Political Economy & Social Choice Fundamentals

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Preference Relations & Ranked Choice Voting

If a voter with options A and B always prefers *either* A over B *or* prefers B over A, and is never indifferent between any of the available options, then that voter's preferences are ***complete***.

Formally, this means that either

$$A \succcurlyeq B \quad \text{or} \quad B \succcurlyeq A\,.$$

In other words, completeness of preferences means that a voter is never undecided and they can always rank their preferences in order.

Preference Relations & Ranked Choice Voting

If a voter prefers A over B and that voter prefers B over C, then by ***transitivity***, that voter also prefers A over C:

$A \succcurlyeq B$ and $B \succcurlyeq C$ together imply that:

$$A \succcurlyeq B \succcurlyeq C$$

Formally, a preference relation is called ***rational*** if it satisfies both conditions of ***completeness*** and ***transitivity***.

Ranked Choice Voting: Effects of Rules

- The rules of how a ranked choice system is designed are crucial

- Consider the following four voters with respectively given rational preferences:

  - V1: $A \succcurlyeq B \succcurlyeq C$
  - V2: $A \succcurlyeq C \succcurlyeq B$
  - V3: $B \succcurlyeq C \succcurlyeq A$
  - V4: $C \succcurlyeq B \succcurlyeq A$

Who wins a ranked-choice election?

Who is the most polarizing candidate?

Voting System Design: Effects of Rules

- **System 1:** 1st choice = 4 points, 2nd choice = 3 points, 3rd choice = 2 points
  - V1: $A \succcurlyeq B \succcurlyeq C$
  - V2: $A \succcurlyeq C \succcurlyeq B$
  - V3: $B \succcurlyeq C \succcurlyeq A$
  - V4: $C \succcurlyeq B \succcurlyeq A$

  Candidate A receives 4+4+2+2 = 12 points
  Candidate B receives 3+2+4+3 = 12 points
  Candidate C receives 2+3+3+4 = 12 points

Voting System Design: Effects of Rules

- **System 2:** 1st choice = 2 points, 2nd choice = 1 point, 3rd choice = 0 points
  - V1: $A \succcurlyeq B \succcurlyeq C$
  - V2: $A \succcurlyeq C \succcurlyeq B$
  - V3: $B \succcurlyeq C \succcurlyeq A$
  - V4: $C \succcurlyeq B \succcurlyeq A$

  Candidate A receives 2+2+0+0 = 4 points
  Candidate B receives 1+0+2+1 = 4 points
  Candidate C receives 0+1+1+2 = 4 points

Voting System Design: Effects of Rules

- **System 3:** 1st choice = 3 points, 2nd choice = 1 point, 3rd choice = 0 points
  - V1: $A \succcurlyeq B \succcurlyeq C$
  - V2: $A \succcurlyeq C \succcurlyeq B$
  - V3: $B \succcurlyeq C \succcurlyeq A$
  - V4: $C \succcurlyeq B \succcurlyeq A$

Candidate A receives 3+3+0+0 = 6 points ---> *Candidate A will win*
Candidate B receives 1+0+3+1 = 5 points
Candidate C receives 0+1+1+3 = 5 points

Voting System Design: Effects of Rules

- **System 4:** 1st choice = 3 points, 2nd choice = 2 points, 3rd choice = 0 points
  - V1: A ≽ B ≽ C
  - V2: A ≽ C ≽ B
  - V3: B ≽ C ≽ A
  - V4: C ≽ B ≽ A

Candidate A receives 3+3+0+0 = 6 points
Candidate B receives 2+0+3+2 = 7 points
Candidate C receives 0+2+2+3 = 7 points

---> *Now Candidates B and C will tie*

Voting System Design: Effects of Rules

- **System 5:**

First rank by the number of first choice votes received, then eliminate last place and count that vote towards whichever is more preferred among the remaining options:

- V1: $A \succcurlyeq B \succcurlyeq C$
- V2: $A \succcurlyeq C \succcurlyeq B$
- V3: $B \succcurlyeq C \succcurlyeq A$
- V4: $C \succcurlyeq B \succcurlyeq A$

→ *Now Candidate A (the polarizing candidate) will win under this system, while B and C tie for second.*

→ *With these four voters and these preferences, B and C will always tie under any system of preference aggregation*

Ranked Choice Voting System: Four Candidates

- **V1:** **A ≽ B ≽ C ≽ D**
- **V2:** **A ≽ C ≽ B ≽ D**
- **V3:** **B ≽ D ≽ C ≽ A**
- **V4:** **C ≽ B ≽ D ≽ A**
- **V5:** **D ≽ C ≽ B ≽ A**

→ *Three of these five voters prefer C over B*
→ *Three of these five voters prefer B over A*
→ *Three of these five voters prefer C over A*

→ *Four of these five voters prefer B over D*
→ *Three of these five voters prefer C over D*

→ *Polarizing Candidate A is the only one with multiple first choice votes, but the other three voters all rank this candidate as their last choice and a majority of the voters rank both candidates B and C above A*

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→ *Four of these five voters prefer B over D*
→ *Three of these five voters prefer C over D*

Points System :

1st choice = 4 points,
2nd choice = 3 points,
3rd choice = 2 points,
4th choice = 1 point

Candidate A receives 4+4+1+1+1 = 11 points
Candidate B receives 3+2+4+3+2 = 14 points
Candidate C receives 2+3+2+4+3 = 14 points
Candidate D receives 1+1+3+2+4 = 11 points

**** While Candidates B and C will tie based on points, with an odd number of voters here we can use simple majority preference as a tie-breaker: since three of the five voters rank C higher than B,* ***Candidate C will be the winner.*** *****

→ *Polarizing Candidate A is the only one with multiple first choice votes, but the other three voters all rank this candidate as their last choice and a majority of the voters rank both candidates B and C above A*

Real World: NYC Ranked Choice Voting System

- All first-choice votes are counted. If a candidate receives more than 50% of first-choice votes, then that candidate wins.

