LABOR MARKET MICROECONOMIC MODELS

Effort/Discipline Model, Hiring/Quitting & Reservation Wage Curve

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SEQUENTIAL OPTIMIZATION: EFFORT/WAGE MICRO LABOR MODEL

- Consider a firm attempting to maximize the financial efficiency of its labor inputs by maximizing employee effort relative to wage expense. A worker's utility is determined by the wage $(\boldsymbol{w})$ earned above the reservation wage $(\boldsymbol{r})$ and the cost of effort level $(\boldsymbol{e})$ exerted.
- Suppose the probability of a worker remaining employed is $\mathbf{e}^{1/2}$ and the cost of effort is $\mathbf{e}^{2}/\mathbf{4}$

  - *Determine the equilibrium wage outcome assuming the above is all public information.*

LABOR/DISCIPLINE MODEL

- Effort = **e**
- Reservation wage = **r**
- Wage = **w**

  - *What is the firm's choice variable?*
  - *What is the worker's choice variable?*

There are diminishing returns from the monotonic concave increasing employment probability function over effort, but the effort cost function is convex increasing.

Q: Why might there be an exponential shape to the cost of effort? Is this realistic?

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- Consider a firm attempting to maximize the financial efficiency of its labor inputs by maximizing employee effort relative to wage expense. A worker's utility is determined by the wage $(\boldsymbol{w})$ earned above the reservation wage $(\boldsymbol{r})$ and the cost of effort level $(\boldsymbol{e})$ exerted.
- Suppose the probability of a worker remaining employed is $\mathbf{e^{1/2}}$ and the cost of effort is $\mathbf{e^2/4}$
  - ➢ *Determine the equilibrium wage outcome assuming the above is all public information.*

❖ The firm moves first, choosing w=**w*** to maximize the effort/wage efficiency (**e/w**) incorporating the worker's best response function. The worker maximizes the benefit of wage (in excess of the reservation wage) times the probability of realizing that wage minus the effort cost of obtaining that wage.

❖ Mathematically, this means the worker maximizes utility function $\mathbf{U = e^{1/2}(w - r) - [e^2/4]}$ by choosing optimal effort level e=**e*** for the given $\boldsymbol{w}$ chosen by the firm. Since the utility function has a monotonic concave increasing benefit term and monotonic convex increasing cost term over the worker's choice variable, it is a non-monotonic concave function, so differentiating with respect to the choice variable and setting equal to zero (First Order Condition) obtains the maximum payoff:

- ❖ This FOC is the worker's best response function: $\mathbf{[e^{-1/2}/\ 2](w - r) - [e/2] = 0} \rightarrow \mathbf{e^* = (w - r)^{2/3}}$
- ❖ Using this best response (e*) the firm maximizes $\mathbf{e^*/w = [(w - r)^{2/3}/w\ ]}$ over its choice variable (w)
- ❖ FOC via quotient rule: $[(2/3)w(w - r)^{-1/3} - (w - r)^{2/3}]/w^2 = 0 \rightarrow \mathbf{[}w^{-2}(w - r)^{-1/3}(w/3 - r)\mathbf{]} = 0 \rightarrow \mathbf{w^* = 3r}$

(assuming $\boldsymbol{w > r}$)

SEQUENTIAL OPTIMIZATION: EFFORT/WAGE MICRO LABOR MODEL

- Consider a firm attempting to maximize the financial efficiency of its labor inputs by maximizing employee effort relative to wage expense. A worker's utility is determined by the wage $(\boldsymbol{w})$ earned above the reservation wage $(\boldsymbol{r})$ and the cost of effort level $(\boldsymbol{c})$ exerted.
- Suppose the probability of a worker remaining employed is $\boldsymbol{\alpha} \ln(\mathbf{c})$ and the cost of effort is $\mathbf{e}^2/\boldsymbol{\beta}$

   ➢ *Determine the equilibrium wage outcome assuming the above is all public information.*

Sequential Optimization: Effort/Wage Micro Labor Model

- Consider a firm attempting to maximize the financial efficiency of its labor inputs by maximizing employee effort relative to wage expense. A worker's utility is determined by the wage (w) earned above the reservation wage (r) and the cost of effort level (x) exerted.
- Suppose the probability of a worker remaining employed is $\alpha \ln(x)$ and the cost of effort is x^2/β

  - *Determine the equilibrium wage outcome assuming the above is all public information.*

- The firm moves first, choosing $w = \mathbf{w^*}$ to maximize the effort/wage efficiency $(\mathbf{x/w})$ incorporating the worker's best response function. The worker maximizes the benefit of wage (in excess of the reservation wage) times the probability of realizing that wage minus the effort cost of obtaining that wage.
- Mathematically, this means the worker maximizes utility function $\mathbf{U} = \ln(\mathbf{x})(\mathbf{w - r}) - [\mathbf{x}^2/\beta]$ by choosing optimal effort level $x = \mathbf{x^*}$ for the given w chosen by the firm. Since the utility function has a monotonic concave increasing benefit term and monotonic convex increasing cost term over the worker's choice variable, it is a non-monotonic concave function, so differentiating with respect to the choice variable and setting equal to zero (First Order Condition) obtains the maximum payoff:

