1. Binomial theorem

$(1 + x)^2 = 1 + 2 \times 1x + x^2$

if $x <<< 1$ then

$(1 + x)^2 = 1 + 2x$

MR* feel

(Carrier + love)^2 = Carrier + 2 love

Because carrier >>> love

$$[x+\Delta x]^n = x^n\left[1 + \frac{\Delta x}{x}\right]^n = x^n\left[1 + n\frac{\Delta x}{x}\right]$$

$\Delta x <<<< x$.

- $(1 - x)^n = 1 - nx$
- $(1 - x)^{-n} = 1 + nx$
- $(1 + x)^{-n} = 1 - nx$

2. Imp formula

$(a + b)^2 = a^2 + b^2 + 2ab$

$(a - b)^2 = a^2 + b^2 - 2ab$

$a^2 - b^2 = (a + b)(a - b)$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

3. AP series

Next term = Previous term + Common difference

$a, a + d, a+2d, a + 3d, a + 4d.....$

Ex 2, 5, 8, 11, 14, 17, so on.

d = Common difference

= n^{th} term $- (n-1)^{th}$ term

$T_n = a + (n-1)d$

(T_n → last term; a → 1st term; n → no. of term; d → Common diff.)

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

(n → no. of terms.)

NOTE:– n = no. of terms not last term.

GP series

Next term = Previous term × Common ratio

a, ar, ar^2, ar^3, ar^4

Ex 16, 8, 4, 2, 1, 1/2, 1/4, so on

$$r \text{ (Common ratio)} = \frac{n^{th} \text{ term}}{(n-1)^{th} \text{ term}}$$

Sum $= \frac{a}{1-r}$, valid when $r < 1$.

Ex– $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, ...$

$$r = \frac{1/4}{1/2} = \frac{1}{2}$$

$$\text{Sum} = \frac{1}{1-\frac{1}{2}} = \frac{1}{1/2} = 2$$

Ex–

$1, -\frac{1}{2}, \frac{1}{4}, \frac{-1}{8}, \frac{1}{16}, \frac{-1}{32}, ...$

$r = -\frac{1}{2}$

$$\text{Sum} = \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{3/2} = \frac{2}{3}$$

4. Quadratic equation

$ax^2 + bx + c = 0$

> a, b, & c are constant in which a can not be zero

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum of roots $= \frac{-b}{a}$, Products of roots $= \frac{c}{a}$

Q. Find roots of equation $x^2 - 5x + 6 = 0$; find value of a, b & c by comparing with $ax^2 + bx + c = 0$

Ans. $a = 1, b = -5$ & $c = 6$

$$x_1 = \frac{-(-5) + \sqrt{(-5)^2 - 4\times1\times6}}{2\times1}$$

$$= \frac{5 + \sqrt{1}}{2} = 3$$

$x_2 = 2$

Q. $x^2 - 4x = 0$

> $x^2 = 4x$
> $x = 4$ wrong

> $x(x - 4) = 0$
> $x = 0$; $x = 4$ correct

Q. $x^2 - 4x + 3 = 0$ then find roots.

Ans. $x^2 - 3x - x + 3 = 0$

$x(x-3) - 1(x - 3) = 0$

$(x - 3)(x - 1) = 0$

$x = 3, x = 1$

5. Logarithms

$\log y^x = \log x$ on the base y

$\log_e x = 2.303 \log_{10} x$

(a) $\log_a (xy) = \log_a x + \log_a y$

(b) $\log\left(\frac{x}{y}\right) = \log x - \log y$

(c) $\log_y x = \frac{1}{\log_x y}$

(d) $\log_e x^{1/n} = \frac{1}{n} \log_e x$

(e) $\log_e x^n = n \log_e x$

(f) $\log_b a \times \log_a b = 1$

(g) $\log_a a = 1$

$\log_e 1 = 0$

$\log_{10} 2 = 0.30$

$\log_{10} 1 = 0$

$\log_{10} 3 = 0.48 \approx 0.5$

$\log_e(\sin 90°) = 0$

$\log_{10} 5 + \log_{10} 20 = 2$

$\log_2 3 = \frac{\log_{10} 3}{\log_{10} 2} = \frac{48}{30}$

✦ Concept of Anti-log

$\log e^x = Y$

By taking Anti-log (convert into concept of power)

$x = e^y$

MR* ka tadka

log → Concept of Power

Base, Power, Result: $2^3 = 8 \Rightarrow \log 2^8 = 3$

Base wahi rahega (Power ⇌ Result interchange hoga)