- If no candidate earns more than 50% of first-choice votes, then counting will continue in rounds:

  - At the end of each round, the last-place candidate is eliminated and voters who chose that candidate now have their vote counted for their next choice.

  - Your vote is counted for your second choice only if your first choice is eliminated. If both your first and second choices are eliminated, your vote is counted for your next choice, and so on.

  - This process continues until there are two candidates left. The candidate with the most votes wins.

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- **V6:** **D ≽ A ≽ B ≽ C**
- **V7:** **D ≽ C ≽ A ≽ B**
- **V8:** **B ≽ C ≽ D ≽ A**
- **V9:** **A ≽ C ≽ B ≽ D**

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→ 3 of 9 list Candidate **A** as their first choice and 3 of 9 list Candidate **B**, so no candidate receives more than 50% and the last-place candidate is eliminated with second choice votes re-allocated

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→ 3 of 9 list Candidate **A** and 3 of 9 list Candidate **B** as their first choice, so no candidate receives more than 50% and the last-place candidate is eliminated with second choice votes re-allocated

→ Candidate **C** only received one first-choice vote, compared to two for **A** and two for **D**, so the second choice of Voter 4 is **B** : now Candidate **B** has **4** of 9 votes, which is still not a majority.

→ The process continues by eliminating the last place candidate, which is now Candidate **D**

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- This process continues until there are two candidates left. The candidate with the most votes wins.

→ Eliminating Candidate **D** (last place in the second round) and re-assigning those two votes to the next remaining choice:

→ Voter 6's next choice is **A**, bringing Candidate **A**'s vote tally up to 4
→ Voter 7's second choice is **C**, who was eliminated in the first round, so this vote is applied to the voter's next choice, which is Candidate **A**

➢ **Candidate A now wins with 5 of 9 votes**
  ➢ If either Voter 6 ***or*** Voter 7 had preferred **B** over **A** then Candidate **B** would have won the election

Paradox: **Arrow's Impossibility Theorem**

- When voters have 3 or more options, no ranked voting system can convert ranked preferences of the individuals into a ***complete*** and ***transitive*** ranking which also satisfies the following conditions: universality, non-dictatorship, Pareto efficiency, and independence of irrelevant alternatives.

- ***No rank-order electoral system can be designed that always satisfies these three "fairness" criteria:***

  - If every voter prefers alternative X over alternative Y, then the group prefers X over Y.

  - If every voter's preference between X and Y remains unchanged, then the group's preference between X and Y will also remain unchanged even if voters' preferences between other pairs do change.

  - Non-dictatorship: no single voter possesses the power to always determine the group's preference.

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  - Non-dictatorship: no single voter possesses the power to always determine the group's preference.

- **Universality** (unrestricted domain)

- **Pareto efficiency** (nobody can be made any better off without making someone worse off)

- **Independence of irrelevant alternatives** (IIA)

Political Economy View of Institutions:

Government = *the monopolist of violence*

** Why do we have governments? **

"Organizing Violence" - Bates, Greif, Singh (2002)

"In stateless societies, coercion is privately provided; violence is employed to engage in, and to defend against, predation. At best, violence results in mere redistribution; being destructive, it more often results in a loss of social welfare. When organized, however, violence can be socially productive; it can be employed to defend property rights, thereby strengthening the incentives to engage in productive activity. To explore how violence can be rendered a source of welfare, the authors develop a model of a stateless society in which people's rights to the product of their labor are secure only if they possess coercive capabilities. Using case materials and formal logic, the authors then compare this outcome with that obtained when private agents reward specialists in violence for defending property rights. In doing so, we plumb the role of the state."

