Constructing Market Demand from Individual Preferences: Complementary Goods

Consider a market with N_d consumers where every consumer i is one of three possible types: $i \in \{\alpha, \beta, \gamma\}$ so $N_d = N_\alpha + N_\beta + N_\gamma$. Consumers have preferences over pizza (z) and beer (b) described by the following type-specific convex utility functions:

$$U_\alpha(z, b) = z * b$$

$$U_\beta(z, b) = z * b^2$$

$$U_\gamma(z, b) = z * b + 3z$$

With \$$M$ of money available for each consumer and product prices P_z and P_b the individual budget constraint is $z \cdot P_z + b \cdot P_b \leq M$ and for simplicity let's assume this is binding, so all money will be spent on these goods to the extent that more consumption increases utility. Utility is monotonically increasing over both goods for all consumer types with the given preferences here, so all money will be spent. Equating the slope of the indifference curves (MRS) with the slope of the budget constraint (MRT or "price ratio") we can obtain the following optimal consumption ratios for types α, β, γ respectively:

$$\text{MRS}^\alpha_{z,b} = -\frac{\frac{\partial U}{\partial z}}{\frac{\partial U}{\partial b}} = \left|-\frac{b}{z}\right| = \text{MRT} = \left|-\frac{P_z}{P_b}\right|$$

$$z \cdot P_z = b \cdot P_b \quad \rightarrow \quad z = \frac{b \cdot P_b}{P_z} \;\; , \;\; b = \frac{z \cdot P_z}{P_b}$$

$$\text{MRS}^\beta_{z,b} = -\frac{\frac{\partial U}{\partial z}}{\frac{\partial U}{\partial b}} = \left|-\frac{b^2}{2bz}\right| = \text{MRT} = \left|-\frac{P_z}{P_b}\right|$$

$$(2bz)P_z = b^2 \cdot P_b \quad \rightarrow \quad z = \frac{b \cdot P_b}{2 \cdot P_z} \;\; , \;\; b = \frac{2z \cdot P_z}{P_b}$$

$$\text{MRS}^\gamma_{z,b} = -\frac{\frac{\partial U}{\partial z}}{\frac{\partial U}{\partial b}} = \left|-\frac{b+3}{z}\right| = \text{MRT} = \left|-\frac{P_z}{P_b}\right|$$

$$(b+3)P_b = z \cdot P_z \quad \rightarrow \quad z = \frac{(b+3)P_b}{P_z} \;\; , \;\; b = \frac{z \cdot P_z}{P_b} - 3$$

Substituting each of these into the budget constraint obtains individual demand for each type:

$$M - z \cdot P_z - \left(\frac{z \cdot P_z}{P_b}\right) \cdot P_b = 0 \quad \rightarrow \quad z^*_\alpha = \frac{M}{2P_z} \;\; , \;\; b^*_\alpha = \frac{M}{2P_b}$$

$$M - z \cdot P_z - \left(\frac{2z \cdot P_z}{P_b}\right) \cdot P_b = 0 \quad \rightarrow \quad z^*_\beta = \frac{M}{3P_z} \;\; , \;\; b^*_\beta = \frac{2M}{3P_b}$$

$$M - z \cdot P_z - \left(\frac{z \cdot P_z}{P_b} - 3\right) \cdot P_b = 0 \quad \rightarrow \quad z^*_\gamma = \frac{M + 3P_b}{2P_z} \;\; , \;\; b^*_\gamma = \frac{M - 3P_b}{2P_b}$$

Suppose there are the following number of consumers of each type: $\{N_\alpha = 5, N_\beta = 10, N_\gamma = 15\}$.