6. Rule of Power

1. If Power of any non-zero number is zero then result will be one.

Ex- $8^0 = 1$

2. Negative Property of exponent (x is non zero number)

$$x^n = \frac{1}{x^{-n}} \Rightarrow \frac{1}{x^n} = x^{-n}$$

$$\frac{1}{10^3} = 10^{-3}$$

3. Product Property of Exponent

$$x^n\, x^m = x^{n+m}$$

$$10^3 \times 10^4 = 10^7$$

4. Division Property

$$\frac{x^n}{x^m} = x^{n-m} \Rightarrow \frac{10^3}{10^2} = 10^{3-2}$$

5. Power of a Power:

$$(x^n)^m = x^{nm}$$

$$(10^2)^3 = 10^6$$

6. $10^2 + 10^3 = 100 + 1000 = 1100$

7. Fractional exponent

$$(x)^{3/2} = (x^3)^{1/2}$$

8. Multiplication with fraction.

$$0.5 = \frac{1}{2} \qquad 1.33 \times 12 = \frac{4}{3} \times 12 = 16$$

$$0.6 = \frac{6}{10} \qquad 16 \times .25 = \frac{1}{4} \times 16 = 4$$

$$0.4 = \frac{4}{10} \qquad 0.75 \times 16 = \frac{3}{4} \times 16 = 12$$

$$0.66 = \frac{2}{3} \qquad 0.33 \times 15 = \frac{1}{3} \times 15 = 5$$

$$1.33 = \frac{4}{3} \Rightarrow 0.75 = \frac{3}{4} \Rightarrow 0.33 = \frac{1}{3}$$

9. Important property

$$2^\infty = \infty \qquad e^\infty = \infty$$

$$1^\infty = 1 \qquad e^{-\infty} = 0$$

$$4^{-\infty} = 0 \qquad e^0 = 1$$

$$(8)^{2/3} = (8)^{(1/3)\times 2} = (2)^{3\times(1/3)\times 2} = 2^2 = 4$$

$$(32)^{3/5} = (2^5)^{3/5} = 2^3 = 8$$

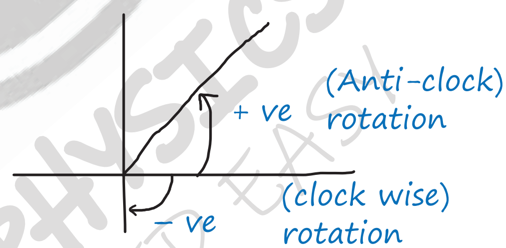
Important roots

$\sqrt{121} = 11$	$\sqrt{400} = 20$
$\sqrt{144} = 12$	$\sqrt{900} = 30$
$\sqrt{169} = 13$	
$\sqrt{196} = 14$	$\sqrt{0.64} = 0.8$
$\sqrt{225} = 15$	$\sqrt{0.16} = 0.4$
$\sqrt{256} = 16$	

7. Trigonometry

Angle
- → Arc = $R\theta$ algebraic function
- → $\sin\theta/\cos\theta/\tan\theta$ Trigo. function
- → Angle have unit radian. but dimensionless.
- → For algebraic function, we always use S.I. unit radian but for trigonometric function we may use rod/degree.
- → $180° = \pi$ rad

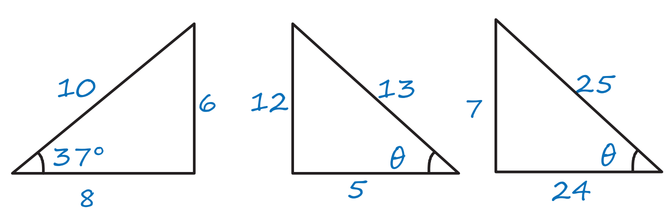
$$1° = \frac{\pi}{180} \text{ rad} \qquad 1 \text{ rad} = \frac{180}{\pi}$$



Q. Total Angle moved by object in π-rotation?

Ans. – $\theta = \pi(2\pi) = 2\pi^2$ rad.

