BASIC GAME THEORY & APPLICATIONS

SPI 615b (Microeconomics)

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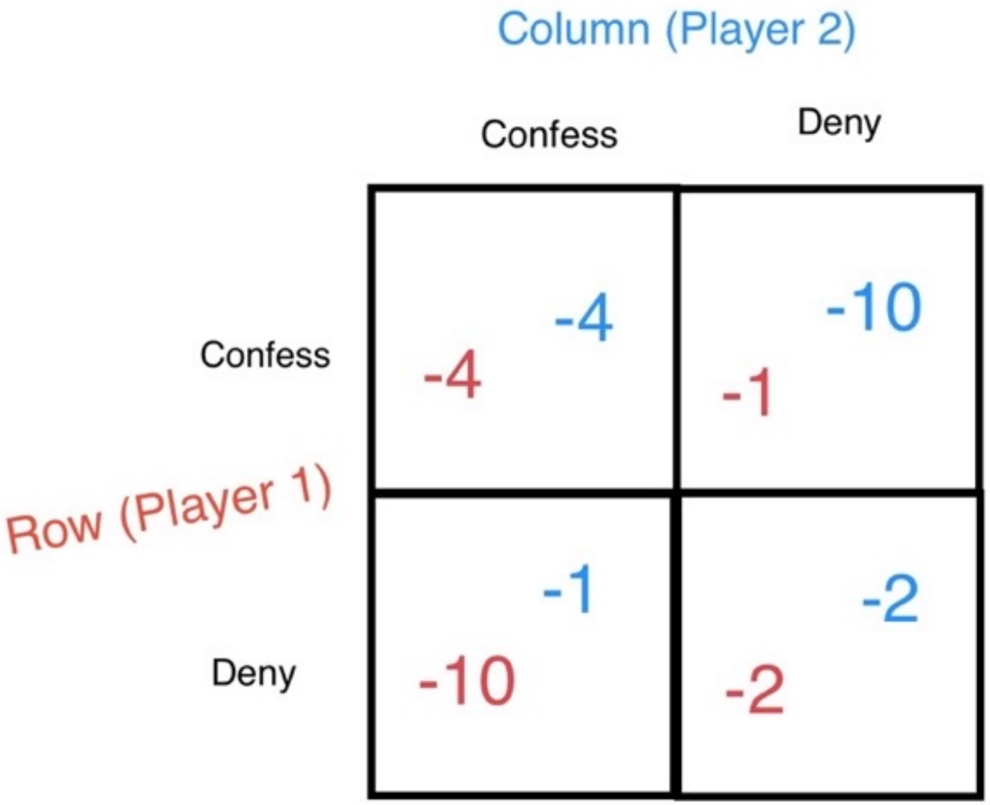
CLASSIC PRISONERS' DILEMMA



Column (Player 2)

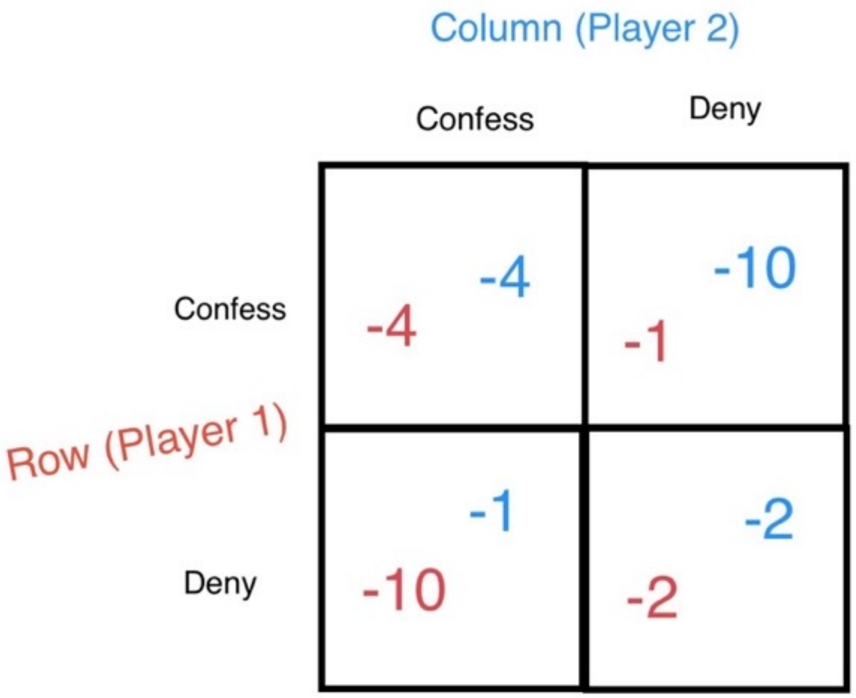
Row (Player 1)

	Confess	Deny
Confess	-4 -4	-10 -1
Deny	-1 -10	-2 -2

Standard Game Theory Assumptions:

- The numbers should always be interpreted as utility payoffs, but with a function for utility over money these could be converted and interpreted in dollar values.
- Each player can only control their own choice of actions and only cares about their own outcomes, and there is no coordination.
- Matrix format indicates simultaneous and uncoordinated decisions, and these adversarial situations are assumed to be played only once unless they are indicated to be a repeated game.

CLASSIC PRISONERS' DILEMMA - DOMINANT STRATEGY NASH EQUILIBRIUM



Column (Player 2)

Row (Player 1)

	Confess	Deny
Confess	-4 -4	-10 -1
Deny	-1 -10	-2 -2

- In this symmetric simultaneous game, both suspects have a better outcome from choosing *Confess* regardless of what the other player does.