Institutional Possibilities Frontier

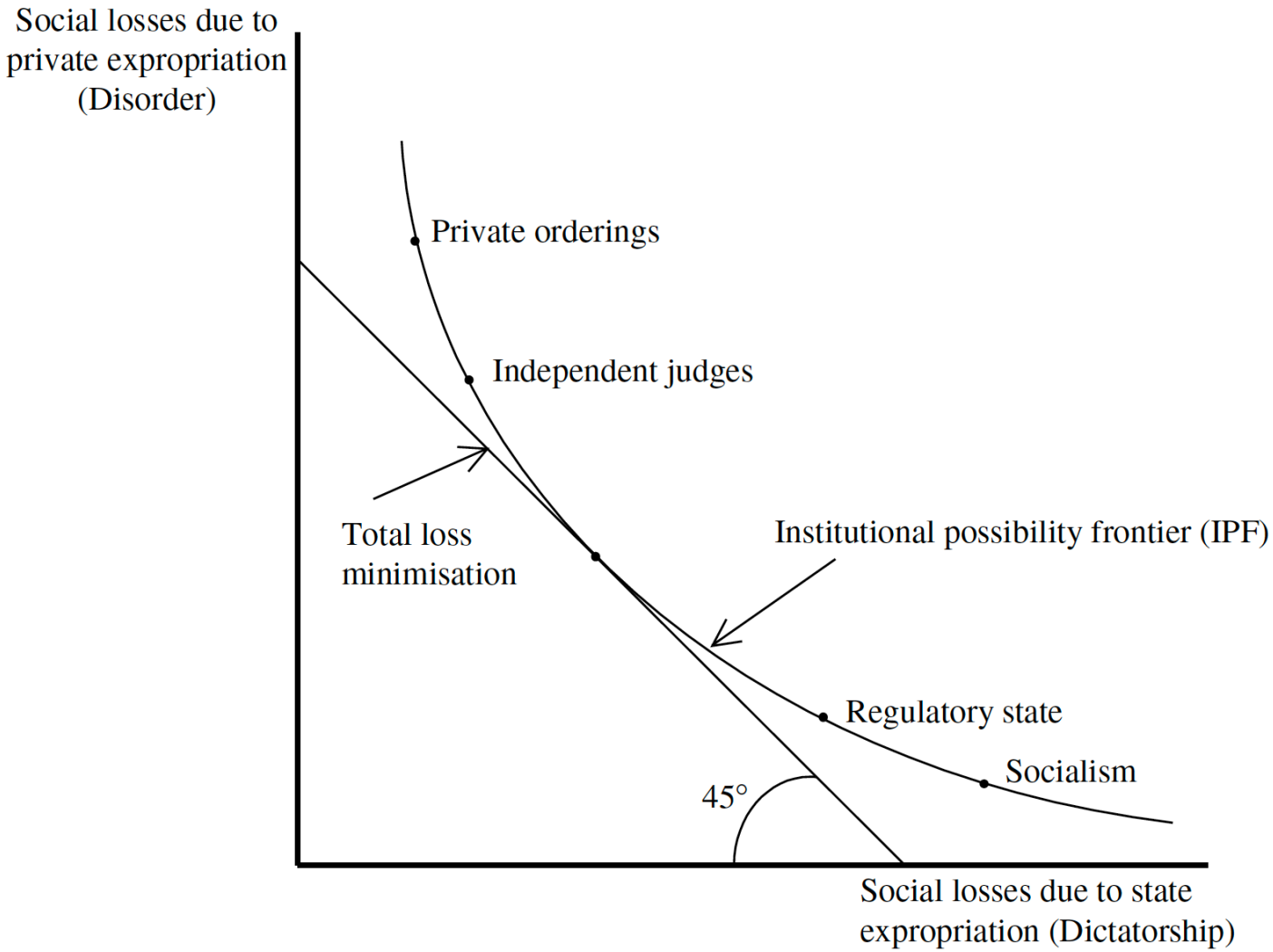

Fig. 1. Institutional possibilities.
Source: Djankov *et al.*, 2003b.

Interpreting the IPF & Iso-cost "Loss curves"

- The linear isocost lines (which are like indifference curves) show the same total cost (or loss, reduction in social surplus, etc.) over a combination of two things: in this case every point on the same isocost line represents the same total disutility
    - This is an illustration of the possible combinations of private and government theft that add up to the same total amount of loss for society
    - This is similar to a budget constraint but represents negative value
- The curved Institutional Possibilities Frontier shows the different attainable arrangements for a society: being closer to the origin, where (0,0) indicates no expropriation at all, is better
- Moving along an IPF curve down and to the right from anarchy (lots of private theft) over towards authoritarianism, we can see that the best point for a society is where its IPF intersects the lowest attainable isocost curve

