# Advanced Microeconomics: Labor Negotiation Models

Effort/Discipline Model, Hiring/Quitting & Reservation Wage Curve

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# LABOR/DISCIPLINE MODEL

- Effort = **e**
- Reservation wage = **r**
- Wage = **w**

- *What is the firm's choice variable?*
- *What is the worker's choice variable?*

# LABOR/DISCIPLINE MODEL - EXAMPLE 1

**Question:** A firm wants to offer a wage $w$ that maximizes the effort level $e$ per dollar. A worker decides her effort level to maximize the expected wage rent (the wage minus the reservation wage, which a worker can get if employed by this firm) minus the effort cost. Assume that worker remains employed with probability $e^{1/2}$. The effort cost is $(1/4)e^2$.

How would you characterize the worker's return to effort?

What is the optimal wage offer of the firm?

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**There are diminishing returns from the monotonic concave increasing employment probability function over effort, but the effort cost function is convex increasing.**

***Q: Why might there be an exponential shape to the cost of effort? Is this realistic?***

# LABOR DISCIPLINE MODEL - EXAMPLE 1 SOLUTION

**Solution:** Suppose that the probability of not being dismissed is $e^{1/2}$ and the cost of effort is $(1/4)e^2$. The objective function of a worker is then $e^{1/2}(w-r)-(1/4)e^2$, where $w$ is the wage and $r$ is the reservation wage. Differentiating this with respect to $e$ and setting to zero, we have $(1/2)e^{-1/2}(w-r)-(1/2)e=0$. Rearranging this,

$$e=(w-r)^{2/3}$$

Now, the firm's objective is to maximize $e/w$ by choosing $w$, which is the effort per wage cost. Since firms expect $e=(w-r)^{2/3}$, firm minimize $(w-r)^{2/3}w^{-1}$. Differentiating this, we have $-(w-r)^{2/3}w^{-2}-2/3w^{-1}(w-r)^{-1/3}=w^{-2}(w-r)^{-1/3}((w-r)-\frac{2}{3}w)$. Thus, as long as firms prefer $w>r$, we must have $w-r-\frac{2}{3}w$ equals zero due to the first order condition. Rearranging this, we have $w=3r$.

# HIRING & QUITTING MODEL


- Firms choose **w** to obtain the optimal employment level
- This curve also shows us the workers' reservation wages

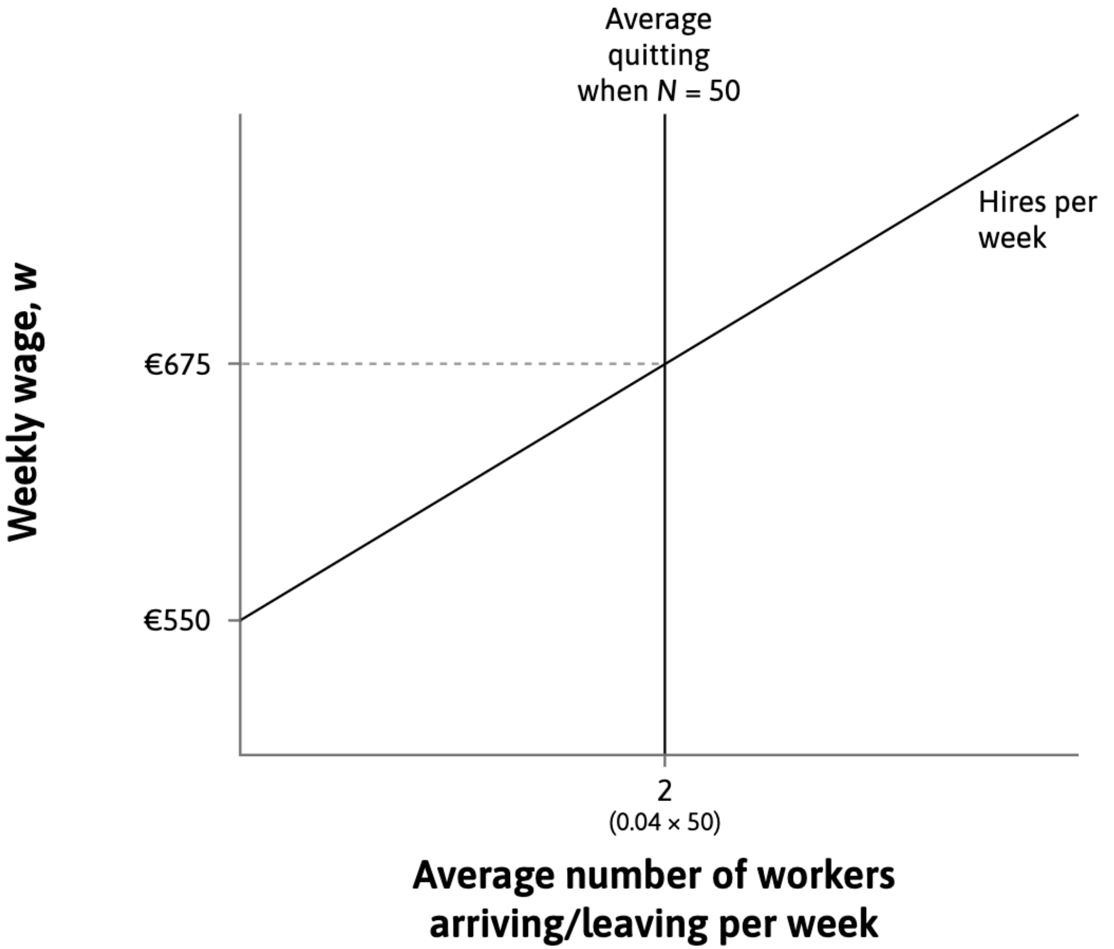
# HIRING & QUITTING MODEL (EXT 6.5)