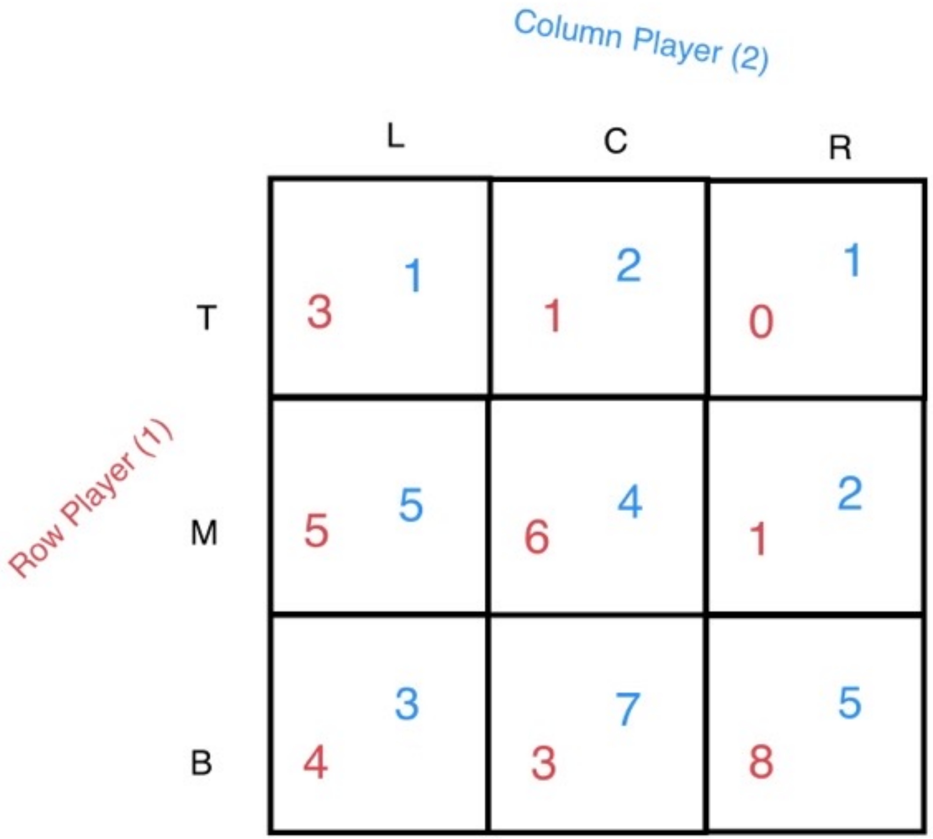
- *Confess* is therefore a **dominant strategy** because it is always the **best response,** and this is true for both players

- The existence of a dominant strategy guarantees the existence of a **Nash Equilibrium**: a situation in which no player involved has any incentive to change their strategy

- The unique Nash Equilibrium that will result as the outcome of this game is {Confess, Confess}

- While a better outcome is possible for both players, it cannot be achieved: each player has an ability to gain by deviating from a situation of {Deny, Deny} and therefore even if they could communicate and agree to this, both would deviate because they do not expect the other player to follow through with the commitment.

MATRIX GAME EXAMPLE 1

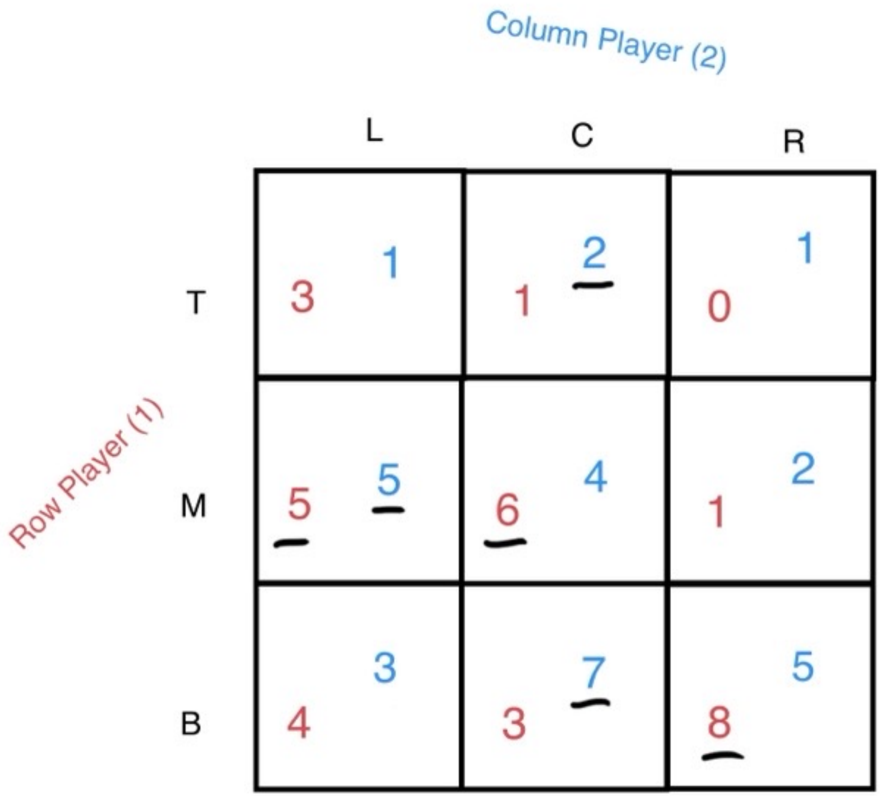


Column Player (2)

Row Player (1)

	L	C	R
T	3 1	1 2	0 1
M	5 5	6 4	1 2
B	4 3	3 7	8 5

MATRIX GAME EXAMPLE 1



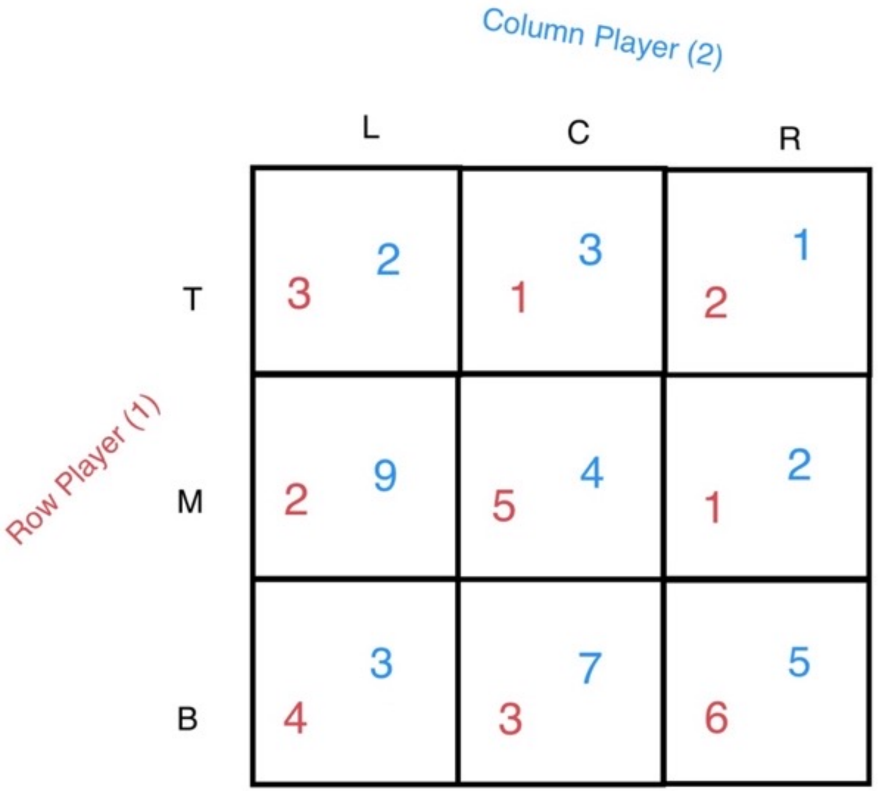
Column Player (2)