  - This FOC is the worker's best response function: $[\mathbf{1/x}](\mathbf{w - r}) - [\mathbf{2x}/\beta] = \mathbf{0} \rightarrow \mathbf{x^*} = [\beta(\mathbf{w - r})/2]^{1/2}$
  - Using this best response (x^*) the firm maximizes $\mathbf{x^*/w} = ([\beta(\mathbf{w - r})/2]^{1/2} / \mathbf{w})$ over its choice variable (w)
  - FOC via product/quotient rule: $[-w^{-2}[\beta(\mathbf{w - r})/2]^{1/2} + w^{-1}(\beta/4)[\beta(\mathbf{w - r})/2]^{-1/2} = 0 \rightarrow \mathbf{w^* = 2r}$

(assuming $w > r$)

HIRING & QUITTING MODEL

- Firms choose **w** to obtain the optimal employment level
- This curve also shows us the workers' reservation wages

HIRING & QUITTING MODEL (EXT 6.5)

- Consider a firm that wants to employ **N** workers
- Proportion **q** of employees will leave each week (**q** = quit rate)
- Firm must hire **qN** workers each week to maintain workforce

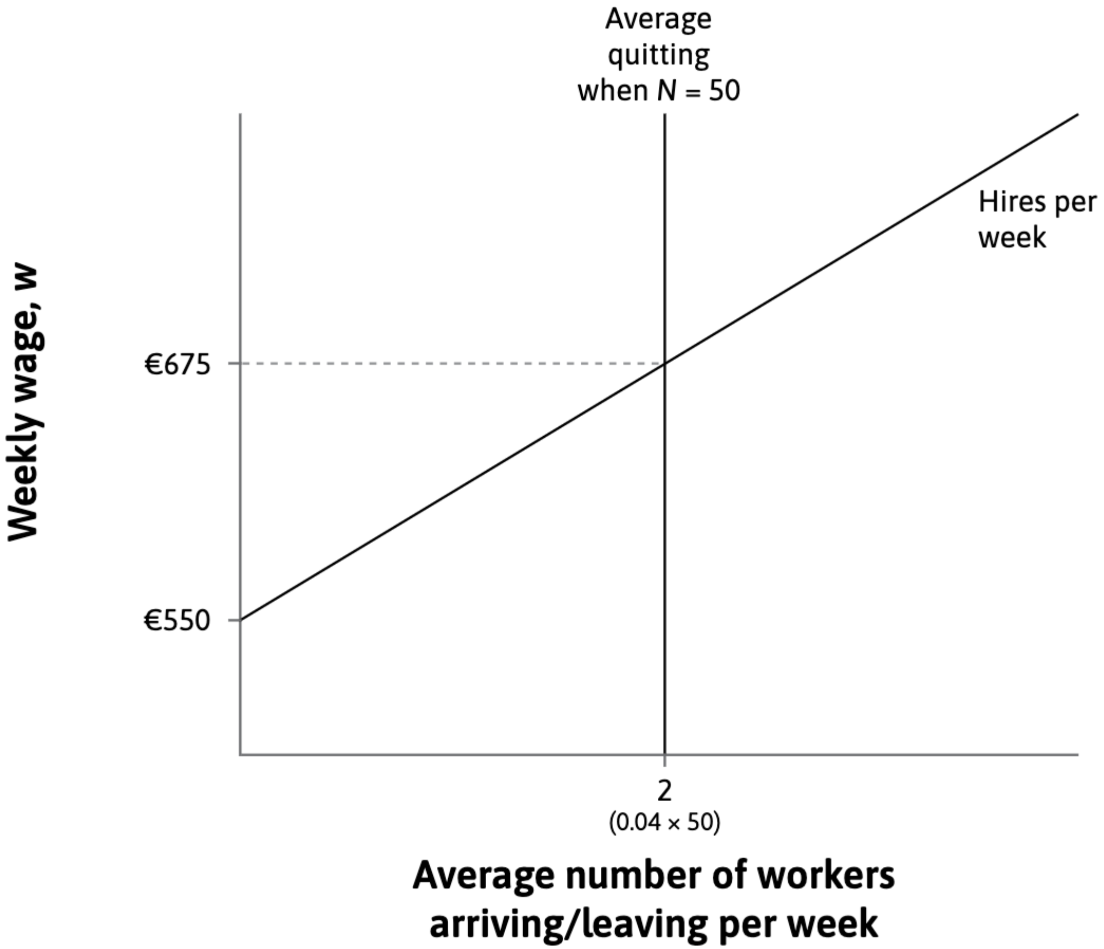
- Suppose the firm encounters **m** qualified matches per week and they will accept a job offer for a sufficiently high wage **w** (greater than the reservation wage)
- This varies across the workers, so we can express the proportion who would accept offer **w** as **P(w)** : thus, to keep employment constant at **N**, the wage must equate the number of incoming workers with the number who quit (maintaining a "steady state")
- Mathematically, this "reservation wage curve equation" is: **mP(w) = qN**

