Multiplayer Games, Strategic Randomization & Mixed Strategy Nash Equilibrium, Collective Action & Coordination Problems

Microeconomics (615)

Andrew Gates (Princeton University)

FOUR PLAYER GAME

	3	A			B	
4	1 \ 2	L	R	1 \ 2	L	R
X	U	(1,2,-1, 1)	(3,3,1,-1)	U	(0,0,0,0)	(2,1,2,1)
	D	(0,1,-3,0)	(4,2,1,3)	D	(2,1,0,3)	(-1,2,0,2)
	1 \ 2	L	R	1 \ 2	L	R
Y	U	(3,4,0,0)	(1,2,1,2)	U	(1,-1,-2,2)	(3,2,2,-1)
	D	(1,2,-1, -1)	(2,-1,3,1)	D	(2,1,1,1)	(0,3,1,0)

FOUR PLAYER GAME

Player 1 chooses Up or Down

Note that the order of payoffs corresponds with the player number

	3	A			B	
4	1 \ 2	L	R	1 \ 2	L	R
X	U	(1,2,-1, 1)	(3,3,1,-1)	U	(0,0,0,0)	(2,1,2,1)
	D	(0,1,-3,0)	(4,2,1,3)	D	(2,1,0,3)	(-1,2,0,2)
	1 \ 2	L	R	1 \ 2	L	R
Y	U	(3,4,0,0)	(1,2,1,2)	U	(1,-1,-2,2)	(3,2,2,-1)
	D	(1,2,-1, -1)	(2,-1,3,1)	D	(2,1,1,1)	(0,3,1,0)

FOUR PLAYER GAME

Player 1 chooses Up or Down & Player 2 chooses Left or Right

Note that L is a dominant strategy in Game ***AY*** but R is a dominant strategy in the other three Games

	3	A			B	
4	1 \ 2	L	R	1 \ 2	L	R
X	U	(1,2,-1, 1)	(3,3,1,-1)	U	(0,0,0,0)	(2,1,2,1)
	D	(0,1,-3,0)	(4,2,1,3)	D	(2,1,0,3)	(-1,2,0,2)
	1 \ 2	L	R	1 \ 2	L	R
Y	U	(3,4,0,0)	(1,2,1,2)	U	(1,-1,-2,2)	(3,2,2,-1)
	D	(1,2,-1, -1)	(2,-1,3,1)	D	(2,1,1,1)	(0,3,1,0)

FOUR PLAYER GAME

Player 1 chooses Up or Down & Player 2 chooses Left or Right

Player 3 chooses Game Column A or B & Player 4 chooses Game Row X or Y

	3	A			B	
4	1 \ 2	L	R	1 \ 2	L	R
X	U	(1,2,-1, 1)	(3,3,1,-1)	U	(0,0,0,0)	(2,1,2,1)
	D	(0,1,-3,0)	(4,2,1,3)	D	(2,1,0,3)	(-1,2,0,2)
Y	1 \ 2	L	R	1 \ 2	L	R
	U	(3,4,0,0)	(1,2,1,2)	U	(1,-1,-2,2)	(3,2,2,-1)
	D	(1,2,-1, -1)	(2,-1,3,1)	D	(2,1,1,1)	(0,3,1,0)

FOUR PLAYER GAME

Nash Equilibria: (U,R,B,X) and (D,R,A,X)

	3	A			B	
4	1 \ 2	L	R	1 \ 2	L	R
X	U	(1,2,-1, 1)	(3,3,1,-1)	U	(0,0,0,0)	(2,1,2,1)
	D	(0,1,-3,0)	(4,2,1,3)	D	(2,1,0,3)	(-1,2,0,2)
	1 \ 2	L	R	1 \ 2	L	R
Y	U	(3,4,0,0)	(1,2,1,2)	U	(1,-1,-2,2)	(3,2,2,-1)
	D	(1,2,-1, -1)	(2,-1,3,1)	D	(2,1,1,1)	(0,3,1,0)

FINDING ALL NASH EQUILIBRIA IN SIMULTANEOUS GAMES:
STRATEGIC RANDOMIZATION USING "**MIXED STRATEGIES**"

"Date Night" Game: He and She must decide where to go on Friday night…

		She: Bar	She: Museum
He	Bar	2, 1	0, 0
He	Museum	0, 0	1, 2

MIXED STRATEGIES: ASSIGNING PROBABILITIES BETWEEN ZERO AND ONE TO THE AVAILABLE OPTIONS

		She: Bar	She: Museum
He	Bar	2, 1	0, 0
He	Museum	0, 0	1, 2

The traditional Nash Equilibria using "pure strategies" in this "Date Night" game are: {Bar, Bar} and {Museum, Museum}.

