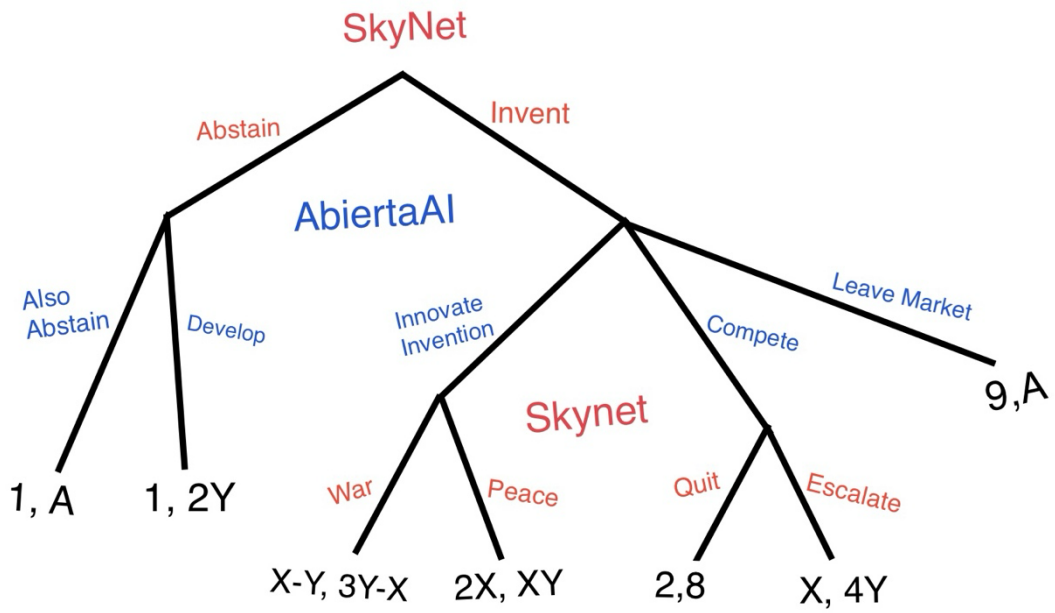
Microeconomics - Advanced Game Theory
Andrew Gates (Princeton University)

Sequential Terminator Game: Multiple Cases of Interacting Unknown Payoff Values

Use the following sequential game tree and assume throughout the entire question that $X \geq 0$ and $Y \geq 0$:



a) Under what conditions will the SPNE outcome of this game be War?

b) Assuming that A=0 (and still assuming that $X \geq 0$ and $Y \geq 0$) find all possible SPNE outcomes of this game.

c) Create a chart illustrating all possible ranges of outcomes with Y as the vertical axis and X as the horizontal axis.

d) Now drop the assumption of A=0 and re-create the same chart showing all possible outcome ranges with the value of A now unknown and allowed to vary.

(Hint: You can depict this with a carefully chosen line representing A in terms of one of the other variables.)

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Conditional Product Offering Decisions with Unknown Parameters

Two competing firms, called Entrant and Incumbent, must non-cooperatively and *simultaneously* choose between four possible approaches to the market. Specifically, each firm has two options for product pricing, an option to incur a cost to develop a different product, and an option to not participate. The Entrant can choose OUT and receive revenue **X** no matter what the Incumbent chooses and the Incumbent can choose LEAVE and receive revenue **L** no matter what the Entrant chooses. The Entrant's other options are NEW, HIGH, and LOW. The Incumbent's other options are INNOVATE, EXPENSIVE, and CHEAP.

The sets of possible *revenues* for each firm are given as follows:

{New, Innovate} = (10,10),	{New, Expensive} = (12,5) ,	{New, Cheap} = (8,4) ,	{New, Leave} = (16,L) ,
{High, Innovate} = (6,13) ,	{High, Expensive} = (7,7) ,	{High, Cheap} = (4,8) ,	{High, Leave} = (11,L) ,
{Low, Innovate} = (7,8) ,	{Low, Expensive} = (8,2) ,	{Low, Cheap} = (3,3) ,	{Low, Leave} = (6,L) ,
{Out , Innovate} = (X,14) ,	{Out , Expensive} = (X,13) ,	{Out , Cheap} = (X,11) ,	{Out , Leave} = (X,L).

If Innovate is chosen, then the Incumbent also incurs some cost $\mathbf{C_I}$ which must be accounted for in determining utility payoffs. If New Product is chosen, then the Entrant also incurs some cost $\mathbf{C_E}$ which must be accounted for in determining payoffs. Payoffs here are defined as revenue minus cost and should be thought of as utility values. You do not need to consider any risk preferences, mixed strategies, collusion, or anything else not explicitly referenced in this question.

a) Draw out the full matrix of utility payoff values.

b) What do **X** and **L** represent conceptually and what factors (conceptually) might affect these values?

c) Suppose **L = 0**. Under what conditions do both firms sell products in the market?

d) Find all pure strategy Nash Equilibria in this game.

Assume now that $\mathbf{C_E = C_I = 10}$ and **L=0**.

e) What is the largest value ($\mathbf{b_{max}}$) that the Incumbent would be willing to bribe the Entrant in exchange for choosing Out instead of entering the market?

f) What is the optimal bribe offer ($\mathbf{b^*}$) that the Incumbent should propose and what will be the result if this occurs?

Consider a *sequential version* of the game where the Entrant first chooses OUT or ENTER, and if there is entry then the Incumbent chooses any of her four possible actions, and then the Entrant must respond with one of his three possible actions. $\mathbf{C_E, C_I, L, X}$ are all unknown.

g) Draw a full extensive form game (complete decision tree diagram) for this scenario.

Now suppose $\mathbf{L < 2}$ and $\mathbf{C_E = C_I = 6}$.

h) Is there any advantage for the entrant from moving first in this sequential version of the game compared to the simultaneous matrix game from before? If yes, quantify the difference.