Row Player (1)

	L	C	R
T	3, 1	1, $\underline{2}$	0, 1
M	$\underline{5}$, $\underline{5}$	$\underline{6}$, 4	1, 2
B	4, 3	3, $\underline{7}$	$\underline{8}$, 5

Underlining to indicate all "best responses" we can see that the unique Nash Equilibrium is {Middle, Left}

MATRIX GAME EXAMPLE 2

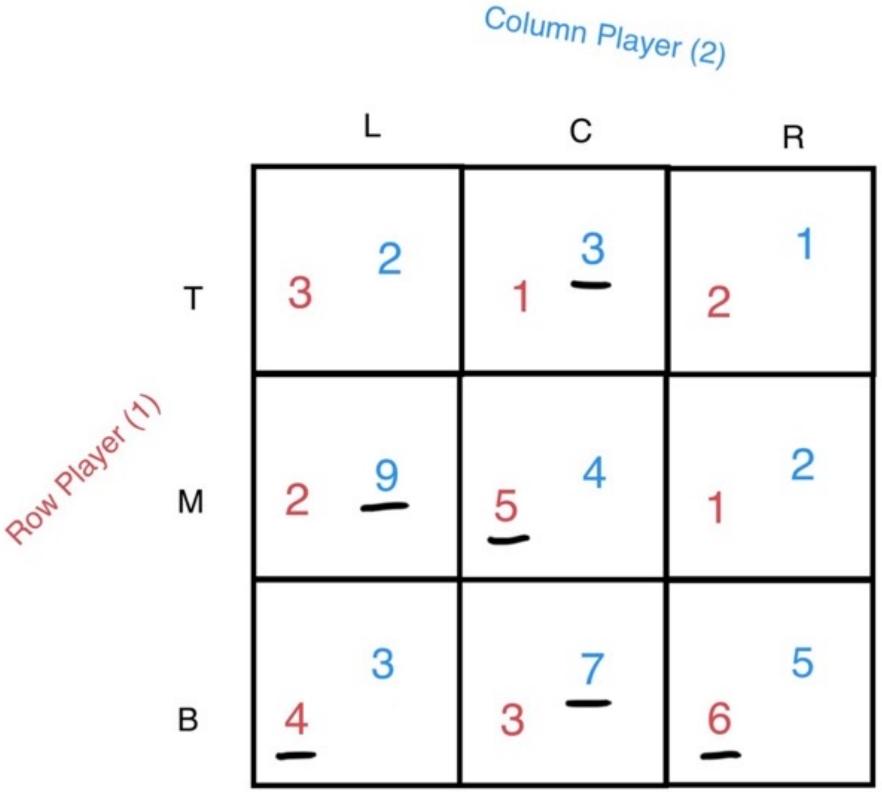


Column Player (2)

Row Player (1)

	L	C	R
T	3, 2	1, 3	2, 1
M	2, 9	5, 4	1, 2
B	4, 3	3, 7	6, 5

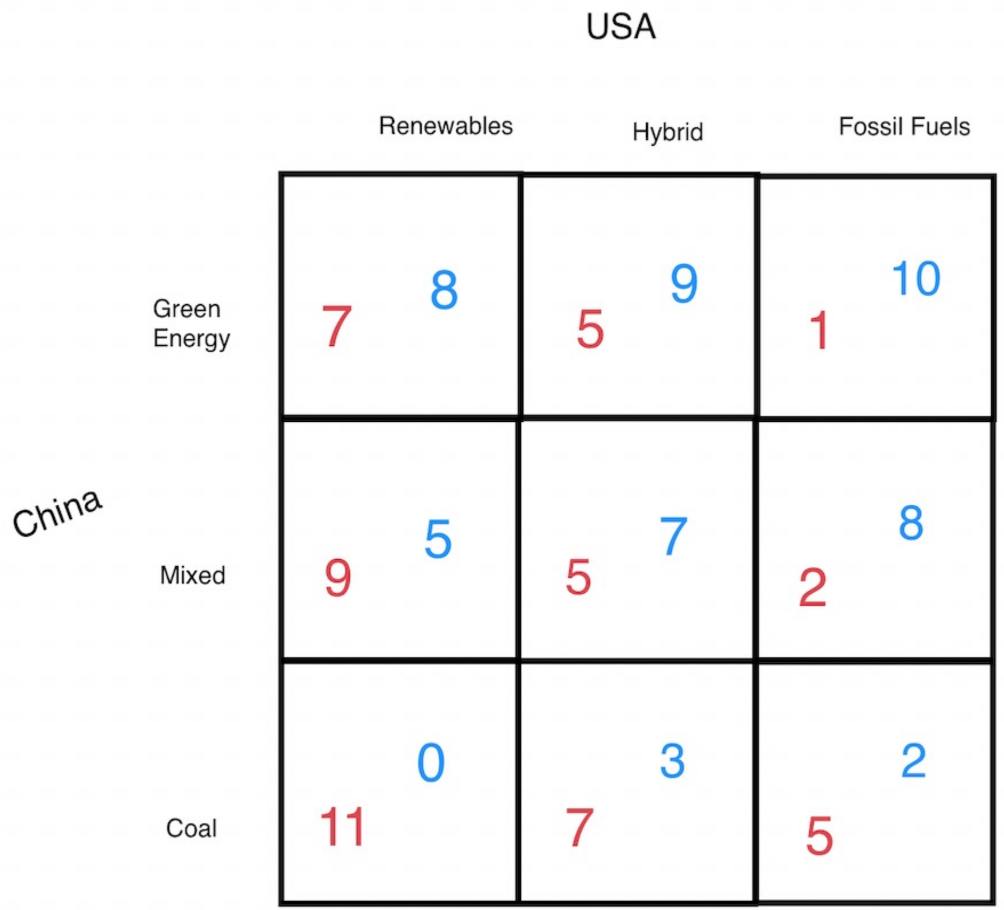
MATRIX GAME EXAMPLE 2



	L	C	R
T	3, 2	1, $\underline{3}$	2, 1
M	2, $\underline{9}$	$\underline{5}$, 4	1, 2
B	$\underline{4}$, 3	3, $\underline{7}$	$\underline{6}$, 5

There is no Nash Equilibrium in this game.