✦ Some Important Triangles



	0°	30°	45°	60°	90°	120°	135°	150°	180°
Sin θ	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
Cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	$-1/2$	$-1/\sqrt{2}$	$-\sqrt{3}/2$	−1
Tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not define	$-\sqrt{3}$	−1	$-1/\sqrt{3}$	0

$\sin\theta = \frac{1}{\text{Cosec }\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

$\sin(90+\theta) = \cos\theta$

$\sin(180-\theta) = \sin\theta$

$\sin(90-\theta) = \cos\theta$

$\cos(180-\theta) = -\cos\theta$

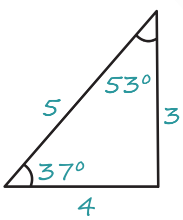
$\cos(90-\theta) = \sin\theta$

$\cos(90+\theta) = -\sin\theta$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$



$\sin\theta = \frac{P}{H}$ $\quad \cos\theta = \frac{B}{H}$

$\tan\theta = \frac{P}{B}$ $\quad \sin 37° = \frac{3}{5}$

$\cos 37° = \frac{4}{5}$ $\quad \sin 53° = \frac{4}{5}$ $\quad \cos 53° = \frac{3}{5}$

$\cos(-60°) = \frac{1}{2} \Rightarrow \sin(-30°) = -\frac{1}{2}$

$\tan(-135°) = -1$

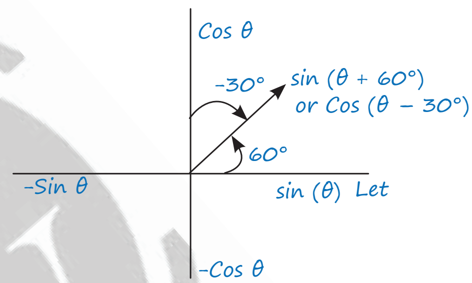
Unique Relation

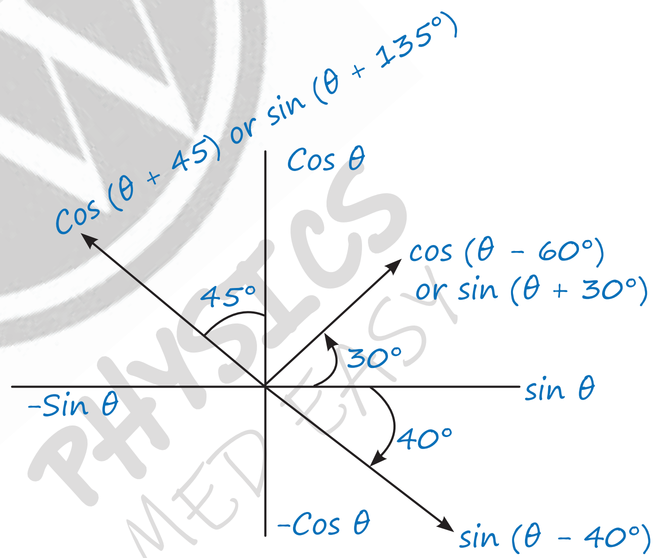
$\sin^2\theta + \cos^2\theta = 1 \quad \Rightarrow \quad 1+\cot^2\theta = \text{Cosec}^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

8. Phasor diagram

Vector representation of trigonometric function





Equation-1	Equation-2	Phase difference
$I = I_0 \sin(\theta + \pi/3)$	$I = I_0 \sin(\theta - \pi/6)$	$\phi = 90°$
$I = I_0 \sin(\theta + \pi/3)$	$I = I_0 \cos(\theta - \pi/6)$	$\phi = 0°$
$I_1 = I_0 \sin(\theta)$	$I = I_0 \cos(\theta + \pi/6)$	$\phi = 2\pi/3$
$I_1 = \sin(\theta - \pi/3)$	$I = I_0 \cos(\theta + \pi/3)$	$\phi = \frac{7\pi}{6} = 210°$
$I_1 = \sin(\theta - 60°)$	$I = I_0 \cos(\theta - 30°)$	$\phi = \frac{2\pi}{3} = 120°$

9. Sin (A + B) = Sin A cos B + cos A sin B

Sin (A − B) = Sin A cos B − cos A sin B

Cos (A + B) = Cos A cos B − Sin A sin B

Cos (A − B) = Cos A cos B + Sin A sin B

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(a) $A = B = \theta$

$\sin (A + B) = \sin 2\theta = 2\sin\theta \cos\theta$

$\cos (A + B) = \cos 2\theta = \cos^2\theta - \sin^2\theta$

(b) $2\cos^2\theta = 1 + \cos(2\theta)$.

$2\sin^2\theta = 1 - \cos(2\theta)$

If Angle is Small:-

$\sin\theta \approx \theta$ $\quad\tan\theta \approx \theta$ $\quad\cos\theta = 1$

Sin (2°) = 2° (wrong)

$$\sin(2^\circ) = 2 \times \frac{\pi \text{rad}}{180^\circ} = \frac{\pi}{90^\circ} \text{rad}$$

$\cos(4^\circ) = 1$

$$\tan 3^\circ = \frac{\pi \text{rad}}{60}$$

10.

