Formal Models of Market Competition

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Introduction

- Formal models of market structure and competition:
  - Monopoly
  - Cournot
  - Stackelberg
- Adverse Selection: Akerlof's lemons numerical example
- Discussion of concepts

Monopoly Profit Maximization

- Market demand is given by: $Q(p) = a - p$ where a is a baseline demand parameter and $0 < p < a$ is the price.
- Remember that demand is given by consumer preferences and beyond the control of firms, even in the monopoly case. Quantity demanded is of course inversely correlated with price, as usual.
- Marginal cost: $0 < c < a$ and assume fixed cost is zero for simplicity.
- We can therefore write profit as the difference between price and cost times the quantity sold:

$$\Pi = (p - c)Q(p)$$

Rewriting the profit function in terms of price, cost, and the demand parameter, we have:

$$\Pi = (p - c)Q(p) = (p - c)(a - p) = ap - ac - p^2 + pc$$

To obtain the maximum, we must differentiate with respect to the choice variable, which in this case is price. Note that a monopolist can control price or quantity, which are totally inter-dependent in this case.

$$\frac{\partial \Pi}{\partial p} = a - 2p + c = 0$$

so monopoly price is

$$p_M = \frac{a + c}{2}$$

$$Q\left(p_M\right) = a - \left(\frac{a+c}{2}\right) = \frac{a-c}{2} \text{ is the monopolist output quantity}$$

Monopoly profit is therefore given by:

$$\begin{aligned} \Pi_M = pq - cq &= \left(\frac{a+c}{2}\right)\left(\frac{a-c}{2}\right) - c\left(\frac{a-c}{2}\right) \\ &= \frac{a^2-c^2}{4} - \left(\frac{2ac-2c^2}{4}\right) = \frac{(a-c)^2}{4} \end{aligned}$$

Cournot Competition

- In this case, competing firms i, j non-cooperatively and simultaneously choose quantity for a homogeneous product. A good example would be oil production.

- The total output in the market is $\sum_i q_i = Q$ and in the case of two firms, for example, the total market quantity would be $Q = q_i + q_j$

- Price is still given by $p = a - Q$ which reflects demand for the whole market, which is again based on consumer preferences with price and quantity inversely related.

- Firm i therefore chooses q_i to maximize profit $\Pi_i = (p - c)q_i$

Firm i seeks to maximize profit by choosing q_i, and we can re-write this as:

$$\Pi_i = (p - c)q_i = \left(a - \left[\sum_i q_i\right] - c\right) q_i = \left(a - \left[q_i + \sum_{j \neq i} q_j\right] - c\right) q_i$$

$$\Pi_i = aq_i - q_i^2 - q_i q_j - cq_i$$

Differentiating profit with respect to quantity, which the the choice variable here, we obtain:

$$\frac{\partial \Pi_i}{\partial q_i} = a - 2q_i - q_j - c = 0$$

- FOC:

$$\frac{\partial \Pi_i}{\partial q_i} = a - 2q_i - q_j - c = 0$$

- With only two identical firms as the simplest case, we can observe that $q_i = q_j = \hat{q}$ in a symmetric pure strategy NE, we can rewrite this as:

$$a - c = 2q_i + q_j = 3\hat{q}$$

and re-arrange to obtain our Nash Equilibrium Cournot quantity outcome for each firm: $\hat{q} = \frac{a-c}{3}$

- Substituting back in for price, we obtain:

$$p = a - Q = a - 2\left(\frac{a-c}{3}\right) = \frac{3a}{3} - \frac{2a}{3} + \frac{2c}{3} = \frac{a+2c}{3}$$

which gives

$$\Pi_i = \Pi_j = (p - c)\hat{q} = \left(\frac{a+2c}{3} - c\right)\left(\frac{a-c}{3}\right) = \left[\frac{(a-c)^2}{9}\right]$$

- Notice that in the Cournot duopoly scenario, we have market quantity:

$$Q = q_i + q_j = 2\hat{q} = 2\left(\frac{a-c}{3}\right) > \frac{a-c}{2} = q_M$$

  so the market has a larger quantity under this Cournot duopoly scenario than it would have with a monopoly.

- Note that Cournot duopoly profit is lower than the monopolist's profit.

Stackelberg Duopoly

- Now consider a sequential version of this quantity competition: one firm (the "leader") moves first, but otherwise the same as Cournot.
- Firm 2 (the "follower") sets q_2 as a function of q_1, and the best response function is:

$$R_2\left(q_1\right) = \left(\frac{a - c - q_1}{2}\right)$$

- Firm 1 correctly predicts this in setting its output first to maximize profit:

$$\max_{q_1}(\Pi_1) = \left(a - \left[\sum_i q_i\right] - c\right) q_i = \left(a - \left(q_1 + R_2(q_1)\right) - c\right) q_1$$

$$= aq_1 - q_1^2 - \left(\frac{aq_1 - cq_1 - q_1^2}{2}\right) - cq_1.$$

- Taking the partial derivative and setting equal to zero, we have:

$$\frac{\partial \Pi_1}{\partial q_1} = 0 = a - 2q_1 - \frac{a}{2} + \frac{c}{2} + q_1 - c$$

and solving this obtains $q_1^* = \frac{a-c}{2}$ and $q_2^* = \frac{a-c-q_1}{2} = \frac{a-c}{4}$