CLIMATE CHANGE SIMULTANEOUS GAME:



USA

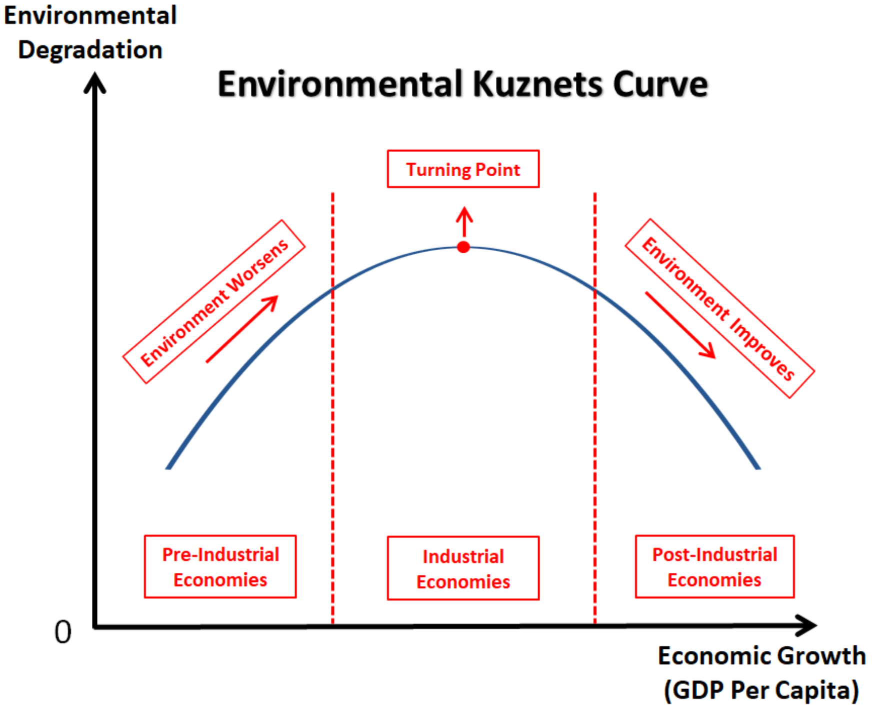
China

	Renewables	Hybrid	Fossil Fuels
Green Energy	7 8	5 9	1 10
Mixed	9 5	5 7	2 8
Coal	11 0	7 3	5 2

CLIMATE CHANGE OUTCOME & REASONING

USA

China	Renewables	Hybrid	Fossil Fuels
Green Energy	7, 8	5, 9	1, $\underline{10}$
Mixed	9, 5	5, 7	2, $\underline{8}$
Coal	$\underline{11}$, 0	$\underline{7}$, $\underline{3}$	$\underline{5}$, 2



The unique NE here is <u>{Coal, Hybrid}</u> : note that Coal is a strictly dominant strategy for China

The inverse parabolic “Environmental Kuznet’s Curve” explains how industrialization increases pollution, but economies with high enough wealth per capita are ultimately willing to pay more to reduce pollution: doing this has a smaller relative cost and increasing relative benefit, so eventually pollution declines

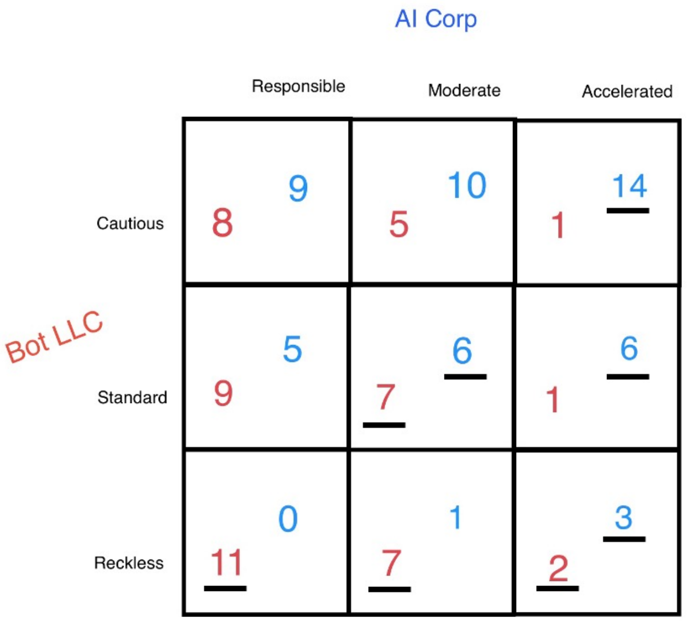
ARTIFICIAL INTELLIGENCE MATRIX GAME

AI Corp

Bot LLC

	Responsible	Moderate	Accelerated
Cautious	9 8	10 5	14 1
Standard	5 9	6 7	6 1
Reckless	0 11	1 7	3 2

ARTIFICIAL INTELLIGENCE MATRIX GAME



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Reckless	0 $\underline{11}$	1 $\underline{7}$	$\underline{3}$ $\underline{2}$

- The two Nash Equilibria here are **{S , M}** and **{R , A}**

- Reckless and Accelerated are both ***weakly dominant***

- Cautious and Responsible are both “strictly dominated” (***never a best response***)

- One NE outcome is much better than the other

This shows us the picture of a contemporary AI race where two firms each want to capture market share and take over the industry: each firm would like the other one to be safe with development while it selfishly pursues an aggressive and dangerous approach.

A key takeaway here is that if government was able to induce the firms to make choices resulting in the “better” NE in the center, then both firms would be better off in the context of this game and overall society would fare better. This might look like subsidies or incentives to make the payoffs in the middle cell become 7.1 and 6.1 or taxes and threats of punishment to make the adjacent payoffs slightly lower for each firm, thus removing the weak dominance of the aggressive and unsafe AI development strategies in either case. An NE is a place where everyone gets “stuck” so it would not be possible to change the outcome once the ”bad” NE scenario occurs.

HOTELLING MODEL: "SELLERS ON THE BEACH"

- Suppose the universe is a one-dimensional closed and bounded 8 mile long beach with a large number of identical and uniformly distributed consumers.
- Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.
- There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.

(A) If the firms must simultaneously and non-cooperatively choose where to locate along the 8 mile beach, what locations will they pick?

SELLERS ON THE BEACH

- Suppose the universe is a one-dimensional closed and bounded 8 mile long beach with a large number of identical and uniformly distributed consumers.
- Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.
- There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.

(B) If you were a benevolent social planner in charge of all decisions for the beach, where would you choose to place the two firms to maximize consumer welfare (social surplus) and why?

What effect would this market intervention have on the utility levels of the two sellers?

SELLERS ON THE BEACH: MOVEMENT CONCEPT

- Suppose the universe is a one-dimensional closed and bounded 8 mile long beach with a large number of identical and uniformly distributed consumers.
- Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.
- There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.
- **Now suppose each firm can move 0.1 miles per day at no cost.**

(C) If Jay begins located at mile marker 1.0 and Dre's starting location is randomly drawn with a uniform probability along the beach, who is more likely to get the most gold coins over time?

