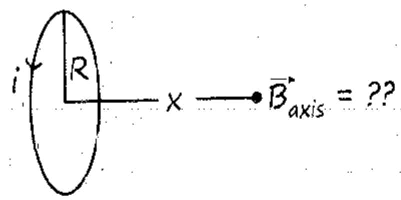
a=b=L
"Square"

$$B_0 = \frac{2\sqrt{2}\,\mu_0 i}{\pi L}$$




$$B_0 = \frac{\sqrt{3}\,\mu_0 i}{\pi L}$$

## Mag. Field on the axis of a current carrying loop :-




$$\boxed{B_{axis} = \frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}}$$

Circular loop :-

$$B_{centre} = \frac{\mu_0 i}{2R}$$

Semi-Circular loop :-

$$B_0 = \frac{\mu_0 i}{4R}$$

Quarter Circle :-

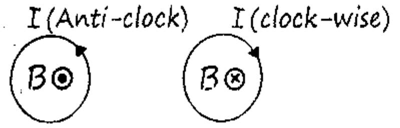
$$B_0 = \frac{\mu_0 i}{8R}$$

o Generalised Formula for Circular Arc :-

Vimp

radian.

$$\boxed{B_Q = \frac{\mu_0 i}{2R}\left[\frac{\theta}{2\pi}\right]}$$

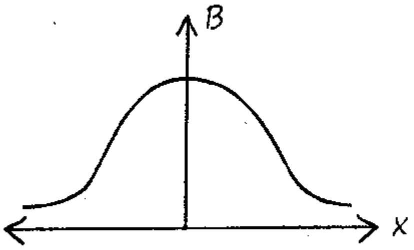



B ⊙ Up Anti B ⊗ down 'c' clock

MR* Anti-Needle up the conductor

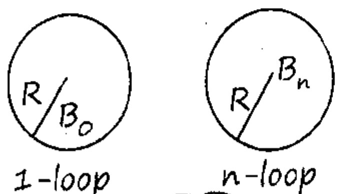
Clock-Needle down the conductor

o Graph of Mag. Field of Current Carrying circular loop :-




## Mag. Field due to looping of wire :-

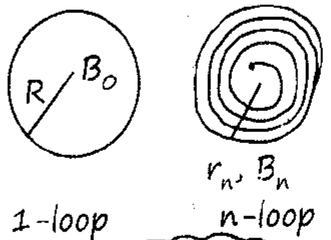
I> "R = Constant."




$B_n = n\, B_{1\ loop}$

II> Same wire rewound !

$L = 2\pi R = const^n$.




$B_n = n^2 B_{1\text{-}loop}$

## Amperes Circuital Law :-

o $\vec{B}$ = due to all inside or outside current.

o Always valid for all type of current.

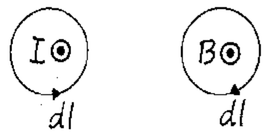
o only applicable when current distribution is symmetric.

o Not a magnetic flux, because here is close line integral.

$\oint \vec{B} \cdot \vec{dl} = \mu_0 i.$

$i$ = enclose current.

o Assume direction in loop and if direction of loop same as magnetic field then current will be positive if opposite then it will be negative.




$\oint \vec{B}.\vec{dl} = +\mu_0 I$ because B also Anti-Clock wise and loop

$\oint \vec{B}.\vec{dl} = -\mu_0 I$ $\vec{B}$ is Anti-Clock but loop is clock

Steps to apply Ampere's circuital law

1. Draw close symmetric Amperian loop, that must be passes through the point where field have to calculate.

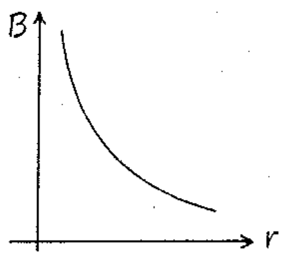
Ex circular, square loop etc

2. Angle between loop and magnetic field must be 0°, 90° or 180°

3. Value of B must be constant at all point of loop.

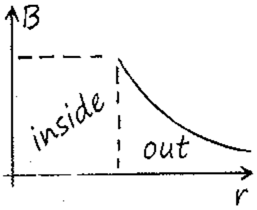
## M.F. Due to a straight infinite current carrying wire :-

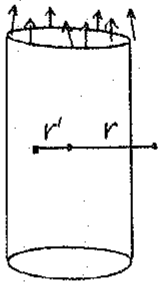
$$\boxed{B = \frac{\mu_0 i}{2\pi r}}$$



## M.F. Due to a infinite Current hollow Cylinder :-

$$B_o = \frac{\mu_0 i}{2\pi r}, \quad B_{in} = 0$$

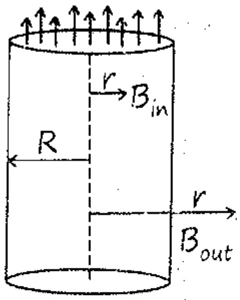



## M.F. Due to infinite Current Carrying solid Cylinder :-

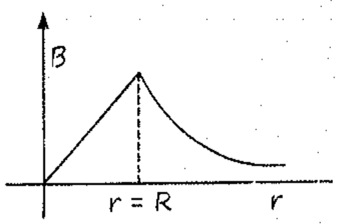
$$B_o = \frac{\mu_0 i}{2\pi r}$$

$$\boxed{B_{in} = \frac{\mu_0 i r}{2\pi R^2}}$$




$$B_{surface} = \frac{\mu_0 i}{2\pi R} \qquad B_{in} = \frac{\mu_0 J r}{2}$$

$$\boxed{\vec{B}_{in} = \frac{\mu_0 \vec{J} \times \vec{r}}{2}} \quad * \; J = \frac{I}{\pi R^2} \; *$$




## M.F. Due to infinite current carrying solid cylinder where $J=J_0\,x$ find mf outside the cylinder :-

$$J = J_0 x$$

$$\int_0^i di = \int_0^R J_0 x.(2\pi x).dx$$

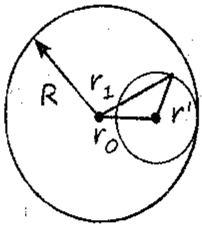
$$\boxed{i = \frac{J_0 2\pi R^3}{3}}$$

$$B(2\pi r) = \mu_0 i_{in}$$

$$B = \frac{\mu_0 J_0 R^3}{3r}$$

## M.F. Inside cavity of solid cylinder :-

$$B = \frac{\mu_0 J r_0}{2}$$



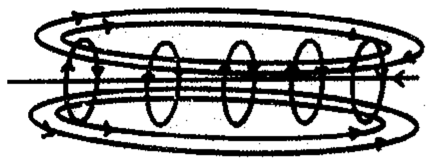

$$\vec{r}_0 + \vec{r}' = r_1$$

$$* \; \vec{r}_0 = \vec{r}_1 - \vec{r}' \; *$$

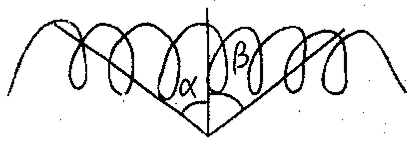
$$*B_{net} = \underbrace{\frac{\mu_0 J r_1}{2}}_{\text{Complete}} - \underbrace{\frac{\mu_0 J r'}{2}}_{\text{Cavity}} \; *$$

- Magnetic field wil be uniform inside cavity.

## Solenoid :-



- Finite Solenoid :-

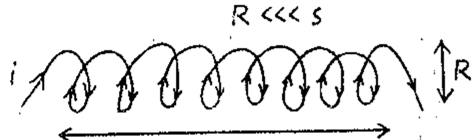



$$B = \frac{\mu_0 n i}{2}[\sin\alpha + \sin\beta]$$

$$B_C = \mu_0 n i, \quad B_{end} = \frac{\mu_0 n i}{2}$$

- Infinite Solenoid :-

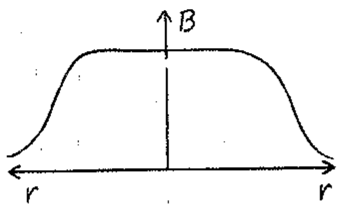
R <<< s



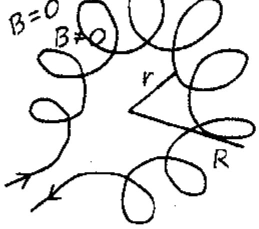

$$\boxed{B = \mu_0 n i = \frac{\mu_0 N i}{L}}$$ N = Total turn, L = Total length

$n \Rightarrow$ Turns per unit length.

$$n = \frac{N}{l} = \frac{\text{Total turns}}{\text{length of solenoid}}$$



## Toroid :-




$$\boxed{B = \frac{\mu_0 Ni}{2\pi R_{avg}}}$$

$$R_{avg} = \frac{R + r}{2}$$

## Magnetic Force :-

Magnetic field rest charge per force nahi lagata

$\vec{F} = qBv\sin\theta = q(\vec{V} \times \vec{B})$

$\theta$ = Angle between $\vec{V}$ & $\vec{B}$.

$\vec{F}\ \perp^{er}\ \vec{V}$ and $\vec{F}\ \perp^{er}\ \vec{B}$.

$\vec{a} \perp^{er} \vec{v}$

only direction will change, speed will remains constant.

K.E = constant

Work done = 0

Power = $\vec{F} \cdot \vec{v} = 0$

∴ M.F. की औकात नहीं है "q" की speed change करे! # Garda visualize.

Lekin $accl^n \neq 0$

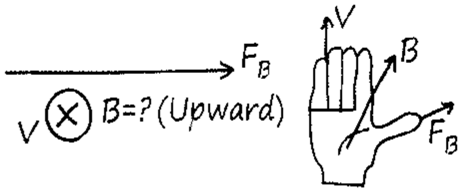
Kyuki $dir^n$ change hokr $\vec{V}$ & $\vec{P}$. change hoga!

M.F. is like Centripetal Force ($F \perp V$)

## Lorentz Force :-

$\vec{F}_m = q[\vec{E} + \vec{V} \times \vec{B}]$

$\vec{F}_m = \vec{F}_{elec.} + \vec{F}_{Mag.}$




MR* Law for direction:-

Place your four finger of right hand along velocity and then slap magnetic field by palm then thumb will represent direction of force.

## Motion of Charge Particle in Magnetic Field :-

1> Charge is projected $II^{el}$ to Mag. Field :-

$\vec{F}_m = 0 \qquad a = 0 \qquad V = Const^n$

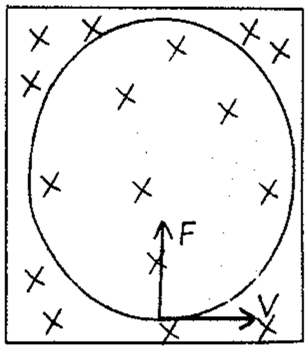
$\boxed{dist^n = vt}$

2> Charge is projected $\perp^{er}$ to Mag. Field :-

$\vec{V}$ = Variable

$F_m = qVB$

$$\boxed{a = \frac{qVB}{m}}$$




- Radius of Circular path :-

$$\boxed{R = \frac{mv}{qB} = \frac{P}{qB} = \frac{\sqrt{2mKE}}{qB} = \frac{\sqrt{2mqV}}{qB}}$$

- Time Period :-

Time taken to complete one rotation.

T does not depends upon speed

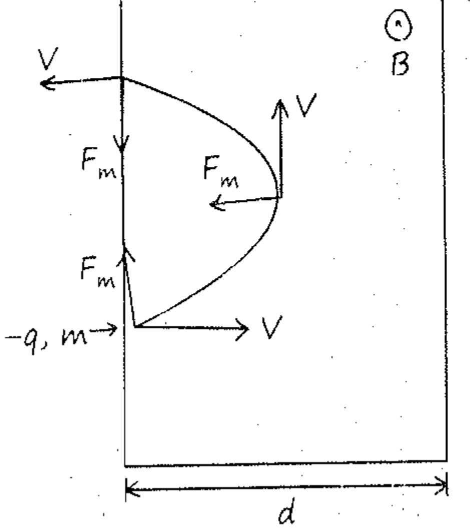
$$T = \frac{2\pi m}{qB} \qquad f = \frac{qB}{2\pi m}$$

$$\boxed{T_\theta = \frac{m\theta}{qB}} \qquad f = \frac{qB}{m\theta}$$

time taken to Rotate $\theta$ angle

3> Charge particle is projected from outside region of M.F. $\perp^{er}$ to Field :-

$R \le d$




$\therefore$ $\boxed{\text{Deviation} = 180°}$

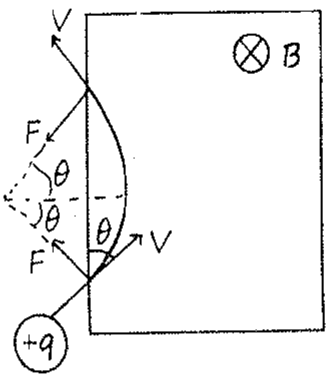
MR**

$$\boxed{F_{avg} = qVB \frac{\sin(\theta/2)}{\theta/2}}$$

$\theta = 180°$

$$F_{avg} = \frac{2qVB}{\pi} \qquad T_\theta = \frac{m\theta}{qB}$$

4> Charge particle is projected $\perp^{er}$ to Mag. Field at some angle with boundary of M. Field :- (M.F. :- inwards)

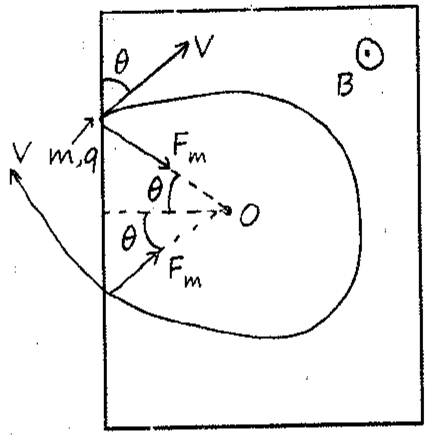



$\boxed{\delta = 2\theta}$

$$T_\theta = \frac{m(2\theta)}{qB}$$

$$F_{avg} = \frac{qVB\sin\theta}{\theta}$$

5> Charge particle is projected $\perp^{er}$ to Mag. Field at some angle with boundary of MF :-




$\boxed{\delta = 2\pi - 2\theta}$

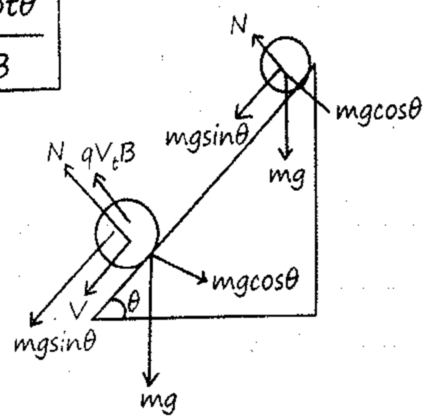
$$T_Q = \frac{m\delta}{qB}$$

$\theta$ = Angle between boundary & velocity !

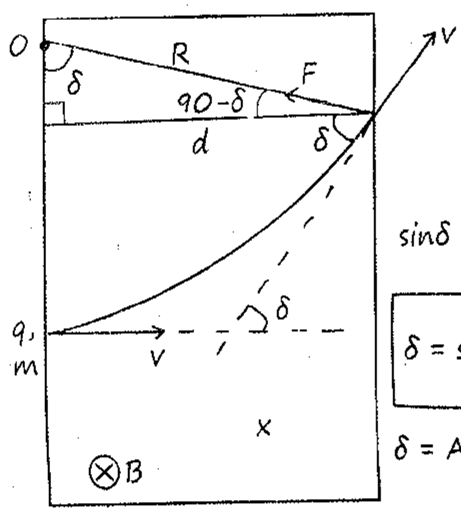
Note :-

- Time req. when "q" loose contact on smooth inclined plane :-
  Magnetic field is outside the plane : B⊙
  $qB\, g\sin\theta t = mg\cos\theta$

$$\boxed{t = \frac{m\cot\theta}{qB}}$$




- Charge particle is projected $\perp^{er}$ to Mag. Field where ($d < R$) then $\delta$ = ?




$$\sin\delta = \frac{d}{R}$$

$$\boxed{\delta = \sin^{-1}\left[\frac{d}{R}\right]}$$

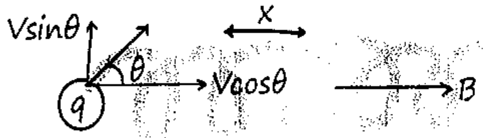
$\delta$ = Ang. deviat$^n$

## Imp Cases :-

1> Object is projected with speed "v" at an angle "θ" from Mag. Field :-

Path of the particle will be helical.

$\theta \neq 0°, 90° \ \& \ 180°$

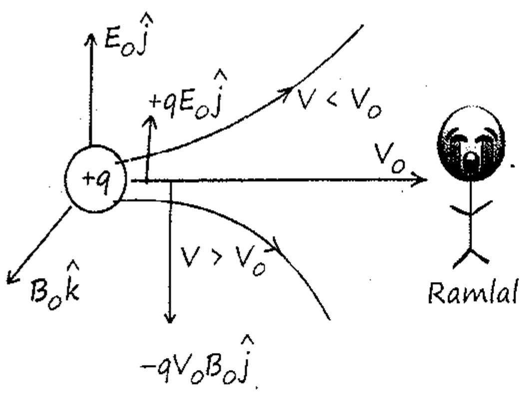



$$R = \frac{mv\sin\theta}{qB} \qquad T = \frac{2\pi m}{qB}$$

$$\boxed{\begin{aligned} \text{Pitch } (x) &= U_x T \\ &= U\cos\theta \cdot \frac{2\pi m}{qB} \end{aligned}}$$

## Velocity Selector :-

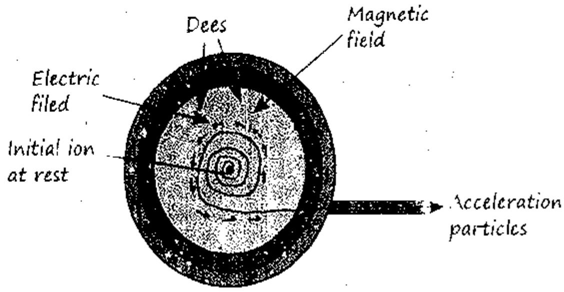
- Magnetic field, Electric field and velocity all are perpendicular to each other.
- Charge velocity which having velocity Vo will passes without deviation because net force on that will be zero.
- Particle which have velocity $V > V_0$ will experience large magnetic force and deviates downward
- Particle which have velocity $V < V_0$ will experience small magnetic force and deviates upward.




$$qE_0 = qV_0B_0 \qquad \boxed{V_0 = \frac{E_0}{B_0}}$$

## Cyclotron :-

- Device use to acceleration charge particle like proton deuteron, α -particle but not for electron, we use betatron for electron.



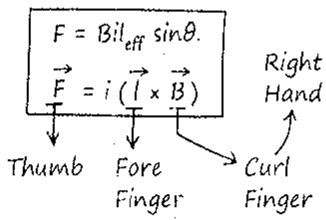

- Electric field used to acceleration and shift provide k.E. to the charge particle.
- Magnetic field used to keep the charge particle inside magnetic field.
- Freq. of oscillator = Freq. of charge particle

$$\boxed{KE_{1rot^n} = 2q\Delta V} \qquad \boxed{f_{req} = \frac{qB}{2\pi m}}$$

$$KE = \frac{1}{2}\frac{q^2B^2R^2}{m} \quad \because \frac{mv^2}{r} = qVB$$

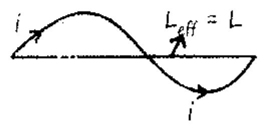
## Magnetic Force on Current Carrying Wire :-

- Magnetic force always perpendicular to the plane of $I\vec{dl}$ and $\vec{B}$.
- Force on close loop of any shape is zero ($l_{eff} = 0$)



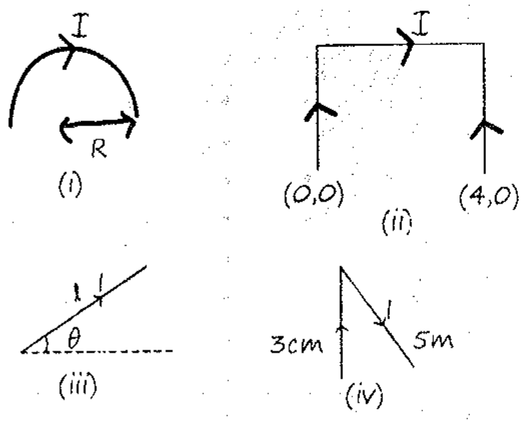

θ = Angle between i & B.

$L_{eff}$ :-



MR*for direction:- Place your four finger of right hand along $I\vec{l}$ and slap magnetic field thumb will represent force.

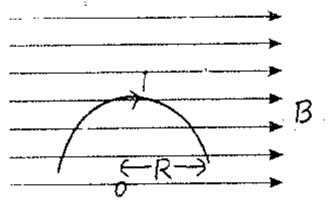
Q. Magnetic force on different wire.



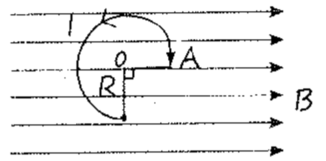

Ans.

(i) $F = 2BIR$ (ii) $F = 4IB$

(iii) $F = BIl$ (iv) $F = 4BI$



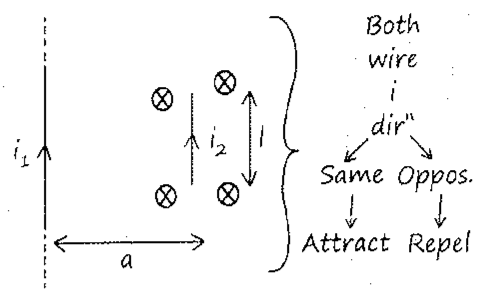

$F = 0$




$F = BIR$
effective length perpendicular to B is R.

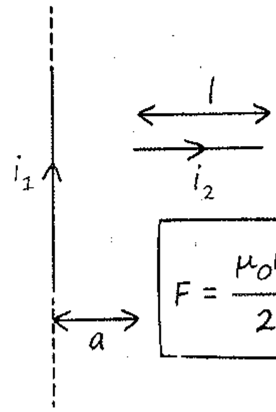
## Force on small current carrying wire due to infinite large current carrying wire :-

Case 1 :-




$$\boxed{F = B_1 i_2 l = \frac{\mu_0 i_1}{2\pi a} i_2 l}$$

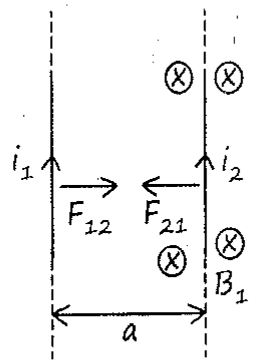
Case 2 :-




$$\boxed{F = \frac{\mu_0 i_1 i_2}{2\pi}\left[\ln\left(\frac{a+L}{a}\right)\right]}$$

#ye sab Mutual Force hai matlab jitna badi-choti wire pr Force lagayega utnahi Force choti-badi pr lagayegi.

Case 3 :-




$F = Bil$

$\frac{F}{l} = Bi$

$$\boxed{\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi a}}$$

"Force per unit length"

## Biot-savart Law in Terms of Velocity of Particle :-

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{qV\sin\theta}{r^2}}$$

$\theta$ = Angle between l & r.

Note :-

$$\frac{F_{MF}}{F_{EF}} = \frac{\mu_0 e^2 V^2}{\cancel{4\pi d^2}} \times \frac{\cancel{4\pi\varepsilon_0 d^2}}{\cancel{e^2}}$$

$$\boxed{\frac{F_{MF}}{F_{EF}} = \frac{V^2}{C^2}} \qquad C^2 = \frac{1}{\mu_0\varepsilon_0}$$

$F_{EF} >>>>>> F_{MF}$

## Circular Current Loop as a Magnetic Dipole :-

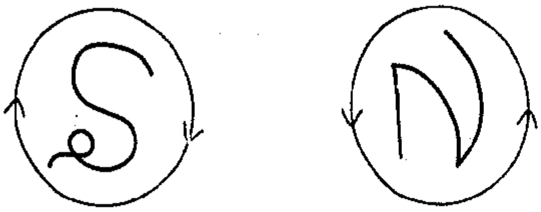
$$B = \frac{\mu_0 i R^2}{2(R^2+X^2)^{3/2}}$$

$$B = \frac{2\mu_0}{4\pi}\frac{R^2\pi i}{X^3} = \frac{2KM}{X^3}$$

$$\boxed{M = i(\pi R^2) = iA}$$

$\to$ Vector $\to$ Amp $m^2$

$\to$ Dir$^n$ along Area vector



CW South-pole $M$:- $\otimes$

ACW North-pole $M$:- $\odot$

Direct$^n$ of $\vec{M}$ is along $\vec{B}$.

$M = NAI \to N$ = no. of turns.

Magnetic Moment of revolving electron :-

$$f = \frac{V}{2\pi R}$$

$$\boxed{M = IA = ef\pi R^2 = \frac{eVR}{2}}$$

Gyromagnetic Ratio:-

- Ratio of Mag. Moment and Angular Momentum:-

$$\boxed{\frac{M}{L} = \frac{e}{2m_e} = \frac{8.8\times10^{10}C}{Kg}}$$

- Bohr Magnet on :-

$$M_{e^-} = \frac{e}{2m_e}\left[\frac{h}{2\pi}\right]$$

$$\therefore M = \frac{e}{2m_{e^-}}[I\omega]$$

\# $I$ = moment of inertia

## Torque on a Current Carrying Loops, Magnetic Dipole :-

$$\vec{\tau} = \vec{M}\times\vec{B} = MB\sin\theta$$
$$\vec{\tau} = BiNA\sin\theta$$

$\theta$ = Angle between M & B.

Torque perpendicular to magnetic moment and magnetic field

Net force = zero

- If angle given from plane of loop

$\tau = MB\sin(90-\theta) = MB\cos\theta$

## Magnetic Potential Energy Stored in Magnetic Dipole :-

$$U = -MB\cos\theta$$
$$U = -BiNA\cos\theta$$

Mag. Field Ka ekhi Udesh hai woh Mag. Dipole Ko Apne Along align Krna Chahta hai.

- Time Period :- $T = 2\pi\sqrt{\frac{I}{MB}}$

$I$ = Moment of Inertia

$M$ = Magnetic moment

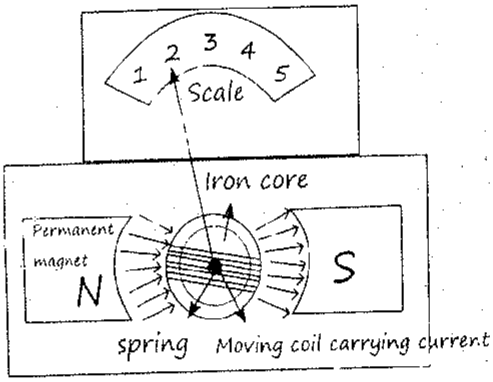
- Work Done to Rotate dipole $W=\Delta U$.
- Work done by M.F. to rotate dipole = $W = -\Delta U$

## Moving Coil Galvanometer :-

Working Principle: Torque on current carrying loop

If No. of turns in moving coil-galvanometer is increase then current sensitivity will increases but voltage sensitivity remains same because resistance will also increase.

$$V_s \propto \frac{N}{R}$$




$\tau = MB = NIAB = C\theta$

Torsional Const$^n$

$\boxed{\tau_{spring} = C\theta}$ $\theta = \dfrac{BiNA}{C}$

Area of Loop

$\boxed{I_S = \dfrac{\theta}{I} = \dfrac{BAN}{C}}$ Current Sensitivity.

Divided by R both sides

$\boxed{V_S = \dfrac{I_S}{R} = \dfrac{BAN}{CR} = \dfrac{\theta}{V}}$ Voltage Sensitivity.

Visualisation 1.

If an electron is not deflected in passing through a certain region of space, can we be sure that there is no magnetic field in that region?

No, electron would not be deflected if $\vec{v}$ and $\vec{B}$ are in the same direction.

Visualisation 2.

If a moving electron is deflected sideways on passing through a certain region of space, can we be sure that a magnetic field exists in that region?

No, the sideways deflection may be due to Electric field as well. In the absence of electric field, the sideways deflection shows the presence of magnetic field in the region.

Visualisation 3.

If a charged particle at rest experiences no electromagnetic force, then the electric field must be zero

or

The magnetic field may or may not be zero

Visualisation 4.

If a charged particle kept at rest experiences an electromagnetic force, then The electric field must not be zero

or

The magnetic field may or may not be zero.

Visualisation 5.

If a charged particle projected in a gravity-free room deflects, then Both fields cannot be zero

or

Both fields can be non-zero

Visualisation 6.

A charged particle moves in a gravity-free space without change in velocity. Possible cases are

$E = 0\ B = 0$ or $E = 0\ B \neq 0$ or $E \neq 0\ B \neq 0$

Visualisation 7.

A charged particle moves along a circle under the action of possible constant electric and magnetic fields. Possible case is

$E = 0, B \neq 0$

Visualisation 8.

A charged particle goes undeflected in a region containing electric and magnetic field. It is possible that

$\vec{v} \parallel \vec{E}, \vec{E} \parallel \vec{B}$ or $\vec{E}$ is not parallel to $\vec{B}$

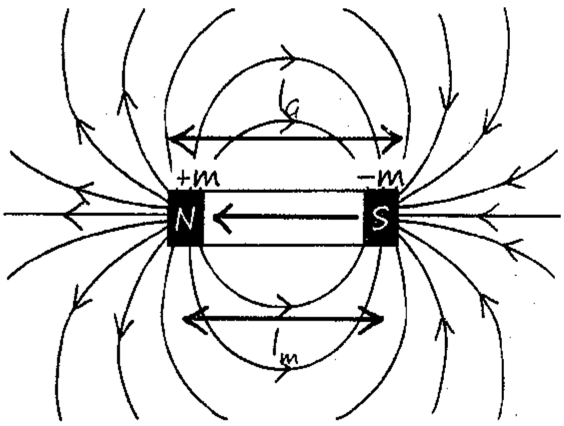
Visualisation 9.

If a charged particle goes unaccelerated in a region containing electric and magnetic fields, then

$\vec{E}$ must be perpendicular to $\vec{B}$ and $\vec{v}$ must be perpendicular to $\vec{E}$.

# Magnetism & Matter

## Bar Magnet :-




$$\boxed{\frac{l_{Mag}}{l_{Geo}} = 0.84}$$

- Magnetic field lines are also called magnetic force line → false because force acts perpendicular to magnetic field.
- Magnetic field lines always from N to S → false, Inside Magnet it is S to N

Properties of Magnetic field lines

- They form closed loop
- They never intersect each other
- Magnetic field lines are crowded near the Pole where magnetic field is strong. and spread apart from each other where field is weak.
- They flow from the South Pole to the north Pole within a magnet and north pole to South Pole outside
- They Comes out and go in at any angle from magnet.

Magnetic Dipole Moment :-

$\vec{M} = m\vec{l} = NIA$  m:- Pole Strength

Direction :- From $S \longrightarrow N$
(Vector)

## Cutting of bar Magnet :-

$m \propto$ Area $\qquad M \propto$ Volume

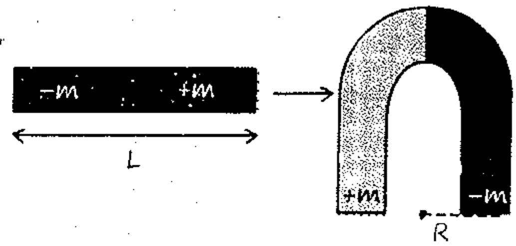
(1) Cut along length $(m/2), l$

$M' = M/2$

(2) Cut $\perp^{er}$ to length $m, l/2$

$M' = M/2$

## Bending of bar Magnet :-



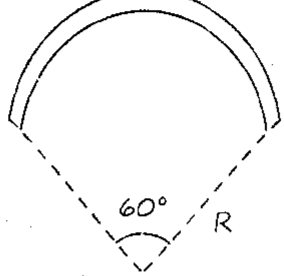

MR***

Job bhi mein Koi Circle dhekhu mera dil dewaana bole....

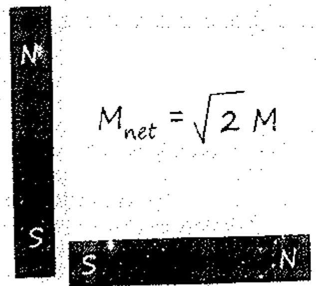
$$\boxed{M' = \frac{M \sin(\theta/2)}{\theta/2}}$$

| Complete Circle | Semi-Circle. |
|---|---|
| $\theta = 2\pi$ | $\theta = \pi$ |
| $M' = 0$ | $M' = \frac{2M}{\pi}$ |

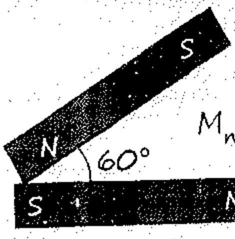
- $M' = \dfrac{M \sin\left(\frac{60°}{2}\right)}{\frac{\pi}{3 \times 2}} = \dfrac{3M}{\pi}$



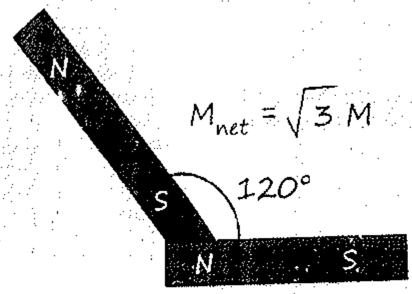
- Two Identical bar Magnet of magnetic Moment M



$M_{net} = \sqrt{2}\,M$



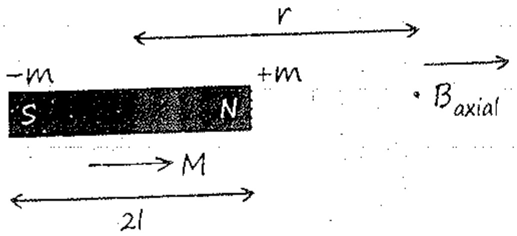
$M_{net} = M$



$M_{net} = \sqrt{3}\,M$

## Mag. Field due to bar Magnet :-

1> On axial point :-

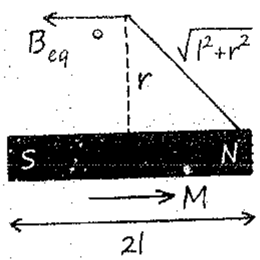



$$B_{axial} = \left(\frac{\mu_0}{4\pi}\right) \frac{2Mr}{(r^2-l^2)^2} = \left(\frac{\mu_0}{4\pi}\right) \frac{2M}{r^3}$$

- Magnetic Field along dipole

2> On Normal Bisector :-

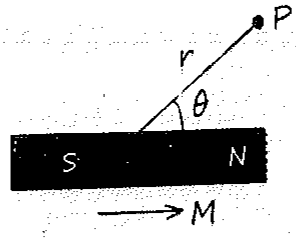
$$B_{eq} = \left(\frac{\mu_0}{4\pi}\right) \frac{2ml}{(r^2+l^2)^{3/2}}$$




$$B_{eq} = \left(\frac{\mu_0}{4\pi}\right) \frac{M}{r^3}$$

- Magnetic Field opposite to dipole.