They both receive payoffs of zero if they do not go to the same place, but each has a preferred location: he prefers the bar and she prefers the museum.

… but what happens if these players are unable to coordinate / communicate / agree on how to play?

In this situation, the best response (with full knowledge of the game shown here but without knowing what the other will do) for each player is to optimally randomize their individual choice of action. This means both will choose a probability distribution (probability of each possible action) that is a best response to the other's optimal probability distribution.

MIXED STRATEGIES: CALCULATING INDIFFERENCE POINTS

He \ She	Bar	Museum
Bar	2, 1	0, 0
Museum	0, 0	1, 2

Let's define his probability of going to the bar as **b** and define her probability of going to the bar as **s**.

He is indifferent between bar and museum when the expected values of his payoff from each option are equal:

EV_{Him} [Bar] = 2(s) + 0(1- s) = 2s

EV_{Him} [Museum] = 0(s) + 1(1- s) = (1 – s)

Equating these (his "point of indifference") : 2s = 1 – s and solving this obtains **s* = 1/3**

MIXED STRATEGIES: CALCULATING INDIFFERENCE POINTS

He \ She	Bar	Museum
Bar	2, 1	0, 0
Museum	0, 0	1, 2

Let's define his probability of going to the bar as **b** and define her probability of going to the bar as **s.**

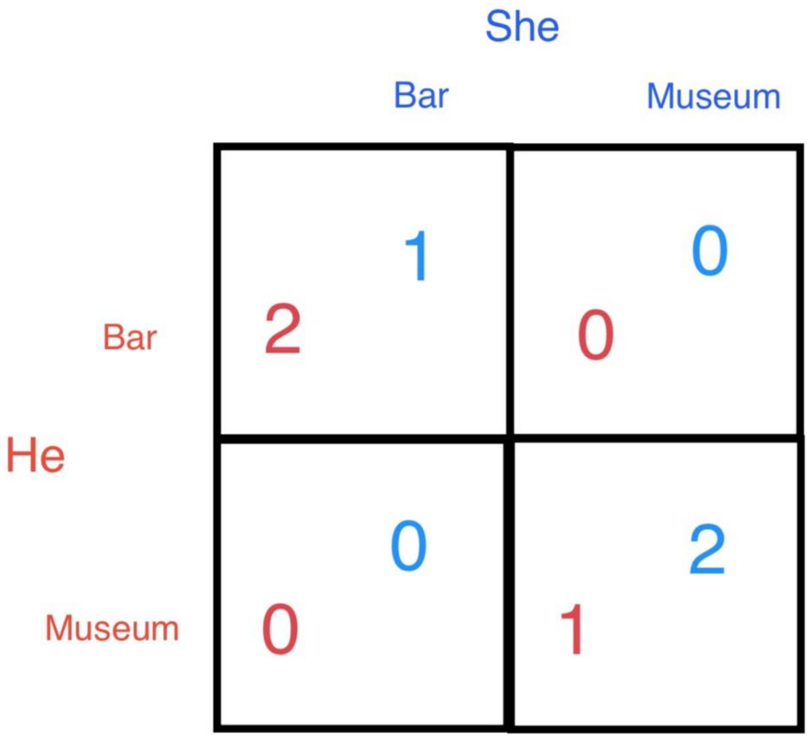
She is indifferent between bar and museum when the expected values of her payoff from each option are equal:

$EV_{Her}[Bar] = 1(b) + 0(1-b) = b$

$EV_{Her}[Museum] = 0(b) + 2(1-b) = 2 - 2b$

Equating these (her "point of indifference"): $b = 2 - 2b$ and solving this obtains **$b^* = 2/3$**

MIXED STRATEGIES: INTUITION OF OPTIMAL PROBABILITIES



His probability of going to the bar is **b* = 2/3**
Her probability of going to the bar is **s* = 1/3**

The unique Mixed Strategy Equilibrium here is that he will randomize his behavior so that he goes to the bar with a 0.667 probability and she will randomize her behavior to go to the museum with a 0.667 probability.

This is a "symmetric" solution to a symmetric game: both players go to their individually preferred location with a 2/3 probability and will go to the other's preferred location with a 1/3 probability.

Neither player can unilaterally achieve a higher payoff by modifying the probability within their individual control, which is one definition of Nash Equilibrium, so they are "stuck" here in this MSE.