Trigonometric function	Maximum Value
$Y = 3\sin\theta$	$Y_{max} = 3$ $Y_{min} = -3$
$Y = 4\sin(5\theta)$	$Y_{max} = 4$ $Y_{min} = -4$
$Y = 3\sin\theta + 4\cos\theta$	$Y_{max} = 5$ $Y_{min} = -5$
$Y = 3\sin\theta + 4\sin\theta$	$Y_{max} = 7$ $Y_{min} = -7$
$Y = 5 - 2\sin\theta$	$Y_{max} = 7$ $Y_{min} = 3$

Q. Force acting on object $F = \frac{4}{3\sin\theta + \cos\theta}$

Then find minimum magnitude of force.

Ans. $$F_{min} = \frac{4}{(3\sin\theta + \cos\theta)_{max}}$$

$$F_{min} = \frac{4}{\sqrt{9+1}} = \frac{4}{\sqrt{10}}$$

11. Sum of 1st n-natural numbers $= \frac{n(n+1)}{2}$

Sum of Squares of 1st n-natural numbers $= \frac{n(n+1)(2n+1)}{6}$

Sum of Cubes of 1st n-natural numbers

$$= \left[\frac{n(n+1)}{2}\right]^2$$

12. Differentiation

DC = − ve

$\frac{dy}{dx}$ = The rate of change in y w.r.t.x

= Slope of y−x graph.

$\frac{d^2y}{dx^2}$ = Double diffⁿ of Y w.r.t x

= The rate of change in $\left(\frac{dy}{dx}\right)$ w.r.t x

= Slope of Slope

= Change in slope w.r.t x

$$\frac{d \sin x}{dx} = \cos x$$

$$\frac{d \tan x}{dx} = \sec^2 x$$

$$\frac{d \cot x}{dx} = -\operatorname{cosec}^2 x$$

$$\frac{d \log_e x}{dx} = \frac{d \ln x}{dx} = \frac{1}{x}$$

$$\frac{d \cos x}{dx} = -\sin x$$

$$\frac{d \sec x}{dx} = \sec x \tan x$$

$$\frac{d \operatorname{cosec} x}{dx} = -\operatorname{cosec} x \cot x$$

$$\frac{d x^n}{dx} = nx^{n-1}$$

Rules :-

1. Addition Rule:-

$Y = A + B \qquad \frac{dy}{dx} = \frac{dA}{dx} + \frac{dB}{dx}$

2. Substraction Rule:-

$Y = A - B \qquad \frac{dy}{dx} = \frac{dA}{dx} - \frac{dB}{dx}$

3. Multiplication Rule:-

$Y = A\,B \qquad \frac{dy}{dx} = \frac{A\,dB}{dx} + \frac{B\,dA}{dx}$

4. Division Rule:-

$$Y = \frac{A}{B} \qquad \frac{dy}{dx} = \frac{B\left(\frac{dA}{dx}\right) - A\left(\frac{dB}{dx}\right)}{B^2}$$

$\frac{d \sin(90°)}{dx} = 0 \qquad Y = t^2 \text{ find } \frac{dy}{dx}$

$\frac{d e^x}{dx} = e^x \qquad \frac{dy}{dx} = \frac{dt^2}{dx} \times \frac{dt}{dt}$

$\frac{d e^2}{dx} = 0 \qquad \frac{dy}{dx} = 2t \frac{dt}{dx}$

The MR*

Outside Inside Rule

$Y = f(z(x))$ = y is function of z and z is a function of x.

$\frac{dy}{dx} =$ (differentiation of outer function keep inside as it is) × (diff^n of Inner fu^n w.r.t x)

Q. $y = \sin(3x)$

$$\frac{dy}{dx} = \cos(3x)\frac{d(3x)}{dx}$$

$$= 3\cos(3x)$$

$Y = e^{(5x)}$ $\frac{dy}{dx} = 5e^{5x}$	$Y = e^{-4x}$ $\frac{dy}{dx} = -4\,e^{-4x}$
$Y = (x^2+4)^3$ $\frac{dy}{dx} = 3\,(x^2+4)^2 \frac{d(x^2+4)}{dx}$ $= 3(x^2+4)^2 \times 2x$	$Y = \sin(4x^2)$ $\frac{dy}{dx} = \cos(4x^2) \times 8x$

