Formal Models of Market Structure / Competition: Monopoly, Cournot Duopoly, Stackelberg Duopoly

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Intermediate Microeconomics / Industrial Organization

2024

Monopoly Profit Maximization

- Quantity demanded in the market is given by: $Q(p) = a - p$ where a is the "baseline demand parameter" and $p \in (0, a)$ is the price.
- Constant marginal cost: $c \in (0, a)$
- Note that (for simplicity) the elasticity (slope) parameter b is normalized to be equal to 1, with fixed costs assumed to be zero.
- The standard assumption is that demand is exogenously determined by consumer preferences: this is accurately observed by the firm(s) but out of their control (so no marketing, manipulating tastes, or otherwise influencing demand via affecting consumer utility functions...)
- Total profit is price minus cost (unit profit) times total quantity sold:

$$\Pi = (p - c)Q(p)$$

Rewriting the profit function in terms of price, cost, and the demand parameter, we have:

$$\Pi = (p - c)Q(p) = (p - c)(a - p) = ap - ac - p^2 + pc$$

To obtain the maximum, we must differentiate with respect to the choice variable, which in this case is price. Note that a monopolist can control price or quantity, which are totally inter-dependent in this case.

$$\frac{\partial \Pi}{\partial p} = a - 2p + c = 0$$

so monopoly price is

$$p_M = \frac{a + c}{2}$$

$$Q(p_M) = a - \left(\frac{a+c}{2}\right) = \frac{a-c}{2} \quad \text{is the monopolist output quantity}$$

Monopoly profit is therefore given by:

$$\Pi_M = pq - cq = \left(\frac{a+c}{2}\right)\left(\frac{a-c}{2}\right) - c\left(\frac{a-c}{2}\right)$$
$$= \frac{a^2-c^2}{4} - \left(\frac{2ac-2c^2}{4}\right) = \frac{(a-c)^2}{4}$$

Cournot Competition

- In this case, competing firms i, j non-cooperatively and simultaneously choose quantity for a homogeneous product. A good example would be oil production.
- The total output in the market is $\sum_i q_i = Q$ and in the case of two firms, for example, the total market quantity would be $Q = q_i + q_j$
- Price is still given by $p = a - Q$ which reflects demand for the whole market, which is again based on consumer preferences with price and quantity inversely related.
- Firm i therefore chooses q_i to maximize profit $\Pi_i = (p - c)q_i$

Firm i seeks to maximize profit by choosing q_i, and we can re-write this as:

$$\Pi_i = (p - c)q_i = \left(a - \left[\sum_i q_i\right] - c\right) q_i = \left(a - \left[q_i + \sum_{j \neq i} q_j\right] - c\right) q_i$$

$$\Pi_i = aq_i - q_i^2 - q_i q_j - cq_i$$

Differentiating profit with respect to quantity, which the the choice variable here, we obtain:

$$\frac{\partial \Pi_i}{\partial q_i} = a - 2q_i - q_j - c = 0$$

- FOC:
$$\frac{\partial \Pi_i}{\partial q_i} = a - 2q_i - q_j - c = 0$$
- With only two identical firms as the simplest case, we can observe that $q_i = q_j = \hat{q}$ in a symmetric pure strategy NE, we can rewrite this as:
$$a - c = 2q_i + q_j = 3\hat{q}$$
and re-arrange to obtain our Nash Equilibrium Cournot quantity outcome for each firm: $\hat{q} = \frac{a-c}{3}$
- Substituting back in for price, we obtain:
$$p = a - Q = a - 2\left(\frac{a-c}{3}\right) = \frac{3a}{3} - \frac{2a}{3} + \frac{2c}{3} = \frac{a+2c}{3}$$
which gives
$$\Pi_i = \Pi_j = (p-c)\hat{q} = \left(\frac{a+2c}{3} - c\right)\left(\frac{a-c}{3}\right) = \left[\frac{(a-c)^2}{9}\right]$$

- Notice that in the Cournot duopoly scenario, we have market quantity:

$$Q = q_i + q_j = 2\hat{q} = 2\left(\frac{a-c}{3}\right) > \frac{a-c}{2} = q_M$$

so the market has a larger quantity under this Cournot duopoly scenario than it would have with a monopoly.

- Note that Cournot duopoly profit is lower than the monopolist's profit.