MIXED STRATEGY EQUILIBRIUM: EXPECTED VALUES

		She: Bar	She: Museum
He	Bar	2, 1	0, 0
He	Museum	0, 0	1, 2

His probability of going to the bar is **b* = 2/3**
Her probability of going to the bar is **s* = 1/3**

This allows us to calculate the probability of each outcome:

Prob.(Bar, Bar) = (2/3)(1/3) = **2/9**

Prob. (Bar, Mus) = (2/3)(2/3) = **4/9**

Prob. (Mus, Bar) = (1/3)(1/3) = **1/9**

Prob. (Mus, Mus) = (1/3)(2/3) = **2/9**

From these probabilities we can calculate the expected payoff for each player by multiplying the payoff in each of the four cells by the probability of that cell being the game's outcome under MSE:

EV_{Him} [b*=2/3, s*=1/3] = 2(2/9) + 0(4/9) + 0(1/9) + 1(2/9) = 2/3

EV_{Her} [b*=2/3, s*=1/3] = 1(2/9) + 0(4/9) + 0(1/9) + 2(2/9) = 2/3

MIXED STRATEGIES: SUBOPTIMAL OUTCOME

He \ She	Bar	Museum
Bar	2, 1	0, 0
Museum	0, 0	1, 2

With the MSE of **b* = 2/3** and **s* = 1/3 ,** both players have an overall expected payoff value of 0.667 from this game.

If they both flipped a fair 50/50 coin to determine their probabilities instead, then each cell would have a 1/4 probability of being the game's outcome. In this case, each of them would have a *higher* overall expected payoff value:

$$2(1/4) + 0(1/4) + 0(1/4) + 1(1/4) = 0.75$$

This outcome is better for both of them, but it cannot be achieved because it is not a Nash Equilibrium: each of them can increase their individual expected payoff (while reducing the other's) by increasing the probability placed on their individually preferred location. If she is playing with s =1/2 , then he could increase his probability to b = 1 to increase his expected payoff to 2(1/2) = 1.0

MIXED STRATEGIES: MSE UNIQUENESS

He \ She	Bar	Museum
Bar	2, 1	0, 0
Museum	0, 0	1, 2

With the MSE of **b* = 2/3** and **s* = 1/3 ,** both players have an overall expected payoff value of 0.667 from this game.

If he chose b = 2/3 with her choosing s =1/2 then his expected payoff would be: $2(2/3)(1/2) + 1(1/3)(1/2) = 5/6$

… this is less than the expected payoff of 1.0 if he chose to always go to the bar (b=1) while she plays s =1/2 , but considering the symmetry of the game, this cannot be a mixed strategy NE solution because she has the same incentive to deviate from the 0.5/0.5 probability choice set towards her individual preference for museum and the result of both always going to their individual preference always obtains 0 for both players: this is strictly individually worse off in both cases and actually the worst possible outcome for both.

MIXED STRATEGIES: UNIQUENESS & CALCULUS LOGIC

He \ She	Bar	Museum
Bar	2, 1	0, 0
Museum	0, 0	1, 2

With the MSE of **$b^* = 2/3$** and **$s^* = 1/3$,** both players have an overall expected payoff value of 0.667 from this game.

An efficient way to show that the unique best response for both players is choosing their individually preferred location with a 2/3 probability is writing each of their payoff values as functions of both probabilities (the choice variables):

His expected payoff $= V_H = 2(b)(s) + 1(1-b)(1-s) = 3(bs) + 1 - b - s$

Her expected payoff $= V_S = 1(b)(s) + 2(1-b)(1-s) = 3(bs) + 2 - 2b - 2s$

Differentiating each person's payoff value with respect to the choice variable within that person's individual control (indicated by color) obtains the following first order conditions:

$$\frac{\partial V_H}{\partial b} = 3s - 1 = 0 \Rightarrow s_H{*} = 1/3 \qquad \frac{\partial V_S}{\partial s} = 3b - 2 = 0 \Rightarrow b_S{*} = 2/3$$

These are unique simultaneous best responses to the opposing player's best response probability whenever neither player uses a "pure strategy" (probability of 0 or 1).

COMPARING OUTCOMES, OPTIMAL SEQUENCE & VALUE OF COORDINATION CAPABILITY

He \ She	Bar	Museum
Bar	2, 1	0, 0
Museum	0, 0	1, 2

With the MSE of **b* = 2/3** and **s* = 1/3**, both players have an overall expected payoff value of **0.667** from optimal strategic randomization in this game.