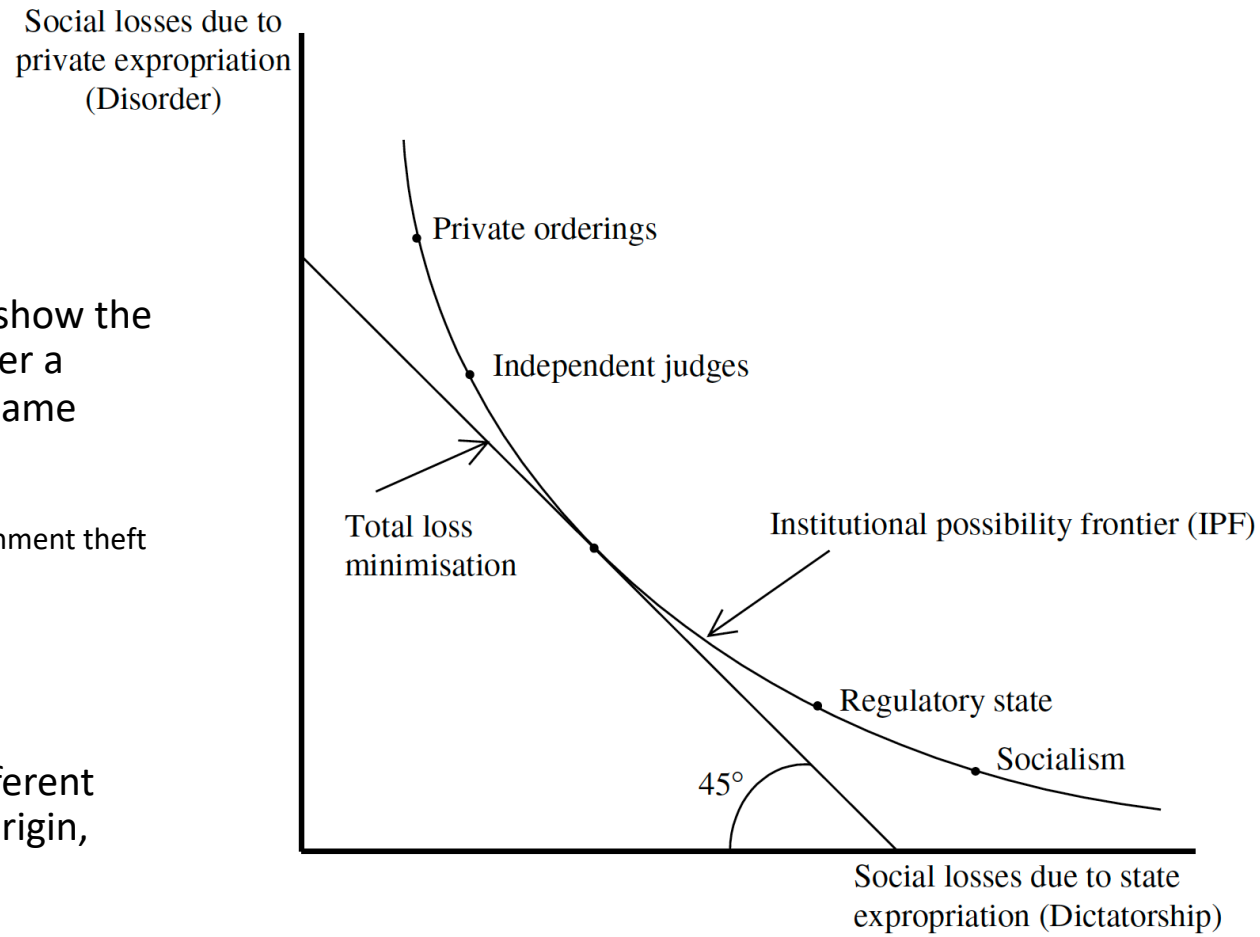

Fig. 1. Institutional possibilities.
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Justification for the existence of government institutions

- The logic of the IPF is that society has lots of losses and inefficiencies with anarchy due to private expropriation (crime, rampant theft, etc)
- Similarly, having a totalitarian state or dictatorship also has lots of losses due to government expropriation ("grabbing hand")
- The most efficient place is somewhere in the middle, with enough government to protect private property ownership but not a powerful enough government to do whatever it wants without accountability
- Different societies have different IPF levels based on many factors, including culture and the preferences of the people

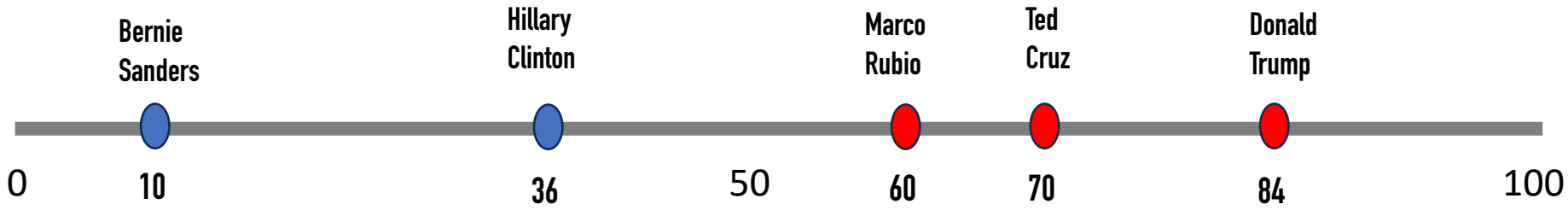
Median Voter Theorem

If voters and candidates are distributed along a 1-dimensional spectrum of preferences or ideology and voters have *single-peaked preferences*, then any voting method satisfying the ***Condorcet Criterion*** will always elect the candidate preferred by the median voter

- The ***Condorcet criterion*** is satisfied if a voting system or election setup always chooses the Condorcet winner when one exists
- The **Condorcet winner** is a candidate preferred by more voters than any other candidate
  - The candidate who wins a majority of the vote in any direct election against any of the other candidates is the Condorcet winner
  - There is not always a Condorcet winner

Hotelling Spatial Competition Model:

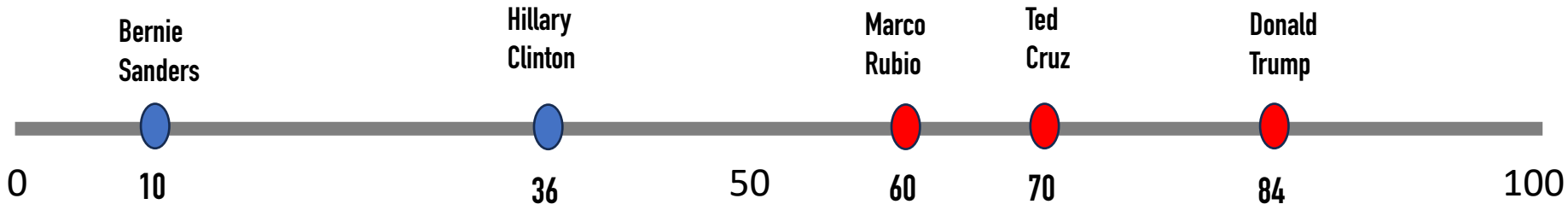
Applications for Political Candidate Locations (1-dimensional Ideology)



- The distance to the halfway point between Sanders and Clinton is (36-10)/2 = 13
  - This means that a voter located at point 23 is exactly indifferent: (10 + 13) = 23 = (36 – 13)
  - Voters at a point to the left of 23 will prefer Sanders and voters to the right of 23 will prefer Clinton

- If the Democratic primary contains all voters from 0 to 50, then Sanders wins 0 to 23 and Clinton wins 23 to 50
  - Sanders receives (23/50) = 46% of the vote and Clinton receives (27/50) = 54% of the vote

Hotelling Spatial Competition Model:
Applications for Political Candidate Locations

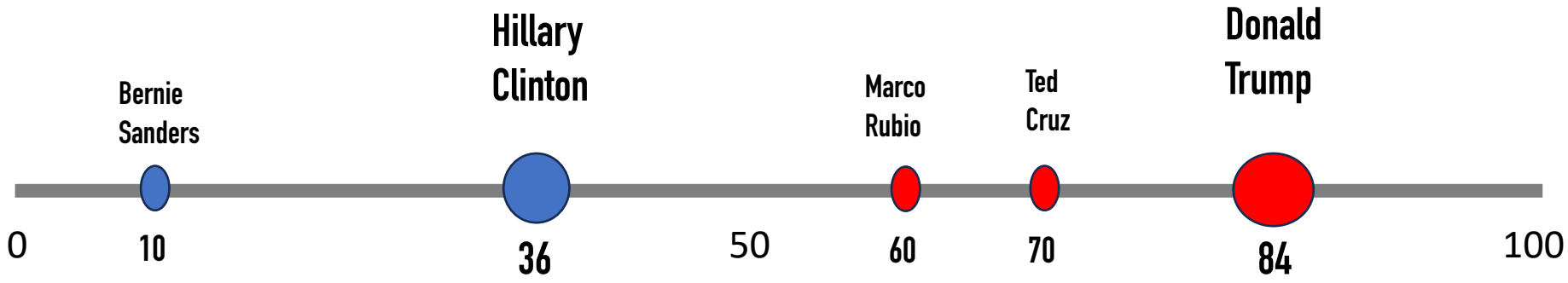


- The distance to the halfway point between Rubio and Cruz is (70-60)/2 = 5
  - This means that a voter located at point 65 is exactly indifferent: (60 + 5) = 65 = (70 – 5)
  - Voters at a point to the left of 65 will prefer Rubio and voters to the right of 65 will prefer Cruz

- The distance to the halfway point between Cruz and Trump is (84-70)/2 = 7
  - This means that a voter located at point 77 is exactly indifferent: (70 + 7) = 77 = (84 – 7)
  - Voters at a point to the left of 77 will prefer Cruz and voters to the right of 77 will prefer Trump

- Rubio wins voters 50 to 65 for (15/50) = 30% of the Republican vote
- Cruz wins voters 65 to 77 for (12/50) = 24% of the Republican vote
- Trump wins voters 77 to 100 for (23/50) = 46% of the Republican vote

Hotelling Spatial Competition Model:
Applications for Political Candidate Locations