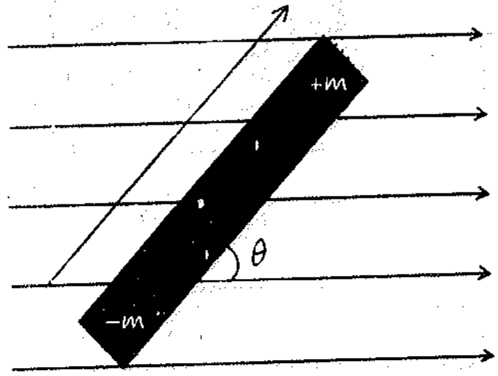
3> Magnetic field due to Dipole at General Point :-




$$B_p = \left(\frac{\mu_0}{4\pi}\right) \frac{M}{r^3} \sqrt{3\cos^2\theta + 1}$$

## Dipole in uniform Mag. Field :-

$\vec{M} = ml$




| | |
|---|---|
| $\vec{\tau} = \vec{M} \times \vec{B}$ | $U = -\vec{M} \cdot \vec{B}$ |
| $\tau = MB\sin\theta$ | $= -MB\cos\theta$ |
| $\theta = 0°$ | $\theta = 0°$ |
| $\tau = 0$ | $U = -MB$ |
| $\theta = 90°$ | $\theta = 90°$ |
| $\tau_{max} = MB$ | $U = 0$ |
| $\theta = 180°$ | $\theta = 180°$ |
| $\tau = 0$ | $U = MB$ |

- $W_{ext} = \Delta U$ $\quad W_B = -\Delta U$
- Stable equilibrium of $\theta = 0°$
- Unstable equilibrium at $\theta = 180°$
- BAR magnet will oscillate in uniform magnetic field about stable equilibrium

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

MR***

Magnetic Field Ka ekhi Udessh hai Ki woh Mag. dipole Ko apni taraf Khich Ke rakhna Chahta hai !

**Analogy :-**

| Electrostatics | Magnetism |
|---|---|
| $\frac{1}{\varepsilon_0}$ | $\mu_0$ |
| Charge $q$ | Magnetic Pole Strength ($m$) |
| Dipole Moment $\vec{p} = q\vec{l}$ | Magnetic Dipole Moment $\vec{M} = m\vec{l}$ |

| | |
|---|---|
| $F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$ | $F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}$ |
| $\vec{F} = q\vec{E}$ | $\vec{F} = m\vec{B}$ |
| Axial Field $\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3}$ | $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{r^3}$ |
| Equatorial field $\vec{E} = \frac{-\vec{p}}{4\pi\varepsilon_0 r^3}$ | $\vec{B} = -\frac{\mu_0}{4\pi}\frac{\vec{M}}{r^3}$ |
| Torque $\vec{\tau} = \vec{p} \times \vec{E}$ | $\vec{\tau} = \vec{M} \times \vec{B}$ |
| Potential Energy U $= -\vec{p} \cdot \vec{E}$ | $U = -\vec{M} \cdot \vec{B}$ |
| Work, W $= pE(\cos\theta_1 - \cos\theta_2)$ | Work, W $= MB(\cos\theta_1 - \cos\theta_2)$ |

MR SPECIAL***

**Force Between**

| Two point Charge | Dipole & Point Charge | Dipole & Dipole |
|---|---|---|
| $F \propto \frac{1}{r^2}$ | $F \propto \frac{1}{r^3}$ | $F \propto \frac{1}{r^4}$ |

## Gauss Law in Magnetism :-

- Isolated Monopoles X

$$\int \vec{B}.\vec{ds} = \phi_B \qquad \oint \vec{B}.\vec{ds} = 0 \text{ (Always)}$$

MR Speical***

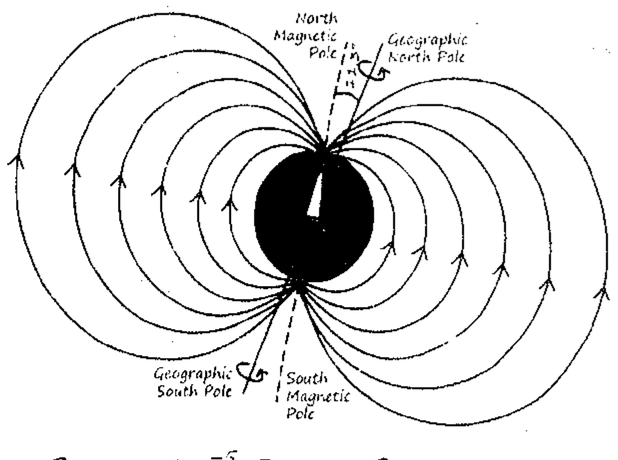
1> $T$ = Same $\Big\}$ When bar magnet cut along length.

2> $T' = \frac{T}{n}$ $\Big\}$ When bar magnet cut $\perp^{er}$ to length.

$n$ = no. of equal cutted part.

$T = \frac{T}{n}$ $\qquad$ $T' = T$

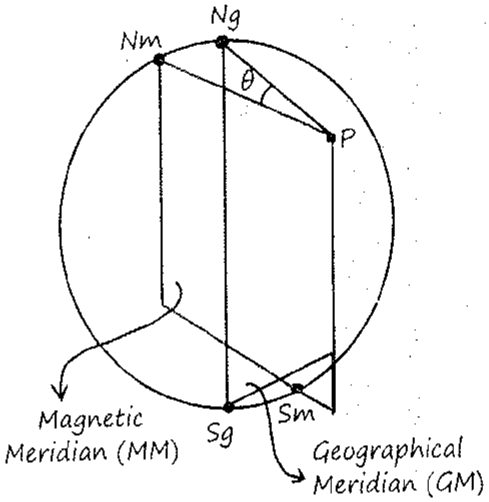
## Earth Magnetism :-




$B_{earth} = 10^{-5}\, T = 0.1$ Gauss.

- Magnetic Field Lines :-

$\parallel^{el}$ :- Equator $\qquad \perp^{er}$ :- Pole.

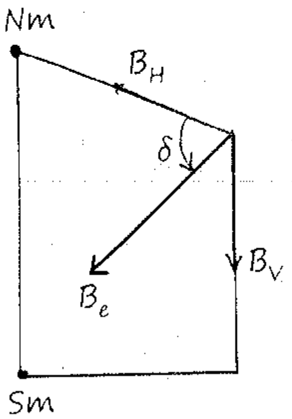



$\theta$ = Angle of declination

Angle made by Magnetic meridian with Geographical Meridian.

- **Angle of dip :-**

  Angle made by Earth's net Mag. Field with horizontal earth surface.
- N-hemisphere = $\delta$ = +ve
- S-hemisphere = $\delta$ = −ve



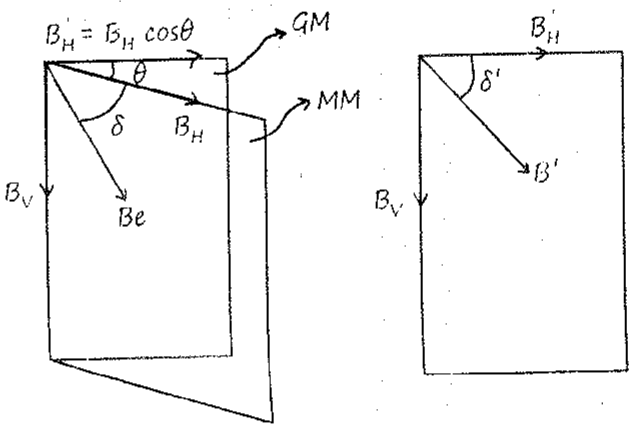

$$\tan\delta = \frac{B_V}{B_H}$$

$$B_e = \sqrt{B_V^2 + B_H^2}$$

$B_V = B_e \sin\delta \qquad B_H = B_e \cos\delta$

Dip Circle :- Plane of Compass.

Apparent Angle of dip :



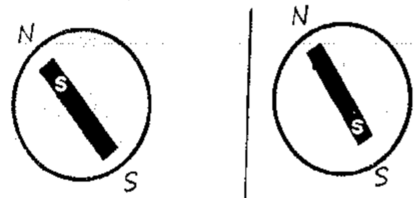

$$\boxed{\tan\delta' = \frac{\tan\delta}{\cos\theta} = \frac{B_V}{B_H'}}$$

**MR Speical*****

Apparent dips when dip circle is placed in two mutually $\perp^{er}$ $dir^n$ are $\theta_1$ & $\theta_2$. What is the Actual dip ($\theta$) at that place :-

$$\boxed{\cot^2\theta_1 + \cot^2\theta_2 = \cot^2\theta}$$

# Neutral Point :-

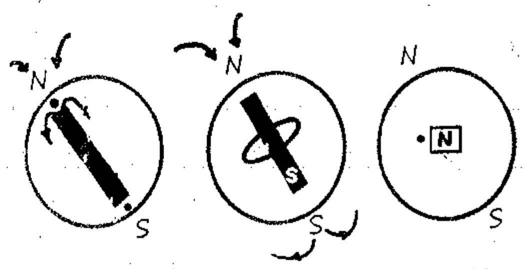



2 = NP are found on the axis

$$\Rightarrow \left(\frac{\mu_0}{4\pi}\right)\frac{2M}{r^3} = B_m$$

∞ = NP are found on equatorial circle

$$\left(\frac{\mu_0}{4\pi}\right)\frac{M}{4\pi} = B_m$$ can be used




2 = NP ∞ = NP 1 = NP

## Vibrational Magnetometer :-

(Oscillational Magnetometer)

$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$

Application -1 :-

To Find Magnetic dipole Moment (M) :-

$$M = \frac{4\pi^2 I}{T^2 B_H}$$

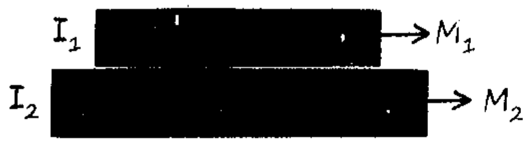
Application -2 :-

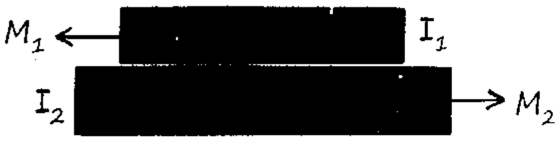
To Find ratio of Mag. dipole moment of two magnet of same size.

$$\frac{M_2}{M_1} = \left(\frac{T_1}{T_2}\right)^2$$

Application -3 :-

To Compare Mag. dipole moment of two magnet of diff. size.



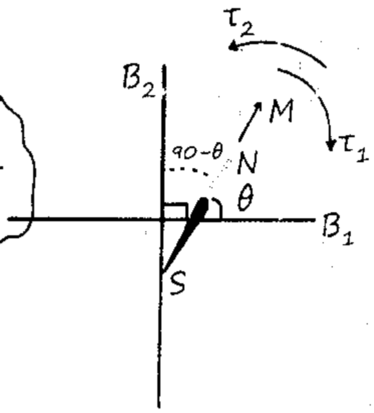

$$\therefore \boxed{\frac{M_1}{M_2} = \frac{T_2^2+T_1^2}{T_2^2-T_1^2} = \frac{f_1^2+f_2^2}{f_1^2-f_2^2}}$$

$$T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1+M_2)B_H}}$$

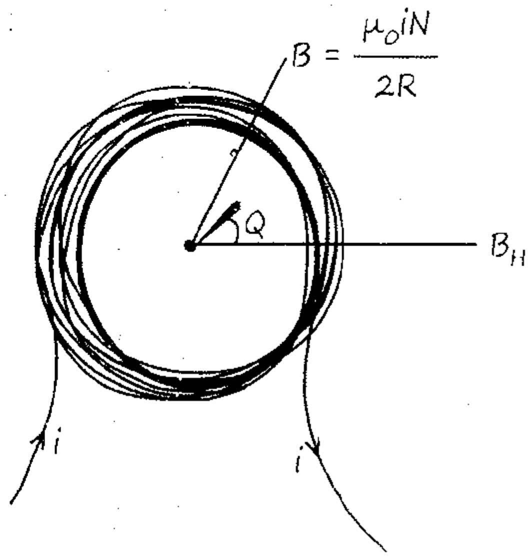
$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1-M_2)B_H}}$$

## Tangent Law :-

$$\tan\theta = \frac{B_2}{B_1}$$




## Tangent Galvanometer :-




$$M \cancel{B_H} \sin\theta = \cancel{M}\left(\frac{\mu_0 iN}{2R}\right)\cos\theta.$$

$$\boxed{\tan\theta = \frac{\mu_0 iN}{2R\,B_H}}$$

$$\underline{K \tan\theta = i} \qquad \left(K = \frac{2R\,B_H}{\mu_0 N}\right)$$

"K = Reduction Factor."

- Magnetic Field Intensity :- **(H)**

  Source की असली औकात

$$H = \frac{B_0}{\mu_0} = \frac{B_m}{\mu_m}$$

- Magnetic Permeability :- **($\mu$)**

$$\mu = \frac{B}{H}$$

$\mu$ → Medium, $H$ → Source

Scalar Unit :- Wb/Amp-m

- Magnetisation & Magnetic Intensity :- (I)

$$\vec{I} = \frac{\vec{M}}{V}$$ → Vector, Medium independent.

↳ $dir^n \parallel^{el}$ to $\vec{M}$

- Magnetic Susceptibility :- ($\chi$)

$$\chi_m = \frac{I}{H}$$ → Scalar → Unit & Dimensionless

$\chi_m \uparrow$ = Easily magnetised.

- Relation Between $\mu_r$ & $\chi_m$ :-

$$\mu_r = 1 + \chi_m$$

## Cause of magnetism :-

Atom → (Nucleus + electron in rotational motion)

#revolving electron Produce magnetic field, (magnetic moment) along axis of Rotation.

#In Paired electron atom, two electrons are in Opposite spin.

$\vec{M}_{atom} = 0$

# In unpaired electron atom.

$\vec{M}_{atom} \neq 0$ and $\vec{M}_{crystal} = 0$ due to random orientation of atoms.

## Materials :-

1> Diamagnetic :-

- Diamagnetic have tendency to Move from region of Stronger to weaker part of external Magnetic Field
  - ∵ They are Magnetised in opposite direction.
- Magnetic field lines are expelled by these substances.

Imp → Diamagnetism is universal property

MR***

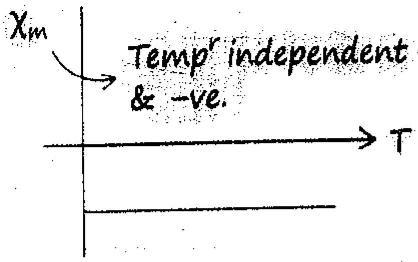
#Magnetise hota hai lenz law (law of inertia ) se.

$\chi_m = -ve$

$B < B_0$

$\mu < \mu_0$

$\mu_r < 1$




2> Paramagnetic :-

Paramagnetic substance

$M_{atom} \neq 0$

$M_{material} = 0$ (in absence of external mag. field)

$M_{material} \neq 0$ (⊕ presence of $B_{ext}$)

Tendency to move from Weak Magnetic field Region to strong magnetic field.

MR***

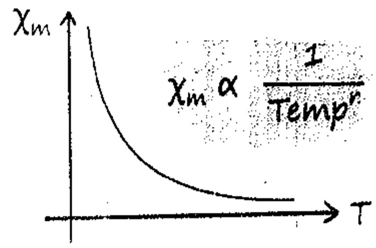
#Magnetise hota hai $\tau_{ext}$ se.

$\chi_m$ = +ve (Small +ve)

$B > B_0$

$\mu > \mu_0$

$\mu_r > 1$




3> Ferromagnetic :-

Ferromagnetic Material

strongly attracted to Magnetic Substance

$\vec{M}_{domain} \neq 0$

$M_{material} = 0$ (∵ Random arrangement of domains)

$M_{material} \neq 0$ (⊕ magnetic field external)

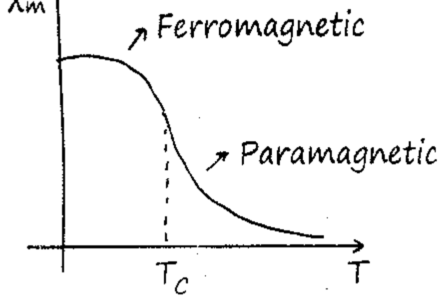
# Domain Formation.

$\chi_m >>> 1$

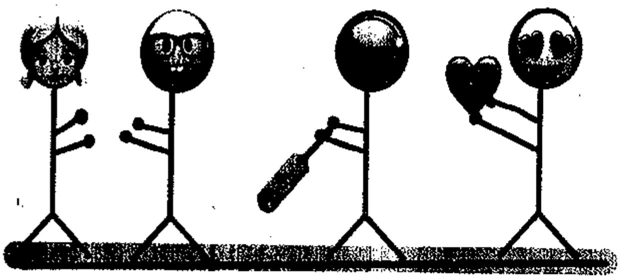
$B >>> B_0$

$\mu >>> \mu_0$

$\mu_r >>> 1$




## MR Speical***




**Curie's Law :-**

$$I = \frac{CB_0}{T}$$ Paramagnetic Sub$^s$.

$$\chi_m = \frac{C\mu_0}{T}$$

**Curie Weiss Law :-**

$$\chi_m = \frac{C}{T - T_c}$$

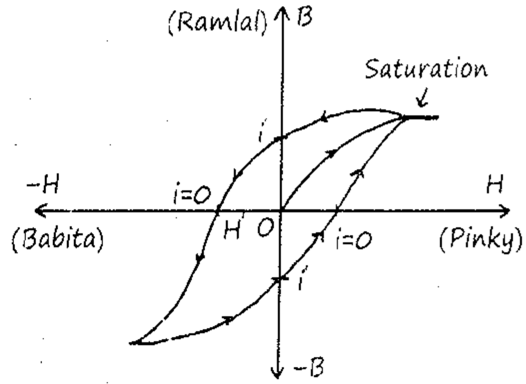
## Hysteresis Loop or B/H Curve :-

## MR Speical***

"Life of Ramlal"

Area under loop $\propto$ कितना कटा Ramlal का

Area under loop $\propto$ Energy loss




| OI' = Retentivity (R) | OH' = Coercivity. (C) |
|---|---|
| Ramlal Pinky Ke Pyaar mein Kitne yaadein bachake rakh paya tha | Babita Ko Jitna Pyaar dikhana pada pinky Ki yaadein Ko mitane Keliye. |

- SOFT IRON :- Small Area.

  Low Retentivity & Coercivity.

- Permanent Mag. :- Large Area.

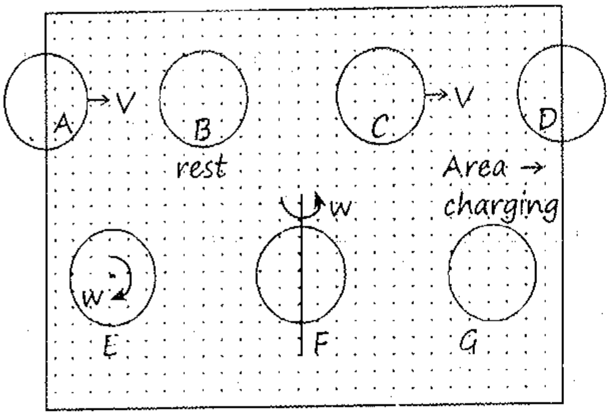
  High Retentivity & Coercivity.

MR

"Everyone including your society, family,friends sirf success ko he salute karte hai"

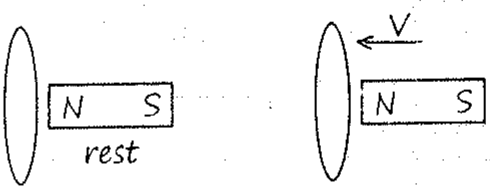
# 27 Electromagnetic Induction

## Faradays Experiment in Uniform Magnetic Field




In loop A, D, F, G → Current induced in loop because of change in magnetic flux.

In loop B, C, E → No current induced, because flux is constant.




No current          Current induced

## Magnetic Flux :-

Counting of magnetic field lines passing through given cross-section area.

- gives the idea of magnetic energy.
- scalar S.I. unit → Tesla-$m^2$ = Weber
  C.G.S. unit → Gauss $cm^2$ = Maxwell

$$\phi = \vec{B}.\vec{A} = BA\cos\theta$$

$\theta$ = Angle between Mag. Field & Area

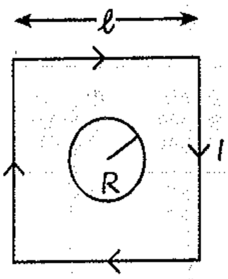
Unit :- 1 Wb = $10^8$ Maxwell

$\vec{B}$ = Variable.

$$\phi = \int \vec{B}.d\vec{A}$$

- If magnetic field $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ and $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ then
- flux $\phi = B_xA_x + B_yA_y + B_zA_z$
- Flux through circular loop ($l >> R$) = $\theta$ Area

$$= \frac{2\sqrt{2}\mu_0 I\pi R^2}{\pi \ell}$$




MR***

$\odot$ = outward = गद्यो की तरफ !

$\otimes$ = inward = गद्यो से दूर !

Note :-

$\phi$ → Variable, Constant

| Variable | Constant |
|---|---|
| $\phi = BA\cos\theta$ | $\phi = BA\cos\theta$ |
| $I_{induced} \neq 0$ | $I_{induced} = 0$ |

Variable due to :-

B → time varying

A → Variable

$\theta$ = Changing

→ Magnitude "Faraday's" Law; Direction "Lenz" Law

## Lenz Law :-

*Joh "i" Ko Paida Krta hh "i" Ushika Oppose Krta hh.

MR* law :-

- Coil is in inertia (Pyaar) of Flux.

External Field — Induced Field B

$\odot B\uparrow \longrightarrow \otimes$ (Then $I_{in}$ = CW)

$\odot B\downarrow \longrightarrow \odot$ (Then $I_{in}$ = ACW)

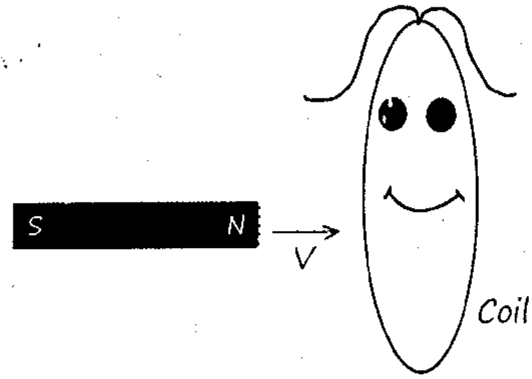
$\otimes B\uparrow \longrightarrow \odot$ (Then $I_{in}$ = ACW)

$\otimes B\downarrow \longrightarrow \otimes$ (Then $I_{in}$ = CW)

*"Up ⊙ Anti, down C"*

- *Coil Na Flux Ko pyaar Krta hai na Uskc hate Krta hai woh bs Change in flux ko oppose Krta hai.*

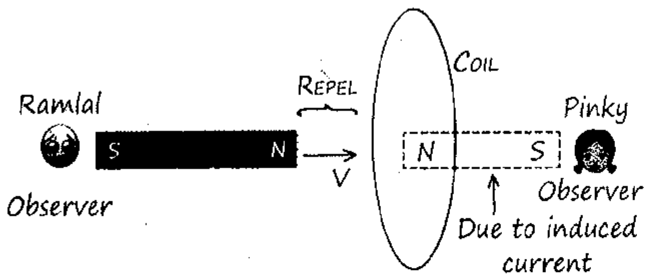
**MR*** law :-




*Coil Kehti hai magnet से :-*

*Pass mt aana repel Karungi durr mt jana attract Karungi aaisehi padhai krte raho Exam Ke baad date Karungi.*

*Case :- I*

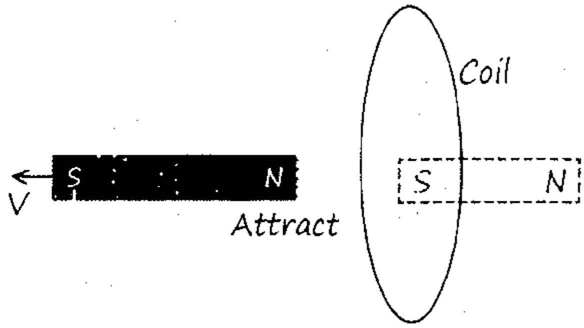



*Current induced in loop*

- w.r.t. Ramlal → ACW
- w.r.t. Pinky → CW

**MR*** → *Aage se Anti then pichhe se clock*

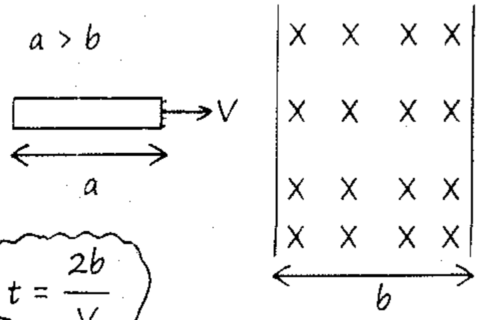
*Case :- II*




## NOTE (AIIMS)

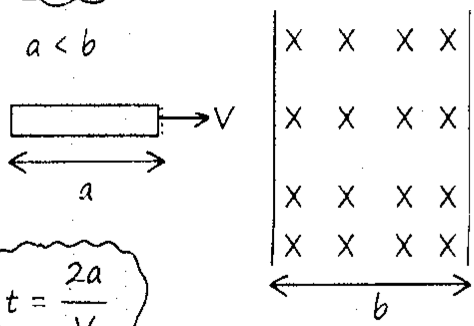
Q. Find time for which current will induced in ractangular loop?

Sol. I> $a > b$

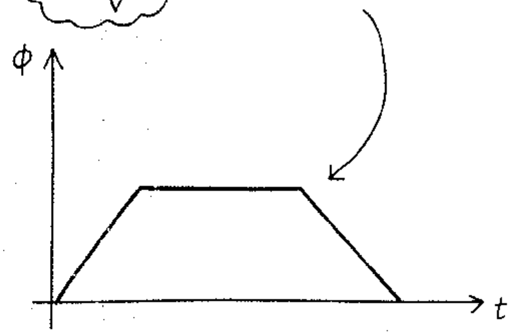


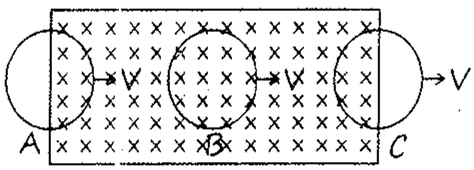

$t = \frac{2b}{V}$

II> $a < b$



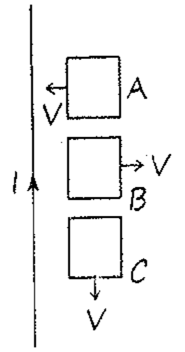

$t = \frac{2a}{V}$




Current induced in loop

Ⓐ → ACW  Ⓑ → 0  Ⓒ → CW

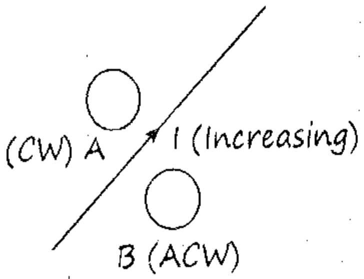



Current induced in loop

$A \rightarrow ACW$

$B \rightarrow CW$

$C \rightarrow Zero$




Faraday's Law of EMI :-

Based on Energy Conserv$^n$

| [Loop] $\varepsilon_{inst.}$ | $\varepsilon_{avg.}$ |
|---|---|
| $\varepsilon_{inst.} = \frac{-d\phi}{dt}$ | $\varepsilon_{avg.} = -\frac{\Delta\phi}{\Delta t}$ |
| $i_{inst.} = \frac{-d\phi}{R.dt}$ | $i_{avg} = -\frac{\Delta\phi}{R\Delta t}$ |
| $\varepsilon_{inst.} = \frac{dBA}{dt}$ | $\Delta Q = -\frac{\Delta\phi}{R}$ |

Q. Radius of circular loop placed perpendicular to magnetic field increasing at rate $r_o$ m/s then find induced emf in loop when radius is r.

Sol. $\text{emf} = -\frac{d\,BA\cos 0°}{dt} = -B\frac{d\,\pi r^2}{dr}$

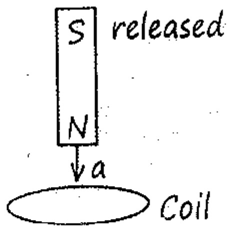
$\text{emf} = -B\pi 2 r_o r$ Volt

MR* Feel :-

We know $\phi = BA\cos\theta$

$$|\text{emf}| = \frac{d\phi}{dt} = \frac{d(BA\cos\theta)}{dt}$$

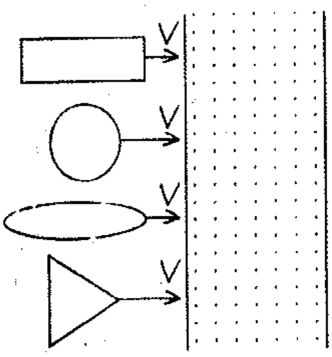
In this formula, three variable B, Area and angle ($\theta$), genrally 3 type ka question aayga B-time dependent, A-time dependent or $\theta$-time dependent.




a = g for non-conducting coil or coil with small cut.

a < g for conducting coil.

Q. In which loop induced emf will be uniform:

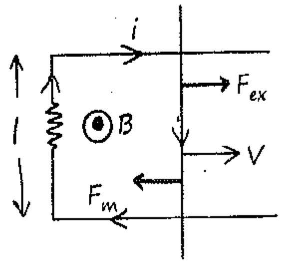



Sol. In rectangular loop because rate of change in flux is constant.

MR* law :-

लोहा हो लकड़ी हो Plastic हो टूटा हुआ तार हो Kuch bhi ho sab mein EMF induced करवाएगा Tera Faraday.

**Rail Problem** :- Rod of length l moving with velocity V.



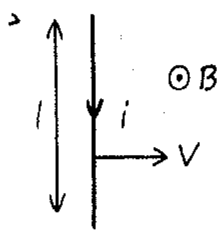

$\varepsilon = BVl$

$i = \frac{BVl}{R}$

$$F_{ext} = F_m = Bil = \frac{B^2l^2V}{R}$$

$$P = F.V = \frac{B^2l^2V^2}{R}$$

Hall Bhaiyaa :- [Rod]




$\hookrightarrow B \perp V \perp l$

$\varepsilon = BVl_{eff.}$

Scalar Triple Product :-

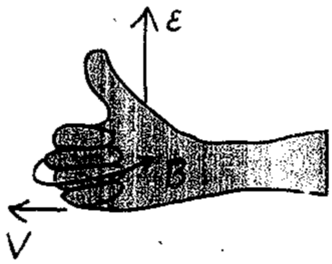
$\varepsilon = (\vec{V} \times \vec{B}) \cdot \vec{l} \qquad \because (F_M = F_E)$

- Direction of EMF :-

  Plam :- Velocity

  Curl Finger :- Magnetic Field

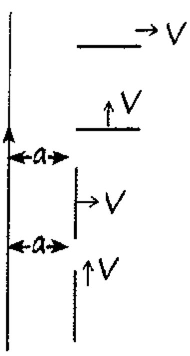
  Thumb :- EMF.




MR* for Direction of higher potential

place four finger along velocity and slap magnetic field thumb will represent higher potential.

- Induced emf in given Rod.

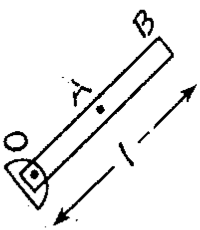



emf = 0

$emf = \frac{\mu_0 iV}{2\pi} \log\left(\frac{a + l}{a}\right)$

$emf = \frac{\mu_0 i\, lV}{2\pi a}$

emf = 0



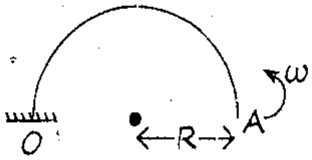

$(emf)_{OB} = \frac{1}{2} B\omega l^2$

$(emf)_{OA} = \frac{1}{2} \frac{B\omega l^2}{4}$

$(emf)_{AB} = \frac{3}{8} B\omega l^2$

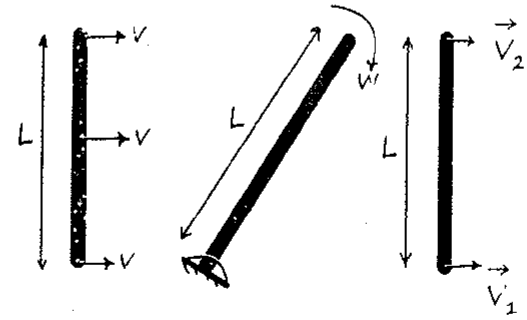
MR* :- *VVimp*

Fire Concent




$|emf|_{OA} = \frac{1}{2} B\omega(2R)^2$

$l_{eff} = 2R$

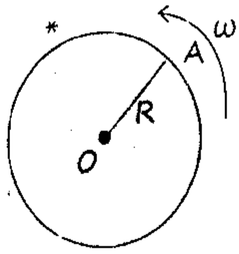



| Translational Motion (T) | Rotational Motion (R) | T + R Motion |
|---|---|---|
| Emotional = BVl | $E_R = \frac{1}{2} B\omega l^2$ | |

$\varepsilon = BL \left[\frac{\vec{V}_1 + \vec{V}_2}{2}\right]$

distⁿ between two points.

- Note :-




$\varepsilon_{OA} = \frac{B\omega R^2}{2}$

Disc. ⟶ Collection of ∞ Rods.

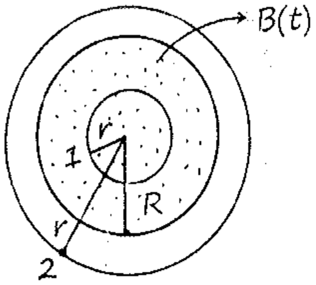
## Induced Electric Field :-

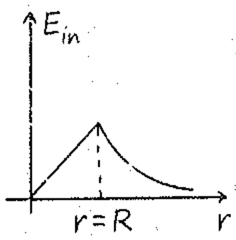
MR* :- $\boxed{\text{Electric Field}}$

| असली | नकली |
|---|---|
| Due to Rest Charge. | Due to time varying M. Field |
| Electrostatic Field. | Induced E. Field |

| असली | नकली |
|---|---|
| Conservative | Non-conservative |
| $\oint \vec{E}.\vec{dl} = 0$ | $\oint \vec{E}.\vec{dl} = A.\dfrac{d\vec{B}}{dt}$ |
| Does not Form Closed loop | Always form Closed loop. (Concentric Circle) |

## Value of induced E. Field :-



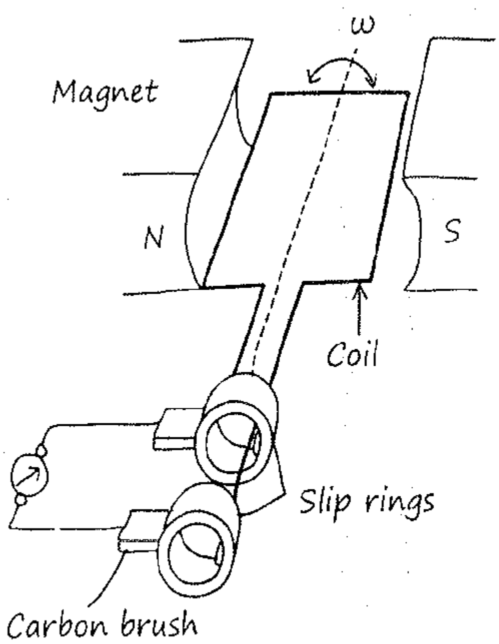

$$\boxed{E_1 = \frac{r\,dB}{2\,.\,dt}} \text{ (in)} \qquad \boxed{E_2 = \frac{R^2}{2r}\frac{dB}{dt}} \text{ (out)}$$

## Ac-generator :-

$\phi$ (flux) = $NAB\cos\theta$ and $\theta = \omega t$

$$\varepsilon = BAN\omega\sin\theta$$

*Convert Mech. Energy into Electric. Energy.