If there was an ability to coordinate, then they would both agree to go to the same place (but which?) and each would obtain a payoff of at least 1.

If this game was repeated, then a "happy healthy relationship" outcome would be for them to alternate equally between {Bar, Bar} and {Museum, Museum} … perhaps coordinating to both choose Bar on odd numbered calendar dates and both choose Museum on even numbered calendar dates. **They would each have an average payoff of 1.5 over time in doing this optimal coordinated strategy sequence.** This could be compared to other formulations of the game to determine the exact value of obtaining the capability to communicate and coordinate to achieve this sequence of optimal cooperative choices.

Furthermore, while we cannot add or directly compare utility values of different colors since these are specific to one individual (unless they are all dollar-valued and/or we have "utility over money" functions to do conversions for both) … the optimal coordinated sequence of alternating 50/50 would be jointly utility-maximizing with equal utility weighting (socially efficient) and also with symmetric altruism functions where each benefits from the other's utility, if we assume this is not already accounted for in the given payoff values.

USA VS. RUSSIA: MSE & INTUITION

USA

	Attack	Peace
Attack	-3000 -3000	0 200
Peace	200 0	0 0

- In this symmetric game, both sides can gain from attacking when the other side chooses Peace
- This incentive to choose Attack must be weighed against the extremely bad outcome for each side if both attack
- The two pure strategy NE outcomes are **{Attack, Peace}** and **{Peace, Attack}** … but this outcome does not make sense and fails to adequately model the world if the game is repeated over and over

USA VS. RUSSIA: MSE & INTUITION

USA

	Attack	Peace
Attack	-3000 -3000	0 200
Peace	200 0	0 0

- If we set ***a*** as the probability of USA choosing attack and ***r*** is the probability of Russia choosing attack, then we can equate the expected payoffs for each to find the mixed strategy Nash equilibrium

- EV_{USA}[Attack] = -3000(r) + 200(1-r) = 200 – 3200r
- EV_{USA}[Peace] = 0(r) + 0(1-r) = 0
- Equating these two expected values and solving obtains **r*= 1/16**
- At this exact point of Russia's attack probability, USA is exactly indifferent between its two options.

USA VS. RUSSIA: MSE & INTUITION

USA		Attack	Peace
	Attack	-3000 (Russia), -3000 (USA)	0 (Russia), 200 (USA)
	Peace	200 (Russia), 0 (USA)	0 (Russia), 0 (USA)

- If we set ***a*** as the probability of USA choosing attack and ***r*** is the probability of Russia choosing attack, then we can equate the expected payoffs for each to find the mixed strategy Nash equilibrium
- $EV_{Russia}[Attack] = -3000(a) + 200(1\text{-}a) = 200 – 3200a$
- $EV_{Russia}[Peace] = 0(a) + 0(1\text{-}a) = 0$
- Equating these two expected values and solving obtains **a*= 1/16**
- At this exact point of America's attack probability, Russia is exactly indifferent between its two options.

USA VS. RUSSIA: MSE & INTUITION

USA		Attack	Peace
	Attack	-3000 (Russia), -3000 (USA)	0 (Russia), 200 (USA)
	Peace	200 (Russia), 0 (USA)	0 (Russia), 0 (USA)

- The unique MSE in this game is **{1/16, 1/16}** , meaning both sides will randomize and commit to attacking with exactly a 1 in 16 chance, which is set before the game begins and then beyond their control as it is literally left up to chance

- This implies that there is a 1/256 probability of both sides choosing Attack every time this game is played

AMERICAN FOOTBALL PLAY-CALLING ZERO-SUM GAME

Defense

Offense	Cover	Line
Rush	7, -7	-1, 1
Pass	0, 0	12, -12

FOOTBALL: MIXED STRATEGY EQUILIBRIUM OUTCOME

Offense \ Defense	Cover	Line
Rush	7, -7	-1, 1
Pass	0, 0	12, -12

- There is **<u>no pure strategy NE</u>**: both teams must strategically randomize their play choices
- If there was a pure strategy NE then it would not be a very entertaining sport

FOOTBALL: SOLVING FOR MIXED STRATEGY EQUILIBRIUM

Defense

Offense	Cover	Line
Rush	7, -7	-1, 1
Pass	0, 0	12, -12

Let’s call **c** the probability that Defense plays Cover:

$EV_{Offense}[Rush] = 7(c) + -1(1 - c) = 8c - 1$

$EV_{Offense}[Pass] = 0(c) + 12(1 - c) = 12 - 12c$

Equating these two expected values and solving obtains **c* = 13/20** as the probability of Cover that makes the Offense indifferent between Rush and Pass.