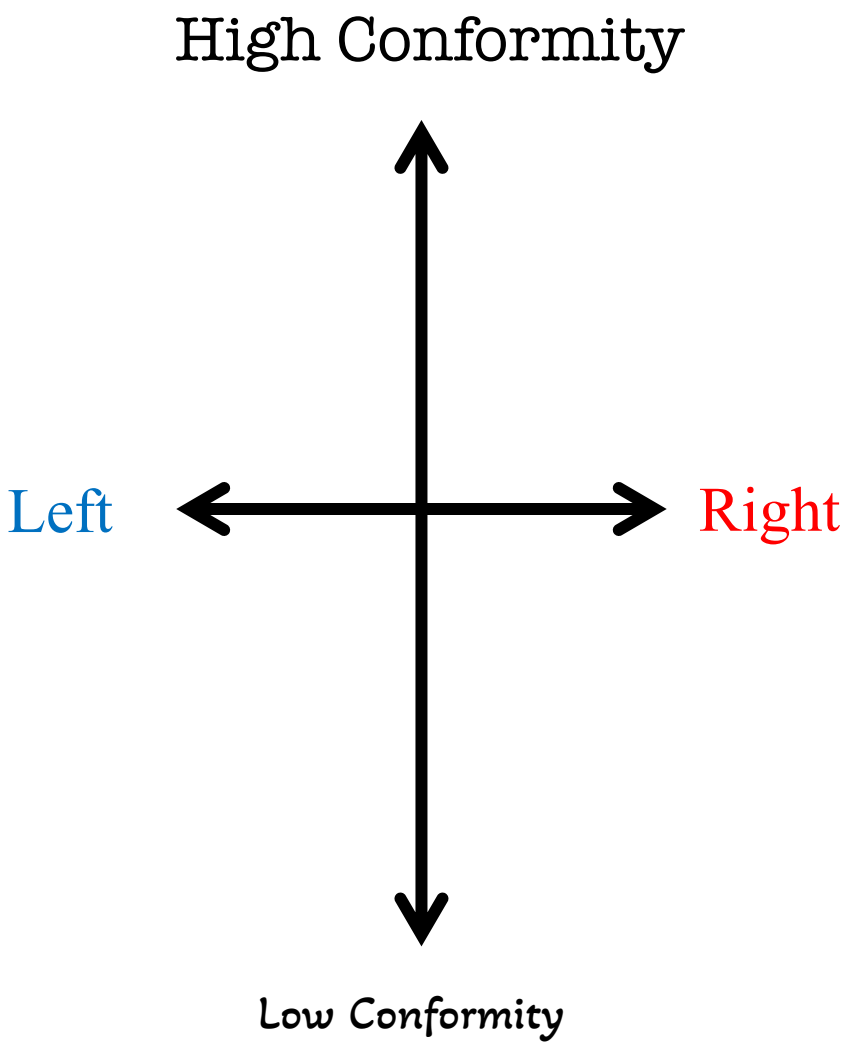


- In a general election matchup we can use the same process: *assuming that the candidates keep their same positions*, the distance to the midpoint between Clinton and Trump is (84-36)/2= 24
    - This means that the general election voter who is indifferent between Clinton and Trump is located at point (36 + 24) = **60** = (84 – 24)
    - In reality, candidates almost always try to re-position themselves in the general election by moving towards the median voter
- The predicted outcome here is a victory for Clinton with 60% of the total vote, including capturing the median voter
    - With a uniform or other symmetric distribution centered at 50, the median voter is at position 50
    - According to these numbers, Rubio would have won against Clinton, Cruz would have lost against Clinton, and Sanders would have lost against any of the other candidates

Alternative Political Ideology Representations: 2-dimensions

- Representing the vertical axis as the degree of conformity: in any totalitarian state (whether “far right” or “far left”) there is almost total conformity and almost no ability to openly oppose the regime or espouse conflicting ideology

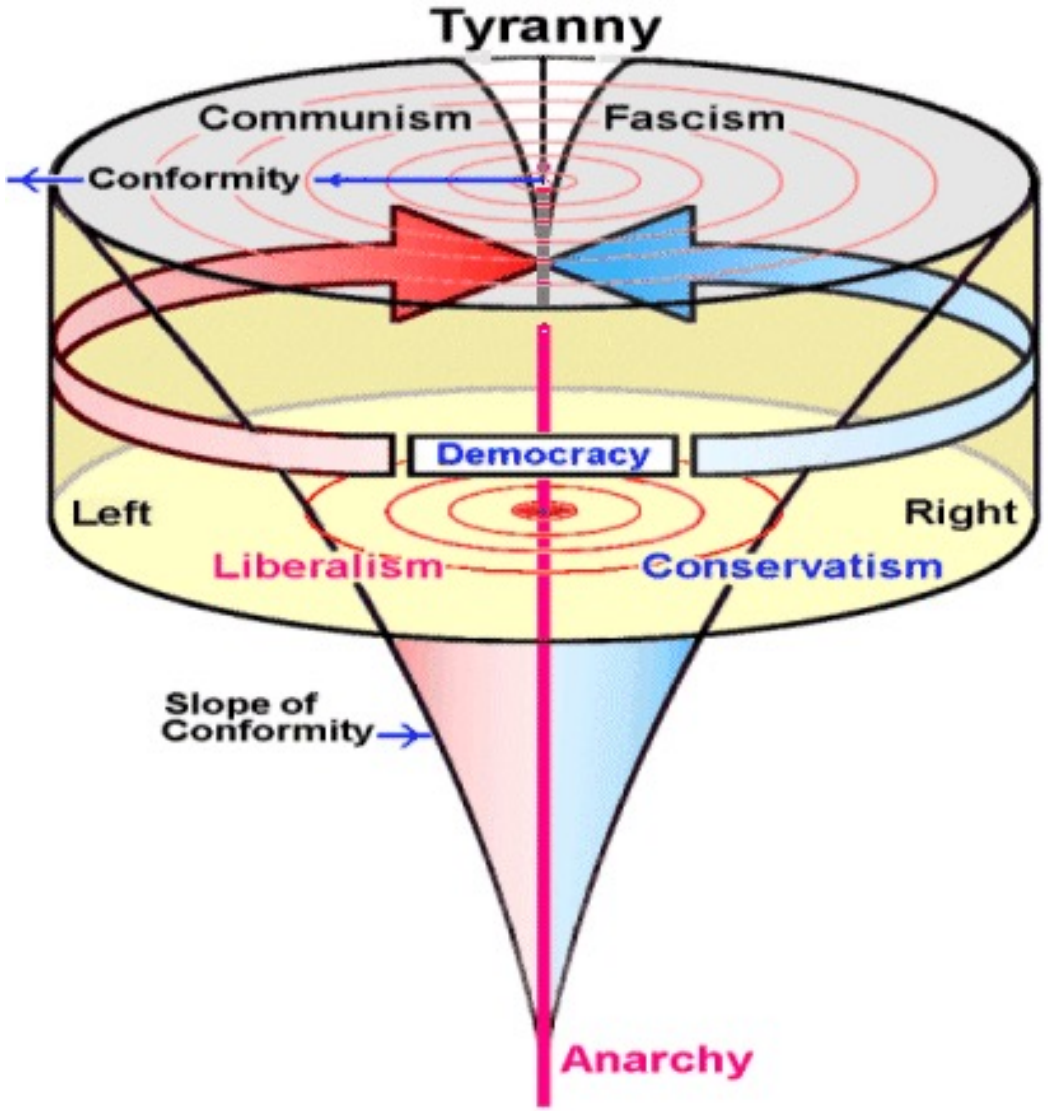
- This model now has “up and down” to represent the degree of conformity as well as “left” and “right”



