
BASIC GAME THEORY & APPLICATIONS

Microeconomics for Policy

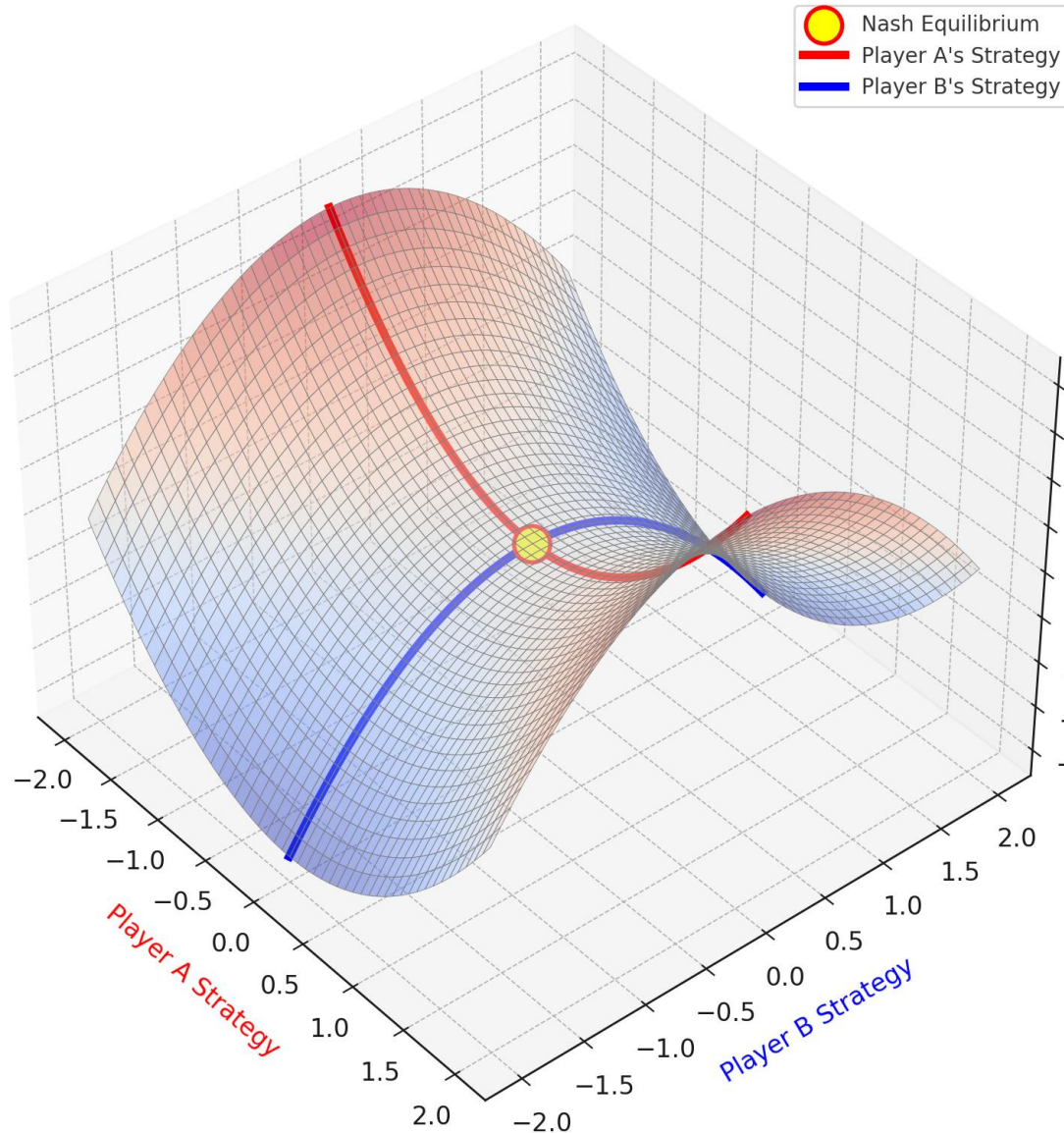
Andrew Gates
Princeton University



MODELING STRATEGIC INTERACTIONS: INTRO TO GAME THEORY

- “Non-Cooperative Games” - John Nash (1951): Princeton Math PhD Dissertation
 - Formalized equilibrium concept for zero sum games without coalitions or communication
 - Built on work from Von Neumann, Morgenstern, Kakutani, Kuhn, Tucker, Gale, Shapley, others
 - Proved that every finite non-cooperative game has at least one equilibrium point, which may be a “pure” or “mixed” strategy (*pure strategies are like binary choices for players’ optimal responses while mixed strategies involve strategic randomization using optimal probabilities...*)

"NON-COOPERATIVE GAMES" - JOHN NASH (1951)



Equilibrium Point:

An n -tuple \mathbf{s} is an *equilibrium point* if and only if for every i

$$(1) \quad p_i(\mathbf{s}) = \max_{\text{all } r_i\text{'s}} [p_i(\mathbf{s}; r_i)].$$

Thus an equilibrium point is an n -tuple \mathbf{s} such that each player's mixed strategy maximizes his payoff if the strategies of the others are held fixed. Thus each player's strategy is optimal against those of the others. We shall occasionally abbreviate equilibrium point by eq. pt.

We say that a mixed strategy s_i uses a pure strategy $\pi_{i\alpha}$ if $s_i = \sum_{\beta} c_{i\beta} \pi_{i\beta}$ and $c_{i\alpha} > 0$. If $\mathbf{s} = (s_1, s_2, \dots, s_n)$ and s_i uses $\pi_{i\alpha}$ we also say that \mathbf{s} uses $\pi_{i\alpha}$.

From the linearity of $p_i(s_1, \dots, s_n)$ in s_i ,

$$(2) \quad \max_{\text{all } r_i\text{'s}} [p_i(\mathbf{s}; r_i)] = \max_{\alpha} [p_i(\mathbf{s}; \pi_{i\alpha})].$$

We define $p_{i\alpha}(\mathbf{s}) \equiv p_i(\mathbf{s}; \pi_{i\alpha})$. Then we obtain the following trivial necessary and sufficient condition for \mathbf{s} to be an equilibrium point:

$$(3) \quad p_i(\mathbf{s}) = \max_{\alpha} p_{i\alpha}(\mathbf{s}).$$

If $\mathbf{s} = (s_1, s_2, \dots, s_n)$ and $s_i = \sum_{\alpha} c_{i\alpha} \pi_{i\alpha}$ then $p_i(\mathbf{s}) = \sum_{\alpha} c_{i\alpha} p_{i\alpha}(\mathbf{s})$, consequently for (3) to hold we must have $c_{i\alpha} = 0$ whenever $p_{i\alpha}(\mathbf{s}) < \max_{\beta} p_{i\beta}(\mathbf{s})$, which is to say that \mathbf{s} does not use $\pi_{i\alpha}$ unless it is an optimal pure strategy for player i . So we write

$$(4) \quad \text{if } \pi_{i\alpha} \text{ is used in } \mathbf{s} \text{ then } p_{i\alpha}(\mathbf{s}) = \max_{\beta} p_{i\beta}(\mathbf{s})$$

as another necessary and sufficient condition for an equilibrium point.