Let’s call **r** the probability that Offense plays Rush:

$EV_{Defense}[Cover] = -7(r) + 0(1 - r) = -7r$

$EV_{Defense}[Line] = 1(r) + -12(1 - r) = 13r - 12$

Equating these two expected values and solving obtains **r* = 12/20** as the probability of Rush that makes the Defense indifferent between Cover and Line.

FOOTBALL: MIXED STRATEGY EQUILIBRIUM OUTCOME

Defense

Offense	Cover	Line
Rush	7, -7	-1, 1
Pass	0, 0	12, -12

There is no pure strategy NE: both teams must randomize, choosing optimal probabilities of actions. Equating the expected payoff values from each of the two strategies for each player over the probabilities of the other's strategy obtains the unique intersecting set of probabilities where neither side can improve its expected payoff by changing its set of probabilities.

With **c** = Pr(cover) and **r** = Pr(rush) the MSE is **r* = 3/5 , c* = 13/20** : the offense will optimally rush 60% of the time and the defense will optimally cover against passing 65% of the time. This is the unique intersection of optimizing probabilities where neither side can do better by changing its specified randomization (odds of each possible play type).

When Defense plays Cover 65% of the time, the Offense would have a lower expected payoff from choosing 59% or 61% or any other possible probability that is not 60%.

When Offense plays Rush 60% of the time, the Defense would have a lower expected payoff from choosing 64% or 66% or any other possible probability that is not 65%.

MIXED STRATEGIES: NOMENCLATURE & SOME INTUITION

Recall the basic definition of a Nash Equilibrium: ***a situation in which no decision-maker can unilaterally achieve a better individual outcome by changing anything within their individual control*** (assuming everyone involved is purely selfish and rational, and in this case and most others also assuming "perfect information" – meaning every player knows all of the available choices and payoffs for every player, or equivalently that every player can see the given matrix or the entirety of the information provided in a problem… and will correctly analyze all of this to understand what others will optimally do in their own self-interest)

Note that the term "mixed strategy" comes from the fact that the equilibrium is comprised of a mixture of two (or more) possible strategies being utilized by at least one player. This means that at least two strategies will have an optimal probability of usage between 0 and 1.

Nash's original paper included proof of the existence of an equilibrium in finite zero-sum games (like the football example) particularly when there is no "pure strategy" Nash Equilibria. The term "pure strategy" means the probability of each possible strategy is exactly equal to 0 or exactly equal to 1.

ONE-TIME SIMULTANEOUS CHOICE TARIFFS: TWO NATION **TRADE WAR** MATRIX GAME

Red Nation (Column)

Blue Nation (Row)	Low	Medium	High
Low	8 8	7 5	9 3
Medium	5 7	6 6	8 4
High	3 9	4 8	2 2

ONE-TIME SIMULTANEOUS CHOICE TARIFFS: TWO NATION TRADE WAR MATRIX GAME

Red Nation (Column)

Blue Nation (Row)	Low	Medium	High
Low	8, 8	5, 7	3, 9
Medium	7, 5	6, 6	4, 8
High	9, 3	8, 4	2, 2

❖ The two pure Nash Equilibria here are **{High , Medium}** and **{Medium , High}**

- Symmetric game: everything identical for both players
- Nothing is strictly or weakly dominant
- Low is "strictly dominated" (***never a best response***)

➢ How do we make sense of this?

ONE-TIME SIMULTANEOUS CHOICE TARIFFS: TWO NATION **TRADE WAR** MATRIX GAME

Red Nation (Column)

Blue Nation (Row)

	Low	Medium	High
Low	8, 8	5, 7	3, 9
Medium	7, 5	6, 6	4, 8
High	9, 3	8, 4	2, 2

❖ The two pure strategy NE outcomes are **{High , Medium}** and **{Medium , High}**

