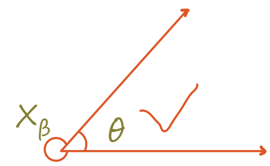
2

Vector

Scalar Quantity	Vector Quantity
❍ Having Magnitude only	❍ Having Magnitude, direction and follow triangle law of vector addition.
❍ Follow simple algebric addition	❍ Can be changed by changing magnitude only, or changing dir^n only or changing both.
❍ Can be changed only by changing its value	
Ex-Speed, time, Mass, Volume, density current, etc.	Ex-Force, Velocity, current density, torque etc.

1. In vector +ve and –ve indicate direction only. Ex- +5N and –5N, same magnitude of force in opposite direction.
2. Angle between vector – When two vectors are placed head to head or tail to tail then smaller angle between vector is called angle between vector.



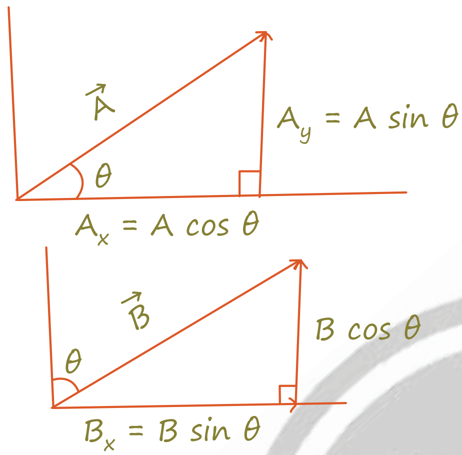
3. Vector can be shifted parallel to itself by keeping magnitude cmd direction fixed.
4. Rotation of vector not allowed it will change meaning of vector.
5. If Angel between $\vec{A}$ and $\vec{B}$ vector is θ then angle between $\vec{A}$ and $-\vec{B}$ is $(180-\theta)$.

Type of Vectors

Type	Magnitude	Direction\Angle
Equal Vector	Same	Same ($\theta = 0$)
Parallels Vector	May or May not same	Same ($\theta = 0$)
Opposite Vector or Negative Vectors	Same	Opposite $\theta = 180°$
Antiparalles Vector	May or May not same	$\theta = 180°$ opposite
Orthogonal	May same	$\theta = 90°$
Zero/Null Vector	Zero	any direction
Unit Vectors	One	$\hat{A} = \frac{\vec{A}}{A}$

- All equal vectors are parallel but all parallels are not equal.
- All opposite (Negative) Vectors are Antiparallel but all antiparallel are not Opposite Vector

Component of Vector (effect of Vector)



Magnitude of Vectors :

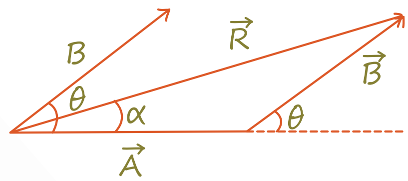
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

If vector is making an angle α, β and γ from x, y and z-axis respectively then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$; $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$\cos\alpha = \frac{Ax}{A}$ $\quad \cos\beta = \frac{Ay}{A}$ $\quad \cos\gamma = \frac{Az}{A}$

Triangle Law of Vector addition



$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

resultant must be in the plane of $\vec{A}$ and $\vec{B}$.

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

If $\theta = 0°$	$\theta = 90°$	$\theta = 180°$
$R_{max} = A + B$	$R = \sqrt{A^2 + B^2}$	$R_{min} = A - B$

$A - B \le R \le A + B$

If $|\vec{A}| = |\vec{B}| = A$ and Angle b/w them θ

$|\vec{R}| = 2A\cos(\theta/2)$ $\quad D = 2A\sin(\theta/2)$

$\theta = 0°$	$\theta = 60°$	$\theta = 90°$	$\theta = 120°$	$\theta = 180°$
$R = 2A$	$R = \sqrt{3}A$	$R = \sqrt{2}A$	$R = A$	$R = 0$
$D = 0$	$D = A$	$D = \sqrt{2}A$	$D = \sqrt{3}A$	$D = 2A$