CLASSIC PRISONERS' DILEMMA

Column (Player 2)

| | Confess | Deny |
|---------|----------|----------|
| Confess | -4 -4 | -1 -10 |
| Deny | -10 -1 | -2 -2 |

Row (Player 1)

Standard Game Theory Assumptions:

- **Non-Cooperative:** Each player can only control their own strategy/choice and only cares about their own payoffs, and there is no coordination... Red player only considers red values, Blue player only considers blue values. Red utility and Blue utility are not comparable to each other...
- The numbers should always be interpreted as utility payoffs, but with a function for utility over money [like $U(m) = m$] these payoff values could be converted and interpreted/used as dollar-valued numbers.
- Matrix format indicates simultaneous and uncoordinated decisions, and these adversarial situations are assumed to be played only once unless there is specific indication that it is a repeated game, which is a more complex topic involving discounting of payoff streams.

CLASSIC PRISONERS' DILEMMA - DOMINANT STRATEGY NASH EQUILIBRIUM

Column (Player 2)

| | | Confess | Deny |
|----------------|---------|---------|--------|
| Row (Player 1) | Confess | -4 -4 | -1 -10 |
| | Deny | -10 -1 | -2 -2 |

- In this simultaneous game, both players achieve a better outcome from choosing *Confess* than choosing *Deny*, regardless of what the other player does.
- The Red player chooses the row, the Blue player chooses the column, and each cannot control the other player's choice: the top row is always a better number than the bottom row and the left column is always a better number than the right column.
- *Confess* is therefore a **dominant strategy** because it is always the **best response**, which is true for both players in this symmetric game.
- The existence of a dominant strategy guarantees the existence of a **Nash Equilibrium**: a situation where no player has any incentive to change their strategy
- The unique Nash Equilibrium that will result is **{Confess, Confess}**
- While a better outcome is possible for both players, it cannot be achieved: each player has an ability to gain by deviating from a situation of {Deny, Deny} and therefore even if they could communicate and agree to this, both would deviate because they do not expect the other player to follow through with the commitment.

AMERICAN FOOTBALL: PLAY-CALLING ZERO-SUM GAME

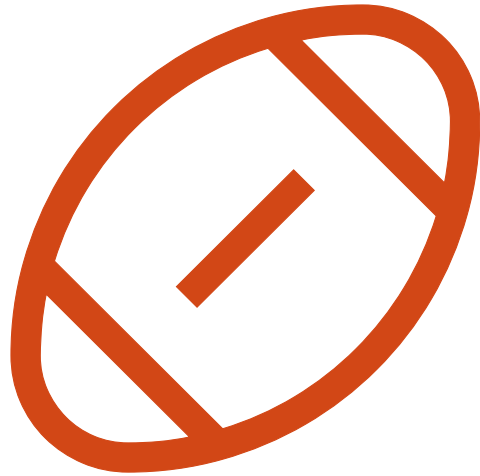


Defense

| | | Cover | Line |
|---------|------|---------|-----------|
| Offense | Rush | 7 -7 | -1 1 |
| | Pass | 0 0 | 12 -12 |



ZERO SUM FOOTBALL GAME: NO (PURE) N.E. OUTCOME

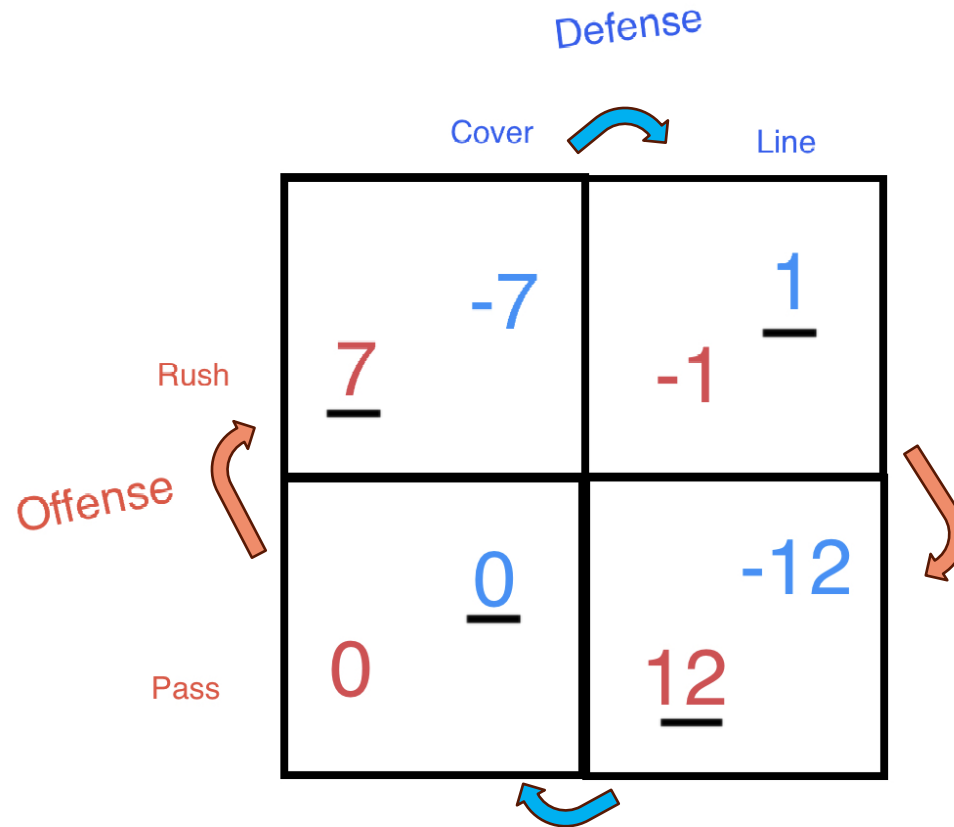


Defense

Cover Line

| | | |
|------|-------------|---------------|
| Rush | <u>7</u> -7 | -1 <u>1</u> |
| Pass | 0 <u>0</u> | 12 <u>-12</u> |

Offense



- **There is no Nash Equilibrium here** (or else it would not be an entertaining sport) ... both teams must strategically randomize their play choices – a more advanced concept called “mixed strategies”.