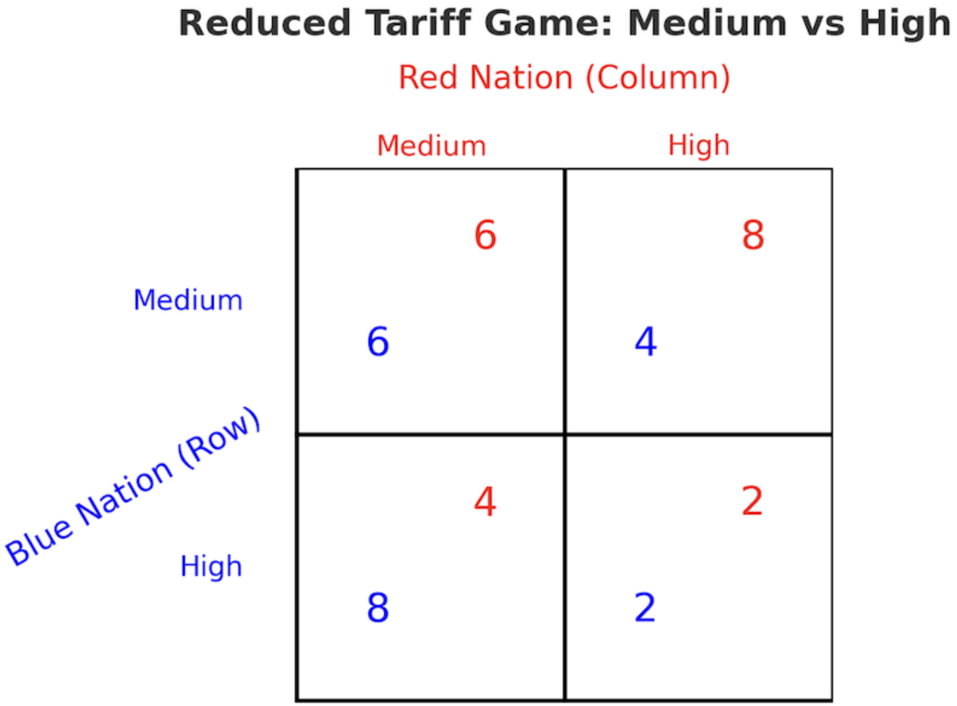
- The best possible outcome for each nation is imposing High tariffs while the other chooses Low tariffs, but Medium is the best response to High for both nations
- **Intuition:** if both nations choose High then both will suffer the worst possible payoff from a trade war, but each nation can benefit individually from imposing high tariffs while the other only implements medium tariffs
  - ***There is a risk/reward trade-off here that determines a mixed strategy equilibrium***

➢ MSE may be a more realistic solution concept than using pure strategies here

➢ Strategic randomization approximates human unpredictability in some ways

ONE-TIME SIMULTANEOUS CHOICE TARIFFS: TRADE WAR MATRIX GAME – MSE

- Eliminating ***Low*** since it is strictly dominated (never the best response to anything) we can analyze this reduced 2x2 game to calculate optimal strategic randomization



- Defining **b** as the probability that blue plays medium and **r** as the probability that Red plays medium, we can solve for the MSE: (b = 0.5, r = 0.5)

- Both countries will essentially flip a coin to decide whether to impose medium or high tariffs: each of them is choosing the payoff-maximizing probability weighting here in response to the other's best response

N-PLAYER INNOVATION GAME (COORDINATION PROBLEM)

Suppose the pharmaceutical industry in Copyland is comprised of n identical firms (with equal market share) which are all equally capable of inventing a new vaccine technology, which is certain to succeed but requires a costly investment in research and testing. All n firms must simultaneously and individually choose whether or not to spend C billion dollars to develop the technology. Firms operate independently and they cannot communicate, collaborate, or share research costs. Copyland has no intellectual property ("IP") or patent protection laws, so if at least one firm chooses to incur cost C to develop the technology, then all firms will obtain benefit $\frac{V}{n}$ billion dollars from being able to sell this new vaccine. Payoffs for each firm are defined as benefit minus individual cost incurred and all firms are completely self-interested. If zero firms develop the vaccine, then payoffs are zero for all firms.

(i) What is the likelihood that this vaccine will be developed? (find a specific mathematical equation $p(n)$ for this probability)

N-PLAYER GAME: INNOVATION

Solution(i):

The players are each of the n firms, the possible actions are Develop or No Investment, and the payoffs are either $\frac{V}{n}$, $\left(\frac{V}{n} - C\right)$, or zero. Let x_i denote the probability that firm i chooses to develop the vaccine. To find a Nash Equilibrium with identical firms (symmetric behavior) in mixed strategies, each of them must randomize between the two possible actions to be indifferent between them, just like in the matrix examples. The probability that some other firm j does not invest in development is $(1 - x_j)$, and the probability that firm j and firm k both do not invest in development is $(1 - x_j) \cdot (1 - x_k)$. Since it is given that all firms are identical, we know that $x_i = x_j = x_k = \ldots = x$ for all n firms. If we have three rival firms j, k, w, then the probability that none of them invest in development is $(1 - x_j) \cdot (1 - x_k) \cdot (1 - x_w) = (1 - x)^3$. So for n firms, the probability that none will invest in developing the vaccine is $(1 - x) \cdot (1 - x) \cdot (1 - x)\ldots$ n times, which we can express as $(1 - x)^n$.