Vector Subtraction

Angle B/w $\vec{A}$ & $\vec{B}$ is θ then $\vec{D} = \vec{A} - \vec{B}$

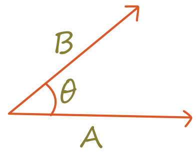
$$|\vec{D}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$\theta = 0°$	$\theta = 90°$	$\theta = 180°$
$D_{min} = A - B$	$D = \sqrt{A^2 + B^2}$	$D = A + B$

$A - B \le D \le A + B$

1. Magnitude of Vector addition and subtraction same at 90°.
2. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ Commutative
3. $n(\vec{A} + \vec{B}) = n\vec{A} + n\vec{B}$ distributive
4. $\vec{A} - \vec{B} \ne \vec{B} - \vec{A}$
5. $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ then $\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$
6. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ is 120°
7. If $|\vec{A}| + |\vec{B}| = |\vec{A} + \vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ is zero.
8. If $\vec{A} + \vec{B} = \sqrt{A^2 + B^2}$ then angle between $\vec{A}$ and $\vec{B}$ is 90°.
9. If $|\vec{A} + \vec{B}| = |\vec{B} - \vec{A}|$ then angle between $\vec{A}$ and $\vec{B}$ is 90°.

Scalar Product (Dot Product)



$\vec{A}\cdot\vec{B} = A(B\cos\theta) = A$(Component of B along A)

$= (A\cos\theta)\,B = B$(Component of A along B)

Component of B along A $= \dfrac{\vec{A}\cdot\vec{B}}{A}$

Component of A along B $= \dfrac{\vec{A}\cdot\vec{B}}{B}$

- Result of dot product is always scalar.

$\hat{i}\cdot\hat{i} = 1 \qquad \hat{j}\cdot\hat{j} = 1 \qquad \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{k} = 0 \qquad \hat{j}\cdot\hat{i} = 0 \qquad \hat{k}\cdot\hat{j} = 0$

$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_xB_x + A_yB_y + A_zB_z$

Application of dot Product

(i) To Find Angle B/W vectors

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

(ii) To check unit vector

If $\vec{A}$ is a unit vector then $\vec{A}\cdot\vec{A} = 1$

(iii) To check perpendicular vector (orthogonal)

If $\vec{A}\cdot\vec{B} = AB\cos 90° = 0$

$\vec{A}\cdot\vec{B} = 0 \qquad (\vec{A} \perp r\ \vec{B})$

(iv) To find component of one vector along other.

$\vec{A}\cdot\vec{B} = A(B\cos\theta)$

$B\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{A}$ = Comp^n of B along A

Cross-Product : [Vector Product]

$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

$\hat{n}$ is direction of $\vec{A}\times\vec{B}$ which is perpendicular to $\vec{A}$ & $\vec{B}$.

$(\vec{A}\times\vec{B})\cdot\vec{A} = 0 \qquad (\vec{A}\times\vec{B})\cdot\vec{B} = 0$

$B\sin\theta = \dfrac{\vec{A}\times\vec{B}}{A}$ = component of B perpendicular of A

$\vec{R} = \vec{A}\times\vec{B}$

Place your finger of right hand along $\vec{A}$ and slap $\vec{B}$ then thumb will represent $\vec{R}$.

$\hat{i}\times\hat{i} = 0 = \hat{j}\times\hat{j} = \hat{k}\times\hat{k}$

$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{j}\times\hat{i} = -\hat{k}$

$\hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{j} = -\hat{i}$

$\hat{k}\times\hat{i} = \hat{j} \qquad \hat{i}\times\hat{k} = -\hat{j}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$= i\,(A_yB_z - A_zB_y)$

$- J(A_xB_z - A_zB_x)$

$+ K(A_xB_y - A_yB_x)$

- Unit vector does not have any unit only have direction and magnitude one.
- Minimum no. of vectors whose resultant can be zero is '2'.
- Minimum no of unequal vectors whose resultant can be zero is 3.
- The resultant of 3 Non-coplaner vectors can't be zero.
- Minimum no of Non-coplaner, vectors whose resultant can be zero is 4.