MATRIX GAME EXAMPLE 1

Column Player (2)

| | L | C | R |
|---|-----|-----|-----|
| T | 3 1 | 1 2 | 0 1 |
| M | 5 5 | 6 4 | 1 2 |
| B | 4 3 | 3 7 | 8 5 |

Row Player (1)



MATRIX GAME EXAMPLE 1

Column Player (2)

| | L | C | R |
|---|-------------------|------------|------------|
| T | 3 1 | 1 <u>2</u> | 0 1 |
| M | <u>5</u> <u>5</u> | <u>6</u> 4 | 1 2 |
| B | 4 3 | 3 <u>7</u> | <u>8</u> 5 |

Row Player (1)

Underlining to indicate all “best responses” we can see that the unique Nash Equilibrium is {Middle, Left}



MATRIX GAME EXAMPLE 2

Column Player (2)

| | L | C | R |
|---------------------|-----|-----|-----|
| Row Player (1) T | 3 2 | 1 3 | 2 1 |
| M | 2 9 | 5 4 | 1 2 |
| B | 4 3 | 3 7 | 6 5 |



MATRIX GAME EXAMPLE 2

Column Player (2)

| | L | C | R |
|---------------------|------------|------------|------------|
| Row Player (1) T | 3 2 | 1 <u>3</u> | 2 1 |
| M | 2 <u>9</u> | <u>5</u> 4 | 1 2 |
| B | <u>4</u> 3 | 3 <u>7</u> | <u>6</u> 5 |

There is no Nash Equilibrium in this game.



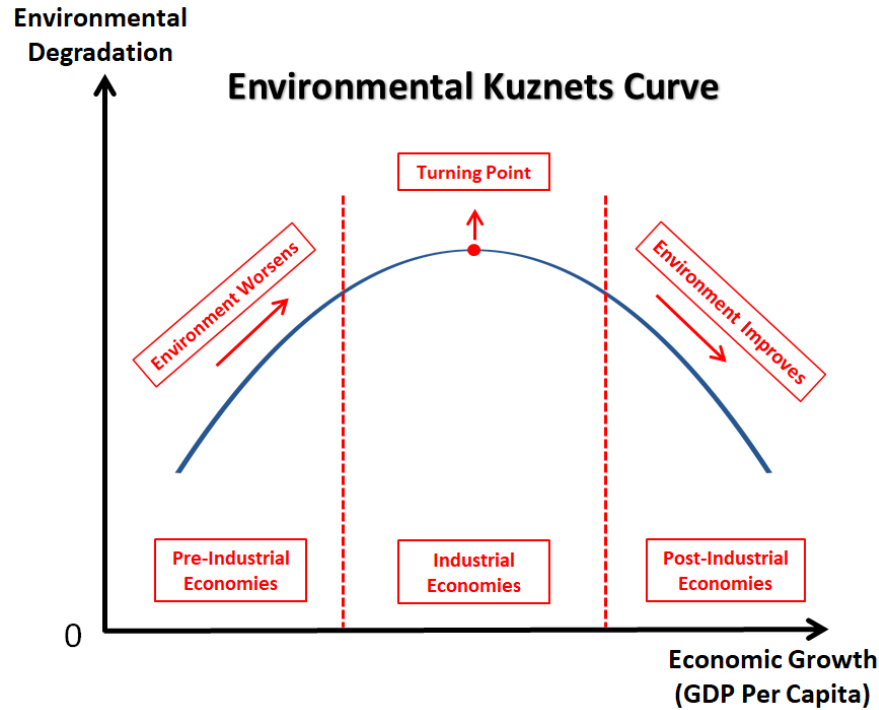
CLIMATE CHANGE SIMULTANEOUS GAME:

| | | USA | | |
|-------|--------------|------------|--------|--------------|
| | | Renewables | Hybrid | Fossil Fuels |
| China | Green Energy | 7 8 | 5 9 | 1 10 |
| | Mixed | 9 5 | 5 7 | 2 8 |
| | Coal | 11 0 | 7 3 | 5 2 |



CLIMATE CHANGE OUTCOME & REASONING

| | | USA | | |
|-------|--------------|-------------|------------|--------------|
| | | Renewables | Hybrid | Fossil Fuels |
| China | Green Energy | 7 8 | 5 9 | 1 <u>10</u> |
| | Mixed | 9 5 | 5 7 | 2 <u>8</u> |
| | Coal | <u>11</u> 0 | 7 <u>3</u> | <u>5</u> 2 |



The unique NE here is {Coal, Hybrid}: note that Coal is a strictly dominant strategy for China

The inverse parabolic “Environmental Kuznet’s Curve” explains how industrialization increases pollution, but economies with high enough wealth per capita are ultimately willing to pay more to reduce pollution: doing this has a smaller relative cost and increasing relative benefit, so eventually pollution declines



ARTIFICIAL INTELLIGENCE MATRIX GAME

AI Corp

| | Responsible | Moderate | Accelerated |
|----------|-------------|----------|-------------|
| Cautious | 8 9 | 5 10 | 1 14 |
| Standard | 9 5 | 7 6 | 1 6 |
| Reckless | 11 0 | 7 1 | 2 3 |

Bot LLC



ARTIFICIAL INTELLIGENCE MATRIX GAME

AI Corp

| | Responsible | Moderate | Accelerated |
|----------|-------------|-------------------|-------------------|
| Cautious | 8 9 | 5 10 | 1 <u>14</u> |
| Standard | 9 5 | <u>7</u> <u>6</u> | 1 <u>6</u> |
| Reckless | <u>11</u> 0 | <u>7</u> 1 | <u>2</u> <u>3</u> |

Bot LLC

- The two Nash Equilibria here are **{S, M}** and **{R, A}**
- Reckless and Accelerated are both ***weakly dominant***
- Cautious and Responsible are both “strictly dominated” (***never a best response***)
- One NE outcome is much better than the other

This shows us the picture of a contemporary AI race where two firms each want to capture market share and take over the industry: each firm would like the other one to be safe with development while it selfishly pursues an aggressive and dangerous approach.