SOLUTION CONT. (N-PLAYER INNOVATION GAME)

Either someone develops the vaccine or nobody develops the vaccine - exactly one of these two statements is always true, so the probability that someone develops the vaccine is therefore equal to one minus the probability that nobody develops the vaccine. The probability that at least one firm will develop is therefore $p(n) = 1 - [(1-x)^n]$. Now consider the point of indifference for firm i in choosing between the two possible actions:

$$(\frac{V}{n} - C) \cdot [1] = \frac{V}{n} \cdot [1 - (1-x)^{(n-1)}]$$

If firm i develops, then it receives a certain payoff $(\frac{V}{n} - C)$ no matter what choices the other firms make.

If firm i does not develop, then its expected payoff is $\frac{V}{n}$ times the probability that at least one of the $(n-1)$ other firms does develop the vaccine technology.

$$(\frac{V}{n} - C) \cdot [1] = \frac{V}{n} \cdot [1 - (1 - x)^{(n-1)}]$$

If firm i develops, then it receives a certain payoff $(\frac{V}{n} - C)$ no matter what choices the other firms make.

If firm i does not develop, then its expected payoff is $\frac{V}{n}$ times the probability that at least one of the $(n - 1)$ other firms does develop the vaccine technology.

We can then use algebra to solve for x:

$$(V - Cn) \cdot [1] = V \cdot [1 - (1 - x)^{(n-1)}]$$

$$1 - (1 - x)^{(n-1)} = \frac{V - Cn}{V}$$

$$1 - (1 - x)^{(n-1)} = 1 - \frac{Cn}{V}$$

$$(1 - x)^{(n-1)} = \frac{Cn}{V}$$

$$(1 - x) = \left(\frac{Cn}{V}\right)^{1/(n-1)}$$

$$x = 1 - \left(\frac{Cn}{V}\right)^{1/(n-1)}$$

SOLUTION CONT. (N-PLAYER INNOVATION GAME)

$$x = 1 - \left(\frac{Cn}{V}\right)^{1/(n-1)}$$

The probability of any one arbitrary firm i developing the vaccine is x, so the probability that at least one of the n firms develops is equal to one minus the probability that zero firms develop:

$$p(n) = 1 - [1 - (x)]^n = 1 - \left[1 - \left(1 - \left(\frac{Cn}{V}\right)^{1/(n-1)}\right)\right]^n = 1 - \left(\frac{Cn}{V}\right)^{n/(n-1)} = 1 - \left(\frac{C}{(\frac{V}{n})}\right)^{n/(n-1)}$$

Notice this depends on the cost-benefit ratio ($\frac{C}{V}$) of investing and the number of firms which are making this simultaneous and independent decision. If the payoff value increases, then the probability of each firm (and therefore at least one firm) choosing to develop will increase. At the same time, this decreases as there is an increase in the number of identical firms splitting the market value, while the individual cost of choosing develop stays the same. As the cost of development goes up and/or as the number of firms increases, the individual and aggregate probabilities of development both decrease. This type of coordination issue is a problematic but unfortunately very common behavior scenario in markets and policymaking.

N-PLAYER INNOVATION GAME (FOUR FIRMS)

Suppose the pharmaceutical industry in Copyland is comprised of n identical firms (with equal market share) which are all equally capable of inventing a new vaccine technology, which is certain to succeed but requires a costly investment in research and testing. All n firms must simultaneously and individually choose whether or not to spend C billion dollars to develop the technology. Firms operate independently and they cannot communicate, collaborate, or share research costs. Copyland has no intellectual property ("IP") or patent protection laws, so if at least one firm chooses to incur cost C to develop the technology, then all firms will obtain benefit $\frac{V}{n}$ billion dollars from being able to sell this new vaccine. Payoffs for each firm are defined as benefit minus individual cost incurred and all firms are completely self-interested. If zero firms develop the vaccine, then payoffs are zero for all firms.

(ii) Suppose now the industry has four identical firms named Phrizer ("P"), Mofirma ("M"), Jensen ("J"), and Zenika ("Z"). If $V = 10$ and $C = 5$, what is the probability that this vaccine will be invented now?

(iii) If the government can offer to subsidize the market, increasing the value of V by some amount S to raise the likelihood of development for the new vaccine technology, precisely how much does it need to spend to meet the policy goal of having at least a $\frac{15}{16}$ probability of development?