Aggregate market demand for pizza (Z_d) and beer (B_d) are the sum of all individual demand for each:

$$Z_d = \sum_i N_i z_i = N_\alpha \cdot z_\alpha + N_\beta \cdot z_\beta + N_\gamma \cdot z_\gamma = 5\left(\frac{M}{2P_z}\right) + 10\left(\frac{M}{3P_z}\right) + 15\left(\frac{M + 3P_b}{2P_z}\right) = \left[\frac{80M + 135P_b}{6P_z}\right]$$

$$B_d = \sum_i N_i b_i = N_\alpha \cdot b_\alpha + N_\beta \cdot b_\beta + N_\gamma \cdot b_\gamma = 5\left(\frac{M}{2P_b}\right) + 10\left(\frac{2M}{3P_b}\right) + 15\left(\frac{M - 3P_b}{2P_b}\right) = \left[\frac{100M - 135P_b}{6P_b}\right]$$

Constructing Market Demand from Individual Preferences: Complementary Goods (DMR)

Consider a market with N_d consumers where every consumer i is one of three possible types: $i \in \{\alpha, \beta, \gamma\}$ so $N_d = N_\alpha + N_\beta + N_\gamma$. Consumers have preferences over pizza (z) and beer (b) described by the following type-specific convex utility functions:

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$$\text{MRS}^\alpha_{z,b} = -\frac{\frac{\partial U}{\partial z}}{\frac{\partial U}{\partial b}} = \left| -\frac{\frac{b^{1/2}}{2z^{1/2}}}{\frac{z^{1/2}}{2b^{1/2}}} \right| = \text{MRT} = \left| -\frac{P_z}{P_b} \right|$$

$$z \cdot P_z = b \cdot P_b \quad \rightarrow \quad z = \frac{b \cdot P_b}{P_z} \ , \ b = \frac{z \cdot P_z}{P_b}$$

$$\text{MRS}^\beta_{z,b} = -\frac{\frac{\partial U}{\partial z}}{\frac{\partial U}{\partial b}} = \left| -\frac{\frac{1}{2}(zb^2)^{-1/2} \cdot b^2}{\frac{1}{2}(zb^2)^{-1/2} \cdot (2zb)} \right| = \text{MRT} = \left| -\frac{P_z}{P_b} \right|$$

$$(2bz)P_z = b^2 \cdot P_b \quad \rightarrow \quad z = \frac{b \cdot P_b}{2 \cdot P_z} \ , \ b = \frac{2z \cdot P_z}{P_b}$$

$$\text{MRS}^\gamma_{z,b} = -\frac{\frac{\partial U}{\partial z}}{\frac{\partial U}{\partial b}} = \left| -\frac{\frac{1}{2}(zb+3z)^{-1/2} \cdot (b+3)}{\frac{1}{2}(zb+3z)^{-1/2} \cdot z} \right| = \text{MRT} = \left| -\frac{P_z}{P_b} \right|$$

$$(b+3)P_b = z \cdot P_z \quad \rightarrow \quad z = \frac{(b+3)P_b}{P_z} \ , \ b = \frac{z \cdot P_z}{P_b} - 3$$

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$$M - z \cdot P_z - \left(\frac{2z \cdot P_z}{P_b} \right) \cdot P_b = 0 \quad \rightarrow \quad z^*_\beta = \frac{M}{3P_z} \ , \ b^*_\beta = \frac{2M}{3P_b}$$

$$M - z \cdot P_z - \left(\frac{z \cdot P_z}{P_b} - 3 \right) \cdot P_b = 0 \quad \rightarrow \quad z^*_\gamma = \frac{M + 3P_b}{2P_z} \ , \ b^*_\gamma = \frac{M - 3P_b}{2P_b}$$

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$$B_d = \sum_i N_i b_i = N_\alpha \cdot b_\alpha + N_\beta \cdot b_\beta + N_\gamma \cdot b_\gamma = 5\left(\frac{M}{2P_b}\right) + 10\left(\frac{2M}{3P_b}\right) + 15\left(\frac{M - 3P_b}{2P_b}\right) = \left[\frac{100M - 135P_b}{6P_b}\right]$$