A key takeaway here is that if government was able to induce the firms to make choices resulting in the “better” NE in the center, then both firms would be better off in the context of this game and overall society would fare better. This might look like subsidies or incentives to make the payoffs in the middle cell become 7.1 and 6.1 or taxes and threats of punishment to make the adjacent payoffs slightly lower for each firm, thus removing the weak dominance of the aggressive and unsafe AI development strategies in either case. An NE is a place where everyone gets “stuck” so it would not be possible to change the outcome once the “bad” NE scenario occurs.



HOTELLING SPATIAL COMPETITION MARKET MODEL: “SELLERS ON THE BEACH”

- Suppose the universe is a one-dimensional (closed and bounded) 8 mile long beach with a large number of identical and uniformly distributed consumers.
 - Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.
 - There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.
- **If these self-interested firms must simultaneously and non-cooperatively choose one time where to locate along the beach, what locations should they pick?**

SELLERS ON THE BEACH: INTERVENTIONS & EFFICIENCY

- Suppose the universe is a one-dimensional (closed and bounded) 8 mile long beach with a large number of identical and uniformly distributed consumers.
 - Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.
 - There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.
-
- **If you were a “benevolent social planner” where would you choose to place the two firms to maximize social welfare (total consumer surplus) and why?**
 - **What effect would this market intervention have on the two sellers?**
 - **Is this a Pareto improvement from the unregulated duopoly outcome?**

SELLERS ON THE BEACH – THREE FIRMS

- Suppose the universe is a one-dimensional (closed and bounded) 8 mile long beach with a large number of identical and uniformly distributed consumers.
- Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.
- There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.

➤ **What is the Nash Equilibrium outcome with three sellers?**

➤ **What effect will this have on consumer utility?**

SELLERS ON THE BEACH – ONE FIRM

- Suppose the universe is a one-dimensional (closed and bounded) 8 mile long beach with a large number of identical and uniformly distributed consumers.
- Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.
- There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.
 - **If the firms merged to become one monopolist seller, where would it locate?**
 - **What effect would this have on consumer utility?**
 - ❖ ***Metaphorically, what would this represent in the context of politics and policy?***

Sellers on the Beach:

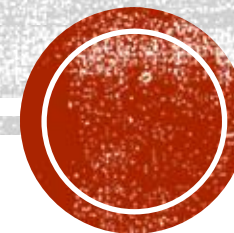
- Suppose the universe is a one-dimensional (closed and bounded) 8 mile long beach with a large number of identical and uniformly distributed consumers.
- Every day each of these consumers wakes up to find one gold coin under their pillow, which they must trade for a daily sustenance package in order to survive. Consumers derive utility only from minimizing their travel distance.
- There are two identical competing firms, called Jay and Dre, with unlimited and costless inventories of sustenance packages. Both firms derive utility (monotonic increasing) only from gold coins, so they want to sell as many sustenance packages as possible.

DISCUSSION

What real-world scenarios could this game represent?

Where do we see firms choose locations in this way?

Where do we observe the opposite location pattern in the real world and why?

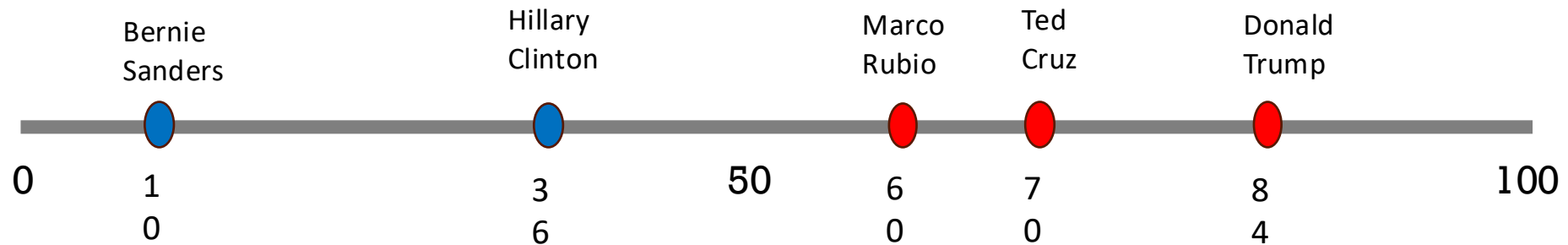


SELLERS ON THE BEACH – CONCLUSIONS & INSIGHTS

- Mandating the socially optimal set of locations does no harm to the two firms and increases social welfare by reducing the average travel distance from 2 miles to 1 mile, even though it harms the consumers closest to the center and is therefore not a Pareto improvement.
- Adding a third firm results in a non-stationary outcome (literally: the firms would endlessly oscillate best responses / leapfrog for position...) so there is no NE, although this competition in the market benefits consumers.
- A monopolist is indifferent across all possible locations: with perfectly inelastic demand it always captures the entire market.
 - Locating at 4 would be no worse than the unregulated duopoly NE outcome of $\{4,4\}$... but there is no incentive here for a monopolist to account for social welfare.
 - Metaphorically, a monopolist choosing 0 or 8 would be like an unaccountable dictator choosing extreme policies that are needlessly harmful social welfare with no benefit for anyone.
- Gas stations are a great example of how this form of competition is apparent in the 2D world.
- We might see the same clustering pattern if we privatized things like fire/police/EMS instead of government placing them in socially efficient locations to minimize emergency response distance.

APPLICATION OF SPATIAL COMPETITION:

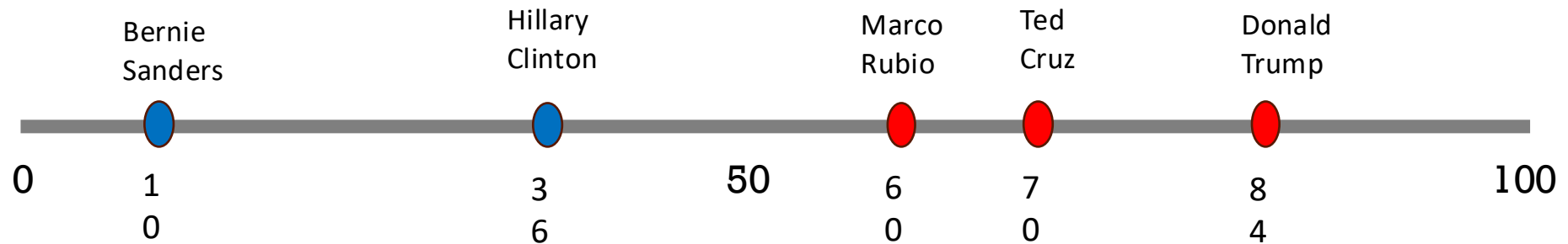
APPLICATIONS FOR POLITICAL CANDIDATE LOCATIONS (1-DIMENSIONAL IDEOLOGY)