$Y = A \sin(wt - kx)$

$\frac{dy}{dx} = A\cos(wt - kx) \times (-k)$

Q. If radius of sphere is increasing $1/\pi$ m/s then find rate of change in volume w.r.t. time when radius is 3m.

Ans. $V = \frac{4}{3}\pi R^3$

$$\frac{dv}{dt} = \frac{4}{3}\pi 3R^2 \frac{dR}{dt}$$

$$= 4\pi R^2 \left(\frac{1}{\pi}\right)$$

$$\left(\frac{dv}{dt}\right) = 4R^2 = 4(3)^2 = 4 \times 9 = 36$$

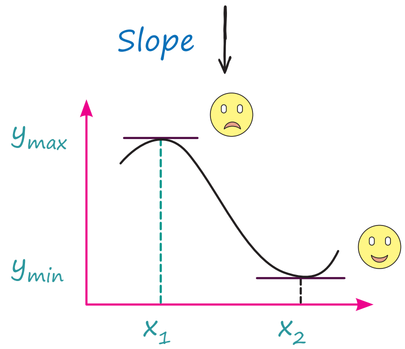
13. Maxima and minima:

MR* for maxima/minima

- For location of maxima/minima put $\frac{dy}{dx}$ (slope) = 0 and find value where x will be max^m/min^m.
- For exact maxima and minima dont check double differentiation. Just put value of x and find y.
- Double differentiation check nahi karna just x ki value put kark y nikala jo y jayda wo maximum y ko kam wo minimum y.

Maxima

$$\frac{dy}{dx} = 0 \qquad \frac{d^2y}{dx^2} = -ve$$



Minima

$$\frac{dy}{dx} = 0 \qquad \frac{d^2y}{dx^2} = +ve$$

Slope ↑

14. Integration:

→ Area under the curve → Inverse of differentiation

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$$ Not valid for $n = -1$

Addition Rule:

$$\int (u + v).\, dx = \int u.dx + \int v.dx$$

$$\int \sin x\, dx = -\cos x + c.$$

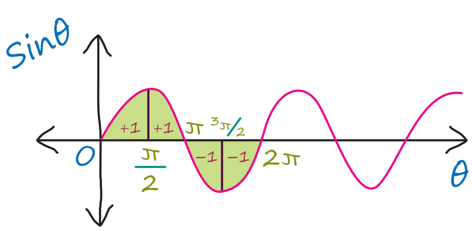
$$\int \cos x\, dx = \sin x + c.$$

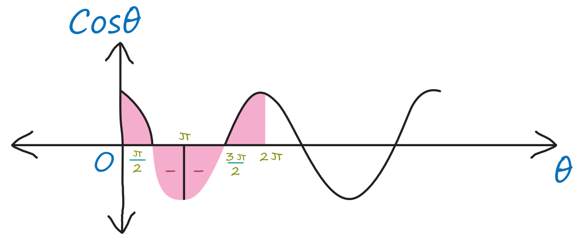
$$\int e^x dx = e^x + c$$

$$\int 1/x\, dx = \ln x + c.$$

$$\int \sec^2 dx = \tan x + c$$

$$\int e^{3x} dx = \frac{e^{3x}}{3} + c.$$





Chain Rule → MR*

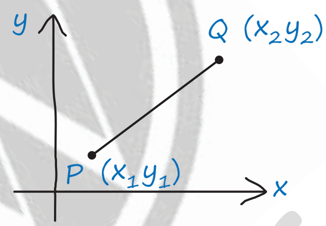
Applicable when power of x is one

$$\int y\, dx = \frac{\text{Integration of outer function keep inside as it is.}}{\text{Coefficient of } (x)}$$

$$\int (2x+3)^4\, dx = \frac{(2x+3)^5}{5[2]} + C$$

$$\int \sin(3x-4)\, dx = \frac{-\cos(3x-4)}{3} + C$$

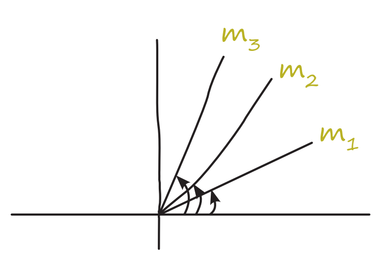
15. Co-ordinate geometry and graph:



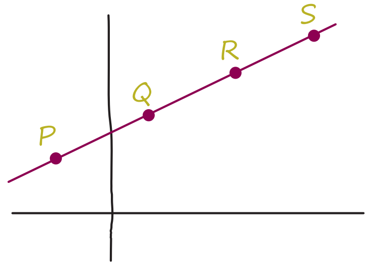
$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\tan\theta = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

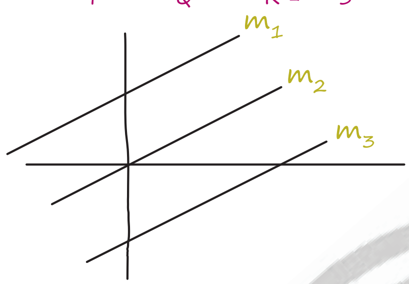
- Slope of straight line remains same at all the point
- If $0° \le \theta < 90°$ then slope is positive
- If $90° < \theta \le 180°$ then slope is negative
- If $\theta = 90°$ then slope is infinite
- If $\theta = 0°$ then slope is zero
- If straight line parallel to x-axis then slope zero



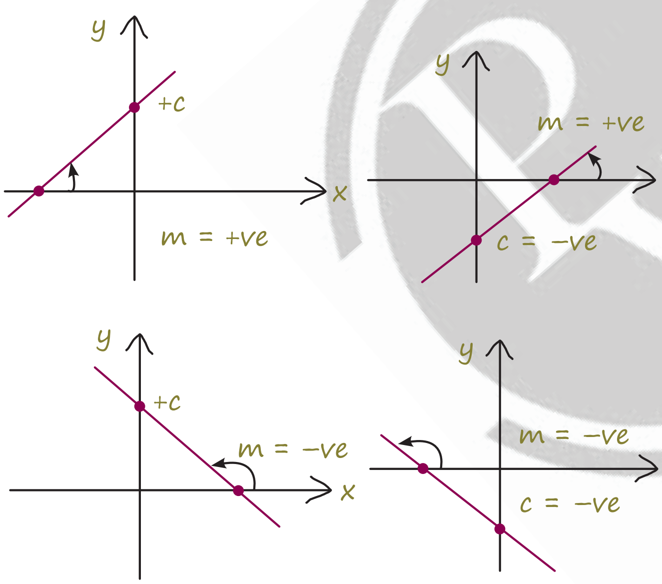
$m_3 > m_2 > m_1$



$m_P = m_Q = m_R = m_S$

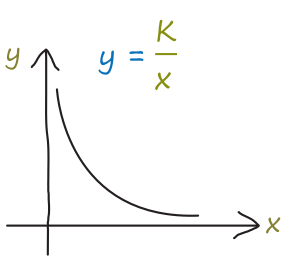


$m_1 = m_2 = m_3$

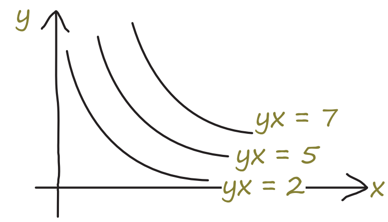


If two straight line perpendicular to each other then product of their slope is −1.

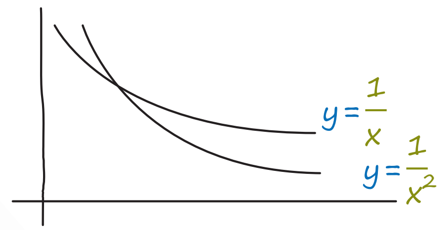
16. Rectangular Hyperbola:

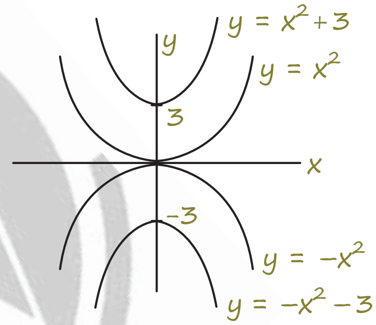


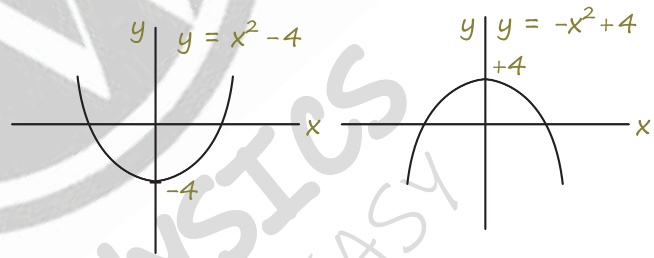
K is value Jitna Jayda graph utna upar shift hoga.