SELLERS ON THE BEACH – THREE FIRMS

- Suppose the universe is a one-dimensional closed and bounded 8 mile long beach with a large number of identical and uniformly distributed consumers.
- Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.
- There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.
- **Now suppose each firm can move 0.1 miles per day at no cost.**

(D) If a third firm joins, now where will the three sellers locate?

What effect will this have on consumer utility?

SELLERS ON THE BEACH – ONE FIRM

- Suppose the universe is a one-dimensional closed and bounded 8 mile long beach with a large number of identical and uniformly distributed consumers.
- Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.
- There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.

(E) If the firms merged to form a monopoly seller, where would it locate?

What effect would this have on consumer utility?

Metaphorically, what would this represent in the context of politics and policy?

Sellers on the Beach:

- Suppose the universe is a one-dimensional closed and bounded 8 mile long beach with a large number of identical and uniformly distributed consumers.

- Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.

- There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.

DISCUSSION

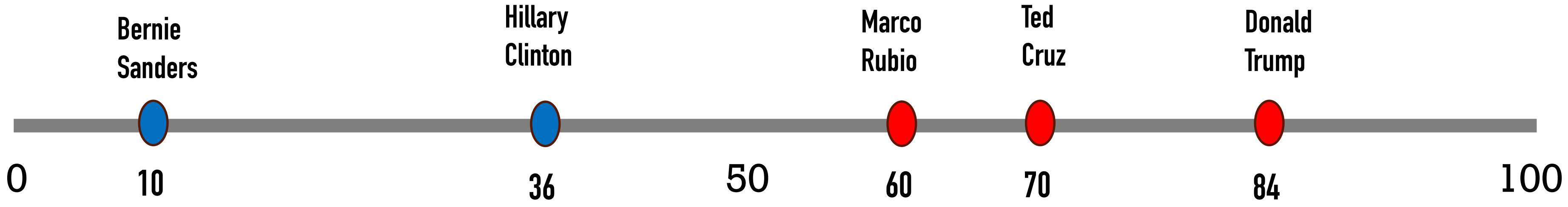
What real-world scenarios could this game represent?

Where do we see firms choose locations in this way?

Where do we observe the opposite location pattern and why?

APPLICATION OF SPATIAL COMPETITION:

APPLICATIONS FOR POLITICAL CANDIDATE LOCATIONS (1-DIMENSIONAL IDEOLOGY)

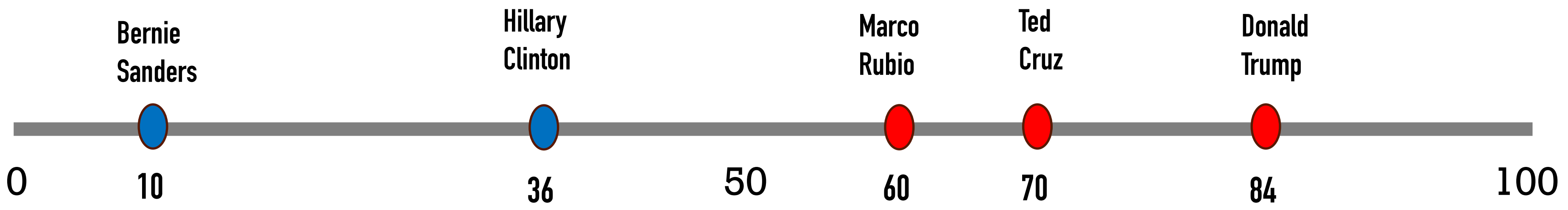


- The distance to the halfway point between Sanders and Clinton is (36-10)/2 = 13
  - This means that a voter located at point 23 is exactly indifferent: (10 + 13) = 23 = (36 – 13)
  - Voters at a point to the left of 23 will prefer Sanders and voters to the right of 23 will prefer Clinton

- If the Democratic primary contains all voters from 0 to 50, then Sanders wins voters from 0 to 23 and Clinton wins voters from 23 to 50
  - Sanders receives (23/50) = 46% of the vote and Clinton receives (27/50) = 54% of the vote

UNDERSTANDING SPATIAL COMPETITION:

APPLICATIONS FOR POLITICAL CANDIDATE LOCATIONS



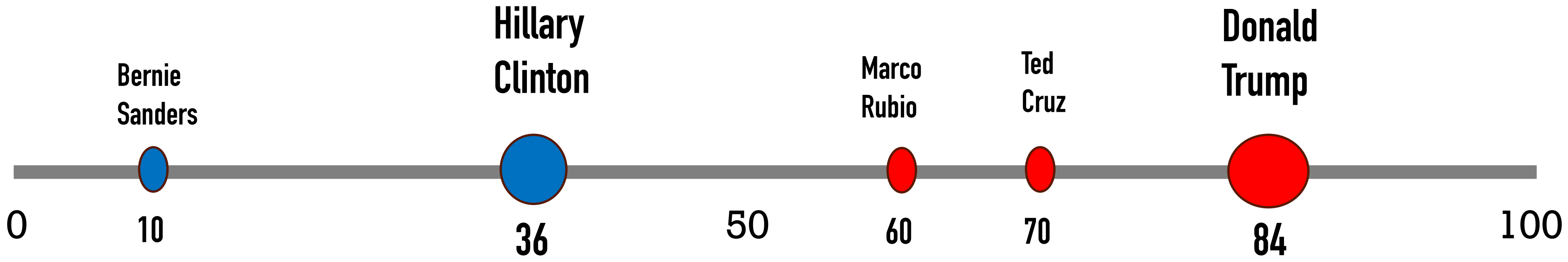
- The distance to the halfway point between Rubio and Cruz is (70-60)/2 = 5
  - This means that a voter located at point 65 is exactly indifferent: (60 + 5) = 65 = (70 – 5)
  - Voters at a point to the left of 65 will prefer Rubio and voters to the right of 65 will prefer Cruz

- The distance to the halfway point between Cruz and Trump is (84-70)/2 = 7
  - This means that a voter located at point 77 is exactly indifferent: (70 + 7) = 77 = (84 – 7)
  - Voters at a point to the left of 77 will prefer Cruz and voters to the right of 77 will prefer Trump

- Rubio wins voters 50 to 65 for (15/50) = 30% of the Republican vote
- Cruz wins voters 65 to 77 for (12/50) = 24% of the Republican vote
- Trump wins voters 77 to 100 for (23/50) = 46% of the Republican vote

IMPLICATIONS OF SPATIAL COMPETITION:

APPLICATIONS FOR POLITICAL CANDIDATE LOCATIONS



- In a general election matchup we can use the same process: *assuming that the candidates keep their same positions*, the distance to the midpoint between Clinton and Trump is (84-36)/2= 24

  - This means that the general election voter who is indifferent between Clinton and Trump is located at point (36 + 24) = **60** = (84 – 24)
  - In reality with a two-party system, candidates usually try to re-position themselves in the general election by moving towards the overall median voter after winning the primary

- The predicted outcome here is a victory for Clinton with 60% of the total vote, including capturing the median voter
  - With a uniform or other symmetric distribution, the median voter is at position 50
  - According to these numbers, Rubio would have won against Clinton, Cruz would have lost against Clinton, and Sanders would have lost against any of the other candidates

TRAGEDY OF THE COMMONS: FISHERS ON A LAKE

- Lake Commons can be freely accessed by any fishers who wants to take out a boat, which costs $c = \$20$ per day. Fish are sold on a large competitive market at price $p = \$10$ per fish. Let b denote the number of boats on Lake Commons on a given day and let x denote the total number of fish caught on Lake Commons per day, which depends on the number of boats: $x = 100\sqrt{b}$. (Remember that profit equals total revenues minus total costs.)