Stackelberg Duopoly

- Now consider a sequential version of this quantity competition: one firm (the "leader") moves first, but otherwise the same as Cournot.
- Firm 2 (the "follower") will set its quantity q_2 as a function of the leader's quantity q_1 to maximize its profit:

$$\max_{q_2}(\Pi_2) = \left(a - \left[\sum_i q_i\right] - c\right) q_i = (a - [q_2 + q_1] - c)\, q_2$$

$$= aq_2 - q_2^2 - q_1 q_2 - cq_2$$

- Differentiating with respect to the follower firm's choice variable (its quantity) we obtain the FOC which will define its best response:

$$\frac{\partial \Pi_2}{\partial q_2} = 0 = a - 2q_2 - q_1 - c$$

- Firm 2 (the "follower") sets q_2 as a function of q_1 to maximize its profit, and its best response function (obtained from the FOC) is:

$$R_2\left(q_1\right) = \left(\frac{a - c - q_1}{2}\right)$$

- Firm 1 (the "leader") correctly predicts this in setting its output first to maximize its own profit by choosing its quantity:

$$\max_{q_1}(\Pi_1) = \left(a - \left[\sum_i q_i\right] - c\right) q_i = \left(a - \left[q_1 + R_2(q_1)\right] - c\right) q_1$$

$$= aq_1 - q_1^2 - \left(\frac{aq_1 - cq_1 - q_1^2}{2}\right) - cq_1.$$

- Taking the partial derivative and setting equal to zero, we have:

$$\frac{\partial \Pi_1}{\partial q_1} = 0 = a - 2q_1 - \frac{a}{2} + \frac{c}{2} + q_1 - c$$

and solving this obtains $q_1^* = \frac{a-c}{2}$ and $q_2^* = \frac{a-c-q_1}{2} = \frac{a-c}{4}$

- With identical firms, the leader produces more and secures more profit, so moving first is advantageous.

$$\begin{bmatrix} q_1^* = \frac{a-c}{2} \\ \\ q_2^* = \frac{a-c}{4} \end{bmatrix}$$

Comparison of Models

$$\left[Q^{\text{total}}_{\text{Stackel}} = \frac{3}{4}(a-c)\right] > \left[Q^{\text{total}}_{\text{Cournot}} = \frac{2}{3}(a-c)\right] > \left[Q^{\text{total}}_{\text{Monopoly}} = \frac{1}{2}(a-c)\right]$$

- Quantity supplied to the market is the lowest with a (private) monopolist, higher with a duopoly, and highest with an asymmetric duopoly Stackelberg scenario where one firm is a first-mover.
- Assuming that government / policymakers and consumers want as close to the free market competitive equilibrium quantity as possible, they benefit from having more players and circumstances less conducive to allowing collusive behaviors.

Discussion & Insights

- Monopoly is worst for consumers, with the smallest total market quantity and highest profits per firm.
- With more firms profits decline and total market output increases.
- Assuming no externalities, it is inefficient to have a quantity below the free market equilibrium level because people who value a product at an amount higher than the cost of production are unable to acquire this product. Remember that demand curve represents "willingness to pay" and supply curve represents "willingness to sell".
- Under-producing certain types of products may create strategic issues (transportation, defense, food...) or involve negative externalities in addition to the market failure caused by the inefficiency.

Extra Practice Questions

Consider a Cournot scenario with J $>$ 2 firms which each have constant unit production cost c and face a market with inverse demand function $p(q) = a - bq$ with $a > c \geq 0$ and demand elasticity parameter $b > 0$

Note that a is still a baseline demand parameter and elasticity b is kept "normalized" (equal to 1 for simplicity) throughout all of the examples.

a) Calculate the monopoly outcome for $J = 1$.

b) Find the quantity and profit outcomes when J=3.

c) Determine what happens as $J \to \infty$ and how this affects consumers.

[Hint: compare to Bertrand price competition...]

Discussion - Competition

- Implications for consumer and producer surplus?
- There are many policy actions to address inefficiencies arising from market power.
- Incidence of subsidy and value capture: what is the problem with government subsidizing production to increase quantity to a level closer to the efficient level?
- We can also evaluate potential market failure scenarios (even with many firms) arising from asymmetric information instead of market power...