Higher-Dimensional Political Ideology Representations

- Representing the vertical axis as the degree of conformity: in any totalitarian state (whether “far right” or “far left”) there is almost total conformity and almost no ability to openly oppose the regime or espouse conflicting ideology

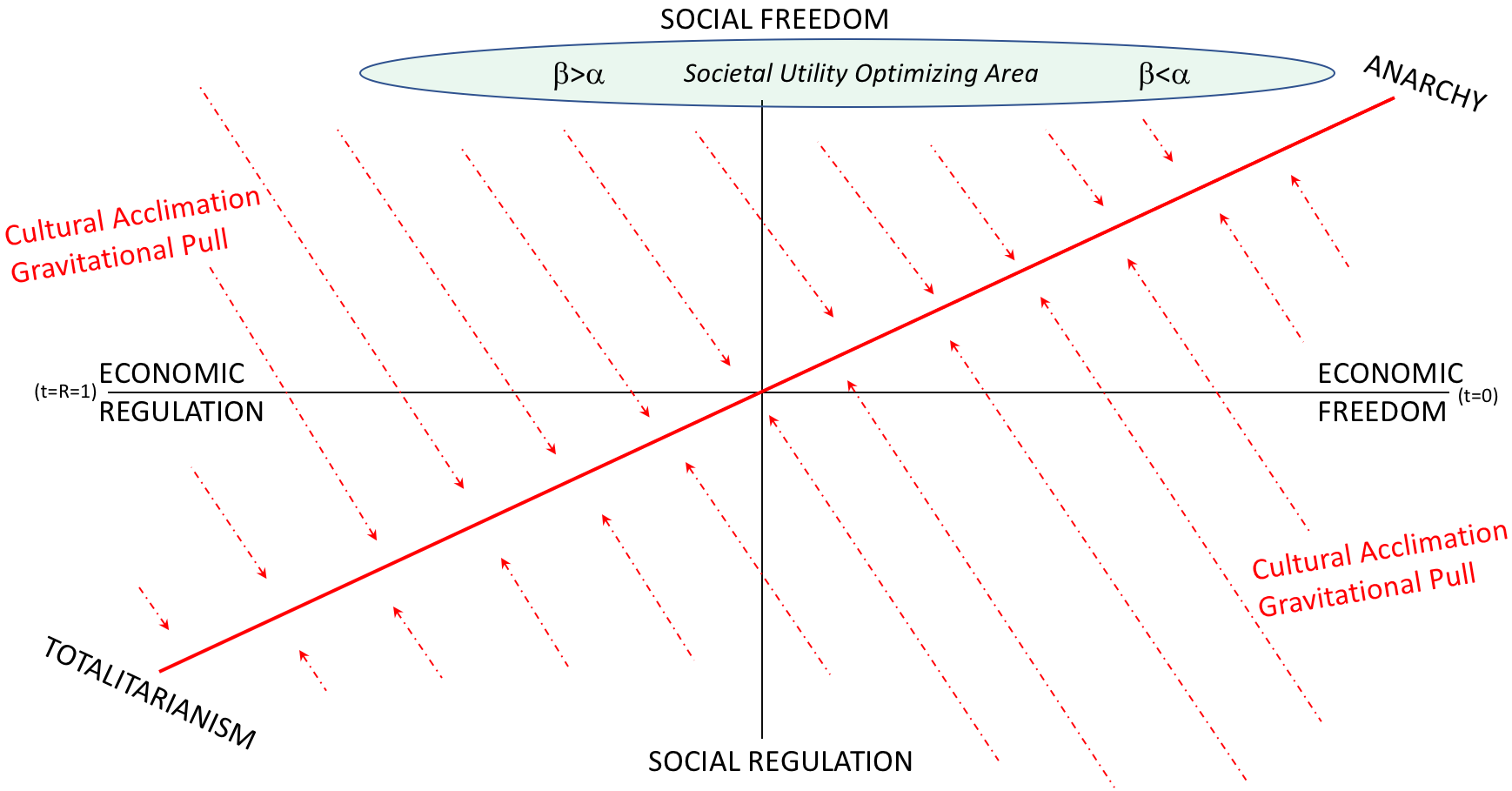
- This type of multi-dimensional model can be parameterized across different formulations of issues to construct higher dimensional representations of how political positions are oriented relative to each other