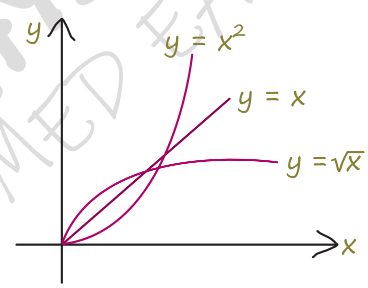


x ka pawer jitna jayda graph utna niche jayga.

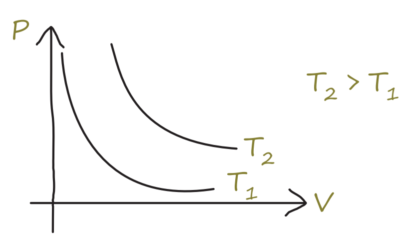




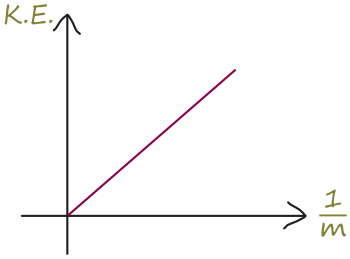




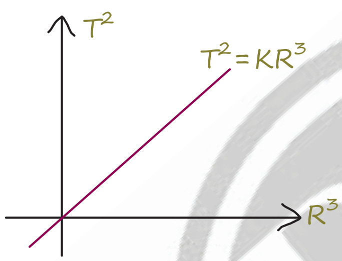
✦ graph for PV = nRT

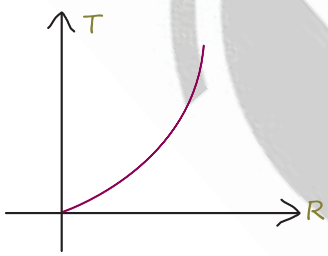


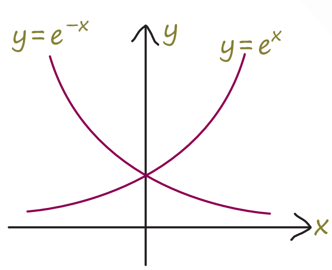
✦ K.E. $= \frac{P^2}{2m}$ graph b/w K.E. and $\frac{1}{m}$ for constant momentum.



MR* → Jisko x- & y-axis pe plot krenge uska power dekhte hai.







17. Equation of Circle

$(x - x_0)^2 + (y - y_0)^2 = R^2$

R is radius & centre is at (x_0, y_0)

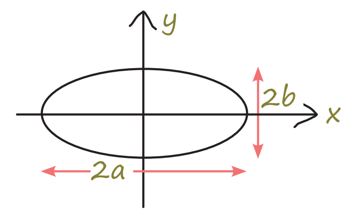
$x^2 + y^2 = 5^2$ centre at (0, 0) R = 5

$(x + 4)^2 + (y - 3)^2 = 49$ centre at (−4, 3)

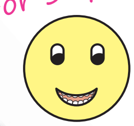
R = 7

18. Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



MR* For Slope



हँसता हुआ रामलाल — Slope always increasing

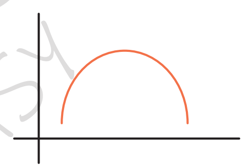
रोता हुआ रामलाल — Slope always decreasing

For magnitude of slope → Now we are talking about value of slope, we will ignore +ve & −ve only consider magnitude.

MR* → Locate where slope is zero

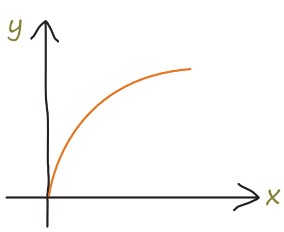
✦ Starting me zero then increasing magnitute of slope.

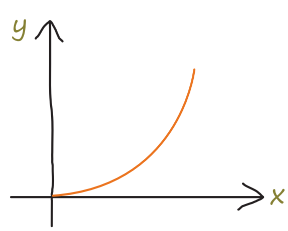
✦ Last me zero then decreasing magnitude of slope and becomes zero.