- The distance to the halfway point between Sanders and Clinton is $(36-10)/2 = 13$
 - This means that a voter located at point 23 is exactly indifferent: $(10 + 13) = 23 = (36 - 13)$
 - Voters at a point to the left of 23 will prefer Sanders and voters to the right of 23 will prefer Clinton
- If the Democratic primary contains all voters from 0 to 50, then Sanders wins voters from 0 to 23 and Clinton wins voters from 23 to 50
 - Sanders receives $(23/50) = 46\%$ of the vote and Clinton receives $(27/50) = 54\%$ of the vote



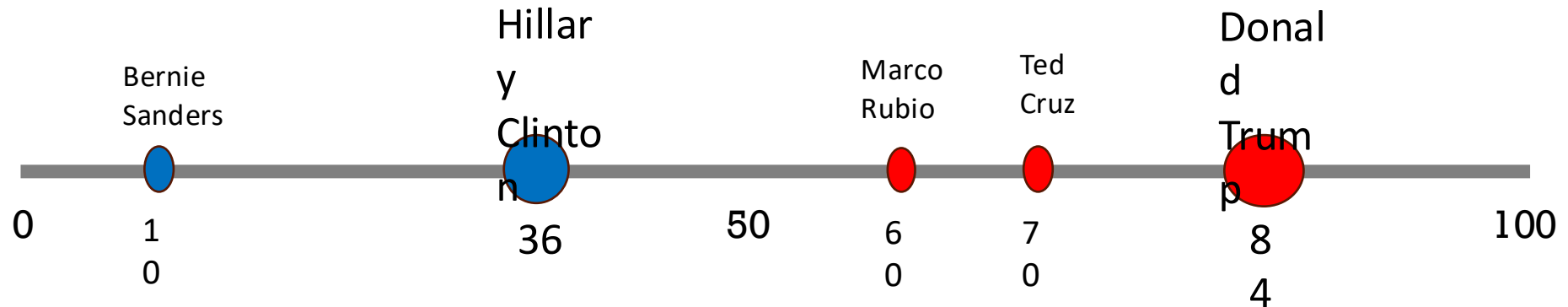
UNDERSTANDING SPATIAL COMPETITION: APPLICATIONS FOR POLITICAL CANDIDATE LOCATIONS



- The distance to the halfway point between Rubio and Cruz is $(70-60)/2 = 5$
 - This means that a voter located at point 65 is exactly indifferent: $(60 + 5) = 65 = (70 - 5)$
 - Voters at a point to the left of 65 will prefer Rubio and voters to the right of 65 will prefer Cruz
- The distance to the halfway point between Cruz and Trump is $(84-70)/2 = 7$
 - This means that a voter located at point 77 is exactly indifferent: $(70 + 7) = 77 = (84 - 7)$
 - Voters at a point to the left of 77 will prefer Cruz and voters to the right of 77 will prefer Trump
- Rubio wins voters 50 to 65 for $(15/50) = 30\%$ of the Republican vote
- Cruz wins voters 65 to 77 for $(12/50) = 24\%$ of the Republican vote
- Trump wins voters 77 to 100 for $(23/50) = 46\%$ of the Republican vote



IMPLICATIONS OF SPATIAL COMPETITION: APPLICATIONS FOR POLITICAL CANDIDATE LOCATIONS



- In a general election matchup we can use the same process: *assuming that the candidates keep their same positions*, the distance to the midpoint between Clinton and Trump is $(84-36)/2= 24$
 - This means that the general election voter who is indifferent between Clinton and Trump is located at point $(36 + 24) = 60 = (84 - 24)$
 - In reality with a two-party system, candidates usually try to re-position themselves in the general election by moving towards the overall median voter after winning the primary
- The predicted outcome here is a victory for Clinton with 60% of the total vote, including capturing the median voter
 - With a uniform or other symmetric distribution, the median voter is at position 50
 - According to these numbers, Rubio would have won against Clinton, Cruz would have lost against Clinton, and Sanders would have lost against any of the other candidates



TRAGEDY OF THE COMMONS: FISHERS ON A LAKE

- Lake Commons can be freely accessed by any fishers who wants to take out a boat, which costs $c = \$20$ per day. Fish are sold on a large competitive market at price $p = \$10$ per fish. Let b denote the number of boats on Lake Commons on a given day and let x denote the total number of fish caught on Lake Commons per day, which depends on the number of boats: $x = 100\sqrt{b}$ (Remember that profit equals total revenues minus total costs.)
 - With free entry (no barriers, no license required, etc)... quantify how many fishers will be active on Lake Commons and how many total fish will be caught.
 - Find the number of active fishers that maximizes profits.

TRAGEDY OF THE COMMONS – UNREGULATED MARKET (NASH EQUILIBRIUM)

- Lake Commons can be freely accessed by any fishers who wants to take out a boat, which costs $c = \$20$ per day. Fish are sold on a large competitive market at price $p = \$10$ per fish. Let b denote the number of boats on Lake Commons on a given day and let x denote the total number of fish caught on Lake Commons per day, which depends on the number of boats: $x = 100\sqrt{b}$
- The unregulated Nash Equilibrium is fishers will **enter until profits are zero**: if there is still any additional profit possible, then someone else will enter... this is a direct application of the “plain English” definition of NE.

