Consumer Choice Framework: Utility-maximizing Consumption Choices

Matt enjoys sushi (S) and cocktails (C) with utility function **$U_M(S,C) = S*C$** precisely describing his consumption preferences. Matt has a $60 gift card to a restaurant where the prices of these two goods are P_S = $5 and P_C = $10.

a) Draw a budget constraint and indifference curves depicting Matt's preferences.

b) Is the budget constraint binding in this problem? Briefly explain.

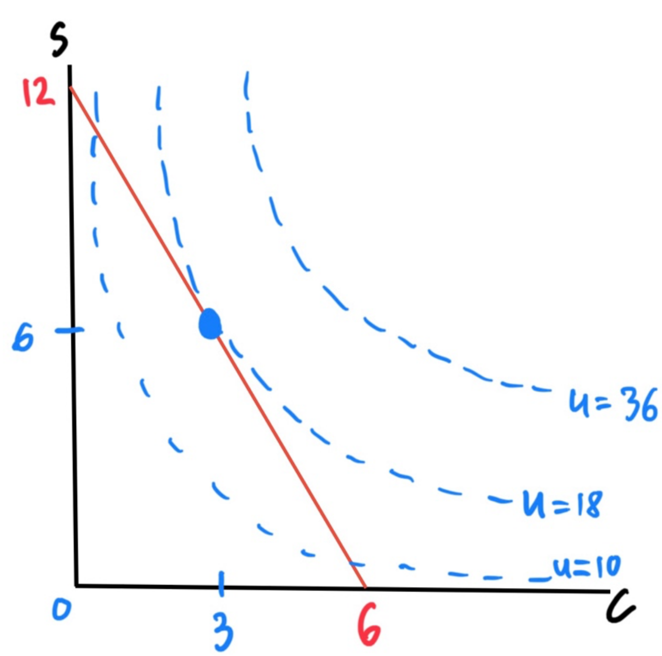
c) Solve Matt's consumer choice optimization problem using a Lagrangian approach.

d) Solve Matt's consumer choice problem using the MRS = MRT approach.

e) Create a table showing each of Matt's possible consumption bundles (integer value allocations only) and the resulting utility from each.

SOLUTION: For constructing the budget constraint, we must find every affordable combination of sushi and cocktails which use all of Matt's purchasing power:

Sushi	Cocktails	Matt's Utility
0	6	**0**
2	5	**10**
4	4	**16**
6	3	**18**
8	2	**16**
10	1	**10**
12	0	**0**



SOLUTION

The budget constraint is binding in this problem, as usual, because Matt can only use the gift card for purchasing these two goods: his utility is monotonically increasing over both, so he will always use up the whole gift card. If it was cash instead of a gift card, the answer would be the same if he placed no value on money but could be different if he also derived utility from the cash, which he might not fully spend in that case.

Using a Lagrangian approach:

$$\mathbf{L}(c, s, \lambda) = c \cdot s + \lambda(60 - 10c - 5s)$$

The first order conditions are:

$$\frac{\partial \mathbf{L}}{\partial c} = s - 10\lambda = 0$$

$$\frac{\partial \mathbf{L}}{\partial s} = c - 5\lambda = 0$$

$$\frac{\partial \mathbf{L}}{\partial \lambda} = 60 - 10c - 5s = 0$$

So from the first two FOCs we have $\frac{c}{5} = \lambda = \frac{s}{10}$ which gives us $s = 2c$ as our optimal consumption ratio. Using the third FOC (budget constraint) we can substitute to obtain the following:

$$60 - 10c - 5(2c) = 0$$
$$c^* = 3, \;\; s^* = 6$$

Using the MRS = MRT approach, we can construct the budget constraint directly: $[\, P_s \cdot s + P_c \cdot c = 100\,]$ and equating the marginal rate of substitution from the utility function with the marginal rate of transformation from the budget constraint, we have the following:

$$\text{MRS} = -\frac{\frac{\partial U}{\partial c}}{\frac{\partial U}{\partial s}} = \frac{-10}{5} = \text{MRT}$$

$$\frac{-s}{c} = -2$$

$$s = 2c$$

Substituting back into the budget constraint with the prices given in the question, we can solve $5 \cdot (2c) + 10 \cdot c = 100$ which also gets $c^* = 3$ and $s^* = 6$. Intuitively, the logic of this approach is that we are finding the one unique place where the slope of the budget constraint (MRT) is equal to the slope of the indifference curves (MRS). Since there are many indifference curves (which all have the same shape) substituting into the budget constraint allows us to identify exactly which indifference curve we are on.