Q1) With free entry (no barriers, no license required, etc)… quantify how many fishers will be active on Lake Commons and how many total fish will be caught.

Q2) Find the number of active fisherman that maximizes profits.

TRAGEDY OF THE COMMONS – UNREGULATED N.E.

- Lake Commons can be freely accessed by any fishers who wants to take out a boat, which costs $c = \$20$ per day. Fish are sold on a large competitive market at price $p = \$10$ per fish. Let b denote the number of boats on Lake Commons on a given day and let x denote the total number of fish caught on Lake Commons per day, which depends on the number of boats: $x = 100\sqrt{b}$

Q1)

- The unregulated Nash Equilibrium is fishers will enter until profits are zero: if there is still any additional profit possible, then the situation is not an NE yet and someone else will enter.
- Profit = Revenue – Cost $= px - cb = 10\,(100\sqrt{b)} - 20b = 0$
- Solving for zero profits:

$$1000\sqrt{b)} = 20b$$

$$b_u = 2500 \text{ and } x_u = 5000$$

TRAGEDY OF THE COMMONS – PROFIT MAXIMIZATION

- Lake Commons can be freely accessed by any fishers who wants to take out a boat, which costs $c = \$20$ per day. Fish are sold on a large competitive market at price $p = \$10$ per fish. Let b denote the number of boats on Lake Commons on a given day and let x denote the total number of fish caught on Lake Commons per day, which depends on the number of boats: $x = 100\sqrt{b}$

Q2)

- To find the profit-maximizing level, we can take the derivative of this concave profit function with respect to the number of boats, set equal to zero, and solve for the peak:

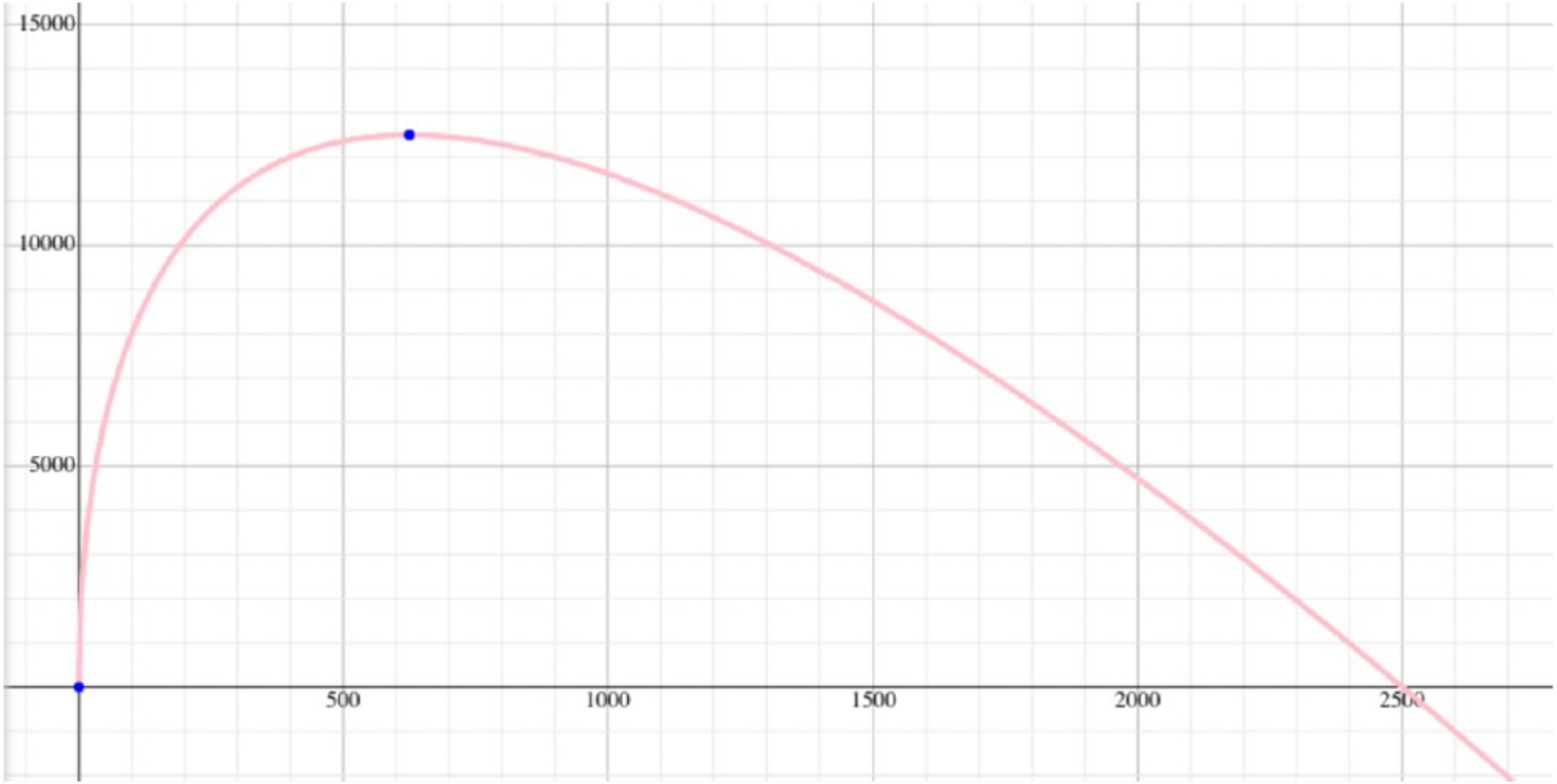
$$\frac{\partial \pi}{\partial b} = 0 = 500b^{\frac{-1}{2}} - 20$$

$$20\sqrt{b} = 500$$

$$400b = 250000$$

$$b^* = 625$$

GRAPHICALLY: PLOT OF PROFIT OVER NUMBER OF BOATS



TRAGEDY OF THE COMMONS – GOVERNMENT INTERVENTION

- Lake Commons can be freely accessed by any fishers who wants to take out a boat, which costs $c = \$20$ per day. Fish are sold on a large competitive market at price $p = \$10$ per fish. Let b denote the number of boats on Lake Commons on a given day and let x denote the total number of fish caught on Lake Commons per day, which depends on the number of boats: $x = 100\sqrt{b}$

Now the government wants to generate revenues so it will charge a *license fee* to fish each day. Suppose a one-day license to fish on Lake Commons now costs $\$\mathbf{f}$ dollars and the government's only objective is tax revenue maximization.