- **Profit = Revenue – Cost** = $px - cb = 10(100\sqrt{b}) - 20b = 0$

Solving for zero profits:

$$1000\sqrt{b} = 20b$$

$$b_u = 2500 \text{ and } x_u = 5000$$

TRAGEDY OF THE COMMONS – PROFIT MAXIMIZATION (MONOPOLIST CHOICE)

- Lake Commons can be freely accessed by any fishers who wants to take out a boat, which costs $c = \$20$ per day. Fish are sold on a large competitive market at price $p = \$10$ per fish. Let b denote the number of boats on Lake Commons on a given day and let x denote the total number of fish caught on Lake Commons per day, which depends on the number of boats: $x = 100\sqrt{b}$
- To find the profit-maximizing level, we can take the derivative of this concave profit function with respect to the number of boats, set equal to zero, and solve for the peak:

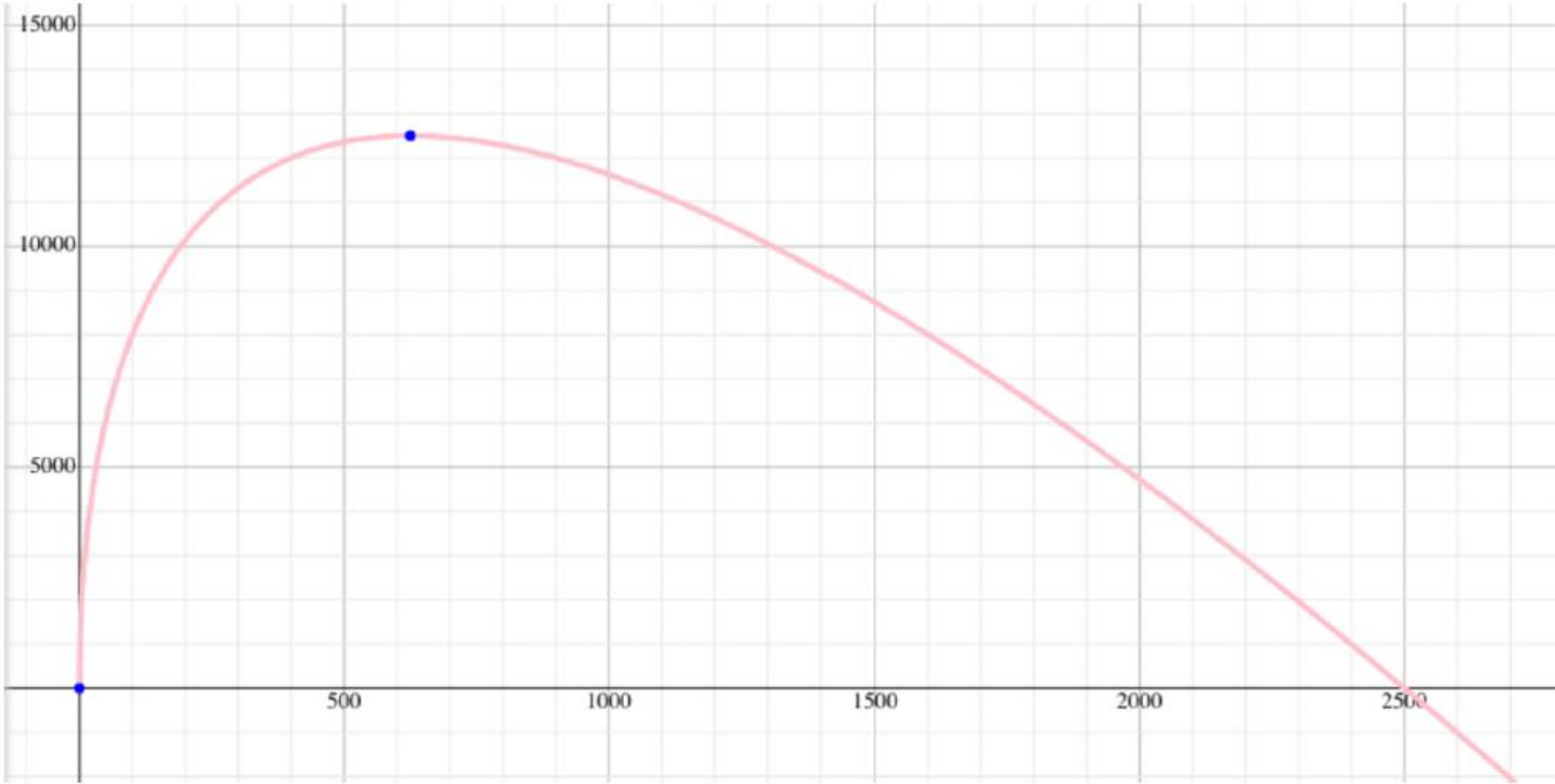
$$\frac{\partial \pi}{\partial b} = 0 = 500b^{-\frac{1}{2}} - 20$$

$$20\sqrt{b} = 500$$

$$400b = 250000$$

$$b^* = 625$$

GRAPHICALLY: PLOT OF PROFIT OVER NUMBER OF BOATS



TRAGEDY OF THE COMMONS – GOVERNMENT INTERVENTION

- Lake Commons can be freely accessed by any fishers who wants to take out a boat, which costs $c = \$20$ per day. Fish are sold on a large competitive market at price $p = \$10$ per fish. Let b denote the number of boats on Lake Commons on a given day and let x denote the total number of fish caught on Lake Commons per day, which depends on the number of boats: $x = 100\sqrt{b}$
- Now the government wants to generate revenues so it will charge a *license fee* to fish each day. Suppose a one-day license to fish on Lake Commons now costs $\$f$ dollars and the government's only objective is tax revenue maximization.
- What license fee will the government set and how will it affect the market?

Using the NE behavior of entry until profits are zero, we can re-write the original zero profit condition to include the fee f as

$$\begin{aligned}10x &= (20 + f)b \\10(100\sqrt{b}) &= 20b + fb \\ \left(\frac{1000}{\sqrt{b}}\right) &= 20 + f \\ b &= \left(\frac{1000}{20 + f}\right)^2\end{aligned}$$

The government will maximize revenue:

$$R = \max_f (f * b) = f * \left(\frac{1000}{20 + f}\right)^2 = \frac{1000000 * f}{400 + 40f + f^2}$$

Using the chain rule for derivatives with multiplication, we can obtain marginal revenue for the government and set it equal to zero to obtain our maximum revenue and then solve for the optimal fee:

Revenue:
$$R = \max_f (f * b) = f * \left(\frac{1000}{20 + f} \right)^2 = \frac{1000000 * f}{400 + 40f + f^2}$$

Marginal Revenue:
$$R' = 1 * \left(\frac{1000}{20+f} \right)^2 + f * \left(\frac{(-1)(2)(1000000)}{(20+f)^3} \right) = 0$$

$$\left(\frac{1000000}{(20+f)^2} \right) = f * \frac{2000000}{(20+f)^3}$$

$$(20 + f)^3 = 2f(20 + f)^2$$

$$20+f = 2f$$

$$f^* = 20$$

$$b_f^* = \left(\frac{1000}{20+f} \right)^2 = 25^2 = 625$$

ENVIRONMENTAL AND POLICY IMPLICATIONS OF THE LICENSE FEE

- Notice that the outcome when a self-interested government sets the fee level to maximize its own revenue is the same outcome as the profit-maximizing level!
 - This is what a monopolist would do with control of the market, but it might also be environmentally efficient in a different situation
- There may be a mitigation of negative externalities to consider here: the license fee will decrease the amount of fishing and prevent depletion of the fish population
- This is a common and extremely important policy scenario that is applicable to other cases of depletable natural resources and over-consumption