## Self-inductance :- (L)

Aaj Kal Ke Rishte Kaha hai itne Ache isliye hm Akele hi hai Ache.

* [redacted] $= \dfrac{\mu_0 N^2 A}{l}$
* $e = -L\dfrac{di}{dt}$
* $\phi = Li$

L (self-inductance) $\propto l$ ($n$ = cos $t^n$)

$\propto 1/l$ ($N$ = cos $t^n$)

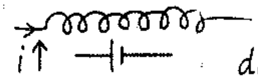
Where $n$ = no of turns per unit length

$N$ = total no of turns

Unit :- Henry (H)

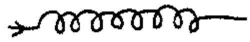
$\phi$ = constant ;- $L_1 i_1 = L_2 i_2$

Direction buddhi से, Magnitude Formula से

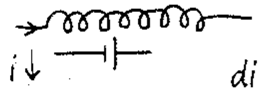



$\varepsilon = L\dfrac{di}{dt}$

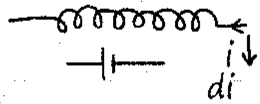
$l$ = cost$^n$



$\varepsilon = 0$




$\varepsilon = L\dfrac{di}{dt}$




$\varepsilon = L\dfrac{di}{dt}$

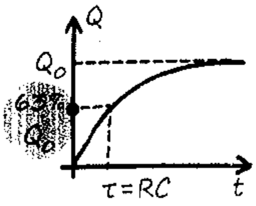
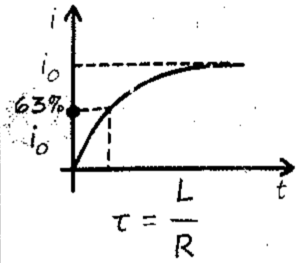
## Energy Store in Inductor :-

$$E = \frac{1}{2}Li^2 = \frac{B_0^{\,2}Al}{2\mu_0}$$

- Energy Stored per unit Volume $= \dfrac{B_0^{\,2}}{2\mu_0}$

## Charging of Inductor :-

| $t = 0$ | $t = \infty$ [Steady State] |
|---|---|
| L = Open Wire<br>C = Simple Wire | L = Simple Wire<br>C = Open Wire |
| ○ $V_L = L\left(\dfrac{di}{dt}\right) = E$<br>○ $V_R = 0$ | ○ $V_L = 0$<br>○ $V_R = E$<br>○ $i = E/R$<br>$\boxed{E_{max} = \dfrac{1}{2}L\left(\dfrac{E}{R}\right)^2}$ |

| Capacitor | Inductor |
|---|---|
| $Q=Q_0[1-e^{-t/RC}]$ | $i=i_0[1-e^{-Rt/L}]$ |
| $\tau = RC$ | $\tau = L/R$ |
| o $t=\infty$ $Q=Q_0$ | o $t=\infty$ $i=i_0$ |
|   |   |

## Graph Between di/dt with Time :-

Rate of Change in Current :-

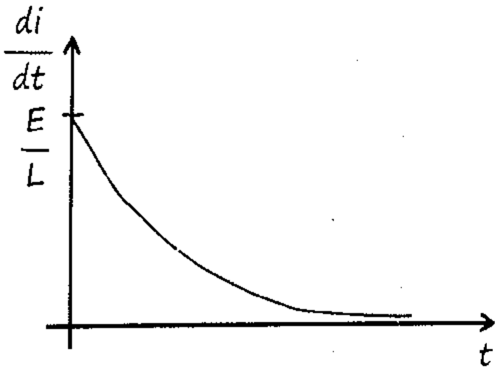
$$\frac{di}{dt} = \left(\frac{E}{L}\right) e^{-t/\tau}$$

$t = 0$

$$\frac{di}{dt} = \left(\frac{E}{L}\right)$$

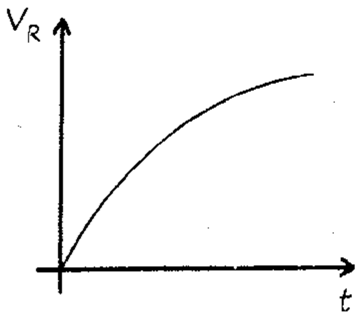
$t = \infty$

$$\frac{di}{dt} = 0$$

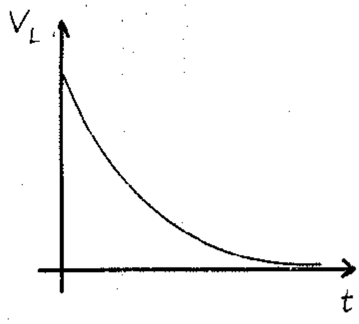



## Graph Between $V_R$ and Time :-

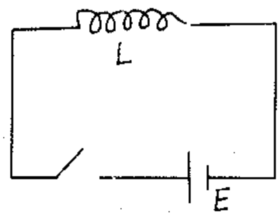
$$V_R = i_0 (1-e^{-t/\tau}).R$$




$$V_L = L\left(\frac{di}{dt}\right) = Ee^{-t/\tau}$$

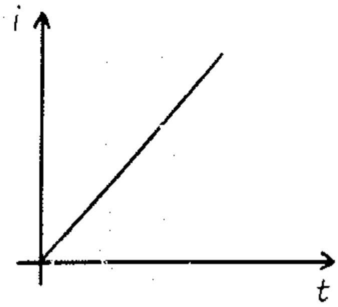



## Current in Inductor as Function of Time :-

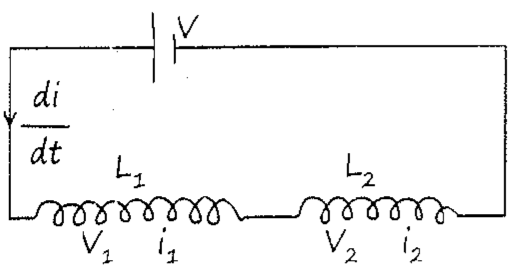



$$i = \frac{E}{L}t$$

Battery be like :- Rasiya Gundo mein Fasgayi.

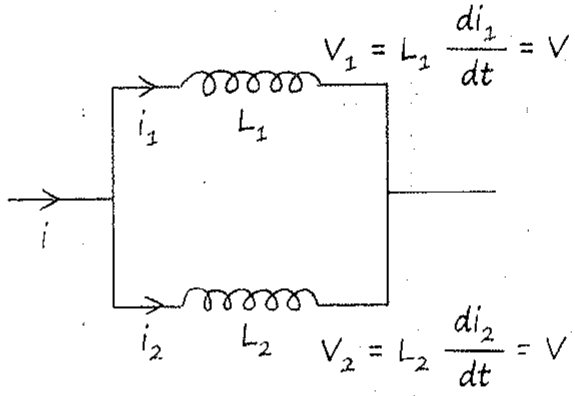



## Series Combination of Inductor :-




$$\boxed{\frac{di}{dt} = \text{same}} \qquad \boxed{L_{eq} = L_1 + L_2}$$

## Parallel Combination of Inductor :-

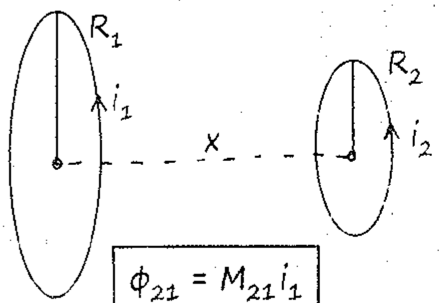



$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

## Mutual Inductance :-

MR* :-

Jitne mein tumhe तंग Karunga utna tum bhi mujhe तंग Krna Jyda kam mt Krna...




$$\boxed{\phi_{21} = M_{21} i_1}$$

Mutual inductance of 2 due to 1.

$$\Rightarrow \boxed{M_{21} = \frac{\mu_0 N_1 N_2 R_1^2 R_2^2 \pi}{2x^3}}$$

## Reciprocity Theorem :-

$$M_{12} = M_{21}$$

| | |
|---|---|
| $\phi_{12} = M_{12} i_2$ | $\phi_{21} = M_{21} i_1$ |
| $\varepsilon_{12} = M_{12} \frac{di_2}{dt}$ | $\varepsilon_{21} = M_{21} \frac{di_1}{dt}$ |

- Note :- Coupling Factor

  $0 \le K \le 1$

  $$\boxed{M = K\sqrt{L_1 L_2}}$$

- $L_1$ & $L_2$ very close $\rightarrow K = 1$.

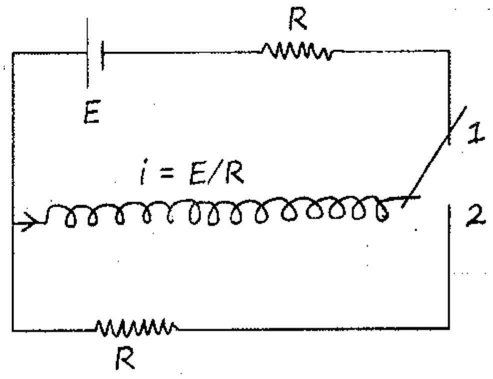
## Series Combination of Inductor (Considering M)

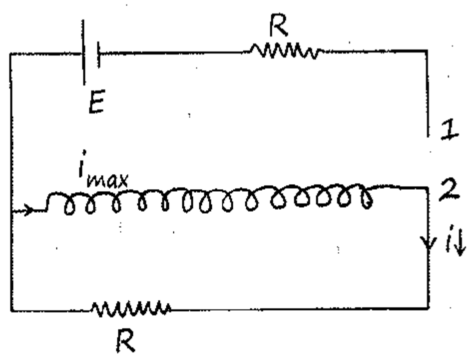
$$L_{eq} = L_1 + L_2 \pm 2M$$

$$\boxed{L_{eq} = L_1 + L_2 \pm 2M}$$

| $L_1$ & $L_2$ are in same order | $L_1$ & $L_2$ are in Opposite order |
|---|---|
| $L_{eq} = L_1 + L_2 + 2M$ | $L_{eq} = L_1 + L_2 - 2M$ |
| [redacted] | [redacted] |

## Discharging of Inductor :-

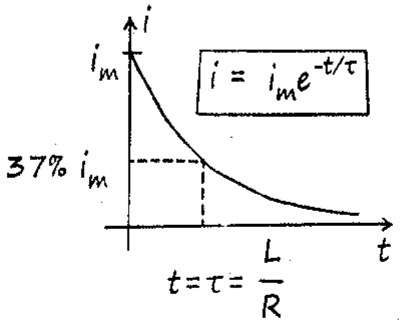


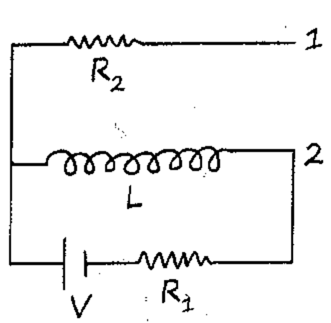

$$i = i_m e^{-t/\tau} \qquad V_L = L\frac{di}{dt} = iR$$

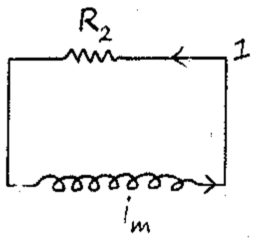
$\tau = \frac{L}{R}$

$\boxed{i = i_m e^{-t/\tau}}$




- **Find total heat loss across $R_2$ when Key is shifted from (2) to (1) :-**




$$\boxed{E_{stored} \text{ or } E_{loss} = \frac{LV^2}{2R_1^2}}$$

$t = \infty$

$$i_m = \frac{V}{R_1}$$

## Eddy Current :-

Agar Kishi तार मे Current Flow करेगा तो उसे Kehte है Current और Plate मे Current Flow करेगा तो Eddy Current.

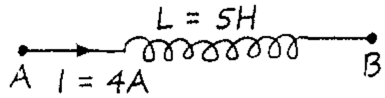
Application :-

1> Magnetic braking in train

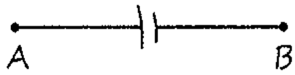
2> Electromagnetic damping.

3> Induction Furnance

4> Electric Power meters.



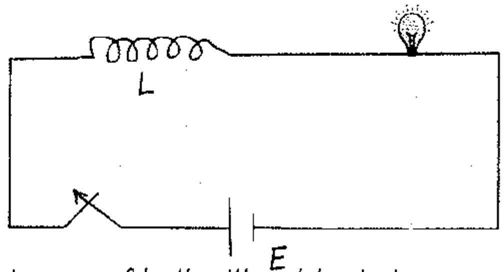

$$\frac{di}{dt} = 2\ A/S \rightarrow i(\uparrow es)$$




$$V_A - \frac{Ldi}{dt} = V_B$$

$$V_A - V_B = \frac{Ldi}{dt} = 5 \times 2 = 10\ V$$

Current is decreasing hence B is at higher potential.

- 

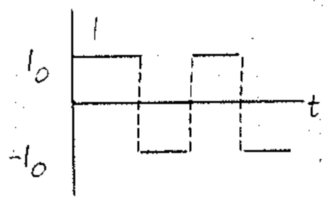

Brightness of bull will suddenly increase when key is just opened.

MR*

"Kushi k liye kam karoge to khushi nahi milegi, lekin khush hokar kam karoge to khushi aur safalta jarur milegi."

## Alternating Current :-

A current of constant magnitude can be A/c current → Yes. It is square wave A/c.



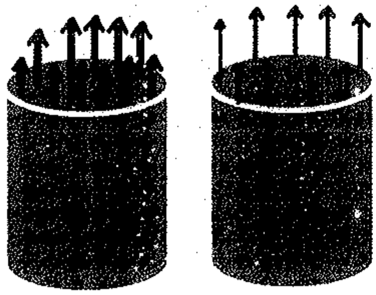

- A Variable current in magnitude only is D/c
  - # A current is varying from +5A to +15A sinusoidally then this current is D/c or (AC+DC) mixture, not A/c because it is not a bidirectional.

A/C → Bidirectional

D/C → Unidirectional (Steady Current)

$i_{D/C} \propto r^{3/2}$

$i_{A/C} \propto$ S. Area.



## Measuring of Current :-

| Moving coil galvanometer | Hot wire ammeter |
|---|---|
| "T" on Current carrying coil. | Heat loss |
| Only D/C | Both D/C & A/C |
| $\theta$ (angle) $\propto i$ | $\theta$ (angle) $\propto H \propto i^2$ |
| Linear Scale | Non-linear Scale |

## Average Value :-

Discrete system

$$i_{avg} = \frac{i_1+i_2+...+i_n}{n}$$

Continuous system

$$\langle i \rangle = i_{avg} = \frac{\int i.dt}{\int dt}$$

## RMS Current :-

$$i_{rms} = \sqrt{\langle i^2 \rangle} = \sqrt{\frac{\int i^2.dt}{\int dt}}$$

## RMS Voltage :-

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{\int V^2.dt}{\int dt}}$$

MR Ratta

- Full Cycle (FC)

  $\langle \sin\theta \rangle = 0$

  $\langle \cos\theta \rangle = 0$

- Half Cycle (HC)

  $\langle \sin\theta \rangle = \frac{2}{\pi}$

  $\langle \cos\theta \rangle = \frac{2}{\pi}$

$$\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = \frac{1}{2} \Big\} \text{Half/Full Cycle}$$

- $\langle 2\sin(\omega t).\cos(\omega t) \rangle_{F.cycle} = \langle \sin(2\omega t) \rangle_{F.cycle} = 0$
- Avg. value of $I_0 \sin(\omega t)$ in half cycle may be zero or $\frac{2i_0}{\pi}$ because it will depends half cycle ki limit kaha se liya hai.

## Alternating Current :-

$$i = i_0 \sin[\omega t + \phi]$$

o Note :-

$$\langle i \rangle_{FC} = 0$$

$$\langle i \rangle_{HC} = \frac{2i_0}{\pi}$$

$$\sqrt{\langle i^2 \rangle} = i_{rms} = \frac{i_0}{\sqrt{2}} \left\} \begin{array}{l} \bullet \text{ Virtual Current} \\ \bullet \text{ Effective Current} \end{array} \right.$$

Reading of Ammeter.

$$H = i_{rms}^2 Rt.$$

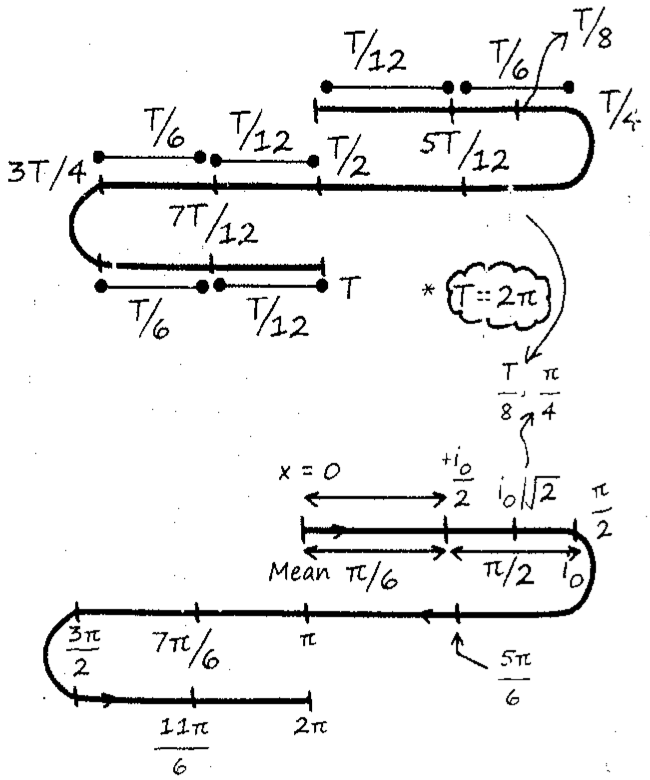
$$V = V_0 \sin(\omega t) = V_0 \sin(2\pi f t)$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

## MR* Table

| | Sinusoidal A/c | Square (A/c) | Triangular | Full wave rectifier | Half wave rectifier |
|---|---|---|---|---|---|
| Avg. Full cycle | 0 | 0 | 0 | $\frac{2i_0}{\pi}$ | $\frac{i_0}{\pi}$ |
| Avg. Half cycle | $\frac{2i_0}{\pi}$ | $i_0$ | $\frac{i_0}{2}$ | $\frac{2i_0}{\pi}$ | $\frac{2i_0}{\pi}$, 0 |
| R.M.S. value | $\frac{i_0}{\sqrt{2}}$ | $i_0$ | $\frac{i_0}{\sqrt{3}}$ | $\frac{i_0}{\sqrt{2}}$ | $\frac{i_0}{2}$ |

## Fire Concept MR*




## Alternating Current

## MR*

(i) $i = \underset{DC}{a} + \underset{AC}{b \sin(\omega t)}$

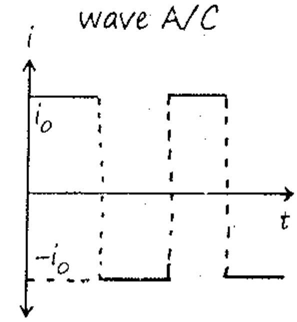
$$i_{rms} = \sqrt{a^2 + \frac{b^2}{2}} = \sqrt{\langle i^2 \rangle}$$

(ii) $i = \underset{AC}{a \sin\omega t} + \underset{AC}{b \cos\omega t}$

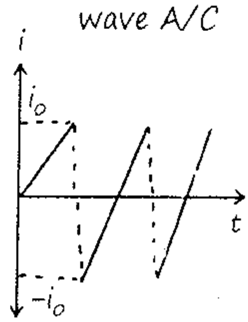
$$i_{rms} = \sqrt{\frac{a^2}{2} + \frac{b^2}{2}} = \sqrt{\langle i^2 \rangle}$$

## Alternating Current :-

Square wave A/C




Triangular wave A/C



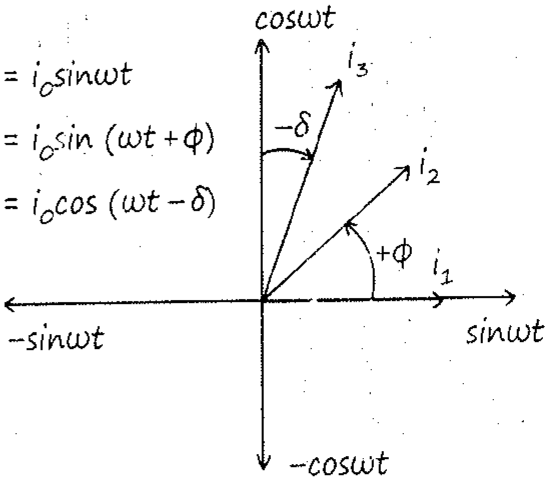
| | |
|---|---|
| $\langle i\rangle_{FC} = 0$ | $\langle i\rangle_{FC} = 0$ |
| $\langle i\rangle_{HC} = i_0$ | $\langle i\rangle_{HC} = i_0/2$ |
| $\sqrt{\langle i^2\rangle_{HC}} = i_0$ | $\sqrt{\langle i^2\rangle_{HC}} = \dfrac{i_0}{\sqrt{3}}$ |

## Representation of A/c Current & Voltage by Phase Diagram :-

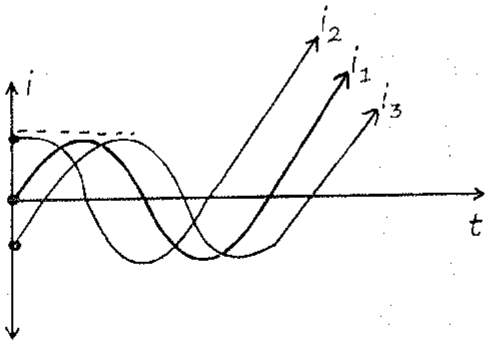
$i_1 = i_0 \sin\omega t$

$i_2 = i_0 \sin(\omega t + \phi)$

$i_3 = i_0 \cos(\omega t - \delta)$



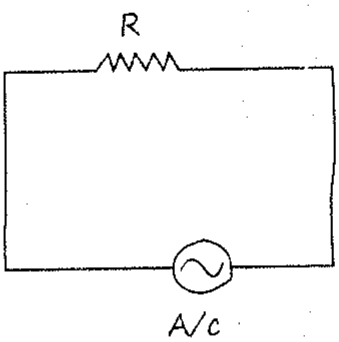

MR*




*Joh jitna niche y-axis pe wo utna piche.*

$*i_3 < i_1 < i_2*$

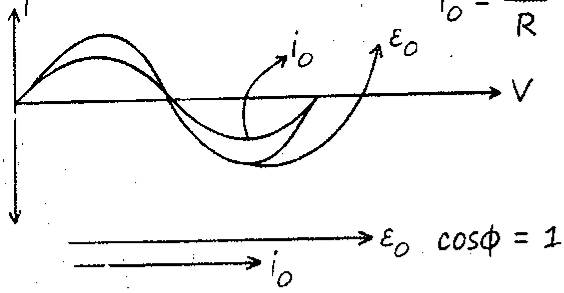
## A/C Source Across Pure Resistance :-




Same phase

$\varepsilon = \varepsilon_0 \sin\omega t \qquad i = i_0 \sin\omega t$

$i_0 = \dfrac{\varepsilon_0}{R}$




$\cos\phi = 1$

o Power drop across R :-

$$\langle P\rangle = \varepsilon_{rms} i_{rms} = \frac{\varepsilon_{rms}^2}{R} = i_{rms}^2 R$$

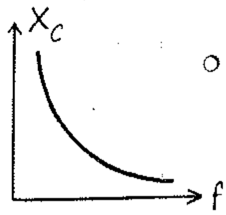
## A/C Source Across Pure Capacitor :-

$*\varepsilon = \varepsilon_0 \sin\omega t$ [redacted]

$*i = i_0 \cos\omega t$ [redacted]

$$*i = \frac{\varepsilon_0}{1/C\omega}\cos\omega t = \frac{\varepsilon_0}{X_C}\cos\omega t$$

$$X_C = \frac{1}{C\omega} = \frac{1}{2\pi f C}$$ } Capacitive Reactance




o Capacitor loves change.

$\boxed{\text{DC – Filter}}$

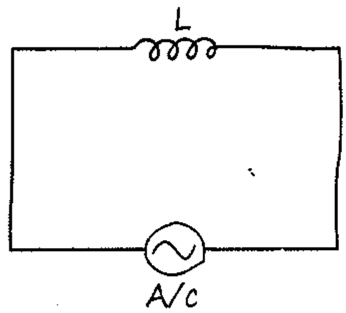
| For D/C | For A/C |
|---|---|
| $f = 0$ | $f = \infty$ |
| $X_C = \infty$ | $X_C = 0$ |
| Capacitor act as Open wire. | Capacitor act as Simple wire. |

o Power drop across pure Capacitor :-

$\cos\phi = 0$

$\langle P\rangle = 0$ $\boxed{\text{Wattless Circuit}}$

## A/C Source Across Pure Inductor :-




$V_L = \varepsilon_0 \sin\omega t$ [redacted]

$$i = \frac{-\varepsilon_0}{\omega L}\cos\omega t = \frac{-\varepsilon_0}{X_L}\cos\omega t$$

- $i_0 = \dfrac{\varepsilon_0}{\omega L}$ - $V_L = iX_L$

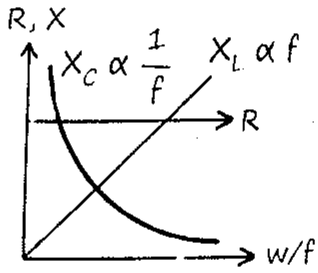
$X_L = \omega L \}$ Inductive Reactance

| For D/c | For A/c of high |
|---|---|
| $f = 0$ | frequency $f = \infty$ |
| $\Rightarrow X_L = 2\pi f L = 0$ | $\Rightarrow X_L = \infty$ |
| $\Rightarrow$ Hence inductor be have as simple wire. | $\Rightarrow$ Hence inductor behave as open wire. |

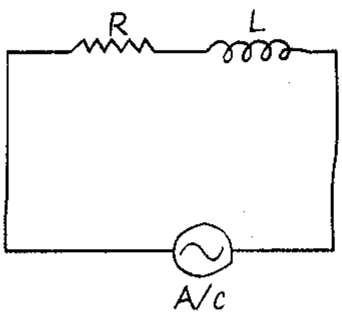
- Power loss across pure inductor

$\langle P \rangle = 0$

$\cos\phi = 0$

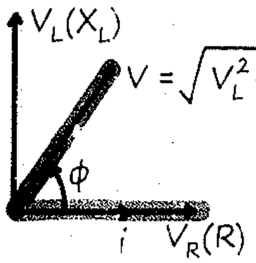



## Series R-L Circuit :-




$V_{net} = V_B = \varepsilon_0 \sin\omega t$

$i = i_0 \sin(\omega t - \phi)$

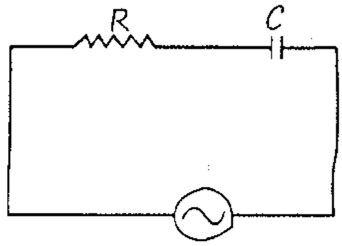



$V = \sqrt{V_L^2 + V_R^2}$ $\boxed{V_{net} \text{ leads } i \text{ by } \phi}$

- $Z = \sqrt{R^2 + X_L^2}$
- $\tan\phi = \dfrac{V_L}{V_R} = \dfrac{X_L}{R}$
- $\cos\phi = \dfrac{V_R}{V_{net}} = \dfrac{R}{Z}$ } Power Factor

$\phi$ = Phase difference.

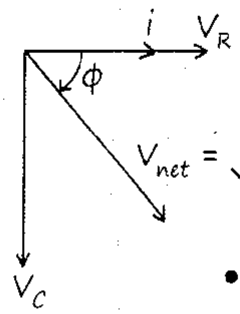
## Series R-C Circuit :-




$i = i_0 \sin(wt + \phi)$

$i_0 = \dfrac{\varepsilon_0}{Z}$

$\varepsilon = \varepsilon_0 \sin(\omega t)$




$\boxed{V_{net} \text{ lags "i" by } \phi}$

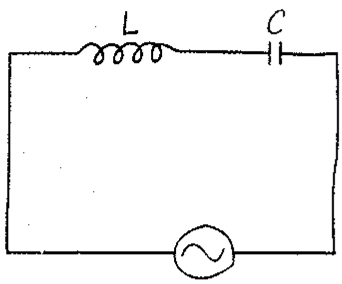
$V_{net} = \sqrt{V_R^2 + V_C^2}$ - $i_0 = \dfrac{\varepsilon_0}{Z}$

- $Z = \sqrt{R^2 + X_C^2}$

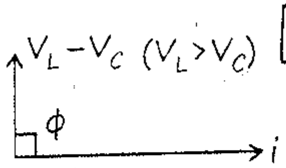
$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R}$$

$$\cos\phi = \frac{V_R}{V_{net}} = \frac{R}{Z}$$

## Series L-C Circuit :-




$\varepsilon = \varepsilon_0 \sin(\omega t)$

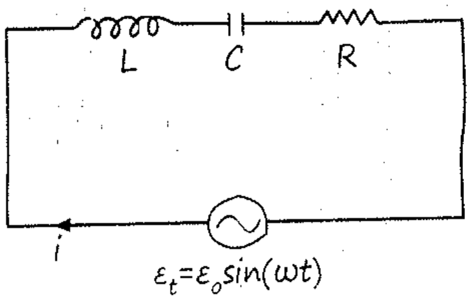



$\boxed{V_{net} \text{ leads } i \text{ by } \phi}$

Net impedence :-

$Z = X_L - X_C \qquad i_o = \varepsilon_o / Z$

## Series LCR Circuit :-

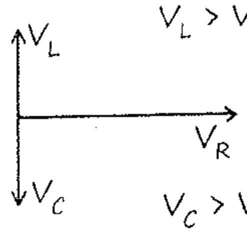



$i = i_L = i_C = i_R$ Instantaneous current

$\varepsilon_t = V_L + V_C + V_R$ Instantaneous voltage

$i_o \neq i_{oL} \neq i_{oC} \neq i_{oR}$ Peak Current

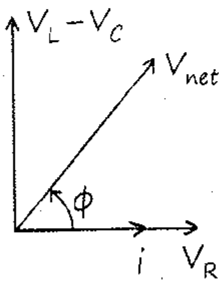
$\varepsilon_o = V_{oL} + V_{oC} + V_{oR}$ Peak voltage




$V_L > V_C \longrightarrow$ Inductive LCR

$V_C > V_L \longrightarrow$ Capacitive LCR

- If $V_L > V_C$ (Inductive LCR)



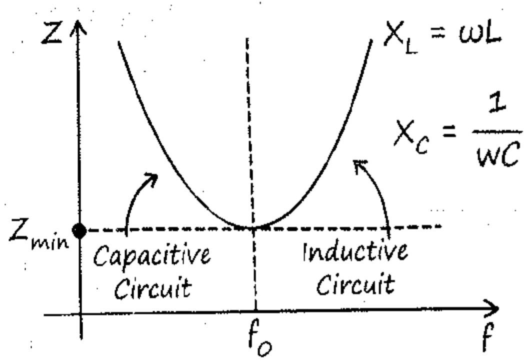

$V_{net} = \sqrt{(V_L - V_C)^2 + V_R^2}$

$\boxed{V_{net} \text{ leads } i \text{ by } \phi}$

- $\tan\phi = \dfrac{V_L - V_C}{V_R} = \dfrac{X_L - X_C}{R}$

- $\cos\phi = \dfrac{V_R}{V_{net}} = \dfrac{R}{Z}$

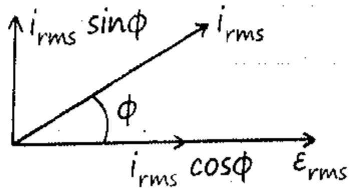
- $Z = \sqrt{(X_L - X_C)^2 + X_R^2}$ • $i_o = \dfrac{\varepsilon_o}{Z}$




If Freq inc from zero then impedence (Z) 1st dec. then inc.