HIRING & QUITTING MODEL

- With N=50, q = 0.04, to maintain a steady state satisfying **mP(w)=qN** we need wage = 675

- Note that this equation implicitly determines **w** as a function of **N**

- If desired employment increased, then the vertical quitting line would shift to the right…



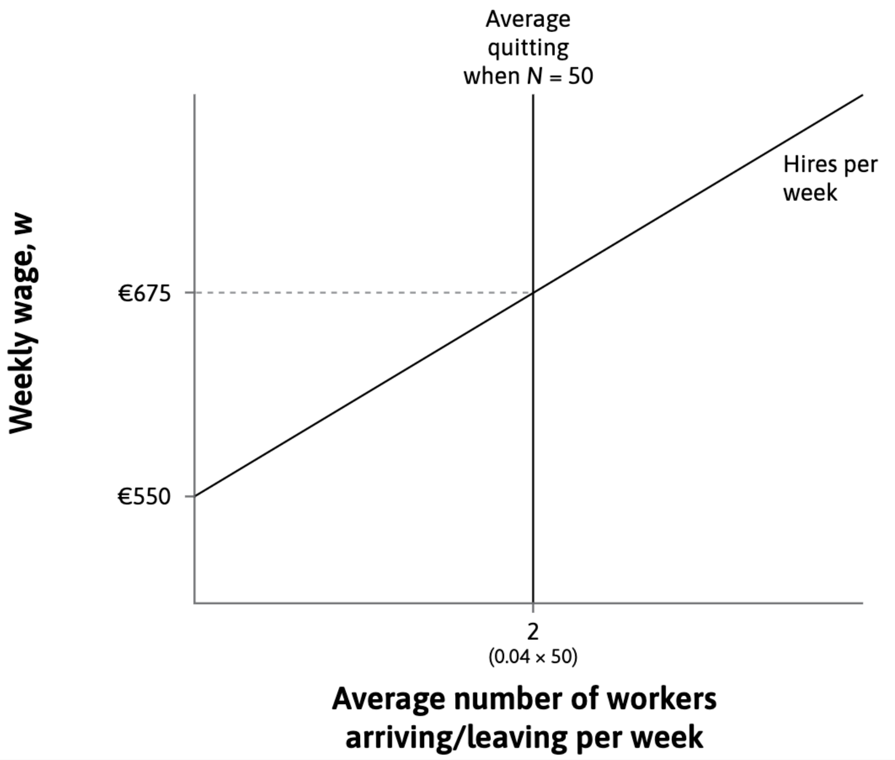
HIRING & QUITTING MODEL (MATHEMATICALLY)

To get **N** as an explicit function of **w** algebraically, we can differentiate and rearrange:

$$N = \frac{mP(w)}{q} \Rightarrow \frac{dN}{dw} = \frac{mP'(w)}{q}$$

With $P'(w) > 0,\ m > 0,\ q > 0$, we know that $\frac{dN}{dw} > 0$.

This confirms that **N** is an increasing function of **w**, which is an essential relationship for this model to work, and it also confirms that **w** is an increasing function of **N**.



HIRING & QUITTING MODEL (MATHEMATICALLY)

We can invert the derivative as follows:

$$\frac{dw}{dN} = \frac{1}{dN/dw} = \frac{q}{mP'(w)} > 0$$

The slope of the line in this new graph to the right is dw/dN : this is flat if wage changes have a large effect on hiring (if $P'(w)$ is big) and this is steep if hiring is not very sensitive to wage changes.

HIRING & QUITTING MODEL (MATHEMATICALLY)

We can also see how changes in **m** or **q** affect the line:

$$N = \frac{mP(w)}{q} \Rightarrow \frac{\partial N}{\partial m} = \frac{P(w)}{q} > 0$$

and

$$\frac{\partial N}{\partial q} = -\frac{mP(w)}{q^2} < 0$$

HIRING & QUITTING MODEL : TAKEAWAYS

- If the firm can find qualified workers to hire at a faster rate with the quitting rate unchanged, then the firm can employ more workers at any given wage and therefore the reservation wage curve shifts up.

- An increase in the quit rate, ceteris paribus, has the opposite effect: if more workers quit each week then the curve shifts down without hiring rate changes.