- With identical firms, the leader produces more and secures more profit, so moving first is advantageous.

$$\begin{bmatrix} q_1^* = \frac{a-c}{2} \\ \\ q_2^* = \frac{a-c}{4} \end{bmatrix}$$

Comparison of Models

$$\left[Q^{\text{total}}_{\text{Stackel}} = \frac{3}{4}(a-c)\right] > \left[Q^{\text{total}}_{\text{Cournot}} = \frac{2}{3}(a-c)\right] > \left[Q^{\text{total}}_{\text{Monopoly}} = \frac{1}{2}(a-c)\right]$$

Discussion

- Monopoly is worst for consumers, with the smallest total market quantity and highest profits per firm.
- As there are more firms , profits decline and total market output increases.
- Assuming no externalities, it is inefficient to have a quantity below the free market equilibrium level because people who value a product at an amount higher than the cost of production are unable to acquire this product. Remember that demand curve represents "willingness to pay" and supply curve represents "willingness to sell".
- Under-producing certain types of products may create strategic issues (transportation, defense, food...) or involve negative externalities in addition to the market failure caused by the inefficiency.

Extra Practice - See Lecture Notes

Consider a Cournot scenario with J > 2 firms which each have constant unit production cost c and face a market with inverse demand function $p(q) = a - bq$ with $a > c \geq 0$ and demand elasticity parameter $b > 0$

Note that a is still a baseline demand parameter and elasticity b is kept "normalized" (equal to 1 for simplicity) throughout all of the examples.

a) Show the monopoly outcome for $J = 1$.

b) Find the quantity and profit outcomes when J=3.

c) Show what happens as $J \to \infty$ and describe the effect on consumers.

[Hint: the market will get more competitive]

Discussion - Competition

- Implications for consumer and producer surplus?
- There are many policy actions to address inefficiencies arising from market power: some of these were discussed last week.
- Incidence of subsidy and value capture: what is the problem with government subsidizing production to increase quantity to a level closer to the efficient level?
- The last example will evaluate a potential market failure scenario arising from asymmetric information / adverse selection instead of market power.

Adverse Selection: Akerlof's Market for Lemons

- Suppose there are two types of new cars available for sale: good cars and low quality "lemons", which account for proportion θ of all cars for sale. Sellers do not distinguish between types. Buyers have no way to observe differences but they know the proportion θ of lemons. After buyers own the car, they eventually ascertain which type it is.
- Assume that: good cars are worth \$2000, lemons are worth \$1000, buyers are risk neutral, and cars do not depreciate over time.

- How do we determine the equilibrium price p_n for new cars?
  - ◇ We can create an equation for price as expected value:

$$p_n = (1 - \theta) * 2000 + \theta * 1000 = 2000 - 1000 * \theta$$

Since dealers sell all cars at the same price, buyers will pay the expected value of a new car.

Akerlof - Used Car Market

- Suppose used car sellers are willing to sell their cars at 20% below the new value. So $p_u^G = \$1600$ and $p_u^L = \$800$.
- Since cars do not depreciate, used car buyers will be willing to pay \$2000 for good used cars and \$1000 for lemons. This means there is a surplus of either \$400 or \$200 from each sale... which is a gain from trade!

- Question: What will be the equilibrium price of used cars?
- Buyers cannot distinguish between types but sellers know the true type. Assuming sellers wish to maximize revenue, for any $p_u \geq \$800$ the owners of lemons will benefit from selling. However, for $p_u < \$1600$ those who own good cars will not want to sell. This means the equilibrium used car market price will depend on θ.

Akerlof - Used Car Equilibrium

- If the share of lemons is high enough, then the good quality cars will not be sold on the market.
- Consider the case where $\theta = 0.5$: The expected value of a used car is

$$p_u^{\theta=0.5} = (1-\theta) * 2000 + \theta * 1000 = 2000 - 1000 * \theta = \$1500$$

- In this case, no owner of a good car will sell, so the price cannot be \$1500. The market essentially unravels as only lemons are for sale, with price $p_u = \$1000$ as the result.

- If $\theta = 0.2$ for example, we now have

$$p_u^{\theta=0.2} = (1-0.2) * 2000 + 0.2 * 1000 = 2000 - 1000 * \theta = \$1800$$

- With the given values and assumptions in this problem, the maximum proportion of lemons in which the market does not unravel is $\theta = 0.4$ with a resulting price of \$1600.

Discussion and Comments

- Which industries do you think face the largest issues with adverse selection?
- What are some real-world cases of asymmetric information arising in markets?
- What solutions exist to mitigate the harmful effects of asymmetric information and adverse selection?
- How does this relate to US policies on car insurance and health insurance?

Discussion and Comments

- Which industries do you think face the largest issues with adverse selection?
  - Insurance is the best example
- What are some real-world cases of asymmetric information arising in markets?
  - Collateralized debt (think about packages of securities in the 2008 financial crisis: even the credit ratings agencies, which exist to improve market information, did not fully understand the risk composition)
- What solutions exist to mitigate the harmful effects of asymmetric information and adverse selection?
  - Consider US laws requiring the purchase of car insurance: the market could unravel if only poor drivers purchased insurance, as this would drive up cost and essentially defeat the whole purpose. Insurance costs do go up for accidents, etc, to address incentives.
  - Obamacare was designed according to similar logic.