- $f = 0 \quad Z = \infty \qquad X_C = \infty \quad X_L = 0$
- $f = \infty \quad Z = \infty \qquad X_C = 0 \quad X_L = \infty$

*Power drop :-




Vimp

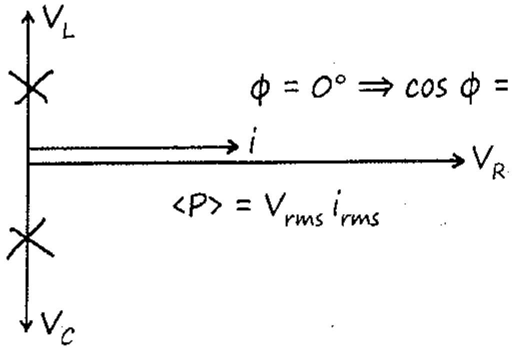
- $\langle P \rangle = \varepsilon_{rms} \, i_{rms} \cos\phi$

Resonance in series LCR circuit :-

$X_L = X_C \qquad V_L = V_C$

$\omega L = \dfrac{1}{\omega C} \Rightarrow \omega = \dfrac{1}{\sqrt{LC}} \Rightarrow f = \dfrac{1}{2\pi\sqrt{LC}}$

$Z_{min} = R \Rightarrow V_{net} = V_R$



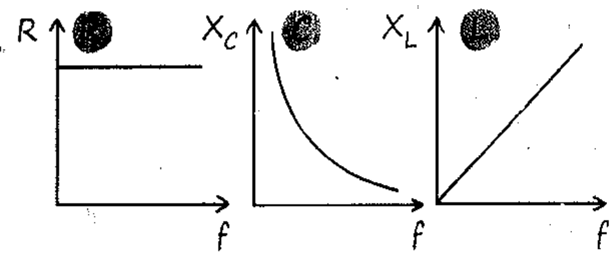

$\phi = 0° \Rightarrow \cos\phi = 1$

$\langle P \rangle = V_{rms}\, i_{rms}$

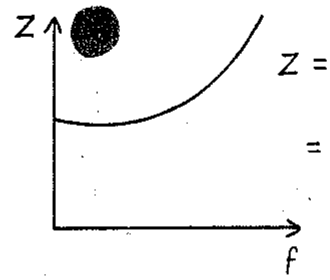
$i_{max} = \dfrac{\varepsilon}{R}$ $\qquad P_{max} = \dfrac{\varepsilon_0^2}{2R}$

## Vimp Graph :- MR*

○

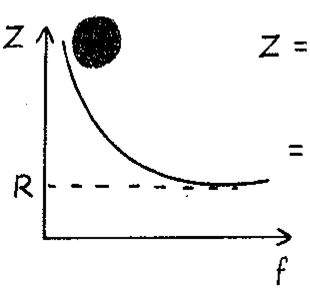



○



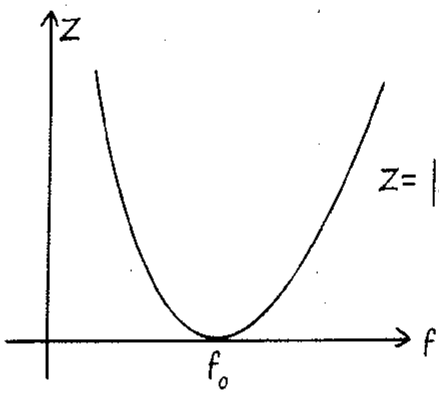

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{(2\pi f L)^2 + R^2}$$

○



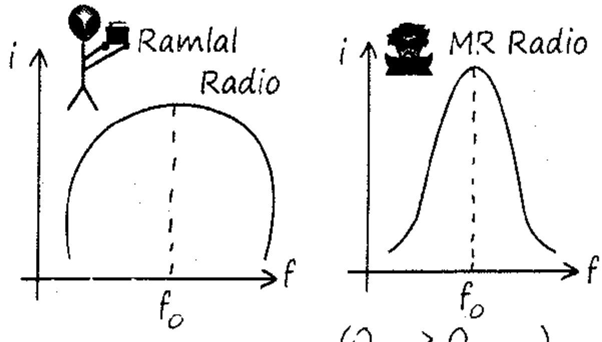

$$Z = \sqrt{X_C^2 + R^2} = \sqrt{\frac{1}{(2\pi f C)^2} + R^2}$$

○ L-C circuit

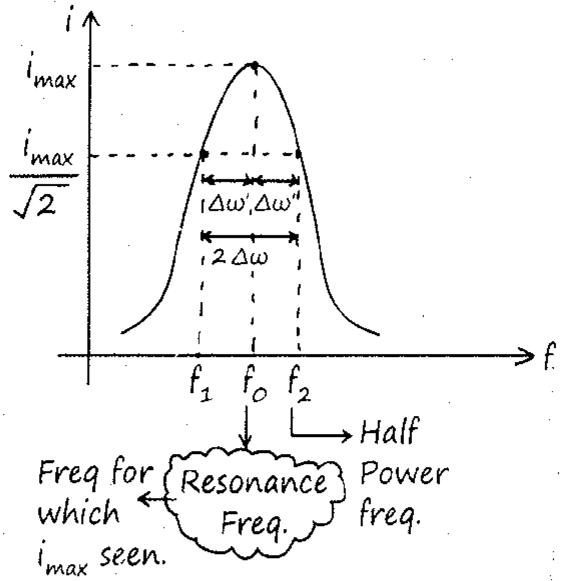



$$Z = |X_L - X_C|$$

## Quality Factor :- (Q)




$(Q_{MR} > Q_{Ramlal})$




$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{2\pi(f_2 - f_1)}$$

$$Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0}{2[2\pi(f_0 - f_1)}$$

$$2\Delta\omega = \frac{R}{L}$$

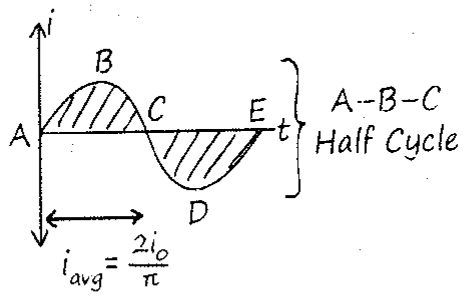
$$Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R} = \frac{X_L}{R} = \frac{X_C}{R}$$

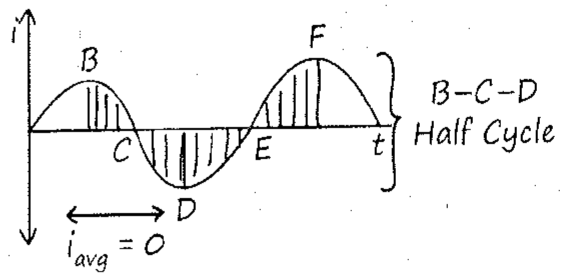
$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R} = \frac{X_L}{R}$$

$$^*Q = \frac{X_L}{R} = \frac{X_C}{R}$$

$$^*Q = \frac{1}{\sqrt{LC}}\,\frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

MR*




## Transformer :-

- Voltage Regulator
- Based on principle of Mutual Inductance.

*$P = const^n$ $\quad$ $iV = const^n$

*$i \propto \frac{i}{V} \propto \frac{i}{N}$ $\quad$ *N = No. of turns.

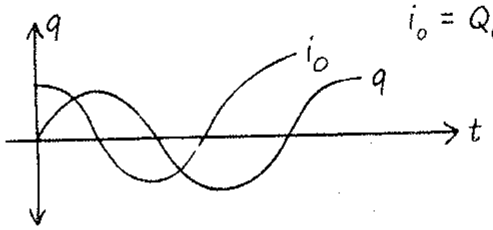
* $\boxed{\frac{i_o}{i_{in}} = \frac{V_{in}}{V_{out}} = \frac{N_P}{N_S}}$

| Step-UP | Step-DOWN |
|---|---|
| $V_o > V_{in}$ | $V_o < V_{in}$ |
| $N_S > N_P$ | $N_S < N_P$ |
| $i_S < i_P$ | $i_S > i_P$ |

## L-C Oscillations :-

$q = Q_o \sin\omega t$ $\quad$ $i = i_o \cos\omega t$

$i_o = Q_o\omega$




$$\frac{d^2q}{dt^2} + \frac{q}{LC} = 0 \Rightarrow \omega = \sqrt{\frac{1}{LC}}$$

$$U_E = \frac{q^2}{2C} = \frac{Q_o^2 \sin^2\omega t}{2C}$$

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}Li_o^2 \cos^2\omega t$$

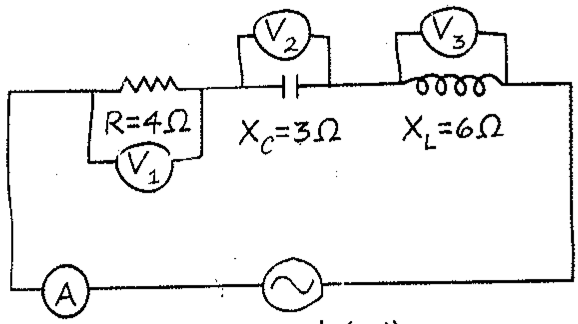
$$\boxed{TE = \frac{Q_o^2}{2C} = \frac{1}{2}Li_o^2 = const^n}$$

## Choke Coil :-

- Control Current in A/C Circuit
- Divide Potential without Power loss.

$(X_L, R)$ Choke Coil. → RL Circuit

| Ideal | Practical | बेकार |
|---|---|---|
| $R = 0$ | $R$ = Low | $R$ = High |
| $X_L \neq 0$ | $X_L$ = High | $X_L$ = Low |
| $\phi = 90°$ | $\phi < 90°$ | $\phi$ = Low |
| $\cos\phi = 0$ | $\cos\phi$ = V.Low | $\cos\phi$ = High |




$\varepsilon = 10\sin(\omega t)$

- $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{4^2 + (6-3)^2}$
- $Z = 5\Omega$
- $i_o = \frac{\varepsilon_o}{Z} = \frac{10}{5} = 2$ Amp
- $i_{RMS}$ (Reading of Ammetre) $= \frac{i_o}{\sqrt{2}} = \sqrt{2}$

- Power factor $\cos\Phi = \frac{R}{Z} = \frac{4}{5}$

  $\Phi = 37°$
- $X_L > X_C \rightarrow$ Inductive circuit

  Voltage will leads by current by 37°
- $i = i_0 \sin(\omega t - \Phi)$

  $i = 2\sin(\omega t - 37°)$

  Reading of $V_1 \Rightarrow$ $V_1 = i_{Rms} R = \sqrt{2}(4)$

  Reading of $V_2 \Rightarrow$ $V_2 = i_{Rms} X_C = 3\sqrt{2}$

  Reading of $V_3 \Rightarrow$ $V_3 = i_{Rms} X_L = 6\sqrt{2}$

RMS voltage across 'R' and 'C'

$$= \sqrt{(4\sqrt{2})^2 + (3\sqrt{2})^2} = 5\sqrt{2}$$

RMS voltage across 'C' and 'L'

$$= 6\sqrt{2} - 3\sqrt{2} = 3\sqrt{2}$$

RMS voltage across 'R' and 'L'

$$= \sqrt{(6\sqrt{2})^2 + (4\sqrt{2})^2} = \sqrt{104}$$

$\langle P \rangle = i_{rms} V_{rms} \cos\Phi$

$$= \sqrt{2} \times \frac{10}{\sqrt{2}} \times \frac{4}{5} = 8W$$

## THE ULTIMATE MR STAR* TABLE

| Circuit | Phase diff. Between i&V | Power factor $\cos\phi = R/Z$ | Impedance (Z) | Who leads! | Power loss |
|---|---|---|---|---|---|
| Pure resistive | 0 | 1 | R | Same Phase | $P = i_{rms}V_{rms}$ |
| Pure Capacitive | $\pi/2$ | Zero | $X_C = \frac{1}{\omega C}$ | Current | Zero |
| Pure inductive | $\pi/2$ | Zero | $Z_L = \omega L$ | Voltage | Zero |
| RL | $0 < \phi < \frac{\pi}{2}$ | b/n 0 to 1 | $Z = \sqrt{R^2 + X_L^2}$ | Voltage | $P = i_{rms}V_{rms}\frac{R}{Z}$<br>$P = i_{rms}V_{rms}\cos\phi$ |
| RC | $0 < \phi < \frac{\pi}{2}$ | b/n 0 to 1 | $Z = \sqrt{R^2 + X_C^2}$ | Current | |
| LC | $\pi/2$ | Zero | $Z = X_L - X_C$ | Depends | |
| Series LCR | $0 \le \phi < \frac{\pi}{2}$ | 1 or b/n 0 to 1 | $Z = \sqrt{R^2 + (X_L - X_C)^2}$ | Depends | $P = i_{rms}^2 R = \frac{V_{rms}^2}{R}$ |

## ELECTRIC OSCILLATION
## EQUIVALENT OF L.C. OSCILLATION

| Mechanical system | Electrical system |
|---|---|
| Mass | Inductance |
| Displacement | Charge |
| Velocity | Current |
| Potential Energy | Electric Energy |
| Kinetic Energy | Magnetic Energy |
| $\frac{1}{K}$ (K=Spring const.) | Capacitance |

Q. In L.C. Oscillation Maximum Current is $I_0$ at t=0 then find current in circuit when magnetic energy is half of electrical energy.

Sol. Total Energy Conserved

$$\frac{1}{2} L I_0^2 = U_e + U_m$$

$$\frac{1}{2} L I_0^2 = 2U_m + U_m$$

$$\frac{1}{2} L I_0^2 = 3\,\frac{1}{2} LI^2$$

$$I = \frac{I_0}{\sqrt{3}}$$

Q. The number of turns in the primary and secondary coils of a step-down transformer are 200 and 50 respectively. If the power in the input is 100 watt at 1A then the output power and current will respectively be

Sol. Power in an transformer (ideal) does not change, the total electric power remains same.

Power = 100 Watt

for current, the proportionality is

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

$$\frac{200}{50} = \frac{1}{I}$$

$$I = \frac{200}{50} = 4\text{Amp.}$$

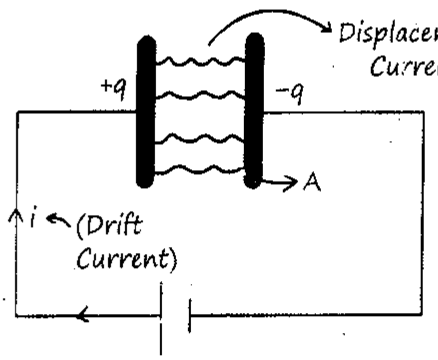
MR*

"Aaj kuch karlo asa ki bhavishya me khud ko kosne ki nobat na aaye."

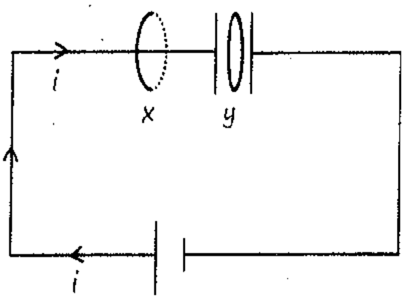
# 29 Electromagnetic Wave

| Charge | E.F. | MF | EM Wave |
|---|---|---|---|
| Rest | ✓ | ✗ | ✗ |
| V = Const$^n$ | ✓ | ✓ | ✗ |
| Accelerated | ✓ | ✓ | ✓ |

## Charging of Capacitor :-




$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$$




According to amperes law

for x: $\int \vec{B}_x \cdot \vec{d\ell} = \mu_0 i$

for y: $\int \vec{B}_y \vec{d\ell} = \mu_0 i \to 0$

hence $B_y = 0$

But maxwell found experimentally

$B_y \neq 0$

and gives reason of this is Displacement current b/w the plate of capacitor.

## Displacement Current :-

$Q = CV$ $\quad \boxed{i_{Drift} = i_{Displacement}}$

$$i = C \cdot \frac{dV}{dt} = \frac{\varepsilon_0 A}{\ell} \frac{dV}{dt}$$

$$i = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d\phi}{dt}$$

- Formed only due to changing E. Field.
- Not exist under steady current ($\phi$ = constant)
- Flows between c/s Area of Capacitor plate.

## Maxwell Equation :- (4 equation)

1> Gauss law of Electrostat.

$$\phi = \oint \vec{E} \cdot \vec{ds} = \frac{q_{in}}{\varepsilon_0}$$

2> Gauss law of Magnetism.

$$\oint \vec{B} \cdot \vec{ds} = 0$$

3> Induced E. Field :-

$$-\oint \vec{E}_{in} \cdot \vec{d\ell} = EMF = -\frac{d\phi}{dt}$$

$$\oint \vec{E}_{in} \cdot \vec{d\ell} = \frac{d\phi}{dt}$$

4> Ampere's Maxwell Law :-

$$\oint \vec{B} \cdot \vec{d\ell} = \mu_0 [i_{Drift} + i_{Disp}]$$

$$= \mu_0 \left[ i_{Drift} + \varepsilon_0 \frac{d\phi}{dt} \right]$$

## Source of EMW :-

1> Accelerating Charge

2> LC-Oscillation

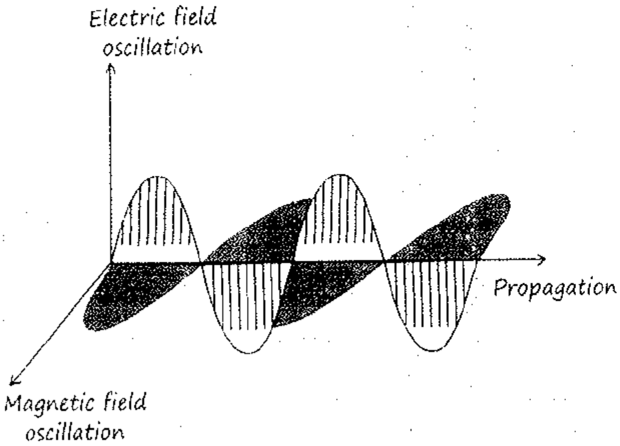
3> Transition of $e^-$ from $n^{th}$ orbit

4> Retardation of $e^-$ when it enters into a target of high At. Weight. (X-ray)

5> De-excitation of nucleus in radioactivity. (Y-ray)

## EM Wave in LC Oscillation :-

- Energy transfer due to oscillation of Electric and magnetic field.
- No Medium required, Non-mechanical wave.
- $Q_{net}$ [EM wave] = 0
- Transverse in nature.
- E&B oscillate perpendicular to each other but in same phase.



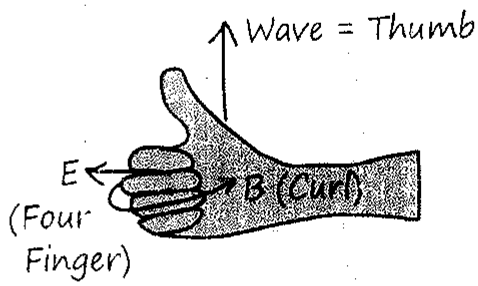

$E_y = E_o \sin\omega t$ $\qquad E_o = Q_o/A\varepsilon_o$

$B_z = B_o \sin\omega t$

$E_o = B_o C$

E.F. & Mag. F are in Same phase

Direction of Wave :- $\vec{E} \times \vec{B}$




## MR* for direction

Direction of wave = $\hat{E} \times \hat{B}$

Place Your four finger (with Palm) of right hand along electric field and Slap magnetic field. thumb will Indicate direction of wave.

## Nature of EMW :-

$$C = \frac{1}{\sqrt{\mu_o \varepsilon_o}} = 3 \times 10^8 \text{ m/s}$$

$$V = \frac{1}{\sqrt{\mu_m \varepsilon_m}} = \frac{C}{\sqrt{\mu_r \varepsilon_r}}$$

$$\boxed{V = \frac{C}{\mu}}$$

$\mu$ = Mag. Permeability

$\mu$ → Refrective index

$\varepsilon$ = Electric Permittivity.

- Angular Frequency :-

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- Angular wave no :-

$$K = \frac{2\pi}{\lambda} = \frac{2\pi v}{C}$$

- Speed of EMW :-

$$\boxed{C = \lambda v = \frac{\lambda}{T} = \frac{\omega}{K}}$$

If Medium is Changed !

$\lambda$ = Changes

f = Same.

$C_1 = \lambda_1 v$

## Energy Density :-

- EM Wave (U) :-

$$(U) = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

$$U_{avg} = \langle U \rangle = \frac{\varepsilon_0 E_0^2}{2} = \frac{B_0^2}{2\mu_0}$$

- Electrostatic ($U_E$) :-

$$U_E = \frac{1}{2}\varepsilon_0 E^2$$

$$\langle U_E \rangle_{avg} = \frac{1}{4}\varepsilon_0 E_0^2$$

- Magnetic ($U_B$) :-

$$U_B = B^2/2\mu_0$$

$$\langle U_V \rangle_{avg} = B_0^2/4\mu_0$$

## Waves :-

Gaddi SUV in My Range

Energy

Y-ray (1 MeV)

X-ray (1 KeV)

UV (10-100 eV)

Visible (1-2 eV)

IR (1 MeV)

Microwave (1 μeV)

Radiowave ($10^{-8}$ eV)

$E\downarrow$

$f\downarrow$

$\lambda\uparrow$

## Intensity of EM Wave :-

$$\boxed{\langle I \rangle = \frac{1}{2}\varepsilon_0 E_0^2 C = \frac{1}{2}\frac{B_0 E_0}{\mu_0}}$$

## Force & Radiation Pressure :-

Surface

| Completely reflecting | Completely absorbing |
|---|---|
| Momentum :- | |
| $\lvert\Delta\vec{P}\rvert = 2P = \frac{2E}{C}$ | $\lvert\Delta\vec{P}\rvert = P = \frac{E}{C}$ |
| Force :- | |
| $F = \frac{\Delta P}{t} = \frac{2E}{Ct}$ | $F = \frac{\Delta P}{t} = \frac{E}{Ct}$ |
| Pressure :- | |
| $P = \frac{F}{A} = \frac{2E}{CAt}$ | $P = \frac{F}{A} = \frac{E}{CAt}$ |
| $P = \frac{2I}{C}$ | $P = \frac{I}{C}$ |
| I = Intensity | $\mu_r = \frac{\mu_m}{\mu_0}$ |
| E = Energy | $\varepsilon_r = \frac{\varepsilon_m}{\varepsilon_0}$ = Dielec. Const$^n$ |

## Poynting Vector :-

Direction :- $\|^{el}$ to wave

Magnitude :- Intensity of EM Wave.

$$\boxed{\vec{S} = \frac{\vec{E}\times\vec{B}}{2\mu_0}}$$

| Type of Radiation | Frequency Range (Hz) | Wavelength Range |
|---|---|---|
| gamma-rays | $10^{20} - 10^{24}$ | $<10^{-12}$m |
| X-rays | $10^{17} - 10^{20}$ | 0.01 nm – 10 nm |
| ultraviolet | $10^{15} - 10^{17}$ | 400 nm – 1 pm |
| visible | $4 - 7.5 \times 10^{14}$ | 750 nm – 400 nm |
| near-infrared | $1\times10^{14} - 4\times10^{14}$ | 2.5 μm – 750 nm |
| infrared | $10^{13} - 10^{14}$ | 25 μm – 2.5 μm |
| microwaves | $3\times10^{11} - 10^{13}$ | 1 mm – 25 μm |
| radio waves | $< 3\times10^{11}$ | > 1 mm |

| | Radiowaves | Microwaves | IR | Visible | UV | X-Ray | Gamma-Rays |
|---|---|---|---|---|---|---|---|
| Production | Accl$^n$ of $e^-$ in Antenna | Klystron, Magneton & Gunn Diode | Heated Bodies | $e^-$ Excitation | Transfer of $e^-$ from E.S. to G.S. | X-ray Tube Innner Shell $e^-$ | Radioactive Nucleus Nuclear Explosion |
| Detection | Receiver's Antenna. | Point Contact Diode | Bolometer, IR Photographic Plate, Photodiode | Human eye, Photographic Films, Photocells | Diodes & Photographic Films. | Gieger Counter Ionisat$^n$ Chamber Photographic Plates | Gieger Counter Ionisat$^n$ Chamber Photographic Plates |
| Uses | FM, AM, TV, Cellular Network | Radar Navigation, Measuring Speed of balls, Microwave Oven (3GHz, Reso. of $H_2O$). | Remotes, Hi Fi System | To See Objects | UV Fitter, Lasik Laser, Sanitization | Diagnostic tool, Radio-therapy. | Radio therapy High level Sanitization. |

**MR***

'बदल जाओ वक्त के साथ,
या फिर वक्त बदलना सीखो।
मजबूरियों को मत कोसो,
हर हाल में चलना सीखो।।'

# 30 Ray Optics

## Light

- It is a form of energy which gives the Sensation of vision.
- Light itself is not visible
- EM wave of $\lambda$ = 380 nm to 700 nm
- [V I B G Y O R] : visible light Ka range
- Speed of light does not depends on Speed of Source and speed of observer.
- Frequency and sense of colour does not depends on medium.
- Intensity, wavelength, speed of light depends on medium.

## Speed of Light

$$C = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \qquad V = \frac{1}{\sqrt{\mu_m \varepsilon_m}}$$

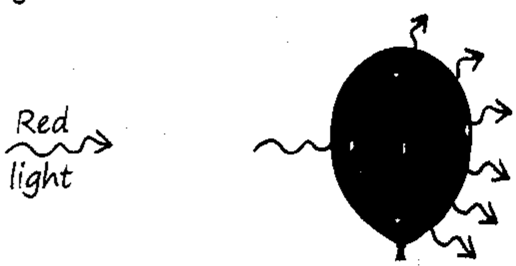
$$V = \frac{C}{\sqrt{\mu_r \varepsilon_r}} = \frac{C}{\mu}$$

$$\therefore \ \mu_1 V_1 = \mu_2 V_2$$

$$V = \lambda f \qquad \lambda \propto \frac{1}{\mu} \propto V$$

MR*

Koi object jis light Ko emit Karega woh waisa dhikega.



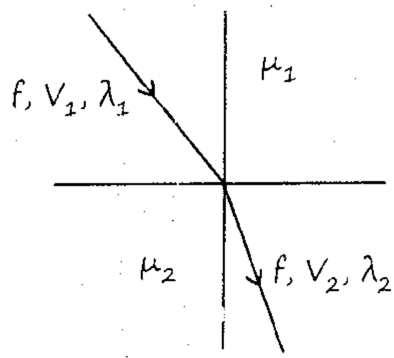

Heat up

$\therefore$ Yellow balloon will burst.

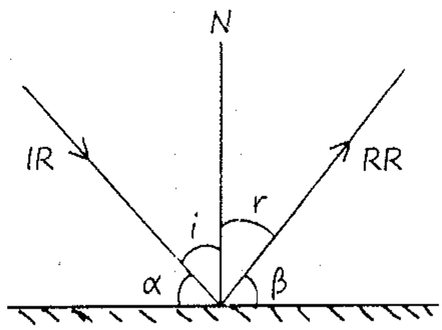
## Plane Mirror

Effect of Reflection & Refraction :-

f = Medium independent, only depend on Source




## Reflection




$\angle i = \angle r$

$\alpha = \beta$

Angle b/n IR & RR is always 2i.

## Deviation from Plane Mirror

(a) By a Mirror :-

$\delta = 180 - 2i$

| Normal Incidence | Grazing Incidence |
|---|---|
| $\delta = 180°$ | $\delta \approx 0$ |

(b) By two inclined Mirror :-

$\delta = 360 - 2\theta$

$\theta$ = Angle b/n Mirror.

## Note :-

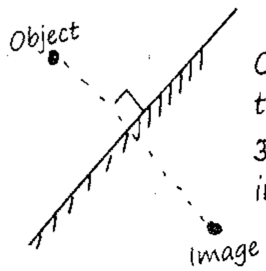
1> If IR rotated "$\theta$" then RR rotates "$\theta$" in Opposite Sense.

2> Plane Mirror is rotated by "$\theta$", reflected ray turns by "$2\theta$" in same sense.

3> If plane minor is rotated by $\theta$ about axis perpendicular to plane then No effect on reflected ray.

## Image Formation by Plane Mirror

### MR*




Object से ek line $\perp^{er}$ to Mirror draw Karo उसी line पर Virtual image hogi.

## Some Properties of Image formation by Plane mirror :-

- Focal length of Plane mirror is Infinite on covering part of mirror, No change in size of Image but brightness will change.
- Real Object → Virtual Image
- Virtual object → Real Image
- Object and Image always of same distance from mirror
- Lateraly inverted image.
- Plane mirror can form Inverted Image of real object

## Height of Mirror :-

(A) To see full height of object :-

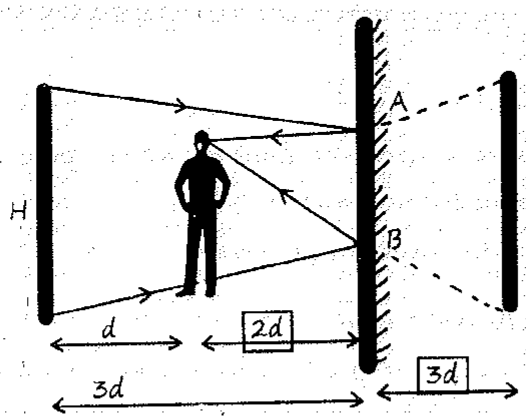
$H_{mirror} = H_{object}/2.$

(B) To see full wall behind object ?

(Observer at Centre).

$$H_{mirror} = \frac{H_{wall}}{3}$$

(C) To see full wall behind object if man is not centre ?




$$\frac{AB}{2d} = \frac{H}{5d} \qquad H_{mirror} = \frac{2H}{5}$$

## Clock System :-

If time in clock → ($H_{hr} : M_{mint} : S_{sec}$)

then time in Image of Clock →

$(11-H)_{hr} : (59-M)_{mint} : (60-S)_{sec}$

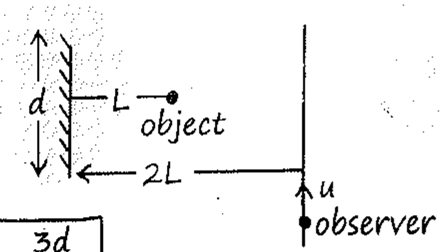
when only HR and MINT is given = $(11-H)_{hr} : (60-M)_{mint}$

## Velocity of Image :-

1> Object is moving $||^{el}$ to plane mirror :-

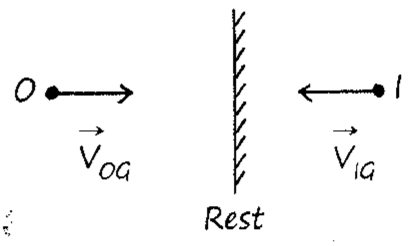
$\vec{V}_{IO} = 0 \qquad \vec{V}_{OG} = \vec{V}_{IG}$

Obserber is moving with constant speed $u$ then time for which observer Can See image of object




$$\boxed{t = \frac{3d}{u}}$$

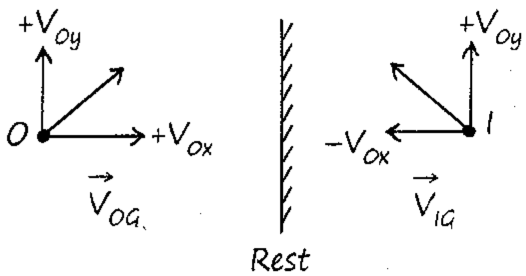
2> Object is moving $\perp^{er}$ to plane mirror :-




$\vec{V}_{IG} = -\vec{V}_{OG}$

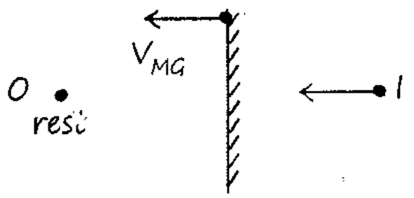
$\vec{V}_{IO} = -2\vec{V}_{OG}$

3> Object is moving at an angle to mirror



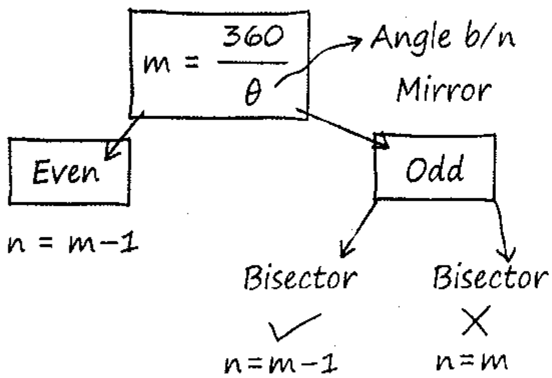

$*V_{IO} = -2V_{OX}$

4> Object at rest & Mirror is moving with $V_{MG}$ :-




$\vec{V}_{IO} = \vec{V}_{IG} = 2\,\vec{V}_{MG}$

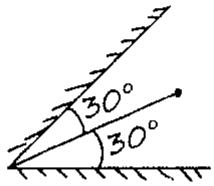
## Image Formation by Two Plane Mirror :-




$m = \frac{360°}{\theta}$ fraction then ush fraction Se choti integer lenge approximate Nahi lena,

Ex $M = 7.8 \rightarrow m = 7$ Image

**MR***




$m = \frac{360°}{60°} = 6$

$n = 5$

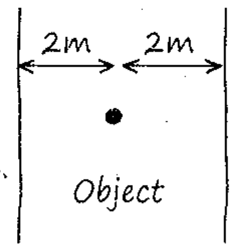
| $M_1$ | $M_2$ |
|---|---|
| 30° | 30° |
| | +60° |
| 90° | 90° |
| | +60° |
| 150° | 150° |

Pahila image Kaha banega

Angle b/n two mirror add Karo.

Ye coincide Karega.

## Infinite Image When Two Plane Mirror is Parallel :-




$m = \frac{360}{0} \Rightarrow n = \infty$

| $M_1$ | $M_2$ |
|---|---|
| 2m | 2m |
| | +4m |
| 6m | 6m |
| 10m | 10m |
| 14m | 14m |
| ⋮ | ⋮ |

Add $dist^n$ b/n two mirror.

## Spherical Mirror :-

| Concave M. | Convex M. |
|---|---|
| o Converging | o diverging. |
| o $f = -ve$ | o $f = +ve$ |
| $U = -ve$ [RO] | $U = -ve$ [RO] |
| $V = -ve$ [RI] | $V = +ve$ [VI] |

o Along Incident ray distance taken as +ve.

## Mirror Equation :-

- $f = \dfrac{R}{2}$
- $\dfrac{1}{V} + \dfrac{1}{U} = \dfrac{1}{f}$

$$\boxed{m_T = \frac{H_I}{H_O} = -\frac{V}{U} = \frac{f}{f-U} = \frac{f-V}{f}}$$

$$\boxed{m_L = \frac{dV}{dU} = -m_T^2}$$

* only valid for small object

Longitudinal Magnification

$$\boxed{m = -\frac{V}{U}}$$

m = +ve
- > 1 = Erect & Mag.
- < 1 = Erect & Diminished.

m = −ve
- > 1 = Inverted Mag.
- < 1 = Inverted & Diminished.

Joh Chahiye uska sign convention nai lete !

## Image Formation by Concave Mirror :-

| Object | Image |
|---|---|
| • ∞ | • f[R, I, −ve] |
| • b/n ∞ & C | • b/n C & f [R, I, −ve] |
| • at C | • at C[R, I, −1] |
| • b/n C & f | • b/n ∞ & C [R, I, −ve↑] |
| • at f | • at ∞. [R, I, −ve] |
| • b/n f & Pole | • Behind Mirrror [V, E, +ve] |

## Image Formation by Convex Mirror :-

| Object | Image |
|---|---|
| • at ∞ | • at focus. [V, E, m = +ve] |
| • b/n ∞ & Pole | • b/n Pole & Focus. |

## MR*

Tum mirror Ko kahi bhi rakho uska "f" change nai hoga lens Ka ho sakta hai.

## Newton's Formula

$f = \sqrt{xy}$

x = Object distⁿ from focus.

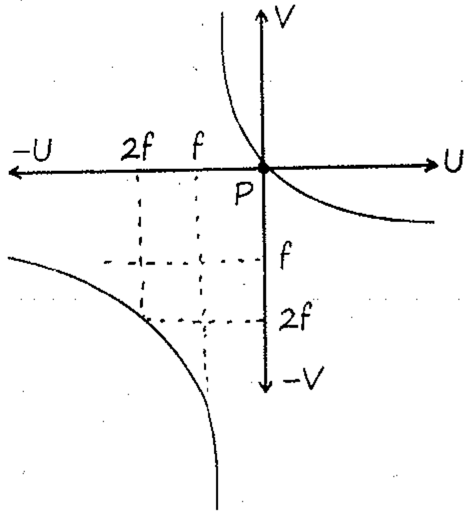
y = image distⁿ from focus.

## Velocity of Image in Case of Concave Mirror :-

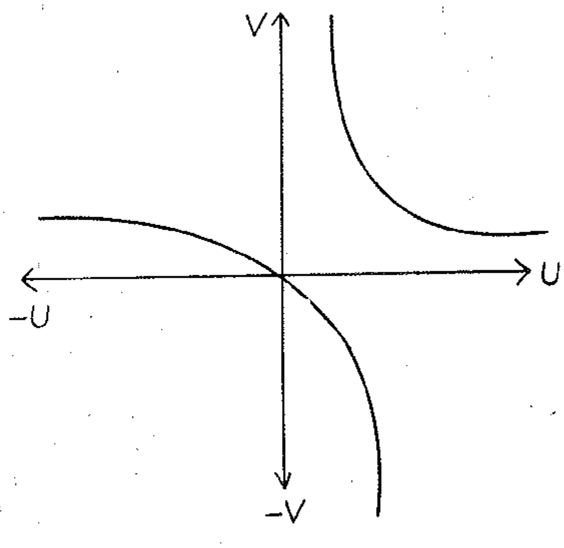
1> Object is moving $\perp^{er}$ to Principle axis :-
- $V_I = m_T V_O$.