Q3) What will the government set for this license fee?
Quantify the effect on the market.

Using the NE behavior of entry until profits are zero, we can re-write the original zero profit condition to include the fee f as

$$\begin{aligned} 10x &= (20+f)b \\ 10(100\sqrt{b}) &= 20b + fb \\ (\frac{1000}{\sqrt{b}}) &= 20 + f \\ b &= \left(\frac{1000}{20+f}\right)^2 \end{aligned}$$

The government will maximize revenue:

$$R = \max_f(f * b) = f * \left(\frac{1000}{20+f}\right)^2 = \frac{1000000 * f}{400 + 40f + f^2}$$

Using the chain rule for derivatives with multiplication, we can obtain marginal revenue for the government and set it equal to zero to obtain our maximum revenue and then solve for the optimal fee:

Revenue:

$$R = \max_{f}(f * b) = f * \left(\frac{1000}{20+f}\right)^2 = \frac{1000000 * f}{400 + 40f + f^2}$$

Marginal Revenue:

$$R' = 1 * \left(\frac{1000}{20+f}\right)^2 + f * \left(\frac{(-1)(2)(1000000)}{(20+f)^3}\right) = 0$$

$$\left(\frac{1000000}{(20+f)^2}\right) = f * \frac{2000000}{(20+f)^3}$$

$$(20 + \mathrm{f})^3 = 2f(20+f)^2$$

$$20+\mathrm{f} = 2\mathrm{f}$$

$$\mathrm{f}^* = 20$$

$$\mathrm{b}_f^* = \left(\frac{1000}{20+f}\right)^2 = 25^2 = 625$$

ENVIRONMENTAL AND POLICY IMPLICATIONS OF THE LICENSE FEE

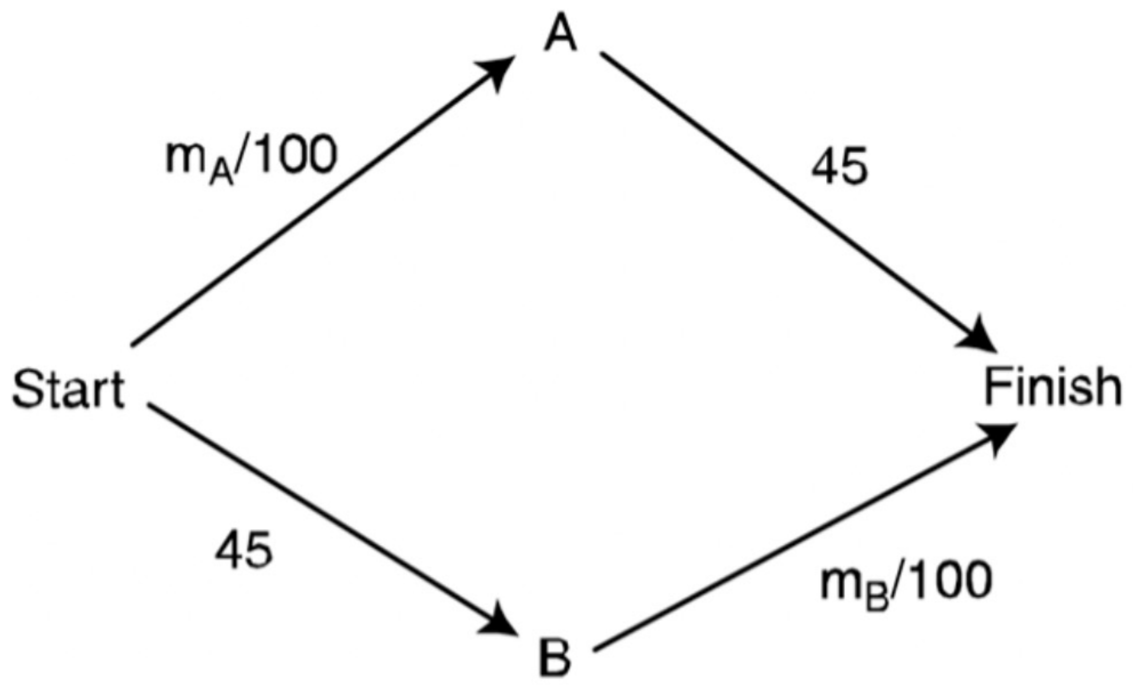
- Notice that the outcome when a self-interested government sets the fee level to maximize its own revenue is the same outcome as the profit-maximizing level!
  - This is what a monopolist would do with control of the market, but it might also be environmentally efficient in a different situation
- There may be a mitigation of negative externalities to consider here: the license fee will decrease the amount of fishing and prevent depletion of the fish population
- This is a common and extremely important policy scenario that is applicable to other cases of depletable natural resources and over-consumption

BRAESS PARADOX – NETWORKS & TRAFFIC

There are **4000** motorists who must commute each morning from **Start** to **Finish** and they must decide whether to drive through either **route A** or **route B**. The route through point A has a travel time (in minutes) equal to 1/100 of the number of motorists $\mathbf{m_A}$ who use tunnel A, plus a fixed 45 minute drive afterwards without traffic. The other route has a fixed 45 minute drive to point B first, plus a travel time of 1/100 of the number of motorists $\mathbf{m_B}$ who use tunnel B. The motorists get utility only from minimizing their individual travel time and there is nothing else you need to consider here.

What is the Nash Equilibrium travel time in this scenario?