Voluntary Contributions (Public Goods Game) – NATO Defense Investments

Three identical countries border a hostile nation and can each choose to improve their shared defense system by incurring a fixed private cost with non-rival and non-excludable benefits. The utility function describing the benefit realized by each of the three countries is $B(\mathbf{n}) = \mathbf{n}^2$, where \mathbf{n} represents the total number of countries choosing to invest. These three self-interested countries each make a one-time binary decision, simultaneously and independently, of whether or not to incur individual private cost C to invest in the shared defense system.

- What level of individual private cost will always result in zero countries choosing to invest?
- Is there any level of individual private cost that guarantees investment?

Voluntary Contributions (Public Goods Game) – NATO Defense Investments

Three identical countries border a hostile nation and can each choose to improve their shared defense system by incurring a fixed private cost with non-rival and non-excludable benefits. The utility function describing the benefit realized by each of the three countries is $B(\mathbf{n}) = \mathbf{n}^2$, where \mathbf{n} represents the total number of countries choosing to invest. These three self-interested countries each make a one-time binary decision, simultaneously and independently, of whether or not to incur individual private cost C to invest in the shared defense system.

- What level of individual private cost will always result in zero countries choosing to invest?

Any individual cost amount greater than 5 precludes any possibility of investment.

The total payoff function is $U(\mathbf{n}) = \mathbf{n}^2 - C$. When all three invest, the payoff is $(3^2 - C)$ and when only two countries invest the payoff is $(2^2 - C)$ so the difference between these is 5, which is the maximum individual cost level that can sustain any probability of investing in this situation. With $C > 5$, each country's best response is to free ride.

- Is there any level of individual private cost that guarantees investment?

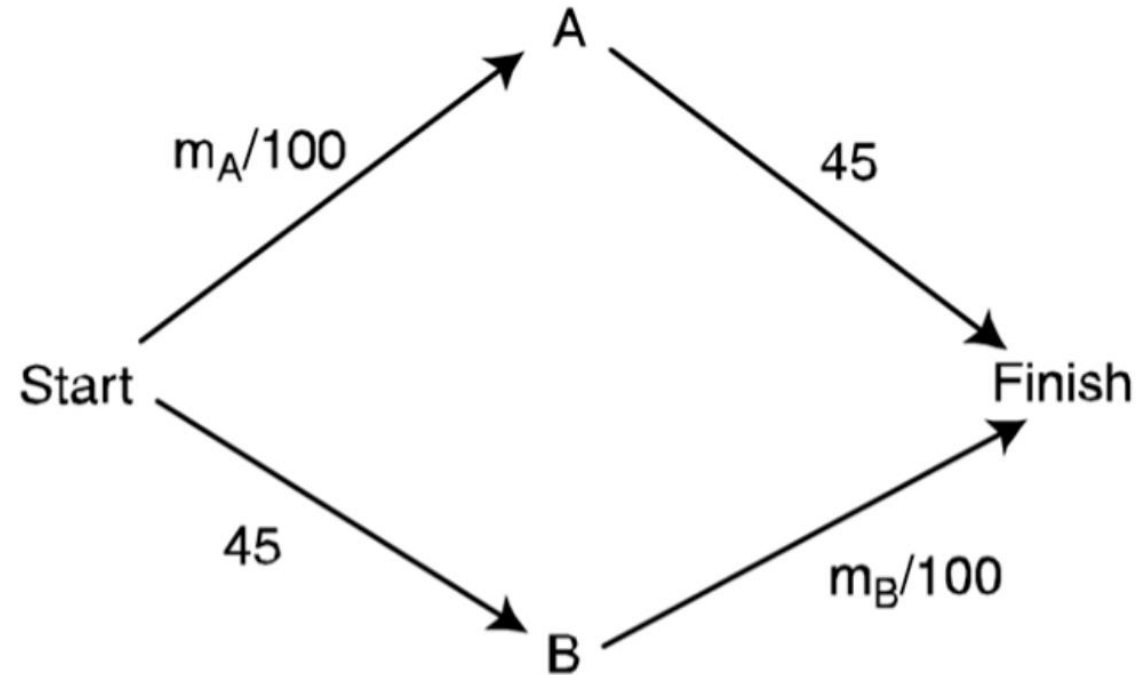
Any individual cost $C < 1$ makes investing a *strictly dominant strategy*, guaranteeing that all three invest.

BRAESS PARADOX – NETWORKS & TRAFFIC

There are **4000** motorists who must commute each morning from **Start** to **Finish** and they must decide whether to drive through either **route A** or **route B**. The route through point A has a travel time (in minutes) equal to $1/100$ of the number of motorists m_A who use tunnel A, plus a fixed 45 minute drive afterwards without traffic. The other route has a fixed 45 minute drive to point B first, plus a travel time of $1/100$ of the number of motorists m_B who use tunnel B. The motorists get utility only from minimizing their individual travel time and there is nothing else you need to consider here.

What is the Nash Equilibrium travel time in this scenario?