2> Object is moving $\parallel^{el}$ to Principle axis :-
- $V_I = m_L V_O = m_T^2 V_O$.

## Graph for Concave Mirror :-




## Graph for Convex Mirror :-



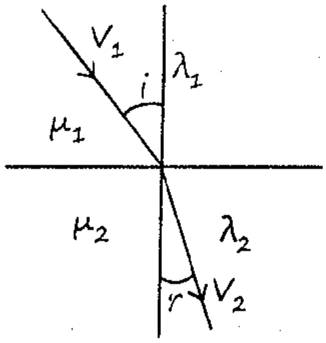
## Refraction :-

1> $R \longrightarrow D \quad \delta = i - r$

2> $D \longrightarrow R \quad \delta = r - i$

Snell's Law :-

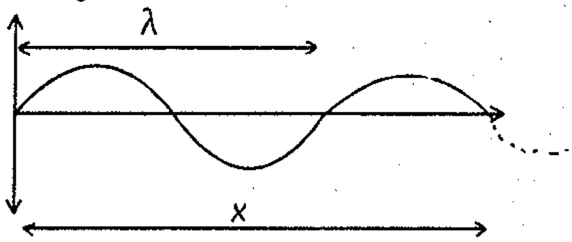
$$\mu_{21} = {}^1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = \frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$




**MR***

Snell's law can be directly applied b/n $1^{st}$ & last medium irrespective of intermediate medium.

"N" Number of wave in a "X" Distance if Wavelength is $\lambda$.




$n\lambda = x$

$$\boxed{n = \frac{x}{\lambda}}$$

## Optical Path :- (d)

*$d_{vacuum}$ & $d_{medium}$ for same time.

$\boxed{d = \mu x}$ $\quad x = d_{medium}$

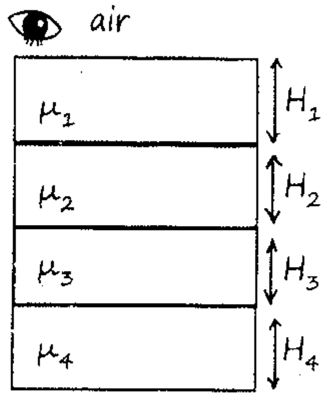
$d = d_{vacuum}$

$$\boxed{t = \frac{\mu x}{C}}$$

## Real & Apparent Depth :-

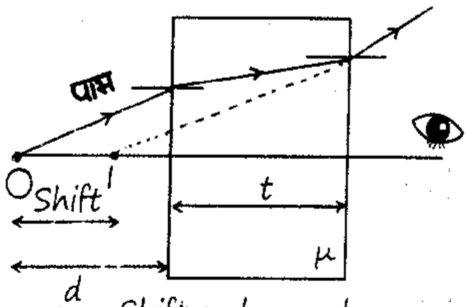
**MR** Special**

$$\boxed{\frac{d_{app}}{\mu_{obr}} = \frac{d_{real}}{\mu_{obj}}}$$




$$\frac{H_{app}}{1} = \frac{H_1}{\mu_1} + \frac{H_2}{\mu_2} + \frac{H_3}{\mu_3} + \frac{H_4}{\mu_4}$$

(A) Diverging ray :-




$$\text{Shift} = d_{real} - d_{app}$$

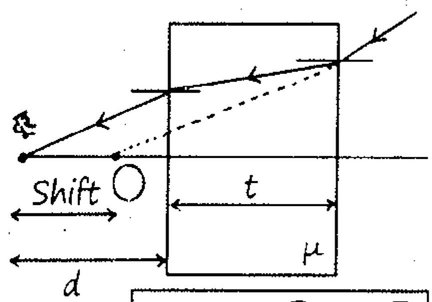
$$= d + t - d - \frac{t}{\mu}$$

$$\boxed{\text{Shift} = t\left[1 - \frac{1}{\mu}\right]}$$

**MR***

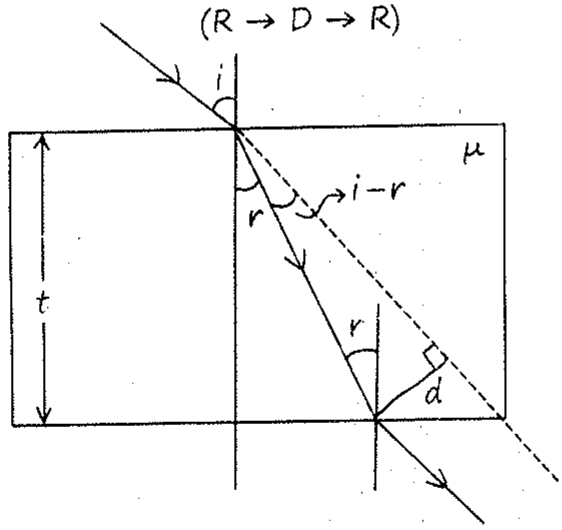
Jab bhi glass pr <u>diverging ray</u> aata hai toh glass ke <u>taraf</u> Image Shift kr jaata hai. Aur <u>Converging</u> ray Ke liye <u>durr.</u>

(B) Converging ray :-




$$\boxed{\text{Shift} = t\left[1 - \frac{1}{\mu}\right]}$$

## Lateral Shift




$$d = \frac{t}{\cos r}\sin(i-r)$$

- If $i \to$ small.

$$d = ti\left[1 - \frac{1}{\mu}\right]$$

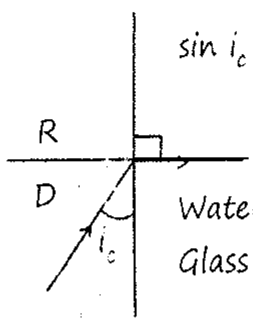
## Total Internal Reflection [$i > i_c$]

- Light must travel from

  D → R Medium.

MR*

$$\sin i_c = \frac{\mu_{Kam}}{\mu_{Jyada}}$$

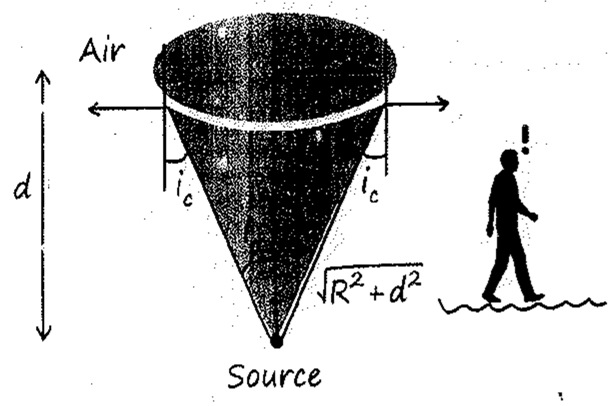



$$\sin i_c = \frac{\mu_2}{\mu_1} = \frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$

Water − Air :- $i_c = 49°$

Glass − Air :- $i_c = 42°$

## Radius of Visibility :-




$$R = \frac{d}{\sqrt{\mu^2-1}}$$

- Water − Air

  $h = \frac{\sqrt{7}}{3}r$

## Velocity of Image in Refraction :-

1> Object is moving parallel to boundary :-

- $V_{IG} = V_{OG}$
- $V_{IO} = 0$

2> Object is moving $\perp^{er}$ to boundary :-

MR*

$$\boxed{\frac{V_{app}}{\mu_{obr}} = \frac{V_{real}}{\mu_{obj}}}$$

## Refraction at Spherical Surfaces :-

MR* One stop solution

$$\boxed{\frac{\mu_{RR}}{V} - \frac{\mu_{IR}}{U} = \frac{\mu_{RR} - \mu_{IR}}{R}}$$

- Normal will pass through Center of Curvature.
- Nature of Surface will be decided by position of object.
- Concave :- R = −ve

  Convex :- R = +ve

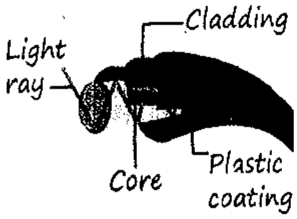
Magnification :-

$$m_T = \frac{H_I}{H_O} = \frac{\mu_{IR} \cdot V}{\mu_{RR} \cdot U}$$

$$m_L = \frac{dV}{dU} = \frac{\mu_{IR} \cdot V^2}{\mu_{RR} \cdot U^2}$$

## OPTICAL FIBER :- [Based on TIR]

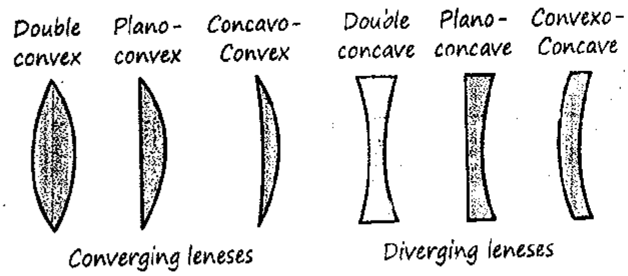
The angle at which ray must be incident so as information gets transmitted :-




$$\boxed{\sin i \le \sqrt{\underbrace{\mu_B^2}_{\text{Bada}} - \underbrace{\mu_C^2}_{\text{Chota}}}}$$

$(\mu_{core} > \mu_{cladding})$

## LENS




## Lens-Maker Equation :-

$$\boxed{\frac{1}{V} - \frac{1}{U} = \left(\frac{\mu_L}{\mu_M} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

## Lens Equation :-

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$$

$$\frac{1}{f} = \left(\frac{\mu_L}{\mu_M} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

## Focal Length of lens :-

- Focal length depends on medium.

$$f = \frac{R}{2(\mu-1)}$$

→ Biconvex (+)

→ Biconcave(−)

$\mu$ = refractive Index of lens w.r.t medium

$$f = \frac{R}{(\mu-1)}$$

→ Planoconvex (+)

→ Planoconcave (−)

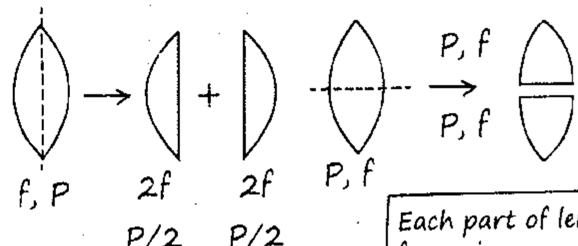
$$\boxed{m_T = \frac{H_I}{H_O} = \frac{V}{U} = \frac{f}{f+U} = \frac{f-V}{f}}$$

$m_T = -ve$ Real Image

$m_T = +ve$ Virtual Image

Small Object $\boxed{m_L = \frac{dV}{dU} = m_T^2}$

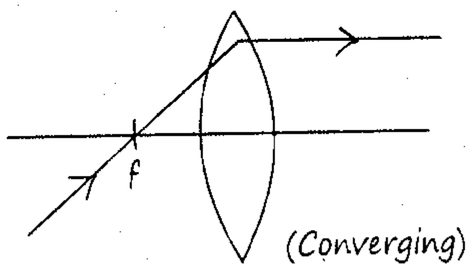
## Cutting of Lens :-



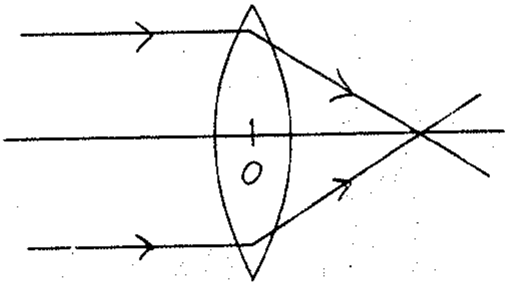

Each part of lens forms image.

## Ray-Diagram :-

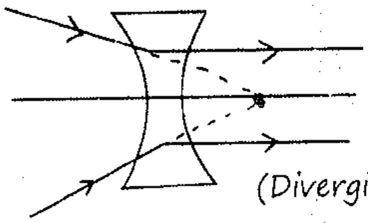
(a) Biconvex lens :- $\boxed{f = +ve}$



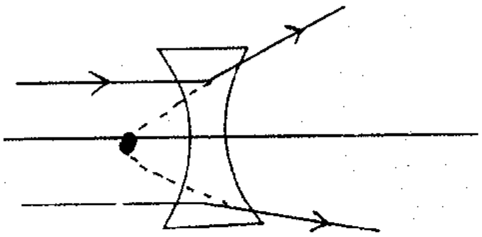

(Converging)




(b) Biconcave lens :- $\boxed{f = -ve}$



(Diverging)



## Nature of Lens Considering R.I. of Surrounding & Lens

- $\mu_L = \mu_M$ Glass Plate.
- $\mu_L > \mu_M$ Same Nature
- $\mu_L < \mu_M$ Opposite Nature

## Image Formation by Equiconvex Lens :-

| Object | Image |
|---|---|
| • ∞ | • f[R, I, –ve] |
| • b/n ∞ & 2f | • b/n 2f & f [R, I, –ve] |
| • at 2f | • at 2f [R, I, –1] |
| • b/n 2f & f | • b/n ∞ & 2f [R, I, –ve↑] |
| • at f | • at ∞ [R, I, –ve↑↑] |
| • b/n f & Pole | • on other side of lens [V, E, +ve] |

Convex lens ⇅ Concave Mirror

(Converging)

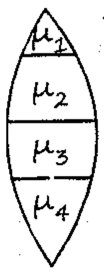
Can a Convex lens be have as diverging lens.
→ Yes, when ($\mu_m > \mu_L$)

## Image Formation by Concave Lens :-

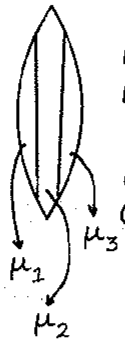
| Object | Iamge |
|---|---|
| • at ∞ | • at focus. [V, E, m = +ve] |
| • b/n ∞ & Pole | • b/n Pole & Focus. |

Concave Lens ⇌ Convex Mirror
(Diverging)

## No. of Image Form :-




No. of image || No. of Materials




No. of image || No. of Object

## Combination of Lens :-

- Power :-

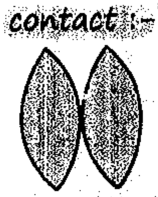
**Lens** $P_L = \dfrac{1}{f_L}$

**Mirror** $P_M = \dfrac{1}{f_M}$

(a) When lens are in **contact** :-

$P = P_1 + P_2$

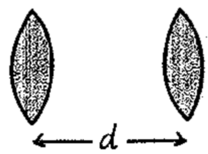
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$



(b) When lens are separated by a distance "d":-

$P = P_1 + P_2 - dP_1P_2$

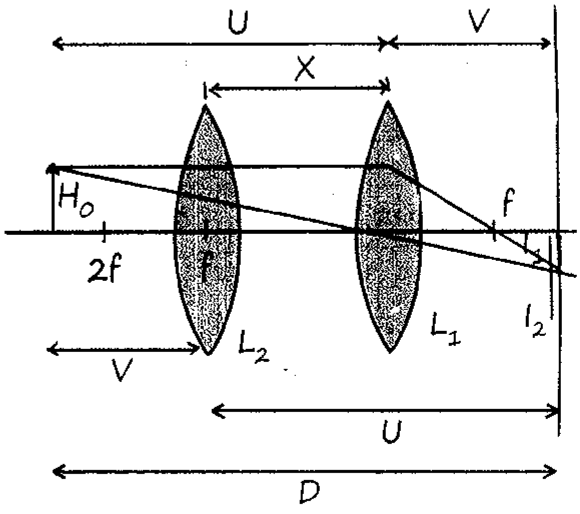
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$




**Take P with sign !**

## Displacement Method :-

Image Ka dist$^n$ Object Ko, Object Ka dist$^n$ Image Ko




$$f = \frac{D^2 - x^2}{4D}$$ D = $dist^n$ b/n Object & Screen.

x = $Dist^n$ b/n two position of lens.

[redacted] $m_1 = \frac{V}{U} = \frac{I_1}{H_O}$

$|f| = \frac{m_1 - m_2}{x}$ $m_2 = \frac{U}{V} = \frac{I_2}{H_O}$

**Combination of a Lens & a Mirror (Silvering of Lens) :-**

**MR***

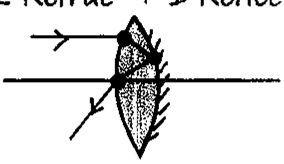
Koi sign mt rakho bs direct lens ka opposite nature mirror Ko dedo.

$$\boxed{\frac{1}{|f_{net}|} = \frac{2}{|f_L|} + \frac{1}{|f_M|}}$$

Convex lens Silvered $\longrightarrow$ Concave Mirror $f = -ve$

Concave lens Silvered $\longrightarrow$ Convex Mirror $f = +ve$

[2 $Refrac^n$ + 1 $Reflec^n$]



Case :-

(a) Convex Lens :-

$$f_{eq} = \frac{R}{2(2\mu - 1)} \Big\} \text{Concave Mirror}$$

(b) Plano-convex lens :-

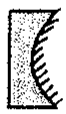
$$f_{eq} = \frac{R}{2(\mu - 1)} \Big\} \text{Concave Mirror}$$

(c) Plano-convex lens :-

$$f_{eq} = \frac{R}{2(\mu - 1)} \Big\} \text{Concave Mirror}$$

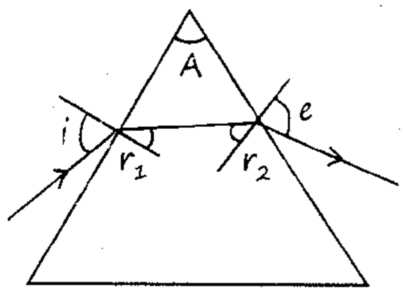
$$f_{eq} = \frac{R}{2(\mu - 1)} \Big\} \text{Convex Mirror}$$

$$f_{eq} = \frac{R}{2\mu} \Big\} \text{Concave Mirror}$$



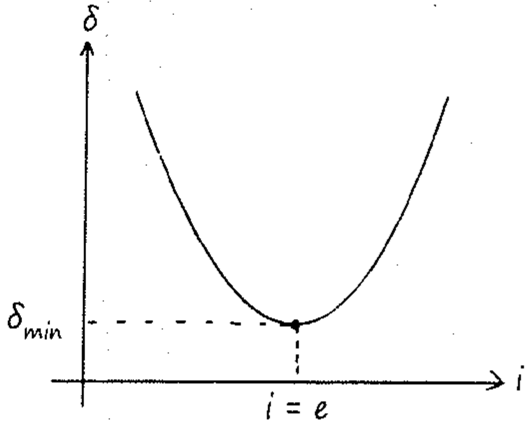
$$f_{eq} = \frac{R}{2\mu} \Big\} \text{Convex Mirror}$$

**Prism**




$A = r_1 + r_2$ $\delta_{Total} = i + e - A$

$$\mu = \frac{\sin i}{\sin r_1} \qquad \frac{1}{\mu} = \frac{\sin r_2}{\sin e}$$



- For Minimum deviation :-

$$r_1 = r_2 = \frac{A}{2} \qquad i = e$$

$$d_{min} = 2i - A \qquad \mu = \frac{\sin\left[\frac{d_m + A}{2}\right]}{\sin\left[\frac{A}{2}\right]}$$

- For thin prism (A = small)

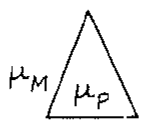
$$d_{min} = A(\mu - 1)$$

A = Refracting Angle.

- Half Angle Formula :-

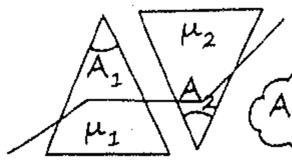
$$\sin\theta = 2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$$

- Prism is placed in medium :-




$$\delta = A\left[\frac{\mu_P}{\mu_M} - 1\right]$$

- Condition for no deviation :-




$$\delta_1 + \delta_2 = 0$$

$$A_1(\mu_1 - 1) = -A_2(\mu_2 - 1)$$

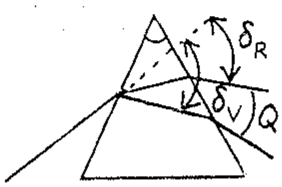
- Dispersion of light :-

$$\mu \propto \frac{1}{\lambda^2} \xrightarrow[\delta\downarrow]{\text{VIBGYOR}} \lambda\uparrow \ \mu\downarrow$$

- Angular dispersion (Q) :-

$$Q = \delta_V - \delta_R$$

$$Q = A[\mu_V - \mu_R]$$




- Mean deviation :-

$$\delta_{mean} = \frac{\delta_V + \delta_R}{2}$$

$$\delta_{mean} = A\left[\frac{\mu_R + \mu_V}{2} - 1\right]$$

$$\delta_{mean} = A[\mu_{mean} - 1]$$

$$\delta_{mean} = A[\mu_{yellow} - 1]$$

- Dispersive Power :- ($\omega$)

$$\omega = \frac{Q}{\delta_{mean}} = \frac{\delta_V - \delta_R}{\delta_{mean}}$$

$$\omega = \frac{\mu_V - \mu_R}{\mu_y - 1} = \frac{\delta_V - \delta_R}{\frac{\delta_V + \delta_R}{2}}$$

- Dispersion without Deviation :-

$$\delta_y = -\delta_y'$$

$$A_1(\mu_1 - 1) = -A_2(\mu_2 - 1)$$

(emergent ray $\|^{el}$ incident ray)

- Deviation without Dispersion :-

$$Q = -Q'$$

$$\delta_V - \delta_R = -[\delta_V' - \delta_R']$$

(White light bahar)

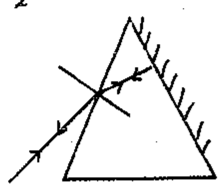
- Total dispersion :-

$$\theta = \theta_1 + \theta_2 = A[\mu - 1](\omega_1 - \omega_2)$$

- Silvering of Prism :-

$$A = r_1 \qquad r_2 = 0$$

$$\mu = \frac{\sin i}{\sin A}$$



$\mu \propto 1/\lambda$

## Optical Instruments :-

### Visual Angle :-

*Angle from horizontal with which a observer sees an object.*

$$\boxed{Q_O = \frac{H_O}{D}}$$

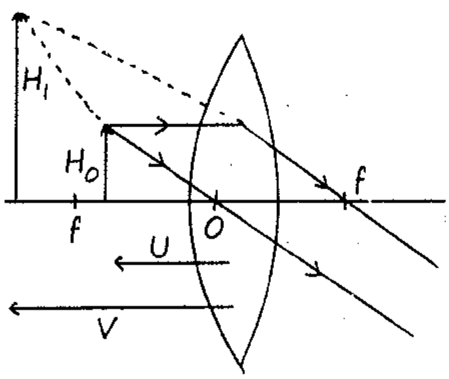
Eye की औकाद !

$D = 25$ cm

### Distance of Distinct Vision

| Myopia | Hypermetropia |
|---|---|
| Concave lens.<br>Near Sightedness | Convex lens.<br>Far-Sightedness |

### Simple Microscope




$$\boxed{M = \frac{D}{U} = D\left[\frac{1}{f} + \frac{1}{V}\right]}$$

**I**

- Stained eye position
- Near point
- $V = D$

[redacted]

**II**

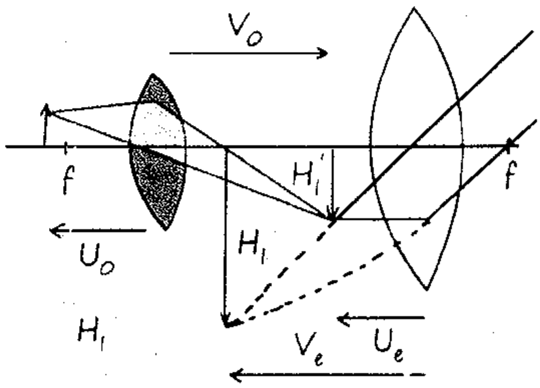
- Relaxed eye position
- Far point
- $v = \infty$.

[redacted]

### Comp

### Compound Microscope

$\boxed{f_o < f_e}$




$$\boxed{\begin{aligned} M &= M_O M_e = \frac{V_O}{U_O}\,\frac{D}{U_e} \\ M &= \frac{DV_O}{U_O}\left[\frac{1}{f_e} + \frac{1}{V_e}\right] \end{aligned}}$$

- Note :-

$$M = \frac{Q_I}{Q_O}$$

**I**

- Stained eye position
- Near point
- $V = D$

$$M = \frac{V_O}{U_O}\left(1 + \frac{D}{f_e}\right)$$

**II**

- Relaxed eye position
- Far point
- $v = \infty$.
- $v_e$ = infinity
  $u_e = f_e$

$$M = \frac{V_O D}{U_O f_e}$$

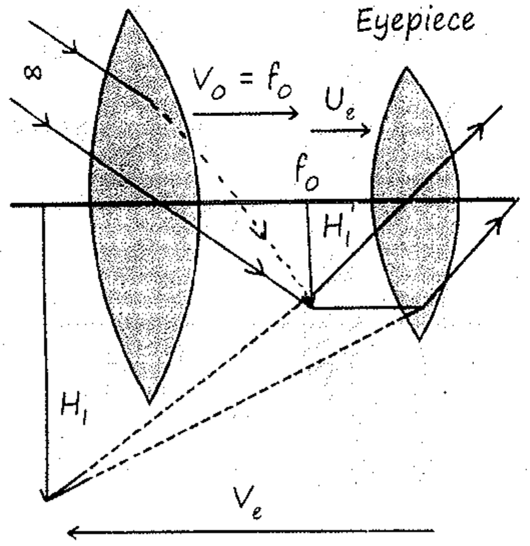
$$M = \frac{L D}{f_o f_e}$$

- Length of Compound Microscope :-

$$L = V_O + U_e \cong V_O$$

In case of far point $L = V_O + f_e$

## Astronomical Telescope




$$\boxed{M = \frac{f_o}{U_e} = f_o\left[\frac{1}{f_e} + \frac{1}{V_e}\right]}$$

**I**

- Stained eye position
- Near point
- $V = D$

$$M = f_o\left[\frac{1}{f_e} + \frac{1}{D}\right]$$

**II**

- Relaxed eye position
- Far point
- $V = \infty$.

$$M = \frac{f_o}{f_e}$$

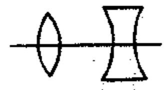
- Length of Telescope :-

  $L = f_o + U_e$

  $L = f_o + f_e$ → For image at ∞.

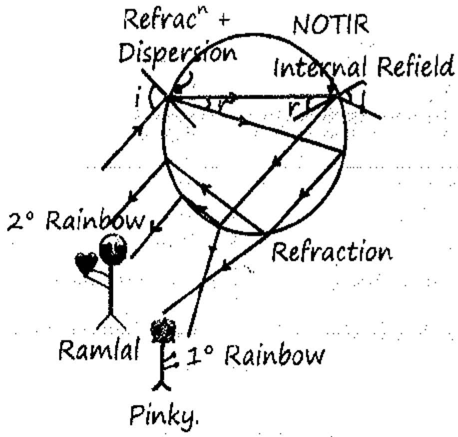
- Galileo Telescope :-

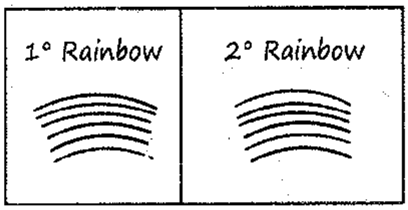
  $L = f_o + f_e$

  Rest Same.

- Rainbow formation :-




- Scattering of Light :-

E.g.:- Blue colour of Sky, Red Sky & sun on the time of sunrise or sunset.

Scatering $\alpha \ \frac{1}{\lambda^4}$

MR*

"Beta duniya gol hai, tuhare kiye gaye mehnat tum tak zarur lautega....mehnat karna mat chhodo...jitna tumhare hath main hai h utna karo, baki bhagwan par chhod do"

# 31 Wave Optics

(Mech + Longitudinal)

Huygen:- Wave

Einstein:- Particle

Maxwell:- EM wave

} Light

(Non Mech + Transverse)

$$\boxed{\frac{2\pi}{\phi} = \frac{\lambda}{\Delta x} = \frac{T}{\Delta t}}$$

## Equation of Wave for Light/ Sound / EM Wave / ac: -

$y = A\sin(\omega t + kx + \phi)$

Wave Sabhi Medium particle Ko SHM Deta hai!

## Newton corpuscular theory:-

**Explain:-**

- Rectilinear propagation
- Reflection
- Refraction of Light

**Can't explain:-**

- Interference, diffraction and polarisation

## Huygens wave theory:-

**Explain:-**

- Rectilinear propagation, Interference, Reflection, Refraction, Diffraction.

**Can't explain:-**

- Polarisation, PEE, Compton effect.

## Types of waves:-

1. **Medium:-**

   Mechanical ✓ EMW ✗

2. **Propagation:-**

   Progressive $\infty$ Stationary Finite

3. **Vibration:-**

   Transverse $\perp$ Longitudinal $\parallel^{el}$

### Wave front

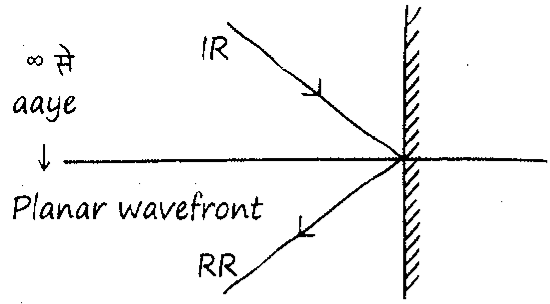
| Spherical | Cylindrical | Plane |
|---|---|---|
| $I \propto A^2 \propto \frac{1}{r^2}$ | $I \propto A^2 \propto \frac{1}{r}$ | $I = A = $ Const$^n$ |

## Intensity of wave:-

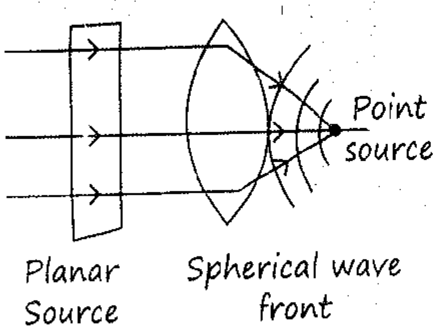
$$I = \frac{1}{2}\rho V A^2 \omega^2$$

## Behaviour of plane wavefront on reflection & refraction:-

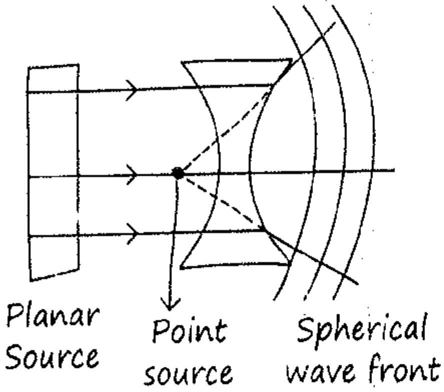
1. **Plane Mirror:-**



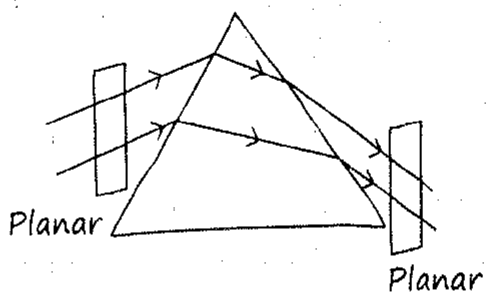
2. **Convex Lens:-**

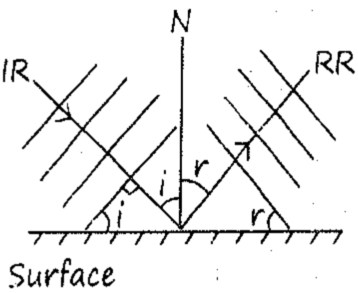



3. **Concave lens:-**




4. **Prism:-**




**Sources:-**

Phase difference

| | |
|---|---|
| Time के साथ constant | Time के साथ Variable |
| Coherent | Incoherent |
| $f_{req}$ = same | |

1. **Incoherent sources:-**

$$\Delta\phi = [\omega_1 - \omega_2]t + [K_1x_1 - K_2x_2] + [\phi_1 - \phi_2]$$

Time Dependent Hai!

2. **Coherent sources:-**

$$\Delta\phi = K(x_1 - x_2] + \phi_1 - \phi_2$$

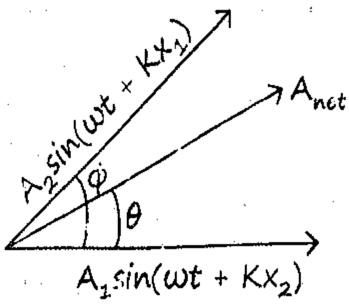
$$\Delta\phi = \frac{2\pi}{\lambda}.\Delta x. \quad \because \quad \phi_1 = \phi_2$$

Single wavelength — Monochromatic source

**Interaction of light:-**

- $\lambda << l_{object}$ = RAY
- $\lambda \approx l_{object}$ = WAVE
- e–/p+/α-particle = PARTICLE (Photon)

**INTERFERENCE** — Req Two coherent sources

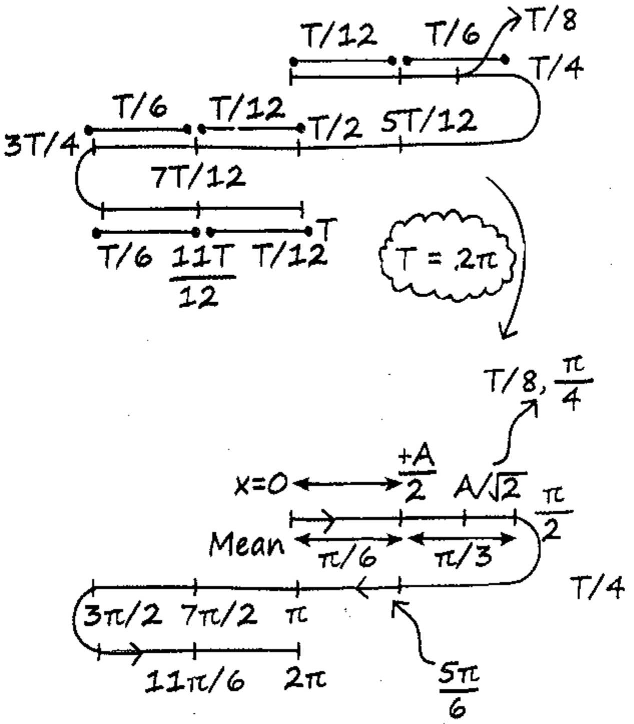



$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

$$\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$

Fire concept MR*




**Intensity:** $I = \frac{1}{2}\rho V A^2 w^2$

$I \propto d \propto A^2 \quad d = \text{slit width}$

$$I_{av} = \frac{I_{min} + I_{max}}{2}$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_2} + \sqrt{I_1}\right)^2}{\left(\sqrt{I_2} - \sqrt{I_1}\right)^2} = \frac{\left(A_2 + A_1\right)^2}{\left(A_2 - A_1\right)^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{\left[\sqrt{\frac{I_2}{I_1}} + 1\right]^2}{\left[\sqrt{\frac{I_2}{I_1}} - 1\right]^2} = \frac{\left[\frac{A_2}{A_1} + 1\right]^2}{\left[\frac{A_2}{A_1} - 1\right]^2}$$

## Constructive Interference

$\phi = 0° \qquad \cos\phi = 1$

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$A_{max} = A_1 + A_2$

$I_1 = I_2 = I_0$

$I_{max} = 4I_0$

$\phi = 2n(\pi) \rightarrow$ even

$\Delta x = n(\lambda) \rightarrow$ integral

$n = 0, 1, 2, 3, \ldots$

MR*

$\lambda = 2\pi$

- YDSE:-

  $Y = n\left(\frac{\lambda D}{d}\right)$ Posit$^n$ of $n^{th}$ Bright.