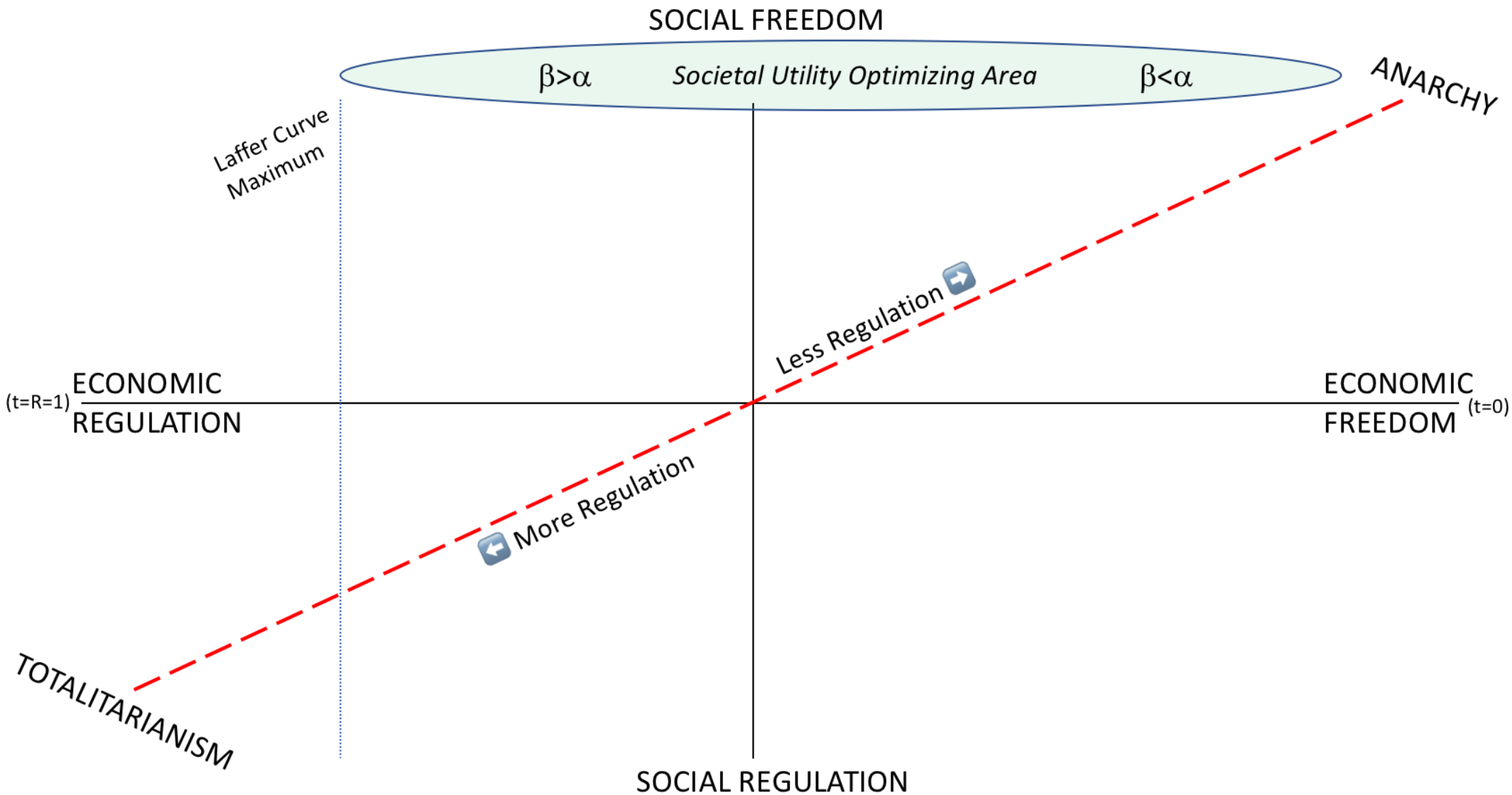
Conceptualizing Multiple Dimensions of Ideology: Economic & Social Issues

$$U_s\{R\} = \alpha(W(R(t)) + \beta R(t) + \delta$$

$$\alpha+\beta = 1 ; \quad W, R, \alpha , \beta > 0 ; \quad R , t \in [0,1]$$



- Red 45 degree line is the "axis of regulation" which is similar to a measure of conformity
- Distance from 45 degree line and longer gravity arrows may increase political instability
- Alpha & Beta characterize the "efficiency/equity trade-off" for a society
- $\beta > \alpha$: implies that a society places more relative value on equality than total economic output (more socialist)
- $\beta < \alpha$: implies the opposite (more capitalist)


SOCIAL FREEDOM
β>α
Societal Utility Optimizing Area
β<α
ANARCHY
Laffer Curve Maximum
Less Regulation
(t=R=1) ECONOMIC REGULATION
ECONOMIC FREEDOM (t=0)
More Regulation
TOTALITARIANISM
SOCIAL REGULATION