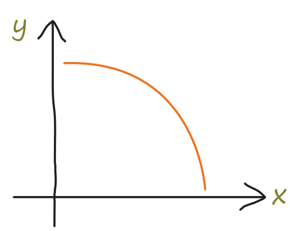


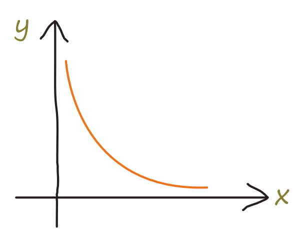
Slope → Increasing
magnitute of slope
1^{st} decreasing then increasing

Slope → Decreasing
magnitute of slope
1^{st} decreasing then increasing





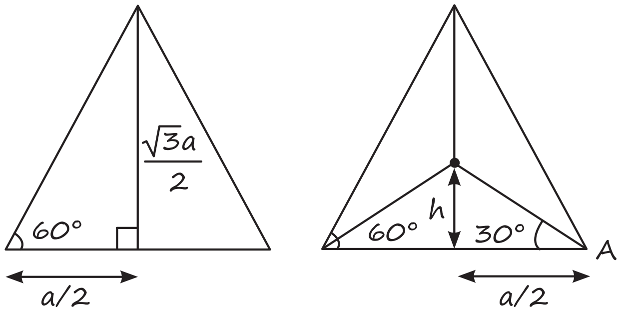




Slope	→ decreasing	increasing	decreasing	increasing
Magnitute of slope	→ decreasing	increasing	increasing	decreasing

19. Some Basic Geometry Shapes:

Equibtral Triangle of side (a)



$$\tan 30° = \frac{h}{a/2}$$

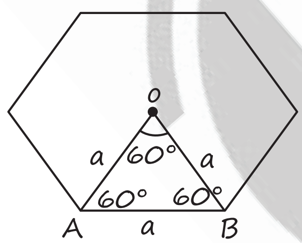
$$h = \frac{a}{2\sqrt{3}}$$

distance from centre to corner

$$= \frac{\sqrt{3}a}{2} \times \frac{2}{3} = \frac{a}{\sqrt{3}}$$

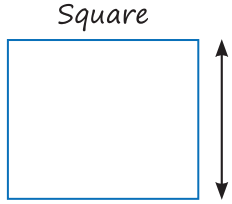
$$\text{Area} = \frac{\sqrt{3}a^2}{4}$$

Hexagonal of side 'a'



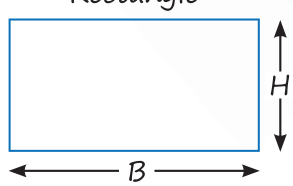
centre to corner
$dist^n = a$

Square



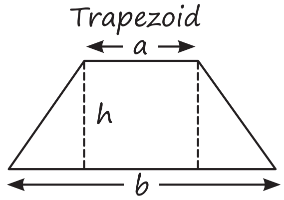
Area = l^2
Perimetre = $4l$

Rectangle

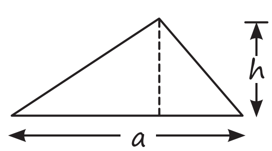


Area = BH
Perimetre = 2(H+B)

Trapezoid

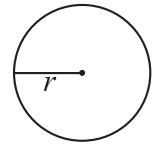


$$\text{Area} = \frac{1}{2}(a + b)h$$

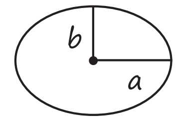


$$\text{Area} = \frac{1}{2}ah$$

Circle

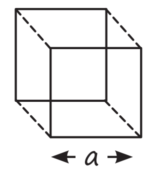


Circumference $= 2\pi r$
Area $= \pi r^2$



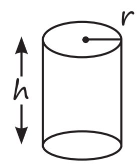
Area $= \pi ab$
Circumference $= \pi r$

Cube



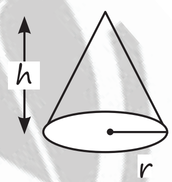
Area $= 6a^2$
Volume $= a^3$

Cylinder



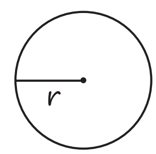
Area $= 2\pi r^2 + 2\pi rh$
Volume $= \pi r^2 h$

Cone



$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

Sphere



Area $= 4\pi r^2$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

20. Average of a varying quantity

If $y = f(t)$ then

$$\langle y \rangle_{Avg} = \frac{\int_{t_1}^{t_2} y\,dt}{\int_{t_1}^{t_2} dt} = \frac{\int_{t_1}^{t_2} y\,dt}{t_2 - t_1}$$

Y may be any physical quantity.

MR* if y is varying linearly then $y_{Avg} = \frac{y_i + y_f}{2}$

MR* If $x+y$ = constant then xy will be maximum for $x = y = \frac{C}{2}$

If sum of two number is constant then product of these two number will be maximum, only when both number are equal.