- NOTE:-

  1> "n" source of same intensity $I_0$. Find $I_{Result^n}$:-

  $I_{net} = nI_0$ (scalar Add$^n$)

  2> "n" source of same intensity $I_0$ then find $I_{max}$:- (Const. Inter.)

  *$I_{max} = n^2 I_0 = \left(n\sqrt{I_0}\right)^2$

## Fringe visibility.-

$$\text{Visibility Ratio} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

$$= \frac{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2 - \left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}} + 1\right)^2 + \left(\sqrt{\frac{I_1}{I_2}} - 1\right)^2}$$

## Destructive inter.:-

$\phi = 180° \qquad \cos\phi = -1$

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$A_{min} = A_1 - A_2$

$I_1 = I_2 = I_0$

$I_{max} = 0, I_{min} = 0$

$\phi = (2n+1)\pi \rightarrow$ odd

$\Delta x = (2n+1)\frac{\lambda}{2} \rightarrow$ odd

$n = 0, 1, 2, 3, \ldots$

- **YDSE:-**

$Y = (2n-1)\dfrac{\lambda D}{2d}$ Posit$^n$ of $n^{th}$ Dark.

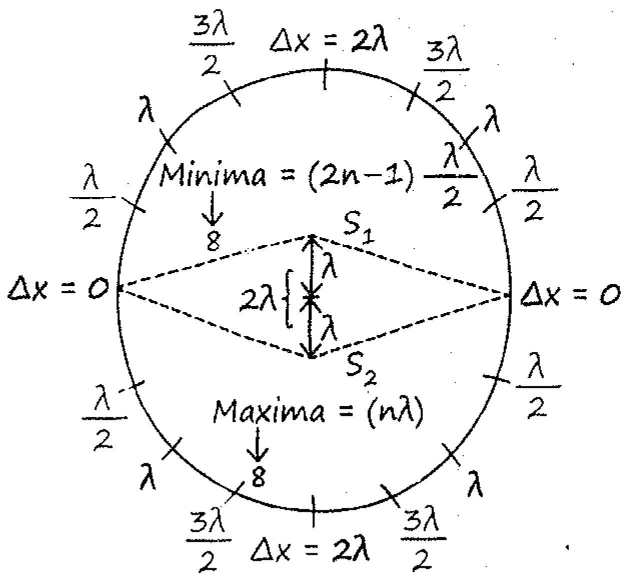
IIT Neet 2016 $\dfrac{I_1}{I_2} = \alpha$

Visibility Ratio $= \dfrac{2\sqrt{\alpha}}{1+\alpha}$

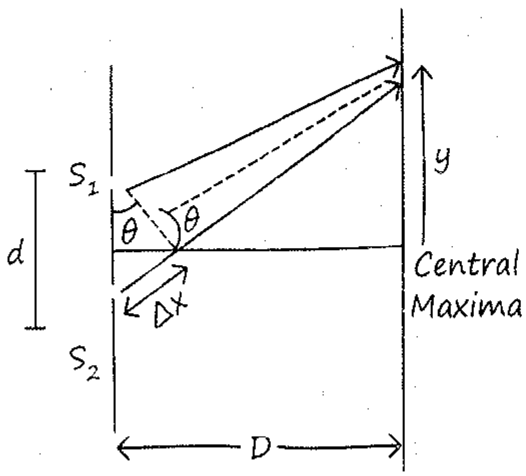
☼ Two wave of "$I_0$" intensity $I_{res.}$ If phase diff is $\phi$:-

$$I = 4I_0\cos^2\frac{\phi}{2}$$

MR*



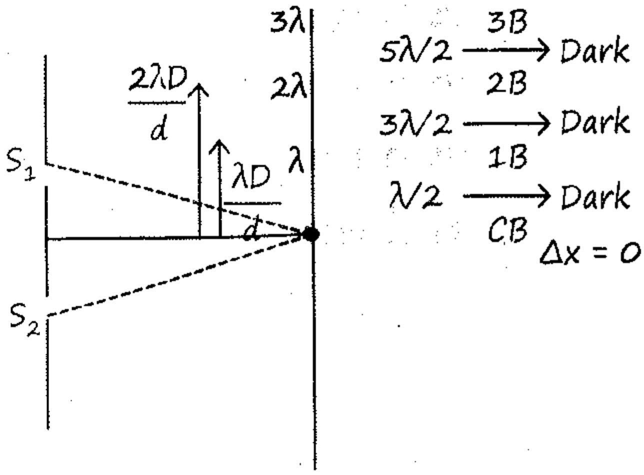

Ydse:-




$d \simeq \lambda$ $\qquad \Delta x = d\sin\theta$

$\qquad\qquad = d\tan\theta$

$$\Delta x = \frac{Yd}{D}$$




**Fringe Width ($\beta$):-**

$\beta = Yn^{th} - Y(n-1)^{th}$

Bright $\qquad$ Bright

$$\boxed{\beta = \frac{\lambda D}{d}}$$

**Angular Fringe Width:-**

$\theta = \dfrac{\beta}{D}$ $\qquad \theta = \dfrac{\lambda}{d}$

MR*

☼ Angular kuch bhi puche aap "D" se divide kardena!

☼ $\mu_{air} > \mu_{vacuum}$

$\beta \propto \dfrac{1}{\mu}$ $\qquad \therefore$ In vacuum, interference with large "$\beta$" seen!

| YDSE in air | YDSE in liq. |
|---|---|
| 1> $\beta = \frac{\lambda D}{d}$ | 1> $\beta' = \frac{\beta}{\mu}$ |
| 2> $\theta = \frac{\lambda}{d}$ | 2> $\theta' = \frac{\theta}{\mu}$ |
| 3> Max:- $Y_n = \frac{n\lambda D}{d}$ | 3> $Y_n' = \frac{n\lambda D}{\mu d}$ |
| Min:- $Y_n = (2n-1)\frac{\lambda D}{2d}$ | $Y_n' = \frac{(2n-1)\lambda D}{2\mu d}$ |
| | $f$ = same<br>$\lambda' = \lambda/\mu$ |

✲ Two light of $\lambda_1 > \lambda_2$ is used in YDSE, then central Maxima & 1st Maxima will :

✲ Central Maxima $\Rightarrow$ At same position.

✲ 1st Maxima:- Not at same place.

$\downarrow$

Jiska $\lambda\uparrow$ $Y\uparrow$ $\because Y = \frac{n\lambda D}{d}$

$\therefore \lambda_1$ = dur rahega!

## White light:-

✲ sabse pahle maxima = violet ka ayega.

✲ sabse pahle dikhega = red.

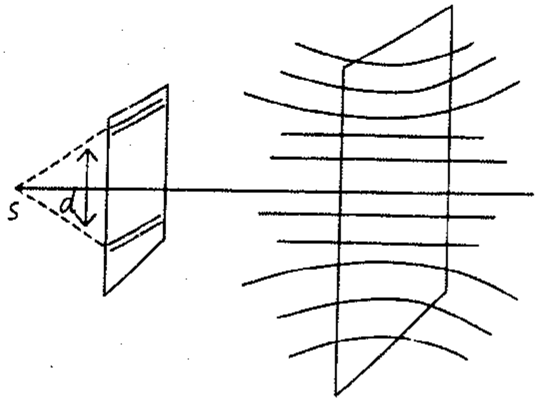
✲ Used to find central maxima.

Fringe

| Nearest | Central | Farthest |
|---|---|---|
| $\downarrow$ | $\downarrow$ | $\downarrow$ |
| Blue | चमकीला<br>White | Red. |

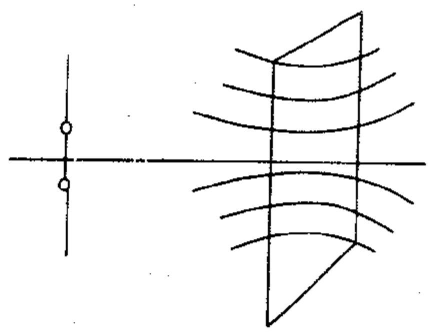
## Shapes of fringes:-

1> Two slit used:-



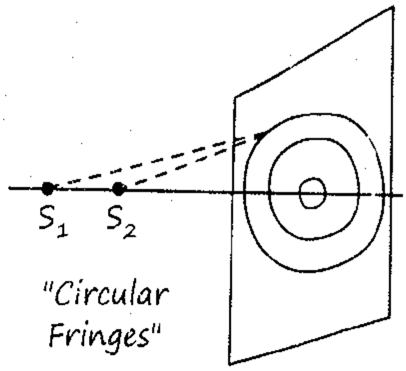

"Hyperbolic" Fringes

2> Two pin holed used:-



"Perfectly Fringes"

3> When two hole is along the line joining of source and screen:-




"Circular Fringes"

## Optical path:-

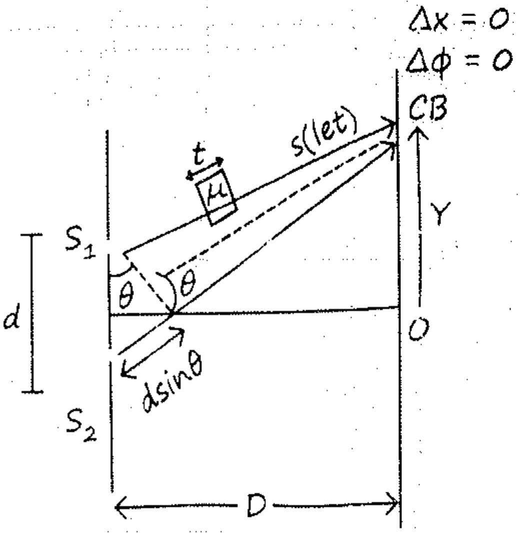
air:- $\Delta\phi = \frac{2\pi}{\lambda}t$

Med$^m$:- $\lambda' = \frac{\lambda}{\mu}$ $\quad\quad \Delta\phi' = \frac{2\pi}{\lambda}(\mu t)$

YDSE when a slab of "$\mu$" R.I. inserted in path of $S_1$:-

$d\sin\theta = t(\mu-1)$

$$\boxed{\frac{dY}{D} = t(\mu-1)} \qquad \beta = \frac{\lambda D}{d}$$




MR*

Jiske path mein slab jayega woh "$\mu t$" jyada chalega agar $\Delta x = 0$ krna hai toh usko niche lao yaneki kam karo toh $S_2$ ke path ko badao dono barabar hojayenge aur $\Delta x = 0$!

**Position of CB:-**

$$\boxed{Y = \frac{Dt(\mu-1)}{d}}$$

Kuch nai Normal YDSE ka

$\lambda = (\mu-1)t$ rakhdo!

## Number of bright & dark fringes:-

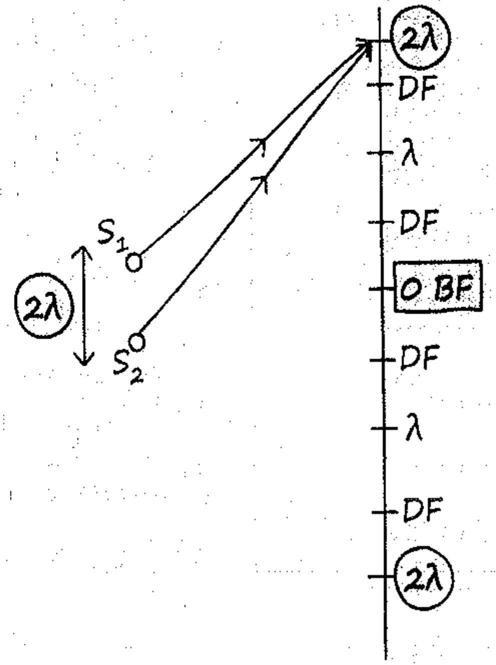
Total No. of:-

(a) Bright Fringe = $\left[\frac{2d}{\lambda} + 1\right]$

(b) Dark Fringe = $2\left[\frac{d}{\lambda} + \frac{1}{2}\right]$

$\frac{d}{\lambda}$ = minimum integer $\left\} \begin{array}{ll} 2.5=2 & 2.9=2 \\ 3.6=3 & \end{array}\right.$

**Majduri se duri MR ★ hai jaruri**




No. of BF = $\frac{2(2\lambda)}{\lambda} + 1 = 5$

No. of DF = $2\left[\frac{d}{\lambda} + \frac{1}{2}\right]$ ← Integer

$= 2\,(2+0.5) = 2\,[2] = 4$

**Polarisation:-**

(Only for T. wave)

$E = cB$ ⟶ negligible compared to E.F

$\therefore$ Restricting of $\vec{E}$ = Polaris$^n$.

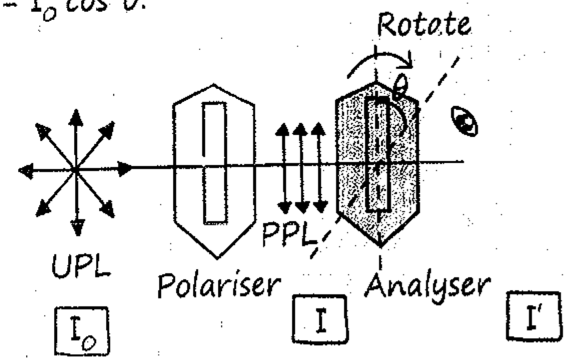
**Law of malus:-**

$I_0$ = Intensity of unpolarised light.

$\theta$ = Angle b/n Analyser & Polarizer.

$I$ = Intensity of light transmitted through the polariser.

*$I = I_0 \cos^2\theta$.*

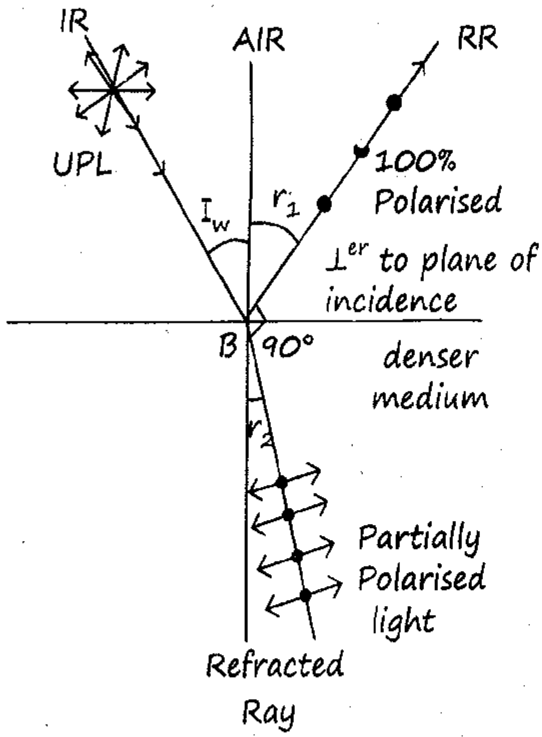


$$I = \frac{I_0}{2} \qquad I' = I\cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

PPL → After polaroid

↓ After Analyser

## Brewster's Law (Polarisation by reflection):




$$i_w = \tan^{-1}(\mu) \qquad \tan i_w = \frac{\mu_2}{\mu_1}$$

## Diffraction:-

Bending of wave around an object.

जोरदार Diffrac$^n$ :- $s \approx \lambda_{wave}$

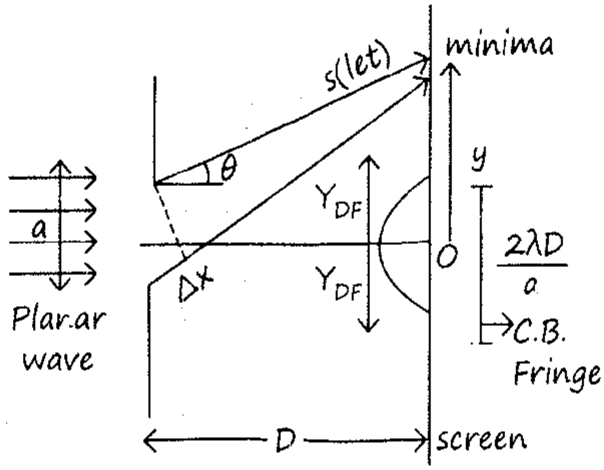
**Fresnel**

- Source & screen at Finite dist$^n$
- Spherical wave front

**Fraunhoffer**

- Source & screen at ∞ dist$^n$
- Planar wave front

## Diffraction:-




उलटा है YDSE का!

Neet

☼ Width of C.B. Fringe $= \frac{2\lambda D}{a}$

☼ Angular width of C.B.F. $= \frac{2\lambda}{a}$

| Minima (D.I) | (C.I) Maxima |
|---|---|
| $a\sin\theta = n\lambda$ | $a\sin\theta = (2n-1)\frac{\lambda}{2}$ |
| $n = 1,2,3,4\ldots$ | $n = 2,3,4\ldots$ |

$$\tan\theta = \frac{Y}{D}$$

☼ $a\sin\theta \approx a\tan\theta$

## Resolving Power:-

Ability of optical instrument to distinguish two neighbouring points.

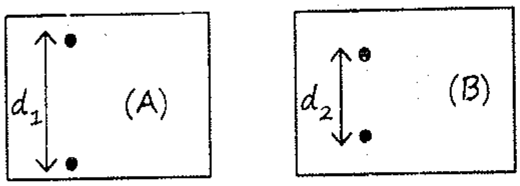
### A. Microscope: (Human eye)

$$R.L. = \frac{1.22\lambda}{2\mu\sin\theta} \qquad R.P. = \frac{1}{R.L}$$

$\mu$ = R.I. of medium b/n object and lens.

$\theta$ = Half angle of cone of light from point object

Numerical aperture = $\mu \sin\theta$



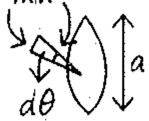

R.P. = B > A      R.L. = A > B

**B. Telescope:-**

R.P = Angular Resolution

$$d\theta = R.L. = \frac{1.22\lambda}{a}$$

R.L. = $d\theta = d_{min}.D$




a = aperture of objective lens
diameter

**Validity of ray-optics:-**

$$Z = \frac{a^2}{\lambda}$$

Iske pahile ray      Iske baad ray

Dist$^n$ of source & screen (Z)

a = width of C.B. Fringe.

**Imp. points:-**

1. 1 inch = 2.54 cm
2. For max R.L. we use max $\lambda$.
3. $\lambda_w = \dfrac{\lambda_{air}}{\mu}$

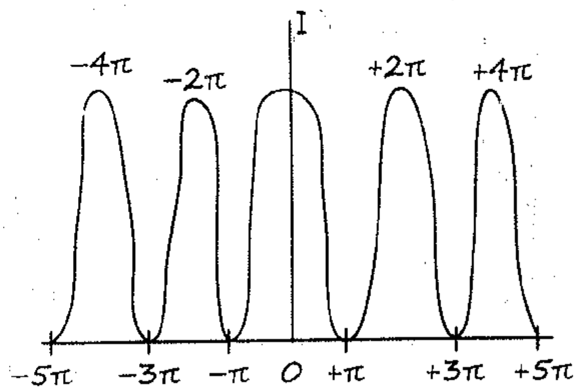
   * $d\theta_w = \dfrac{d\theta_a}{\mu}$ *
4. If two coherent source of equal intensity are taken then fringe visibility is 100%. because $I_{min} = 0$.

**Doppler's effect of light:-**

$$r' = r\left[\frac{1 \pm \frac{V_r}{C}}{\sqrt{1-\left(\frac{V_r}{C}\right)^2}}\right] \Rightarrow \lambda' = \frac{\lambda}{1+\frac{V_r}{C}}$$

$$\frac{\Delta r}{r} = \frac{\Delta\lambda}{\lambda} = \pm\frac{V_r}{C}$$

**Interference:-**




**Interference due to thin filim.**

\# For reflected light:

Maxima – $2\mu t \cos r = (2n + 1)\dfrac{\lambda}{2}$

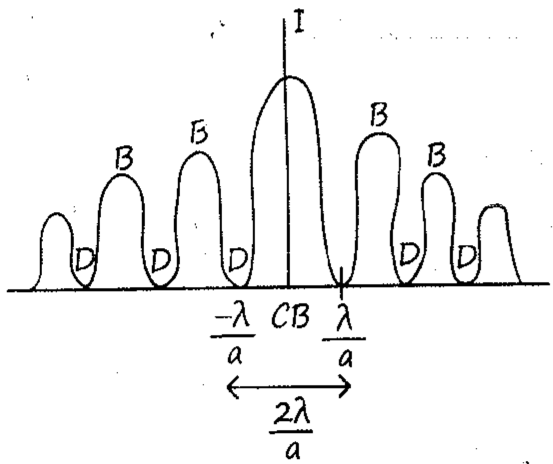
Mininima – $2\mu t \cos r = n\lambda$

\# For transmitted light:

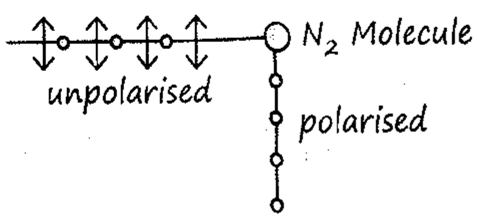
Maxima -- $2\mu t \cos r = n\lambda$

Minima – $2\mu t \cos r = (2n-1)\dfrac{\lambda}{2}$

**Diffraction:-**




**Polarisation by scattering**



# 32 Dual Nature

**Dual Nature of light:**

Newton → Light is a Farticle (corpuscule)
→ Explain reflection / refraction

Huygen → Light is a mechanical wave
→ Ether medium Propsed by Huygen
→ Explain Diffraction / refraction.

Maxwell → Light is a Non-mechanical transverse wave.
→ No medium required.

de Broglie → Nature loves Symmetry
→ Light have dual Nature.

Davission & Geomer → $e^-$ is a Wave $exp^m$ verification

G.P. thomson → verification of electron as wave.

**1. Photon:**

$$E = h\nu = \frac{hc}{\lambda} = \frac{2\times10^{-25}J}{\lambda} = \frac{12400\ eV}{\lambda(A^\circ)}$$

$$v_{photon} = \frac{1}{\sqrt{\mu_0\varepsilon_0}},\ (v_{photon})_{Med.} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

independent of frequency

**Energy of light beam:**

$E=nhf=\frac{nhc}{\lambda}$ n= no. of Photons

**2. Properties of Photon:**

- charge and rest mass =0
- Momentum of Photon $P = \frac{h}{\lambda}$
- Moving mass of photon $m= \frac{h}{\lambda C} = \frac{E}{c^2}$
- Photon does not deviate in electromagnetic field but deviate in gravitational field.

**3. Power of light:**

$$P = \frac{E}{t} = \frac{nh\nu}{t}$$

No. of photons emitted per unit time $= \frac{n}{t}$

$$= \frac{P\lambda}{hc}$$

**4. Intensity of light:**

$$I = \frac{nE}{At} = \frac{nh\nu}{At} = \frac{nhc}{\lambda At} = \frac{Power}{Area}$$

if source is same, frequency is same.

$I \propto$ no. of photons.

Q. If Intensity and frequency both becomes double then no. of Photon will be??

Ans. n = remains same

$I \propto nf$

Q. For a given source. Intensity becomes double then no. of Photon wiil?

Ans. becomes double $\boxed{I \propto n}$

**5. Fractional change in frequency of photon when it travels "h" distance on earth surface:**

$$\frac{\Delta\nu}{\nu} = \frac{gh}{C^2} \qquad h\nu_i + \frac{h\nu_i}{c^2}gh = h\nu_2 + 0$$

**6. Radiation pressure:**

*Complete Absorption : ($\rho$ = 1)

$$P = \frac{I}{C}[2-\rho]$$

$$P = \frac{I}{C}$$

$$F = PA = \frac{IA}{C} = \frac{Power}{C}$$

*Complete Reflective : $\sigma$ = 1

$$P = \frac{I}{C}[\sigma+1]$$

$$P = \frac{2I}{C}$$

$$F = \frac{2IA}{C} = \frac{2Power}{C}$$

## 7. Photoelectric effects:

- Conversion of light energy to electrical energy.
- Explained by Einstein
- Instantaneous process, time lag b/w falling of photon and emiting of electron is $10^{-9}$ sec.
- Efficiency = $\frac{\text{no of electron}}{\text{no of photon}} = 10^{-3}$ to $10^{-4}$.
- One to one intraction one photon can eject only one electron.

$\phi$ = Material Properties = $h\nu_0 = \frac{hc}{\lambda_0}$

where $\nu_0$ and $\lambda_0$ are thereshold frequency and wavelength.

$E_{photon} = \phi + (KE)_{max}$

$h\nu = h\nu_0 + eV_0$

$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + \frac{1}{2} MeV^2_{max}$

For PEE :

$\nu_{light} \geq \nu_0$ [Jyada hogi]

$\lambda_{light} \leq \lambda_0$ [kam hogi]

$(KE)_{max} = eV_0$

$V_0$ = Stopping Potential

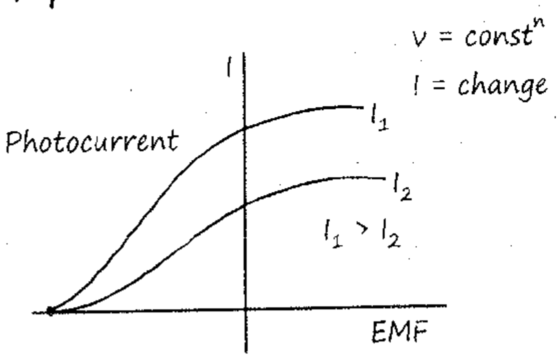
MR*

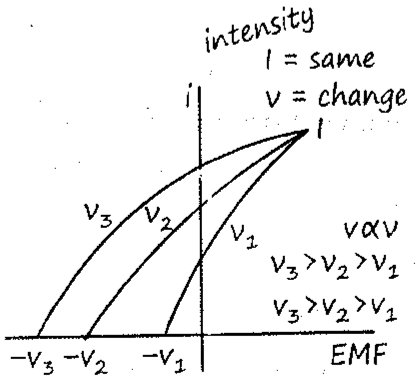
$\boxed{\text{Intensity} \propto \text{no. of Photon} \propto \text{no of electron} \propto \text{Photocurrent}}$

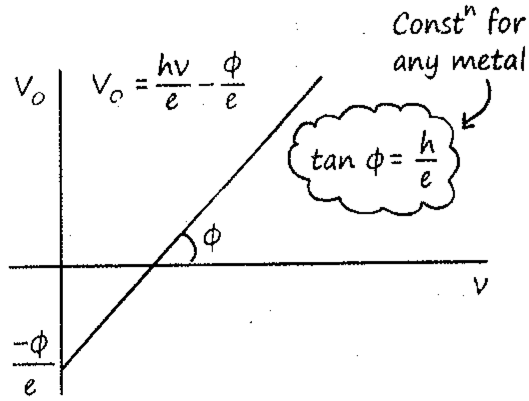
$\boxed{\text{Frequency} \propto \text{energy of Photon} \propto \text{energy of } e^- \text{ (K.E)} \propto \text{stopping potential}}$

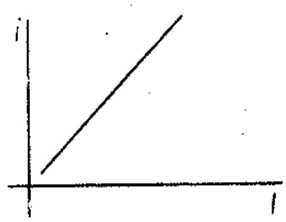
+ Photocurrent depends on Intensity not on frequency.
+ Stopping Potential depends on frequency not on Intensity

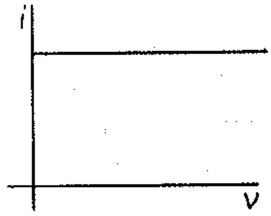
## 8. Graphs:

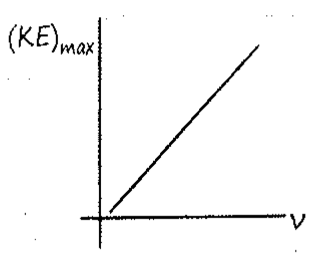


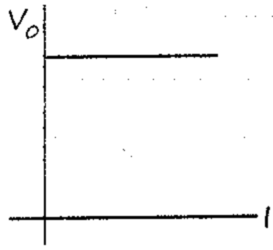



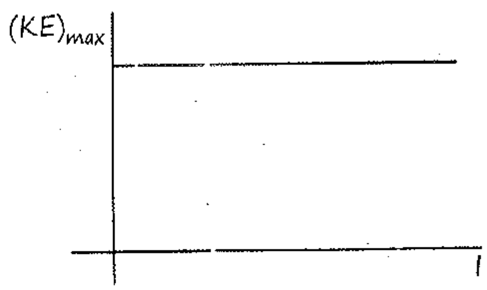



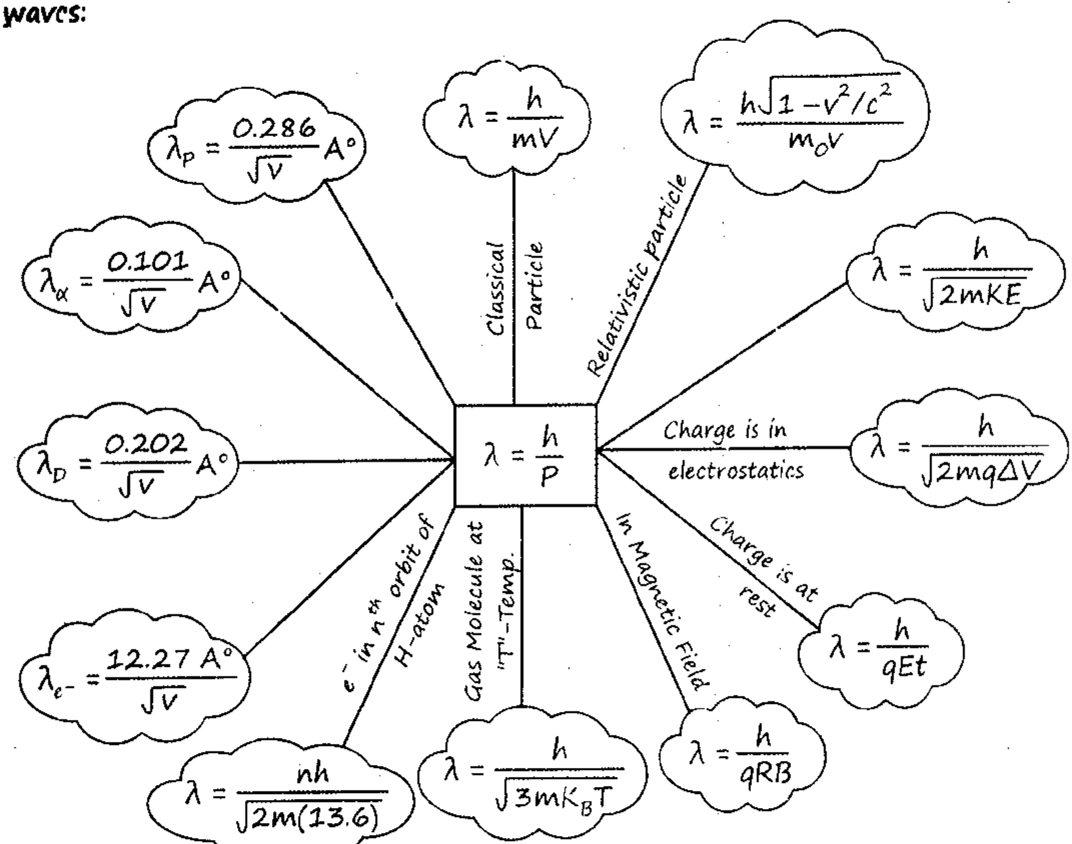




+ If distance b/w Source and plate becomes double then stopping potential remains same but photocurrent becomes one fourth.
+ If frequency becomes double then K.E of electrons becomes more than double.

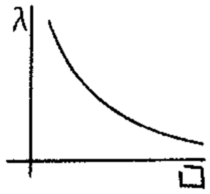
Q. Saturation Current and stopping Potential are $I_o$ and $V_o$ when frequency of light is 1.5 times of threshold frequency. Now if frequency becomes half then saturation current and stopping potential will?

**Ans. Both becomes zero, Photoelectric effect hoga hi Nahi**

**9. Matter waves:**




Vimp. $\lambda$ v/s $\square$ = Always Reactangular hyperbola
: $K_B = 1.38 \times 10^{-28}$




Wavelength of revolving $e^-$ for Bohr's orbit:

$n\lambda = 2\pi r$

**10. Compton Effect:**

E = 2ev  Visible Region

**Photoelectric Effect:**

E = 10eV  X-ray Region

$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{h}{m_eC}(1-\cos\phi)$, $\phi$ = scattering angle

$\frac{h}{m_eC} = 2.44$ pm

Pair Production: E = 1.02 MeV

**11. X-ray:**

(Neutral)  E = KeV

Intensity $\alpha$ Tube Current    Frequency $\alpha$ Tube Voltage

X-RAY

HARD: $\lambda = 1\ A^\circ$ (E↑)    SOFT: $\lambda = 4\ A^\circ$ (E↓)

CONTINUOUS (Retard$^n$ of $e^-$)    CHARACTERISTICS (Collision b/n accelerated $e^-$ and bonded $e^-$)

$$\lambda_o \propto \frac{1}{V_o} \text{ and } \lambda_o = \frac{hc}{\left(\frac{h^2}{\lambda^2(2m)}\right)} = \frac{\lambda^2 2mc}{h}$$

**Inverse Phenomena of Photoectric effect**

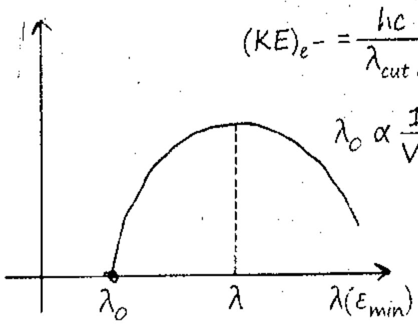
X-ray target atom → High atomic number, High melting Point, High Conductivity

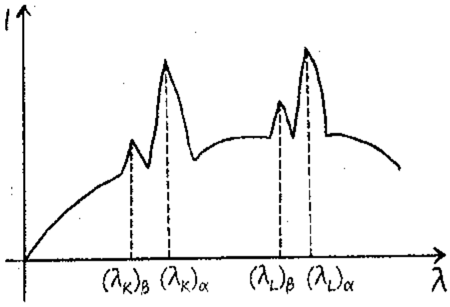
X-RAY

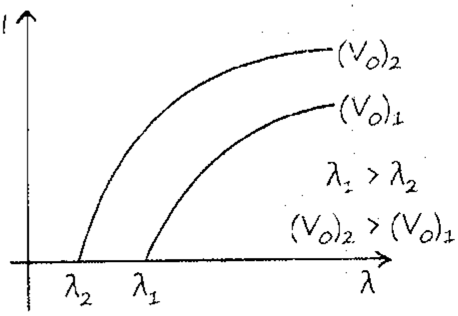
- CONTINUOUS (Retard$^n$ of $e^-$)
- CHARACTERISTICS (Collision b/n accelerated $e^-$ and bonded $e^-$)

$$(KE)_{e^-} = \frac{hc}{\lambda_{cut\ off}}$$

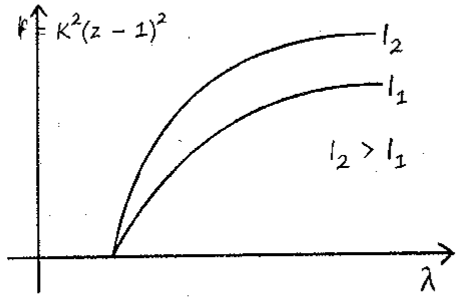
$$\lambda_0 \propto \frac{1}{V_0}$$




**For K-Series:**




Q. Wavelength is $\lambda$ for $K_\alpha$ line for atomic no $z = 43$ then find wavelength for $K_\alpha$ line for atomic no $z = 29$.