Q. If $|\vec{A}\times\vec{B}| = \sqrt{3}\,\vec{A}\cdot\vec{B}$ then angle between $\vec{A}$ and $\vec{B}$ is?

Sol^n. $AB\sin\theta = \sqrt{3}\,AB\cos\theta$

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

- Division of vector with vector is not possible
- Division of magnitude of vector is possible
- Vector can be divided by scalar.
- If vector multiplied by positive scalar then magnitude change direction remains same.
- If vector multiplied by negative scalar then magnitude change direction becomes opposite.

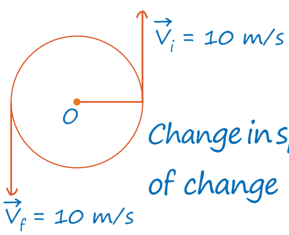
- **Scalar triple Product :**

  $R = (\vec{A} \times \vec{B}) \cdot \vec{C}$ Result R will be scalar and R will be zero if any of these two vector becomes parallel.

Q. Ramlal is moving with velocity 6m/s along east and pinky with 6 m/s at 30° east of north then relative of pinky w.r.t Ramlal.

Sol. $\vec{V}_{PR} = \vec{V}_P - \vec{V}_R$ same vector ka subtraction at 60° $|\vec{V}_{PR}| = 6$ m/s

Q.



Change in speed = 0 magnitude of change in velocity = 20 m/s

Q. If $\vec{A} = 0.6\hat{i} + \beta\hat{j}$ is a unit vector then find value of β.

Sol^n $|\vec{A}| = 1$ if A is unit vector

$$\sqrt{(0.6)^2 + \beta^2} = 1$$

$$\beta^2 + 0.36 = 1$$

$$\beta = \sqrt{0.64} = 0.8$$

Q. Two force 10N and 6N acting then find resultant of these two force may be?

Sol^n $10 - 6 \le R \le 10 + 6$

R will be 4N to 16N

Q. The angle which a vector $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ makes with x, y and z-axis

$\cos\alpha = \dfrac{Ax}{A}$ $\qquad \cos\beta = \dfrac{Ay}{A}$ $\qquad \cos\gamma = \dfrac{Az}{Z}$

$\alpha = 60°$ $\qquad \beta = 60°$ $\qquad \gamma = 45°$

Q. In which of the following combination of three force resultant will be zero.

(a) 3N, 7N, 8N

(b) 2N, 5N, 1N

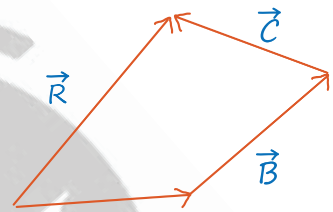
(c) 3N, 12N, 7N

(d) 4N, 5N, 10N

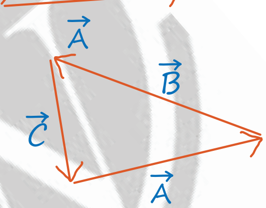
Sol^n Sum of two smaller must be greater or equal to (3rd).

- Polygon Law of vector addition

  Start tail of next vector from head of previous vector and so on.



$\vec{A} + \vec{B} + \vec{C} = \vec{R}$



$\vec{A} + \vec{B} + \vec{C} = 0$

- Angle between $(\vec{A} \times \vec{B})$ and $(\vec{A} + \vec{B})$ is zero

Q. Force acting on object $\vec{F} = 5\hat{i} + 3\hat{j} - 7\hat{k}$ position vector $\vec{r} = 2i + 2j - k$ then find torque ?? (NEET 2022)

$$\vec{\tau} = r \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 5 & 3 & -7 \end{vmatrix}$$

$$\hat{i}(-14 - (-3) - \hat{j}(-14 - (-5) + \hat{k}(6 - 10)$$

$$-11\hat{i} + 9\hat{j} - 4\hat{k}$$

MR*

"अपनी पढाई छोड़ जो तेरे पीछे चला आयेगा।
वो खुद का ना हो सका, तेरा क्या हो जायेगा।।"