- Consider a firm that wants to employ **N** workers
- Proportion **q** of employees will leave each week (**q** = quit rate)
- Firm must hire **qN** workers each week to maintain workforce

- Suppose the firm encounters **m** qualified matches per week and they will accept a job offer for a sufficiently high wage **w** (greater than the reservation wage)
- This varies across the workers, so we can express the proportion who would accept offer **w** as **P(w)** : thus, to keep employment constant at **N**, the wage must equate the number of incoming workers with the number who quit (maintaining a "steady state")
- Mathematically, this "reservation wage curve equation" is: **mP(w) = qN**

# HIRING & QUITTING MODEL

- With N=50, q = 0.04, to maintain a steady state satisfying **$mP(w)=qN$** we need wage = 675
- Note that this equation implicitly determines **w** as a function of **N**
- If desired employment increased, then the vertical quitting line would shift to the right…



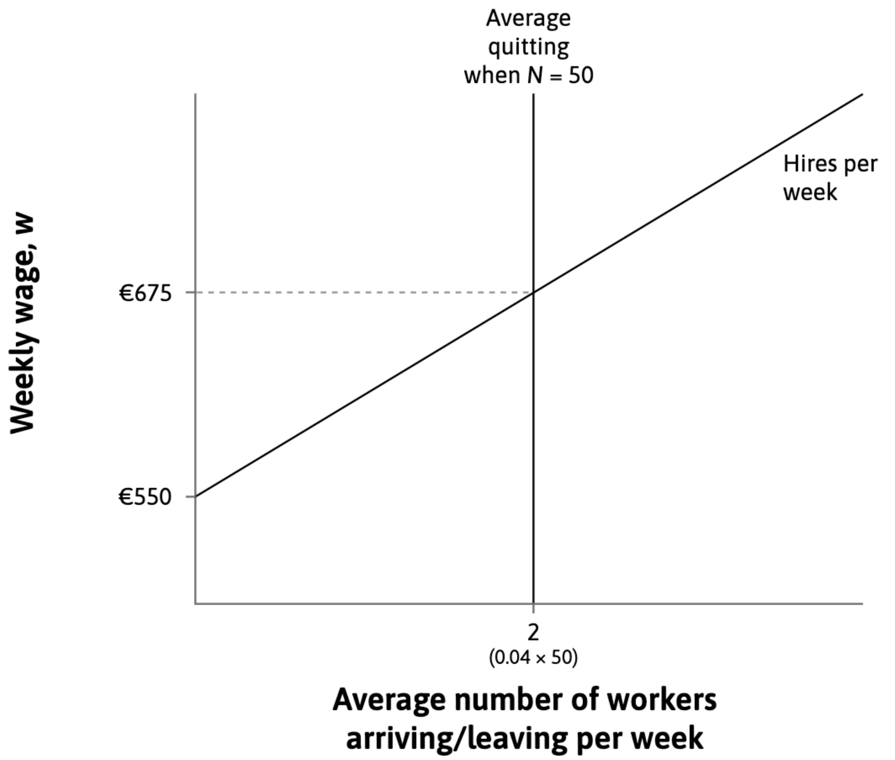
# HIRING & QUITTING MODEL (MATHEMATICALLY)

To get **N** as an explicit function of **w** algebraically, we can differentiate and rearrange:

$$\mathrm{N} = \frac{\mathrm{mP(w)}}{\mathrm{q}} \Rightarrow \frac{\mathrm{dN}}{\mathrm{dw}} = \frac{\mathrm{mP'(w)}}{\mathrm{q}}$$

With $P'(w) > 0,\ m > 0,\ q > 0$, we know that $\frac{dN}{dw} > 0$.

This confirms that **N** is an increasing function of **w**, which is an essential relationship for this model to work, and it also confirms that **w** is an increasing function of **N**.



# HIRING & QUITTING MODEL (MATHEMATICALLY)

We can invert the derivative as follows:

$$\frac{dw}{dN} = \frac{1}{dN/dw} = \frac{q}{mP'(w)} > 0$$


The slope of the line in this new graph to the right is dw/dN : this is flat if wage changes have a large effect on hiring (if $P'(w)$ is big) and this is steep if hiring is not very sensitive to wage changes.

# HIRING & QUITTING MODEL (MATHEMATICALLY)

We can also see how changes in **m** or **q** affect the line:

$$N = \frac{mP(w)}{q} \Rightarrow \frac{\partial N}{\partial m} = \frac{P(w)}{q} > 0$$

and

$$\frac{\partial N}{\partial q} = -\frac{mP(w)}{q^2} < 0$$

# HIRING & QUITTING MODEL : TAKEAWAYS

- If the firm can find qualified workers to hire at a faster rate with the quitting rate unchanged, then the firm can employ more workers at any given wage and therefore the reservation wage curve shifts up.

- An increase in the quit rate, ceteris paribus, has the opposite effect: if more workers quit each week then the curve shifts down without hiring rate changes.