Sol. $\frac{\lambda}{\lambda^1} = \left(\frac{29-1}{43-1}\right)^2$

$\frac{\lambda}{\lambda^1} = \left(\frac{28}{42}\right)^2 = \left(\frac{2}{3}\right)^2$

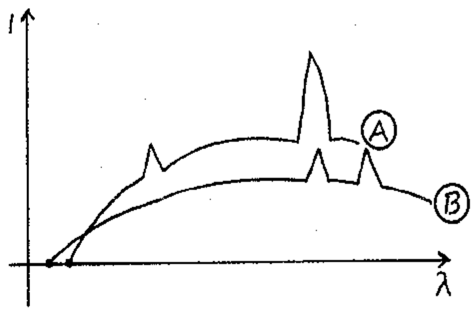
$\frac{\lambda}{\lambda^1} = \frac{4}{9}$ $\quad \lambda^1 = \frac{9}{4}\lambda$

**Heisenberg uncertainty principle:**

$\Delta x.\Delta p \geq \frac{h}{4\pi}$ $\qquad \hbar = \frac{h}{2\pi}$

$\Delta L.\Delta Q \geq \frac{\hbar}{2}$ $\qquad \Delta t.\Delta E \geq \frac{\hbar}{2}$

Graph is given for two different atom A and of atomic no. $Z_A$ and $Z_B$ at cut-off voltage $V_A$ and $V_B$




$V_A < V_B$ cut-off voltage

$Z_A > Z_B$ atomic no.

**MR***

"Kismat sath de ya na de lekin kabiliyat jarur sath deti hai."

# 33 Atoms

## Postulates of Thomson's atomic model :-

Postulate 1: An atom consists of a positively charged sphere with electrons embedded in it

Postulate 2: An atom as a whole is electrically neutral because the negative and positive charges are equal in magnitude

Thomson atomic model is compared to watermelon. Where he considered:

- Watermelon seeds as negatively charged particles
- The red part of the watermelon as positively charged

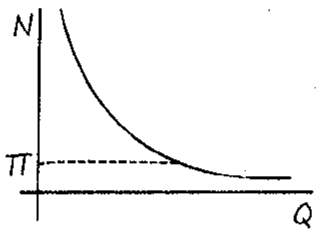
## Limitations of Thomson's atomic model :-

- It failed to explain the stability of an atom because his model of atom failed to explain how a positive charge holds the negatively charged electrons in an atom. Therefore, This theory also failed to account for the position of the nucleus in an atom
- Thomson's model failed to explain the scattering of alpha particles by thin metal foils
- No experimental evidence in its support

## Rutherford Alpha Particle Scattering Exp :

No. of Scattered α-Particle

$$N \propto \frac{1}{\sin^4(\theta/2)}$$




## Distance of closest approach

$$\underbrace{\frac{1}{2}mv^2}_{\text{bombarding Particle}} = \frac{Kq_1q_2}{r_0} = \frac{2KZe^2}{r_0}$$

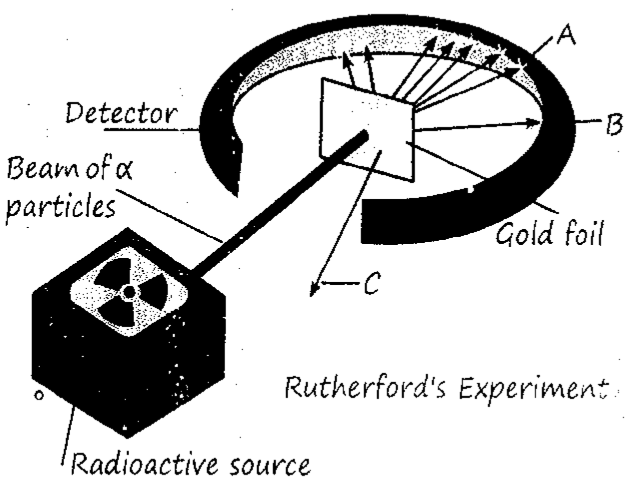
α-particle

**Impact Parameter (b) :**

$$\cot(\theta/2) = \frac{2b}{r_0}$$

$$b = \frac{2KZe^2}{\frac{1}{2}mv^2} \cdot \frac{\cot(\theta/2)}{2}$$

1. Most of the positively charged alpha particles went undeflected through the foil. This shows that most of the space in an atom is empty.
2. Few positively charged alpha particles deflected through small and large angles. This shows that there is presence of positive center in the atom. This positive center is known as nucleus.
3. Very few positively charged alpha particles bounced back. This is because the nucleus is very dense and does not allow the alpha particles to pass through it.
4. The volume occupied by the nucleus is negligible compared to the total volume of the atom. This shows that radius of atom is much higher than that of the nucleus.




Rutherford's Experiment

Q. What are the drawbacks of the rutherford atomic model?

Rutherford's atomic model failed to explain the stability of electrons in a circular path. He stated that electrons revolve around the nucleus in a circular path, but particles in motion would undergo acceleration and cause energy radiation. Eventually, electrons should lose energy and fall into the nucleus. But it never happens.

## Bohr's Atomic Model :

Postulate 1 : $\frac{mv^2}{r} = \frac{KZe^2}{r^2}$

Postulate 2 : $mvr = \frac{nh}{2\pi}$

Radius of $N^{th}$ Orbit :

$$r = \frac{\varepsilon_0 h^2}{\pi m e^2 z} n^2 = 0.53 \frac{n^2}{z}$$

Velocity of $N^{th}$ Orbit :

$$V_n = \frac{nh}{2\pi m r_n} = \frac{e^2}{2\varepsilon_0 h} \frac{z}{n}$$

$$V_n = 2.18 \times 10^6 \frac{z}{n} \text{ m/s}$$

Energies of $N^{th}$ orbit :

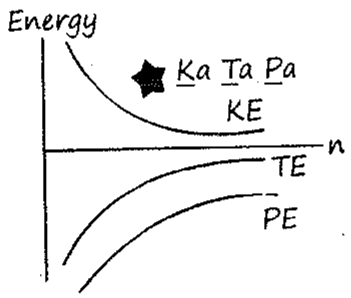
$$KE = \frac{1}{2} mVn^2 = \frac{me^4 z^2}{8\varepsilon_0^2 h^2 n^2}$$

Potential: Kinetic : Total energy $= -1 : \frac{1}{2} : -\frac{1}{2}$

$= -2 : 1 : -1$

* T.E. $= E_n = -13.6 \frac{z^2}{n^2}$ eV

* $V_1 = \frac{E_n}{e}$ = Volt




Special Relations :

- Time Period :

$$T = \frac{2\pi r}{V} \qquad T \propto \frac{n^3}{z^2}$$

- Acceleration :

$$a_c = \frac{v^2}{r} \qquad a_c \propto \frac{z^3}{n^4}$$

- Angular Velocity :

$\omega = 2\pi/T$

$\omega \propto z^2/n^3$

- Angular freq$^n$ :

$$w = 2\pi f \qquad f \propto \frac{z^2}{n^3}$$

- Current :

$$i = \frac{q}{t} \qquad i \propto \frac{z^2}{n^3}$$

- Magnetic dipole Moment :

$$M = i.A = \frac{eV}{2\pi r} . \pi r^2 = \frac{eVr}{2}$$

$M \propto n'$

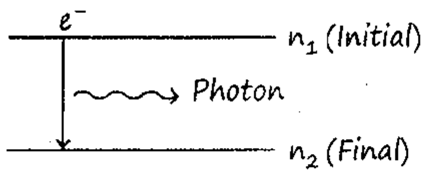
**Relation B/N Magnetic Moment & Angular Momentum :**

Magnetic Field $B \propto \frac{1}{n^5}$

$$M = \frac{eVr}{2} = \frac{eL}{2m} \; (L = mvr)$$

$$\vec{M} = \frac{-e}{2m} \vec{L}$$

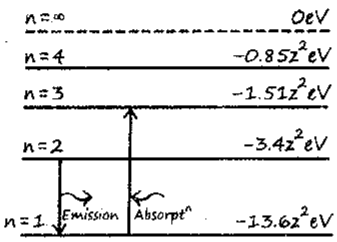
## Radiation Energy :




$$\bar{v} = \frac{1}{\lambda} = \frac{13.6\, z^2}{hc}\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right] \qquad R = 1/hc$$

$$* \quad R = \frac{me^4}{8\varepsilon_0^2 h^3 c} = 1.097 \times 10^7/m$$

## Energy Level Diagram :




* No. of Spectral Line :

$$N = \frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2} \quad \big(n_2 \to n_1\big)$$

$$N = \frac{n(n-1)}{2} \quad \big(n_2 \to G.S.\big)$$

## Recoiling of an atom :

* Momentum :

$$P = \frac{h}{\lambda} = Rz^2 h\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

* Recoil Energy :

$$E = \frac{P^2}{2m} = \frac{h^2}{2m\lambda^2}$$ m : mass of Recoil atom.

## H-Spectrum :

$$\lambda_{min} = E_{max} \big(n_2 = \infty \to n_1 = 1\big) \quad \left[\begin{matrix}\text{Limit} \\ \text{Line}\end{matrix}\right]$$

$$\lambda_{max} = E_{min} \big(n_2 = 2 \to n_1 = 1\big) \quad \left[\begin{matrix}1^{st} \\ \text{Line}\end{matrix}\right]$$

- **Lyman Series : [UV]**

$$\lambda_{min} = \boxed{\frac{1}{R} = 912\ A^\circ}$$

$$\lambda_{max} = \frac{4}{3R} = 1216\ A^\circ$$

- **Balmer Series : [Visible]**

$$\lambda_{min} = \frac{4}{R} = 3648\ A^\circ$$

$$\lambda_{max} = \frac{36}{5R} = 6565\ A^\circ$$

- **Paschen Series : [IR]**

$$\lambda_{min} = \frac{9}{R} = 8208\ A^\circ$$

$$\lambda_{max} = \frac{144}{7R} = 18761.1\ A^\circ$$

- **Brackelt Series : [IR]**

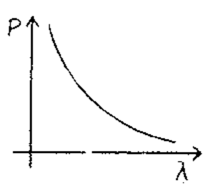
$$\lambda_{min} = \frac{16}{R} = 14592\ A^\circ$$

$$\lambda_{max} = \frac{400}{9R} = 40533\ A^\circ$$

- **Bohr's Quantum Condition from Debroglie Hypothesis :**

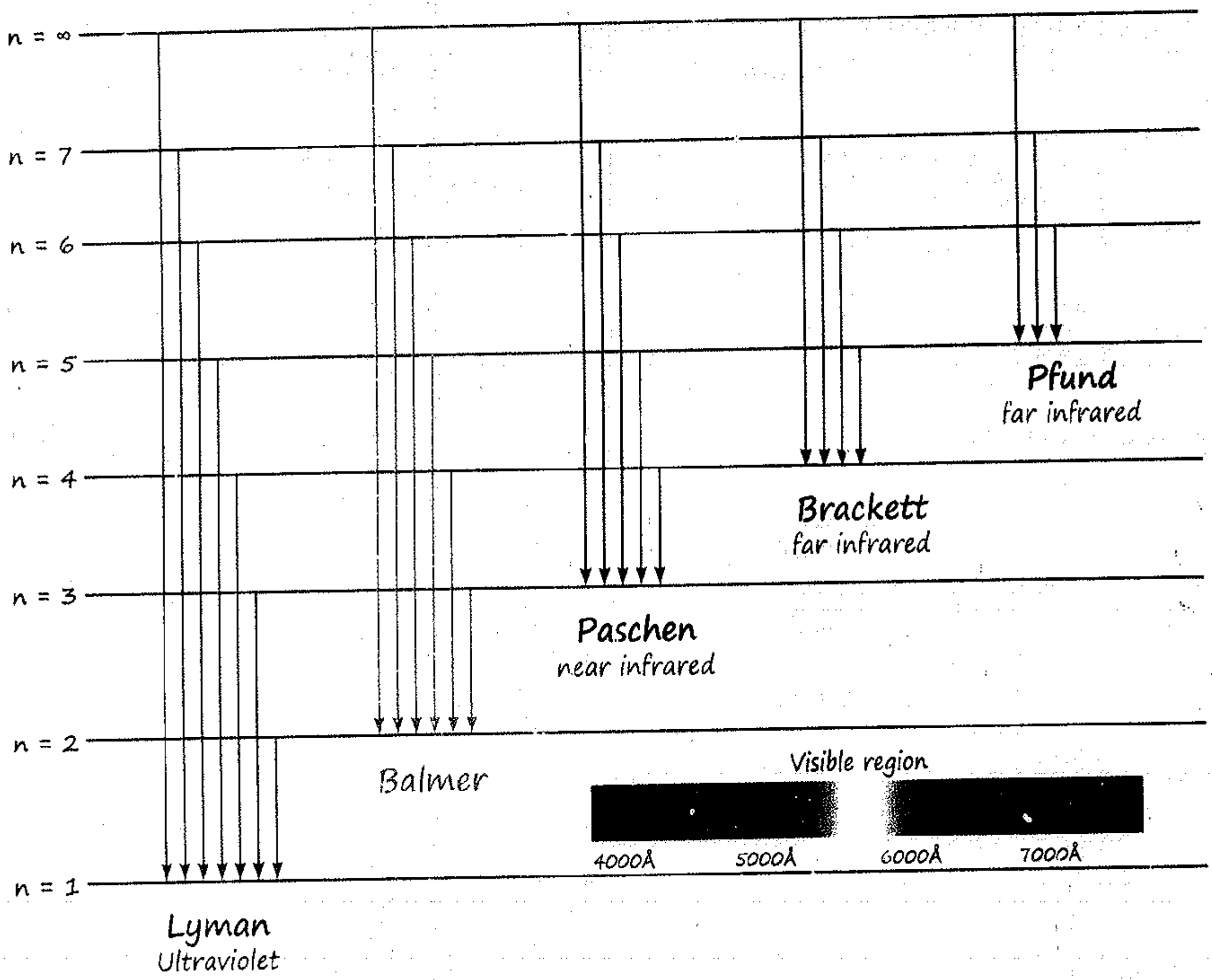
$$2\pi r = n\lambda = \frac{nh}{mv}$$

$$L = \frac{nh}{2\pi}$$




- When H-atom is raised from the ground state to $3^{rd}$ excited state then → potential energy increases and K.E. decreases.

## Spectral Series of Atom




**MR***

"Kaam karo aisa ki pehchan ban jaye...
chalo to aisa ki nishaan ban jaye..., are
zindagi to har koi kaat leta h yaha...., agar dam
hai to jiyo aise ki misaal ban jaye."

# 34 Nuclei

$m_p = 1.67 \times 10^{-27}$ Kg = 1.007 amu

$m_n = 1.67 \times 10^{-27}$ Kg = 1.0087 amu

$m_e = 9.1 \times 10^{-31}$ Kg

$m_\alpha > m_n > m_p > m_e$

**Rest Mass Energy :**

$E = mc^2$

$$1 \text{ amu} = \frac{931.5 \text{ MeV}}{C^2} = \frac{m_6C^{12}}{12}$$

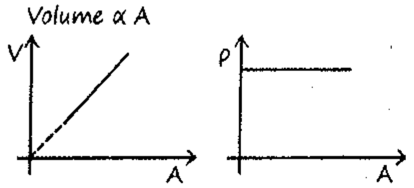
**Nuclear Size :**

$R = R_0 A^{1/3}$ $\quad R_0 = 1.2 f_m$

**Nuclear Density :**

$\rho = 10^{17}$ Kg/m$^3$ Constant

Volume $\alpha$ A



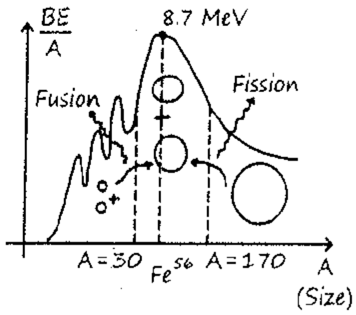

**Nuclear Binding Energy (Mass Defect) :**

$$\text{Mass defect} = \left(\begin{matrix}\text{Mass of all the}\\ \text{nucleons of nucleus}\end{matrix}\right) - \left(\begin{matrix}\text{Mass of}\\ \text{nucleus}\end{matrix}\right)$$

$$\Delta m = [ZM_p + (A-Z)M_n - M_{nucleus}]$$

$BE = \Delta mc^2$

Stability $\alpha \frac{B.E.}{A}$ (Binding energy per mass number)




- As mass number increases then 1st stability increase then decreases.
- Nucleons of lower mass number fuse for stability and release energy.
- Nucleons of higher mass number break (fission) for stability.

**Nuclear Force :**

→ Short range, Non-central, Non conservative

- Weak nuclear force → shortest range ($10^{-16}$m), repulsive, Mediated by Boson
- Strong nuclear force → Range $10^{-15}$m, attractive, mediated by Meson

  Always attractive.

  $F_{NN} = F_{PP} = F_{NP}$

  Range = 1 Fm

  $F_{Nuclear} = 100\ E_{electrostatics}$ !

+ Value :

  $Q = [BE_P - BE_R]$

  Q = +ve (Exo) $\quad$ Q = −ve(Endo)

  $BE_P \uparrow$ = Energy Release

  $Q = [M_{Reactant} - M_{Product}]\ c^2$

Q. Which is more stable $_4X^6$ and $_5Y^{12}$ having B.E. 24eV and 36eV respectively.

Sol. $\left(\frac{B.E.}{A}\right)_X = \frac{24}{6} = 4ev$

$\left(\frac{B.E.}{A}\right)_Y = \frac{36}{12} = 3ev$ (X is more stable than Y)

**Q.** $A \longrightarrow B+D$

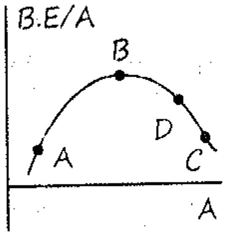
Energy released

$C \longrightarrow D+B$

Energy released

$2B \longrightarrow C$

Energy absorbed




Q. Value of reaction

$A+B \longrightarrow C+D+Q$

$Q = [(m_A+m_B) - (m_C+m_D)] \times c^2$

$Q = [B.E(C) + B.E(D)] - [(B.E(A) + B.E.(B)]$

- If $[m_A+m_B] > (m_C+m_D)$
- If $[B.E.(C+D)] > B.E.(A+B)$

$\longrightarrow$ energy released

- If $m_A+m_B < m_C+m_D$
- If $B.E.(C+D) < B.E.(A+B)$

$\longrightarrow$ energy absorbed

## Law of Radioactive Decay :

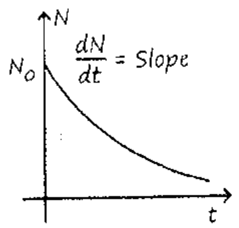
Number of nucleon becomes less and less but always some left.

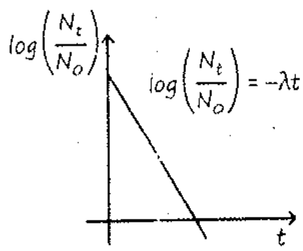
Rate of decay $\alpha$ remaining no. of nucleons

$$\frac{-dN}{dt} = \lambda N$$

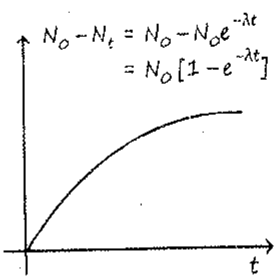
$\lambda$ = decay const$^n$ — Material prop.

$$N_t = N_0 e^{-\lambda t}$$




**No. of nucleons decayed :**



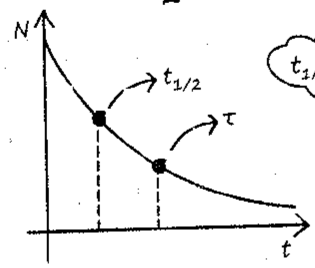

$$N_t = \frac{N_0}{(2)^n}$$

**Mean Life :**

$\tau = \frac{1}{\lambda}$ No. of nucleons becomes 37% of initial.

## Half-Life :

No. of Nucleons $\longrightarrow \frac{1}{2} N_0$ ; $t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$




$t_{1/2} = \ln 2\, \tau$

**Activity :**

(Rate of disintegrat$^n$ of a sample).

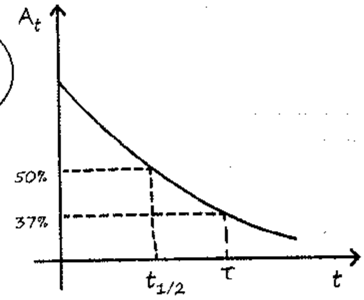
1Bq = 1 decay/s

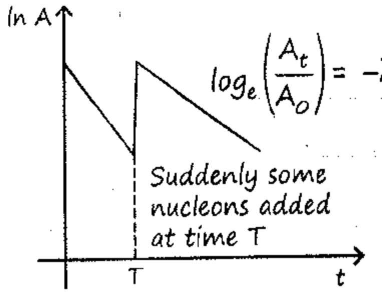
1 Rutherford = $10^6$ dps

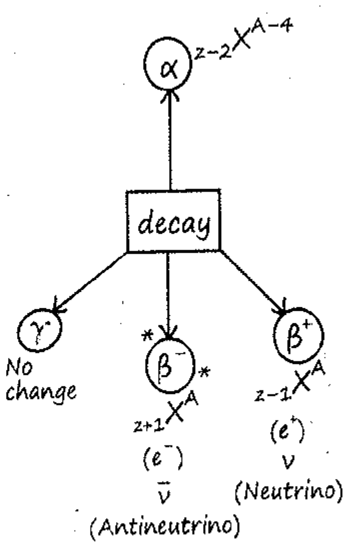
1 Curie = $3.7 \times 10^{10}$ Bq

$A_t = \lambda N_t$

$A_t = A_0 e^{-\lambda t}$



## $\alpha$-decay :

- Recoil velocity of daughter Nuclei :

  Q = Released energy in $\alpha$-decay

  A = Mass no. of original nuclei

$$\boxed{\vec{V}_D = \frac{-4\vec{V}_\alpha}{(A-4)}}$$

- $KE_\alpha = \left[\frac{A-4}{A}\right]Q$
- $KE_D = \left[\frac{4Q}{A}\right]$

## $\beta$-decay :

- Q-value in terms of Atomic Mass :
- $\beta^- : Q = [M_R - M_P]c^2$ } same!
- $\beta^+ : Q = [M_R - M_P - 2M_e]\,c^2$

## Parallel Disintegration :

In parallel disintegration

$$\lambda_{eq} = \lambda_1 + \lambda_2$$

$$(ty_2) = \frac{t_1 t_2}{t_1 + t_2}$$

$$A \xrightarrow{(t_{1/2})_1} B \qquad A \xrightarrow{(t_{1/2})_2} C$$

$$(t_{1/2})_{eq} = \frac{t_1 t_2}{t_1 + t_2}$$

**–: Nuclear Bomb :–**

| H-bomb | Atom Bomb |
|---|---|
| (Fusion) | (Fission) |
| $\rightarrow E = 26.7$ MeV | $E = 200$ MeV |
| Uncontrolled | Controlled! |
| Chain $rx^n$ | |

**–: Nuclear Reactor :–**

$n = 2.5$ nucleons/fission

$$K = \frac{\text{Rate of Prod}^n \text{ of nucleon}}{\text{Rate of loss of neutron}}$$

| | |
|---|---|
| $K = 1$ | Critical |
| $K < 1$ | Controlled |
| $K > 1$ | Uncontrolled |

**–: Power of Reactor :–**

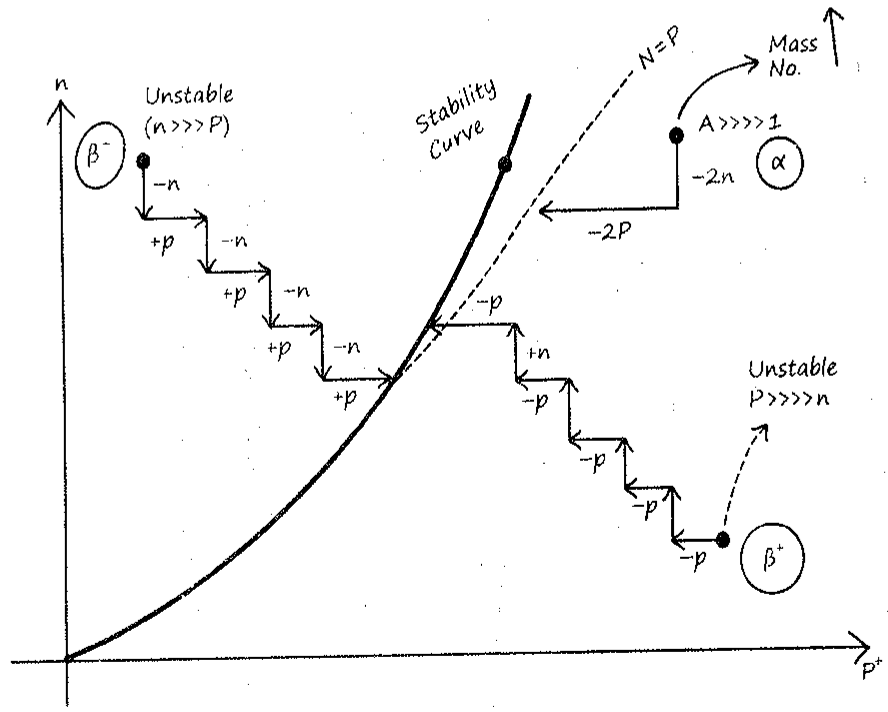
$$P = \frac{nE}{t} = \frac{nMc^2}{t}$$

MR. ★★★

| Time | No. of nuclei at "t" $N_t$ (Remaining) | No. of dis-integrated Nuclei $(N_o - N_t)$ | $\frac{N_t}{N_o - N_t}$ $\left[\frac{\text{Remain}}{\text{decay}}\right]$ | $\frac{N_o}{N_t}$ $\left[\frac{\text{initial}}{\text{remain}}\right]$ | $\frac{N_o}{N_o - N_t}$ $\left[\frac{\text{initial}}{\text{decay}}\right]$ |
|---|---|---|---|---|---|
| $t = 0$ | $N_o$ | 0 | – | – | – |
| $t = 1_{t/2}$ | $N_o/2$ | $N_o/2$ | 1 : 1 | 2 : 1 | 2 : 1 |
| $t = 2_{t/2}$ | $N_o/4$ | $3N_o/4$ | 1 : 3 | 4 : 1 | 4 : 3 |
| $t = 3_{t/2}$ | $N_o/8$ | $7N_o/8$ | 1 : 7 | 8 : 1 | 8 : 7 |
| $t = 4_{t/2}$ | $N_o/16$ | $15N_o/16$ | 1 : 15 | 16 : 1 | 16 : 15 |
| | | $= N_o - \frac{N_o}{2^n}$ | $(1 : 2^n - 1)$ | $(2^n : 1)$ | |

$$nt_{1/2} - \frac{N_o}{2^n} = \frac{N_o}{2^{t/T}}$$

- Neutron v/s Proton graph :

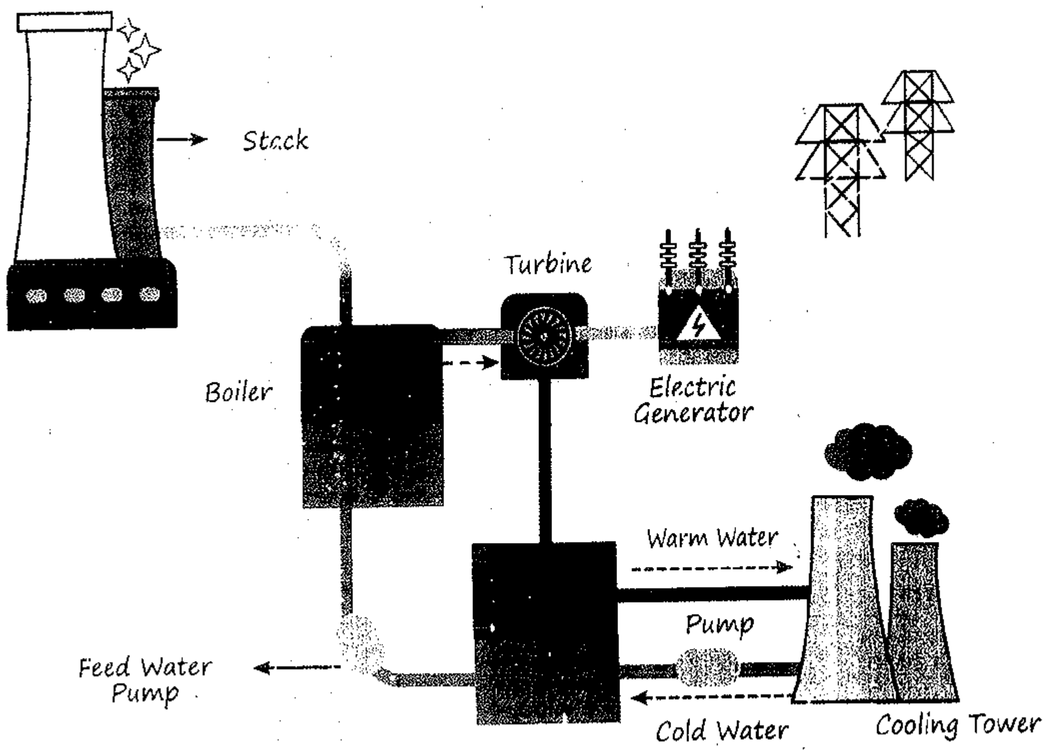



Condition of fusion

1. High temperature = $10^7$K

2. High Pressure

# Thermal Power Plant



## IMP PYQ

Q. Radioactive material A has decay constant 8λ and material B has decay constant λ. Initially, they have same number of nuclei. After what time, the ratio of number of nuclei of material B to that A will be $\frac{1}{e}$?

A ⟶ $N_o$ $\quad N^{+} = N_0 e^{-8\lambda t}$ $\quad 1 = -7\lambda t$

B ⟶ $No$ $\quad N^{-} = N_0 e^{-\lambda t}$ $\quad t = \frac{1}{7\lambda}$

$e = e^{-8\lambda t + \lambda t}$

$e = e^{-7\lambda t}$

Q. A radioactive nucleus of mass M emits a photon of frequency v and the nucleus recoils. The recoil energy will be

(a) $h^2v^2/2\,Mc^2$

(b) zero

(c) hv

(d) $Mc^2 - hv$

**Ans. (a)**

Momentum of a photon

$$p = \frac{hv}{c}$$

Hence, recoil energy, $E = \frac{p^2}{2M}$

$$\therefore \quad E = \frac{\left(\frac{hv}{c}\right)^2}{2M} \quad \text{or} \quad E = \frac{h^2v^2}{2Mc^2}$$

Q. The activity of a radioactive sample is measured as $N_0$ counts per minute at t = 0 and $N_0/e$ counts per minute at t = 5 min. The time (in minute) at which the activity reduces to half its value is

(a) $\log_e 2/5$ (b) $\frac{5}{\log_e 2}$

(c) $5 \log_{10} 2$ (d) $5 \log_e 2$

**Ans. (d)**

Fraction remains after n half lives

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T}$$

$$\text{Given } N = \frac{N_0}{e} \Rightarrow \frac{N_0}{eN_0} = \left(\frac{1}{2}\right)^{5/T}$$

$$\text{or } \frac{1}{e} = \left(\frac{1}{2}\right)^{5/T}$$

Taking log on both sides, we get

$$\log 1 - \log e = \frac{5}{T}\log\frac{1}{2}\,; \quad -1 = \frac{5}{T}(-\log 2)$$

$$\Rightarrow T = 5\log_e 2$$

Now, let t′ be the time after which activity reduces to half

$$\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{t'/5\log_e 2} \quad \Rightarrow \quad t' = 5\log_e 2$$

Q. Two radioactive materials $X_1$ and $X_2$ have decay constants 5λ and λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of $X_1$ to that of $X_2$ will be $\frac{1}{e}$ after a time

Save Time Trick $t = \frac{\text{Power of}}{\Delta\lambda}$

(a) λ (b) $\frac{1}{2}\lambda$

✓~~(c)~~ $\frac{1}{4\lambda}$ (d) $\frac{e}{\lambda}$

Q. In compound $X(n, \alpha) \rightarrow {}_3Li^7$, the element X is

(a) ${}_2He^4$ (b) ${}_5B^{10}$

(c) ${}_5B^9$ (d) ${}_4Be^{11}$

**Ans. (b)**

The given nuclear reaction can be written as

$${}_ZX^A + {}_0n^1 \longrightarrow {}_3Li^7 + {}_2He^4$$

Conservation of mass number gives,

$$A + 1 = 7 + 4 \Rightarrow A = 10$$

Conservation of charge number/Atomic No. gives,

$$Z + 0 = 2 + 3 \Rightarrow Z = 5$$

Hence, Z = 5, A = 10 corresponds to boron (${}_5B^{10}$).

Q Atomic weight of boron is 10.81 and it has two isotopes ${}^{10}_5B$ and ${}^{11}_5B$. Then, the ratio of atoms of ${}^{10}_5B$ and ${}^{11}_5B$ in nature would be

(a) 19 : 81 (b) 10 : 11

(c) 15 : 16 (d) 81 : 19

**Ans. (a)**

Let $n_1$ and $n_2$ be the number of atoms in ${}^{10}_5B$ and ${}^{11}_5B$ isotopes.