BRAESS PARADOX: TRAFFIC ROUTE DIAGRAM

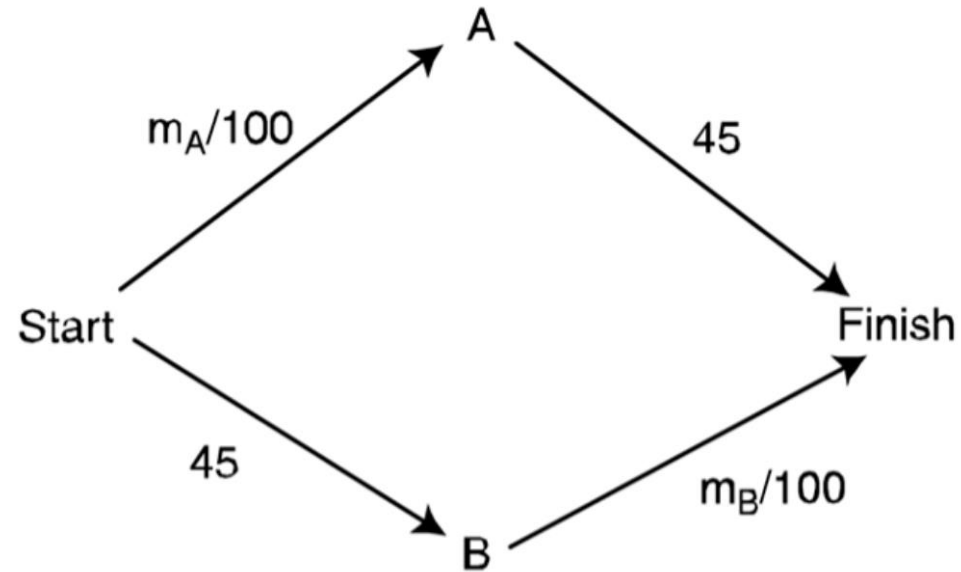


Drivers will always want to take the faster route: this game works over time as well as simultaneously.

In a Nash Equilibrium, the travel time will be the same for both routes.

Note that mathematically we have: $m_A + m_B = 4000$

BRAESS PARADOX – INITIAL SETUP OUTCOME

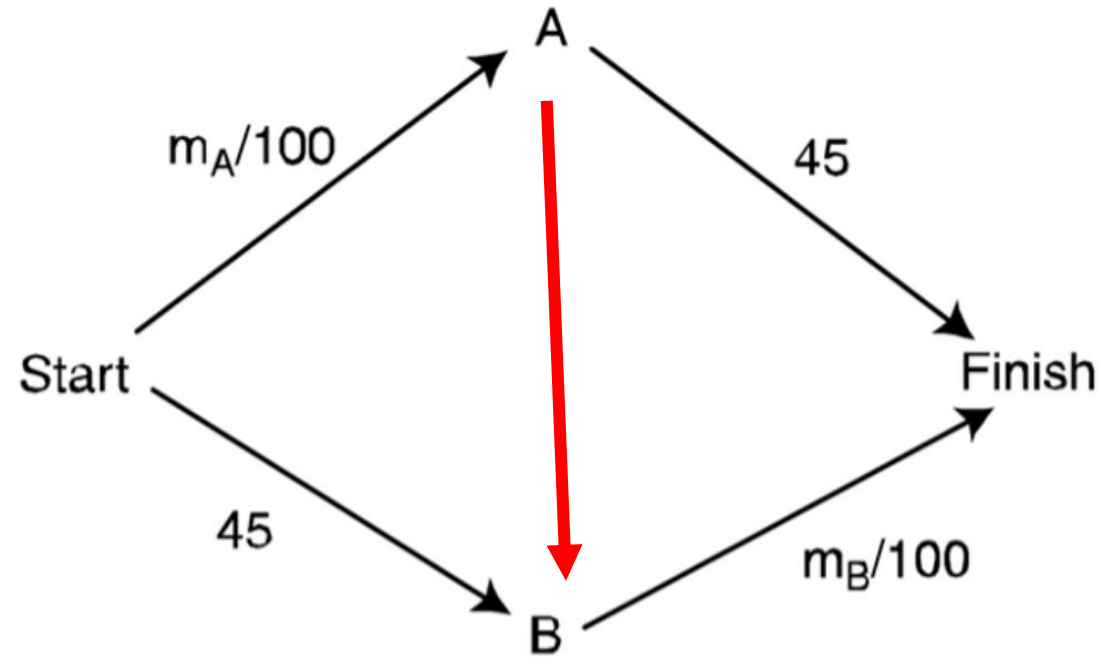


The Nash Equilibrium outcome is 2000 motorists will go to each route:

$$m_A / 100 + 45 = 45 + m_B / 100$$

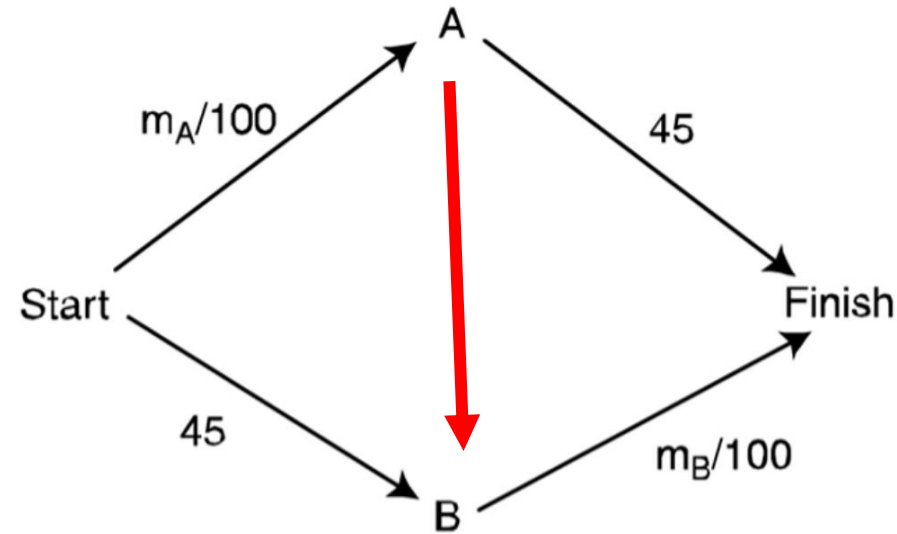
Equilibrium travel time is $45 + 20 = 65$ minutes

BRAESS PARADOX



Now what will happen if we add an instant **shortcut** from point A to point B ?

BRAESS PARADOX – INEFFICIENT OUTCOME



With the shortcut, motorists will change their routes to A to save time until there is nothing more to gain from doing so:

$$\text{Mathematically: } [m_A / 100 + m_B / 100] < [45 + m_B / 100]$$

Now all 4000 motorists will take the shortcut route of Start through A through B to Finish, and the equilibrium total travel time increases to **80 minutes**.

BRAESS PARADOX — REGULATORY INTERVENTIONS

- Whether physical, online, or electrical traffic, there can often be improvements to the situation
 - This is why tunnels and exit lanes on roads often have physical barriers to prevent lane switching
 - On the internet, there can be similar routing tactics used to prevent issues (technically complicated)
 - Power grid systems involve somewhat similar regulation of energy distribution, including limitations on days when extreme weather could cause blackouts (imagine everyone trying to use the maximum level of air conditioning on the hottest day of the year)