SOLUTIONS: N-PLAYER INNOVATION GAME (FOUR FIRMS)

(ii) Suppose now the industry has four identical firms named Phrizer ("P"), Mofirma ("M"), Jensen ("J"), and Zenika ("Z"). If $V = 10$ and $C = 5$, what is the probability that this vaccine will be invented now?

(iii) If the government can offer to subsidize the market, increasing the value of V by some amount S to raise the likelihood of development for the new vaccine technology, precisely how much does it need to spend to meet the policy goal of having at least a $\frac{15}{16}$ probability of development?

Solution(ii):

The probability is zero if the individual benefit is smaller than the individual cost of development.

Solution(iii): For there to be any chance of vaccine development in Copyland, the individual benefit for the firms must be larger than the cost of development, so $\frac{V+S}{n} > C$. With four firms and a cost of 5, we need $V + S > 5 * 4$ so with $V = 10$ the government must subsidize the market by at least 10 billion to have any chance of someone developing the vaccine.

N-PLAYER INNOVATION GAME (POLICY INTERVENTION)

Suppose the pharmaceutical industry in Copyland is comprised of n identical firms (with equal market share) which are all equally capable of inventing a new vaccine technology, which is certain to succeed but requires a costly investment in research and testing. All n firms must simultaneously and individually choose whether or not to spend C billion dollars to develop the technology. Firms operate independently and they cannot communicate, collaborate, or share research costs. Copyland has no intellectual property ("IP") or patent protection laws, so if at least one firm chooses to incur cost C to develop the technology, then all firms will obtain benefit $\frac{V}{n}$ billion dollars from being able to sell this new vaccine. Payoffs for each firm are defined as benefit minus individual cost incurred and all firms are completely self-interested. If zero firms develop the vaccine, then payoffs are zero for all firms.

(iii) If the government can offer to subsidize the market, increasing the value of V by some amount S to raise the likelihood of development for the new vaccine technology, precisely how much does it need to spend to meet the policy goal of having at least a $\frac{15}{16}$ probability of development?

Solution(iii): For there to be any chance of vaccine development in Copyland, the individual benefit for the firms must be larger than the cost of development, so $\frac{V+S}{n} > C$. With four firms and a cost of 5, we need $V + S > 5 * 4$ so with $V = 10$ the government must subsidize the market by at least 10 billion to have any chance of someone developing the vaccine.

To obtain a $\frac{15}{16}$ probability of the vaccine being invented, we need

$$p(n) = \frac{15}{16} = 1 - \left(\frac{Cn}{V+S}\right)^{4/3}$$

so rearranging this yields:

$$\left(\frac{Cn}{V+S}\right)^{4/3} = \frac{1}{16}$$

and therefore

$$\frac{Cn}{V+S} = \frac{1}{8}$$

To obtain a $\frac{15}{16}$ probability of the vaccine being invented, we need

$$p(n) = \frac{15}{16} = 1 - \left(\frac{Cn}{V+S}\right)^{4/3}$$

so rearranging this yields:

$$\left(\frac{Cn}{V+S}\right)^{4/3} = \frac{1}{16}$$

and therefore

$$\frac{Cn}{V+S} = \frac{1}{8}$$

When $n = 4$ and $C = 5$, we need $V + S = 160$ so if $V = 10$ then the government must subsidize the market by $S = 150$ to achieve this sufficiently high probability of the vaccine technology being developed by at least one of the four firms.

With $n = 4, C = 5, V + S = 160$ we have the probability of development:

$$p(4) = 1 - (1-x)^4 = 1 - \left(\frac{5}{\frac{160}{4}}\right)^{4/(4-1)} = 1 - \left(\frac{1}{8}\right)^{4/3} = \frac{15}{16}$$

With these values, the probability of each firm i developing would be:

$$x = 1 - \left(\frac{20}{160}\right)^{1/(4-1)} = 1 - \left(\frac{1}{8}\right)^{1/3} = 1 - \frac{1}{2} = \frac{1}{2}$$

The probability that none of the four firms develop is $(\frac{1}{2})^4 = \frac{1}{16}$

PHARMA INNOVATION GAME: DISCUSSION QUESTIONS

- What are some of the reasons why governments establish intellectual property laws and protections for patent rights?
  - Are there drawbacks to this?
  - What are the effects on consumers?
- Do you see any pros/cons of centralized vs. decentralized industry organization and implications for decision-making?
- What would be a more efficient mechanism for the government to use money to intervene in this situation?