Atomic weight

$$= \frac{n_1 \times (\text{At. wt. of } {}^{10}_{5}B) + n_2 \times (\text{At. wt. of } {}^{11}_{5}B)}{n_1 + n_2}$$

or $10.81 = \dfrac{n_1 \times 10 + n_2 \times 11}{n_1 + n_2}$

or $10.81\, n_1 + 10.81\, n_2 = 10\, n_1 + 11\, n_2$

or $0.81\, n_1 = 0.19\, n_2$

or $\dfrac{n_1}{n_2} = \dfrac{0.19}{0.81} = \dfrac{19}{81}$

Q. The activity of a radioactive sample is measured as 9750 counts/min at $t = 0$ and as 975 counts/min at $t = 5$ min. The decay constant is approximately

(a) 0.922/min (b) 0.691.min

(c) 0.461/min (d) 0.230/min

Ans. (c)

According to law of radioactivity

$$\frac{N}{N_0} = e^{-\lambda t} \quad \ldots(i)$$

$$\Rightarrow \frac{N_0}{N} = e^{\lambda t}$$

[N = final concentration
$N_0$ = initial concentration
$\lambda$ = decay constant]

Taking logarithm on both sides of Eq. (i), we have

$$\log_e\left(\frac{N_0}{N}\right) = \log_e (e^{\lambda t}) = \lambda t \log_e e = \lambda t$$

As we know that, $\log_e x = 2.3026 \log_{10} x$

Making substitution, we get

$$\lambda = \frac{2.3026 \log_{10}\left(\frac{9750}{975}\right)}{5}$$

[$\because N_0 = 9750$ counts/min and $N = 975$ counts/min]

$$= \frac{2.3026}{5} \log_{10} 10 = \frac{2.3026}{5}\ \text{min}^{-1}$$

$$= 0.461\ \text{min}^{-1}$$

Q. Nuclear fission can be explained by

(a) proton-proton cycle

(b) liquid drop model of nucleus

(c) independent of nuclear particle model

(d) nuclear shell model

Ans. (b)

Q. Which of the following is used as a moderator in nuclear reactors ?

(a) Plutonium

(b) Cadmium

(c) Heavy water $D_2O$

(d) Uranium

Ans. (c)

Q. Energy released in the fission of a single ${}_{92}U^{235}$ nucleus is 200 MeV. The fission rate of a ${}_{92}U^{235}$ filled reactor operating at a power level of 5 W is

(a) $1.56 \times 10^{-10}\ s^{-1}$

(b) $1.56 \times 10^{11}\ s^{-1}$

(c) $1.56 \times 10^{-16}\ s^{-1}$

(d) $1.56 \times 10^{-17}\ s^{-1}$

Ans. (b)

$$P = nE/t \quad \left(\frac{n}{t} = P/E\right)$$

$$\text{Fission rate} = \frac{\text{total nuclear power}}{\text{energy produced/fission}}$$

Here, total nuclear power = 5W

Energy released per fission = 200 MeV

$$\therefore \text{ Fission rate} = \frac{5}{200 \text{ MeV}}$$

$$= \frac{5}{200 \times 1.6 \times 10^{-13}} \quad [\because 1 \text{ MeV J}]$$

$$= 1.56 \times 10^{11}\ s^{-1}$$

MR*

"Parinde ruk mat, tujhme jaan baki hai,
manjil dur hai abhi, bahut udan baki hai"

# 35 Semiconductor

## Types of Material :

1. **Conductor :**

   CB & VB Overlap

   $\sigma = 10^2$ to $10^8$ $Sm^{-1}$

2. **Semi-Conductor :**

   $E_g < 3eV$ Si & Ge

   $\sigma = 10^5$ to $10^{-6}$ $Sm^{-1}$

   $E_g = (E_{CB})_{min} - (E_{VB})_{max}$

   At 0 k current can flow through conductor but can not flow through semi-conductor.

3. **Insulators :**

   $E_g \geq 3eV$

   $\sigma = 10^{-11}$ to $10^{-19}$ $Sm^{-1}$

4. **Intrinsic Semi-Conduct :**

   [Pure] $\longrightarrow$ Si & Ge

   At T = 0 K insulator.

## Electric Current in Intrinsic Semi-Conductor :

- **Intrinsic Semi Conductor :**

  $n_e = n_h = n$ = no. of intrinsic charge density

  $\vec{J} = \sigma\vec{E}$

  $J = en(\mu_e + \mu_h)E$ ; $i = i_e + i_h$

  $\sigma = en(\mu_e + \mu_h)$

  $i_e > i_h$ $\mu_e > \mu_h$

  $\mu = \dfrac{e\tau}{m}$ $(M_{eff})_{hole} > (M_{eff})_{e^-}$

* Condition for intrinsic Semi-Conductor :

$n_i = n_e = n_h$ ; $\sigma = en_i(\mu_e + \mu_h)$

- **Extrinsic Semi-Conductor :**

  intrinsic S.C. + impurities = E.S.C

Impurities

| Trivalent | Pentavalent |
|---|---|
| P-type (+) | N-type (–) |
| $n_h >>> n_e$ | $n_e >>> n_h$ |
| "B-Family" | "N-Family" |
| Neutral | Neutral |
| $n_h \geq N_A$ | $n_{e^-} \geq N_D$ |
| $N_A$ = no. of acceptor ion density | $N_D$ = no. of donor ion density |
| Fixed –ve ion | Fixed +ve ion |
| Acceptor 1e⁻ accept udhar lega Ge/Si se phir bond bnayga | Donor 1e⁻ donate kar ke Ge/Si se bond bna lega |

## Law of Mass Action :

$$n_i^2 = n_e . n_h$$

N-Type — P-Type

$n_e \approx N_D$ (Donar) $\quad$ $n_h \simeq N_A$ (Acceptor)

$$n_h = \frac{n_i^2}{n_e} = \frac{n_i^2}{N_D} \qquad n_e = \frac{n_i^2}{n_h} = \frac{n_i^2}{N_A}$$

Q. In a semiconductor, the number density of intrinsic charge carriers at 27°C is $1.5 \times 10^{16}$ $m^{-3}$. If the semiconductor is doped with impurity atom, the hole density increases to $4.5 \times 10^{22}$ $m^{-3}$. The electron density in the doped semiconductor is

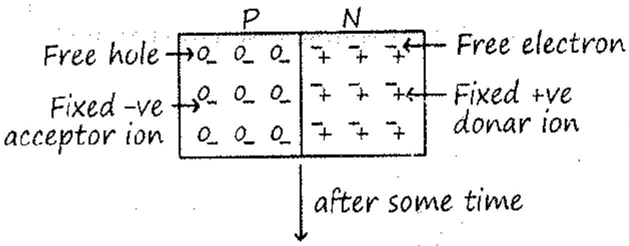
Ans. $n_e n_h = n_i^2$

$$\Rightarrow n_e = \frac{n_i^2}{n_h} = \frac{(1.5 \times 10^{16})^2}{4.5 \times 10^{22}}$$

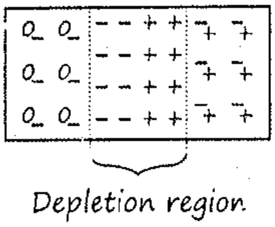
$$\Rightarrow n_e = \frac{1.5 \times 1.5 \times 10^{32}}{4.5 \times 10^{22}}$$

$$\Rightarrow n_e = 5 \times 10^9 \; m^{-3}$$

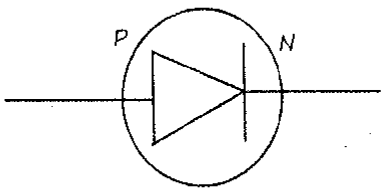
## P-N Junction Diode :



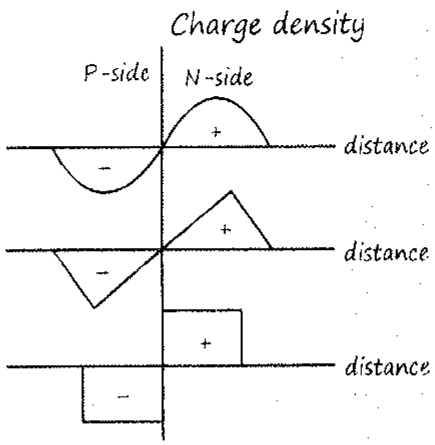

Diffusion current from P to N due to {some electron diffuse from N to P side & some hole diffuse from P to N side}

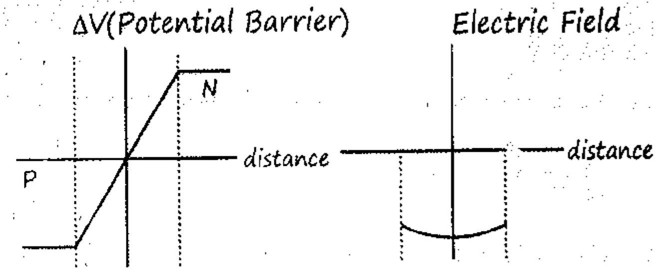



Potential barrier ⟶ Formed due to immobile fixed acceptor and donor ion




- Electric Field formed from N to P side within Depletion region :
- Width of depletion $\propto \dfrac{1}{\text{doping level}} \propto \text{Temp}^r$
- Drift current now from N to P due to electric field
- At equilibrium $i_{Drift} = i_{Diffusion}$
- **Graphs :**




- **Knee Voltage : ($V_B$)**

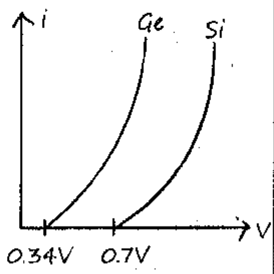
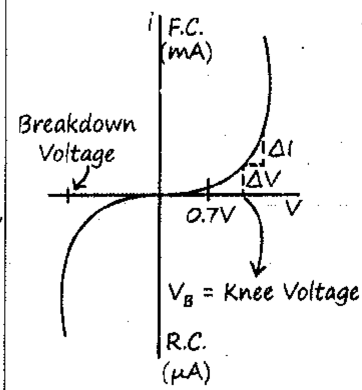
– $V_{min.}$ to flow $e^-$ & hole pairs

– "V" above wiz. current rises rapidly.

Si = 0.7 V = 0.7 eV

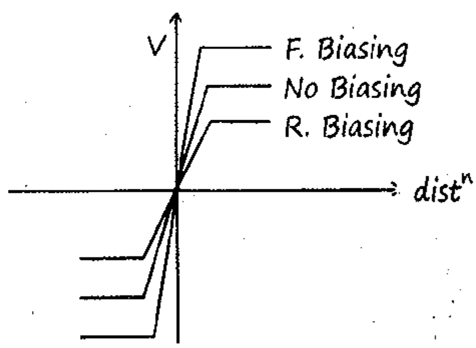
Ge = 0.34 V = 0.34 eV

- **Biasing :**

| Forward | Reverse |
|---|---|
| $i = ✓$ | $i = 0$ |
| P ⟶ N<br>High Low | P ⟶ N<br>Low High |
|  |  |
| | $R_B = \dfrac{\Delta V}{\Delta I}$ |
| | Depl$^n$ layer width "0.5μm to 1μm" |



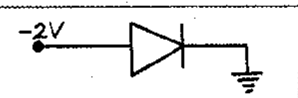
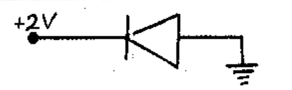
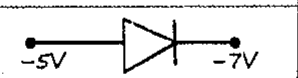
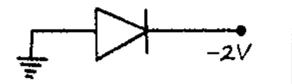
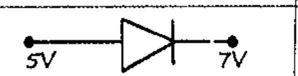


- 

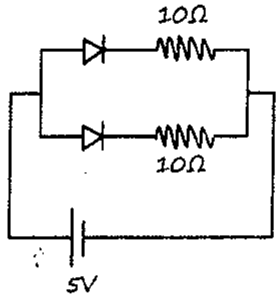
- **MR Feel Table :**

| Biasing | Electric Field | Potential $diff^n$ | Width of Depletion | $i_{Diffusion}$ | $i_{Drift}$ |
|---|---|---|---|---|---|
| Forward | ↓ | ↓ | ↓ | ↑ | ↓ |
| Reverse | ↑ | ↑ | ↑ | ↓ | ↑ |

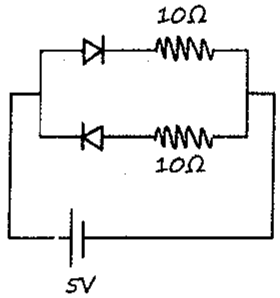
| **Zener Diode** | **Avalanche Breakdown** |
|---|---|
| In high doped semi-C | In low doped semi-C |
| In reverse bias as V↑ the $e^-$s hole become free due to breaking of co-valent bond | In revese bias at vary high voltage |
| Reversible | Not reversible |

| Diode | Biasing |
|---|---|
|  | R.B. |
|  | R.B. |
|  | F.B. |
|  | F.B. |
|  | R.B. |

Q. Find the current through the battery in each of the circuits shown in figure.




(i)




(ii)

Ans. In fig. (i) Both diodes are forward biased. Thus the net diode resistance is O.

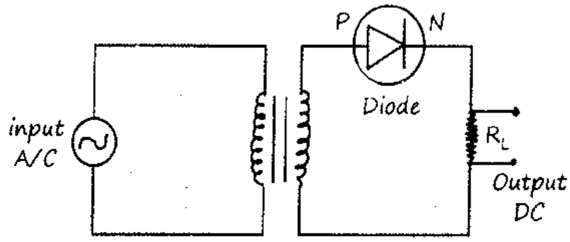
$$i = \frac{5}{(10+10)/10\cdot 10} = \frac{5}{5} = 1A$$

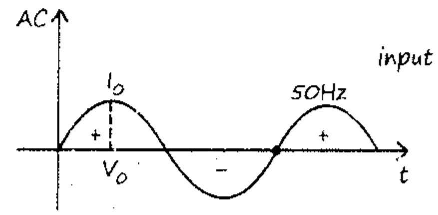
In fig (ii) One diode is forward biased and other is reverse biased. Current passes through the forward biased diode only.

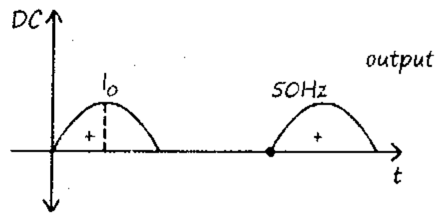
$$i = \frac{V}{R_{net}} = \frac{5}{10+0} = 0.5A$$

## Rectifier :

1. **Half-wave rectifier :** (1 Diode use)




$$i_{DC} = \frac{i_o}{\pi} \qquad V_{DC} = \frac{V_o}{\pi}$$

$$i_{rms} = \frac{i_o}{2} \qquad V_{rms} = \frac{V_o}{2}$$

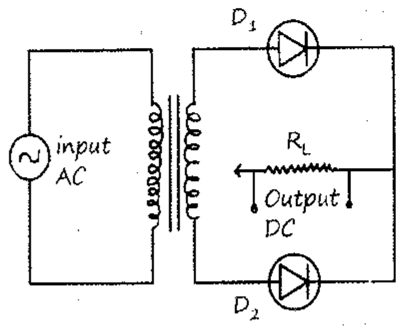
$$\eta = \frac{P_o}{P_i} \times 100 \qquad P = iV = \frac{V^2}{R}$$

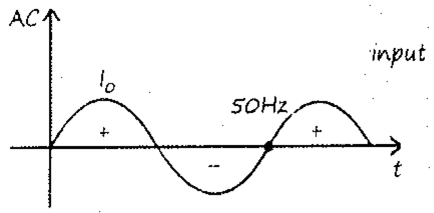
Ripple Factors : $Rectific^n$ Ke बाद Rehne wala AC.

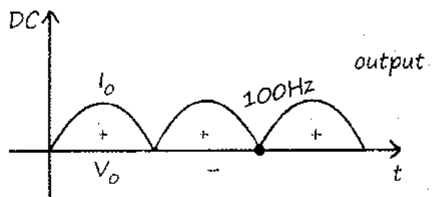
$$r = \frac{i_{AC}}{i_{DC}} = 1.21 \qquad \eta \neq 100\%$$

2. ***Full wave rectifier :***

- ***Center tapped rectifier : (2-Diode use)***




Ripple Factor = 2x $f_{input}$

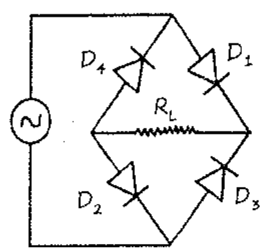
$\eta = P_o/P_i \times 100$

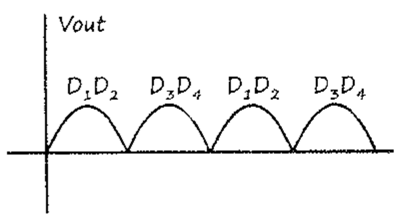
$i_{avg} = \frac{2i_0}{\pi}$ $\quad V_{avg} = \frac{2V_0}{\pi}$

$i_{rms} = \frac{i_0}{\sqrt{2}}$ $\quad V_{rms} = \frac{V_0}{\sqrt{2}}$

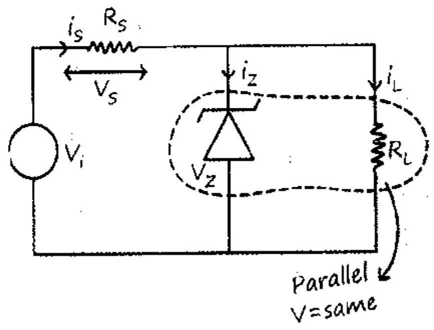
- When capacitor is connected parallel with load resistance then output voltage remains constant.

- ***Bridge rectifier : (4-Diode use)***




## Zener Diode as a Regulator :



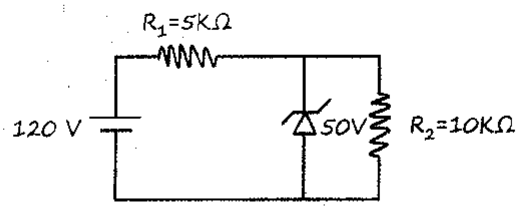

$i_S = i_Z + i_L$ $\qquad V_Z = i_L R_L$

$i_S = \frac{V_S}{R_S}$ $\qquad V_i = V_S + V_Z$

$V_{out} = V_Z = i_L R_L =$ Constant

When diode is working.

Q. For the circuit shown below, the current through the zener diode is:




Ans. Assuming Z.D. does not undergo breakdown,

current in circuit = $\frac{120}{15000}$ = 8mA

Voltage drop across diode = 80V > 50 V.

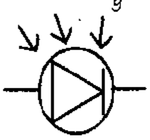
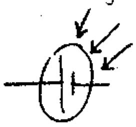
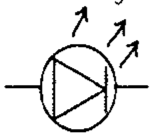
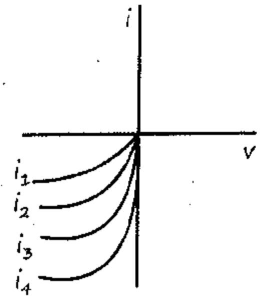
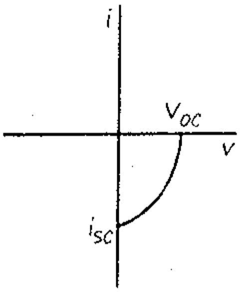
The diode undergo breakdown.

Current in $R_1 = \frac{70}{5000}$ = 14 mA

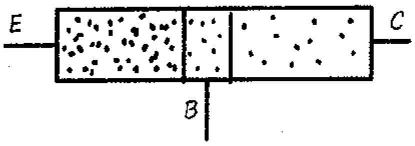
Current in $R_2 = \frac{50}{10000}$ = 5 mA

Current through diode = 9 mA

## Application of Diodes :

| Photodiode | Solar Cell | LED |
|---|---|---|
| $h\nu > E_g$ | $h\nu < E_g$ | $h\nu \le E_g$ |
|  |  |  |
| Reverse Biased | No Biasing | Forward Biasing |
|   |   |   |
| $i_4 > i_3 > i_2 > i_1$<br>Intensity ↑ Photo Current ↑<br>• Act as Light Sensor | | Ga – As = Infrared<br>Ga – As – P → Red, Yellow<br>Ga, P → Red, Green |

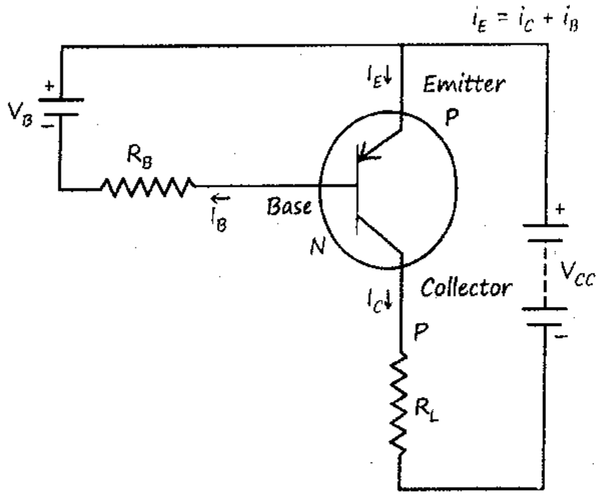
## Transistor :




Dopping : E > C > B

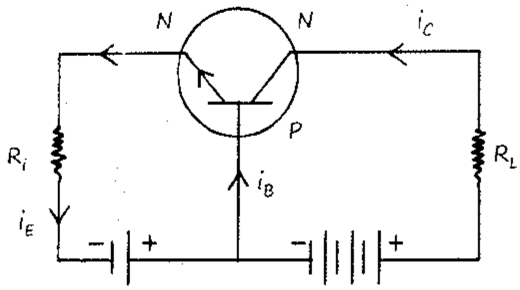
Size : C > E > B

**1. P-N-P Transistor :**



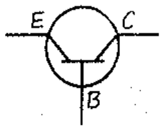

**2. N-P-N Transistor :**

$i_E = i_C + i_B$

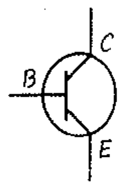



## Circuit Configuration :

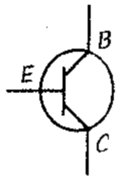
**1. CB Configuration :**




**2. CE Configuration :**




**3. CC Configuration :**

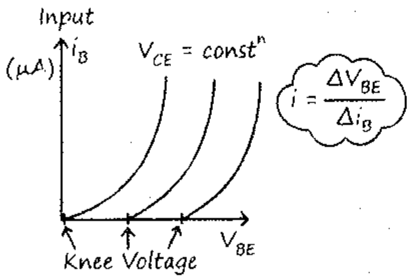


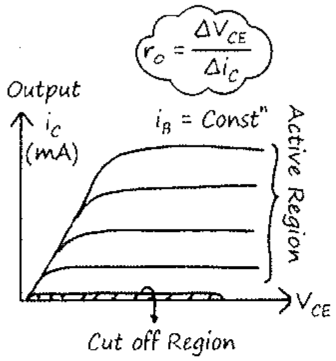
## Applications of Transistor :

- **Relation b/n $\alpha$ & $\beta$ :**

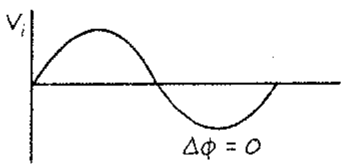
$$\alpha = \frac{\beta}{1+\beta} \; ; \; \beta = \frac{\alpha}{1-\alpha}$$

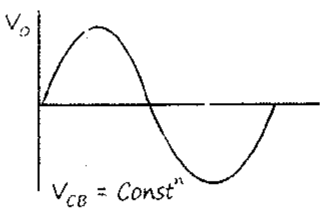
- **Transistor Chr. of CE Configuration :**




## Transistor as CE Amplifier :




**(a) Current Gain ($\alpha$) :**

$$\alpha_{DC} = \frac{I_C}{I_E} \qquad \alpha_{AC} = \frac{\Delta I_C}{\Delta I_E}$$

$0 < \alpha < 1$

$\alpha_{max} = 0.98$

**(b) Resistonce Gain :** $\frac{R_L}{R_i}$

**(c) Voltage Gain :** $(A_V)$

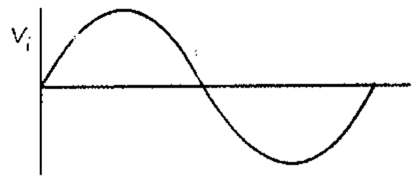
$$\frac{V_o}{V_{in}} = A_V = \frac{I_C R_L}{I_E R_i} = \alpha \times R_{gain}$$

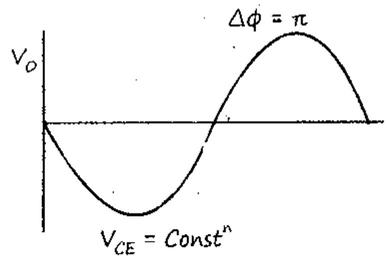
**(d) Power Gain :**

$$P_{gain} = \frac{I_C^2 R_L}{I_E^2 R_i} = \alpha^2 \times R_{gain}$$

V.imp

## Transistor as a CB Amplifier :




**(a) Current Gain ($\beta$) :**

$$\beta_{DC} = \frac{I_C}{I_B} \qquad \beta_{AC} = \frac{\Delta I_C}{\Delta I_B}$$

$\beta$ = 50 to 500 ; $\beta >>>> 1$

**(b) Resistance Gain :** $\frac{R_L}{R_i}$

**(c) Voltage Gain :** $(A_V)$

$$A_V = \frac{I_C R_L}{I_E R_i}$$

$= \beta \times R_{gain}$

**(d) Power Gain :**

$$P_{gain} = \frac{I_C^2 R_L}{I_B^2 R_i}$$

$= \beta^2 \times R_{gain}$

## Transistor as a Switch :

**Input :**

$V_{BB} = V_{BE} + I_B R_i$

When input = 0 Output = 0

i.e. OFF

$V_i = 0$ $\quad I_B = 0$ $\quad \boxed{V_{BB} = V_{BE}}$

**Output :**

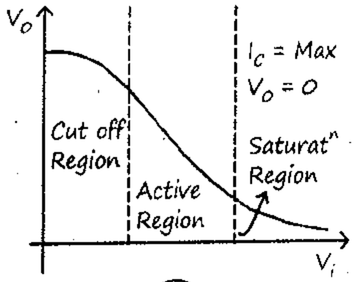
$V_{CE} = V_{CC} - I_C R_L$

$i_c \uparrow$ at one stage $I_C R_L \simeq V_{CC}$

$V_{CE} = V_O = \text{zero}$

i.e. ($I_C$) Current is Max!

## Transfer Characteristics Curve :




Transistors Uses

Switch ⟶ Cut off / Saturⁿ Region

Amplifier ⟶ Active

Oscillator ⟶ Active

## Logic Gates :

MR and Ramlal ne bank me joint account open kiya, dono ko different atm password mila. Atm me dono ka password match hone ke bad paisa milega then atm me Koin sa gate use huaa hai..?

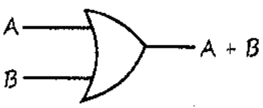
Ans. AND gate

MR audi car se ja raha hai, Ramlal apna truck le ke nikla MR ko takkar marne, Jo aage ya piche khi se takkar mar skta hai, air bag open karne ke liya car me koin sa gate use hoga ....?

Ans. OR gate

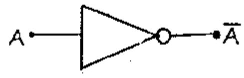
- **Fundamental Gates :**

(1) OR-Gate :




(2) AND-Gate :


(3) NOT-Gate :




Single output

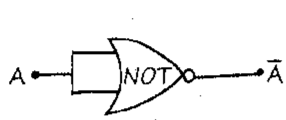
Single input

- **Universal Gate :**

(1) NAND-Gate :

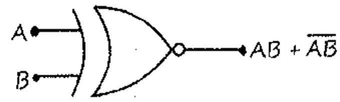


(2) NOR-Gate :

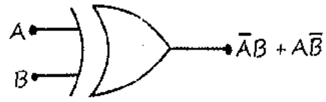




- **Exclusive Gates :**

(1) XNOR :



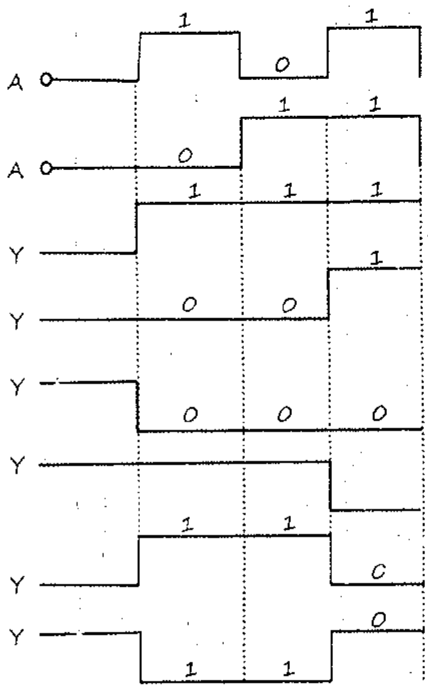

(2) XOR :




- **Truth table for all gate**

| A | B | OR | NOR | AND | NAND | X-OR | X-NOR |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |

- **Time Scale for different gate**




- **Basic Boolean Exp :**

| | |
|---|---|
| $0 + A = A$ | $0.A = 0$ |
| $1 + A = 1$ | $A.A = A$ |
| $A + A = A$ | $1.A = A$ |
| $A + \overline{A} = 1$ | $A.\overline{A} = 0$ |
| | $\overline{\overline{A}} = A$ |

- **De Morgan Principle :**

$$\overline{A + B} = \overline{A}\,\overline{B}$$

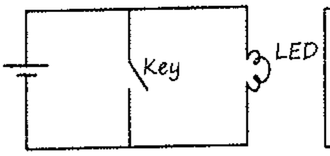
$$\overline{AB} = \overline{A} + \overline{B}$$

- **Special Case :**

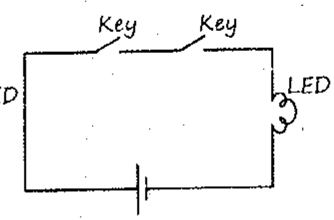
$$\overline{\overline{A} + \overline{B}} = AB$$

$$\overline{\overline{A}\,\overline{B}} = A + B$$

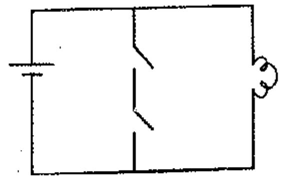
- **Electrical equivalent circuit :**



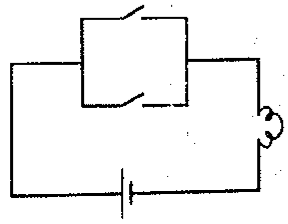

(NOT gate)



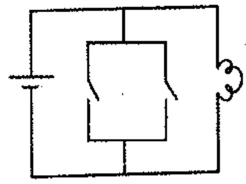

(AND gate)



(NAND gate)



(OR gate)



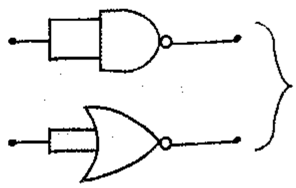
(OR gate)

- **Formation of Different gates using NAND gate:**

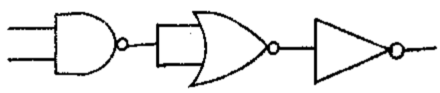
| Gate | NOT | AND | OR | NOR |
|---|---|---|---|---|
| No. of NAND gate required | 1 | 2 | 3 | 4 |

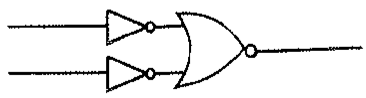
- **Formation of Different gates using NOR gate:**

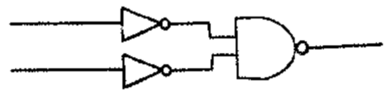
| Gate | NOT | AND | OR | NOR |
|---|---|---|---|---|
| No. of NOR gate required | 1 | 3 | 2 | 4 |

- 

Single input NAND and Single input NOR gate will behave as NOT gate

-  NAND gate

-  AND gate

-  OR gate

* **Some PYQ :**

Neet 2013

**Transconductance :** $(g_m)$

$$V_{gain} = \beta \frac{R_L}{R_i} \qquad G = \frac{\beta}{R_i} \cdot R_L$$

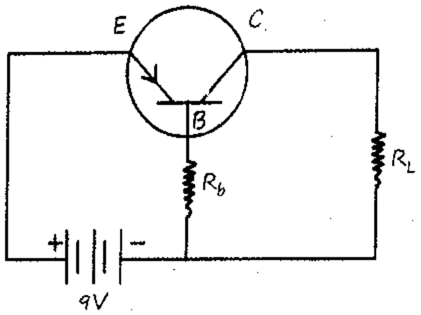
$$G = g_m R_L \qquad g_m = \frac{\Delta I_C}{\Delta V_B} = \frac{\Delta I_C}{\Delta I_B R_i}$$

Q. The device that can act as a complete electronic circuit is

(a) Junction diode
(b) Integrated circuit
(c) Junction transistor
(d) Zener diode

**Ans. (b)**

Q. In a transistor circuit shown in figure, if the base current is 35μA, then the value of resistor $R_b$ is




Ans. By using, $V_b = i_b R_b$

$$R_b = \frac{V_b}{i_b} = \frac{9}{35 \times 10^{-6}} = 257\ k\Omega.$$

Q. For a transistor amplifier power gain and voltage gain are 7.5 and 2.5 respectively. The value of the current gain will be:

Ans. Power gain = Voltage gain × Current gain

7.5 = 2.5 × Current gain

Current gain = 3

Q. A change of 2 mV in base-emitter voltage causes a change of 1 μA in the base current. The input resistance of the transistor is

Ans. $\Delta V_{be} = 2\ mV = 2 \times 10^{-3}\ V$

$\Delta I_b = 1\ \mu A = 10^{-6}$

For transistors, the input resistance is

$$R_i = \frac{\Delta V_{be}}{\Delta I_b}$$

$$R_i = \frac{2 \times 10^{-3}}{10^{-6}}$$

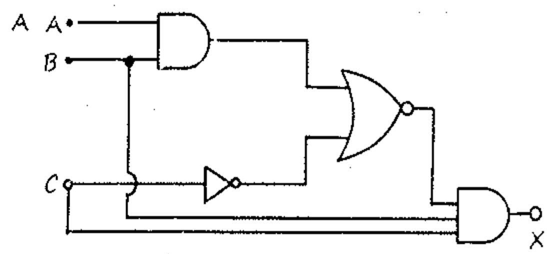
$$R_i = 2 \times 10^{3} = 2k\Omega$$

Q. A change of 8 mA in the emitter current brings a change of 7.9 mA in the collector current. The change in base current required to have the same change in the collector current is

Ans. In transistors,

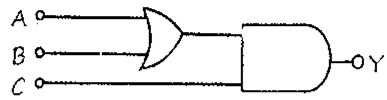
$$I_e = I_p + I_c$$
$$\Delta I_e = \Delta I_b + \Delta I_c$$
$$\Delta I_b = \Delta I_e - \Delta I_c$$
$$\Delta I_b = 8 - 7.9 = 0.1\ mA$$

Q. Find the output boolean function for the logic circuit.




Ans. $X = \overline{A}BC$

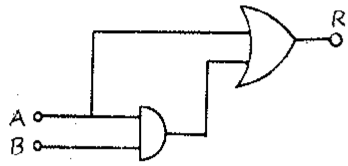
Q. To get output Y = 1 for the following circuit, the correct choice for the input is:




(a) A = 1, B = 0, C = 0
(b) A = 1, B = 1, C = 0
(c) A = 1, B = 0, C = 1
(d) A = 0, B = 1, C = 0

Ans. (c)

Q. A combination of logic gates is shown in the circuit. If A is at 0 V and B is at 5V, then the potential of R is:




(a) 0 V
(b) 5 V
(c) 10 V
(d) Any of these

Ans. (a)

**MR**

" Manjil Mile na mile yah to kismat ki bat h
hum koshish hi na kare ye to galt bat hai "