BRAESS PARADOX: TRAFFIC ROUTE DIAGRAM

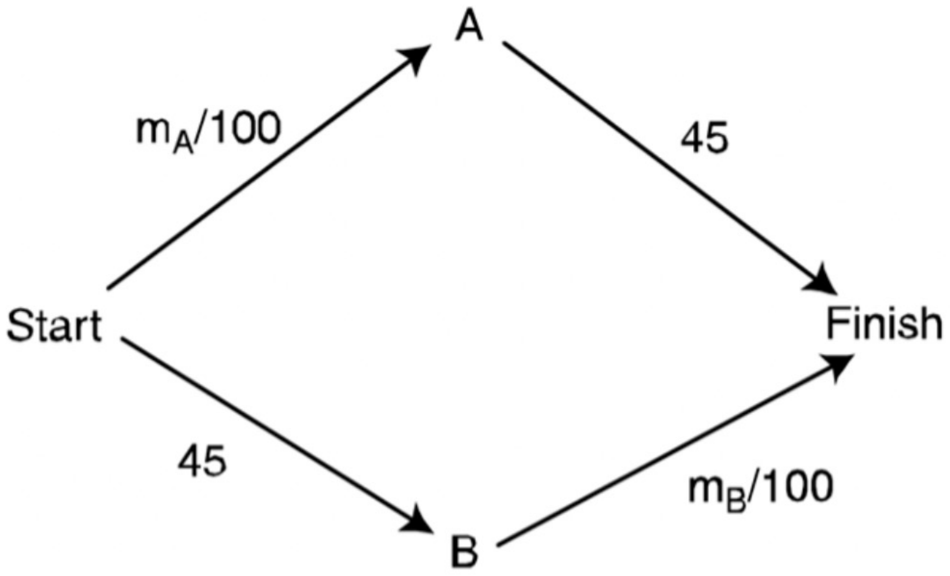


Drivers will always want to take the faster route: this game works over time as well as simultaneously.

In a Nash Equilibrium, the travel time will be the same for both routes.

Note that mathematically we have: $\mathbf{m_A + m_B = 4000}$

BRAESS PARADOX – INITIAL SETUP OUTCOME

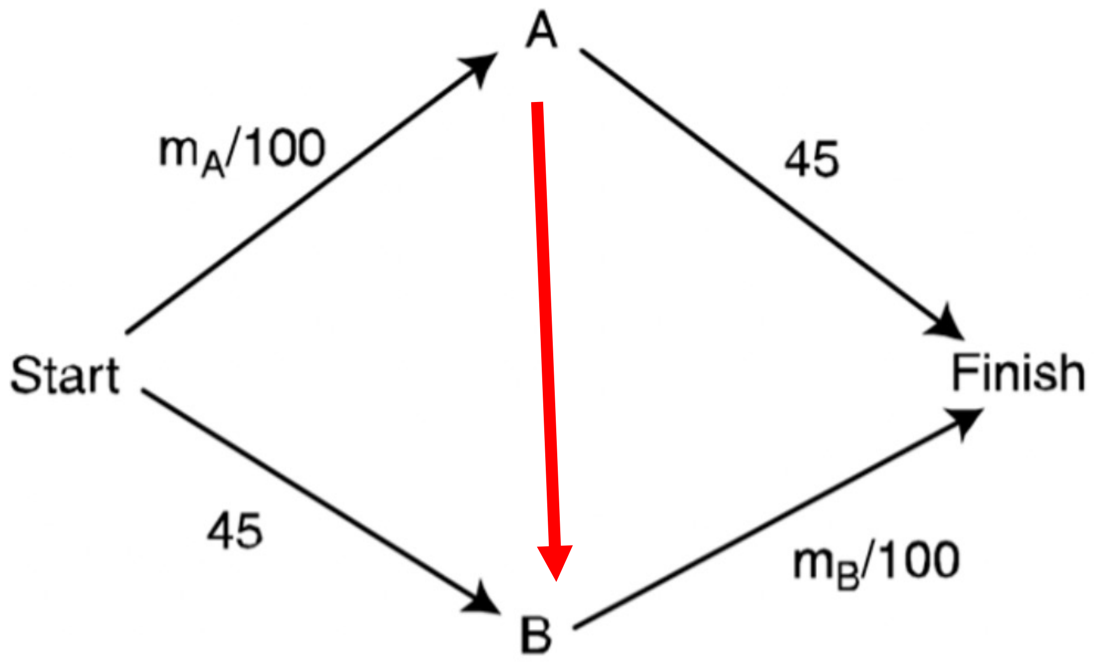


The Nash Equilibrium outcome is 2000 motorists will go to each route:

$$\mathbf{m_A / 100 + 45 = 45 + m_B / 100}$$

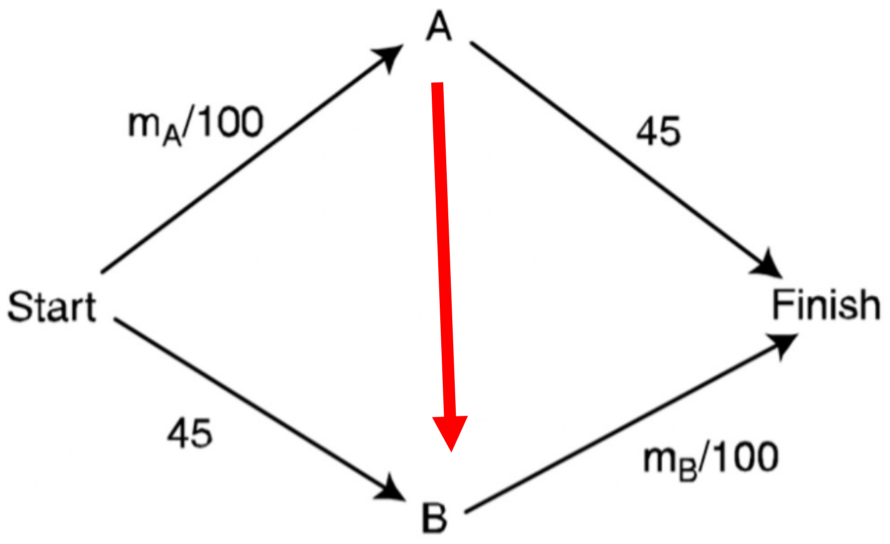
Equilibrium travel time is 45 + 20 = 65 minutes

BRAESS PARADOX



Now what will happen if we add an instant shortcut from point A to point B ?

BRAESS PARADOX – INEFFICIENT OUTCOME



With the shortcut, motorists will change their routes to A to save time until there is nothing more to gain from doing so:

Mathematically: $[\mathbf{m_A / 100 + m_B / 100}] \mathbf{<} [\mathbf{45 + m_B / 100}]$

Now all 4000 motorists will take the shortcut route of Start through A through B to Finish, and the equilibrium total travel time increases to ***80 minutes***.

BRAESS PARADOX – REGULATORY INTERVENTIONS

- Whether physical, online, or electrical traffic, there can often be improvements to the situation
  - This is why tunnels and exit lanes on roads often have physical barriers to prevent lane switching
  - On the internet, there can be similar routing tactics used to prevent issues (technically complicated)
  - Power grid systems involve somewhat similar regulation of energy distribution, including limitations on days when extreme weather could cause blackouts (imagine everyone trying to use the maximum level of air conditioning on the hottest day of the year)